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USER'S GUIDE FOR A PROGRAM PACKAGE FOR SIMULATION OF SELF TUNING REGULATORS

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User´s guide for a program package for simulation of self-tuning regulators

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A program package has been developed for simulation of self-tuning regulators with different structures. The program is based on the interactive simulation program package SIMNON which makes it possible to let the self-tuning regulators control a wide class of systems, e.g. continuous systems. This guide describes the program structure and discusses briefly the implemented algorithms. A number of examples illustrate the use of the program package.

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USER´S GUIDE FOR A PROGRAM PACKAGE FOR SIMULATION OF SELF-TUNING REGULATORS

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TABLE OF CONTENTS

1. INTRODUCTION

During the last few years self-tuning regulators have received a great deal of attention and also proved to be quite useful in many industrial applications, Åström et al (1977). This program package is designed for simulation studies of self-tuning regulators with different structures. The program is based on the interactive simulation program package SIMNON, Elmquist (1975), and it thus includes all the facilities available in SIMNON.

The class of regulators considered can be thought of as composed of three parts, a parameter estimator, a linear controller and a block which determines the controller parameters from the estimated ones, see Fig. 1.1. Within this structure there are many different possibilities, depending on the control and estimation schemes used.

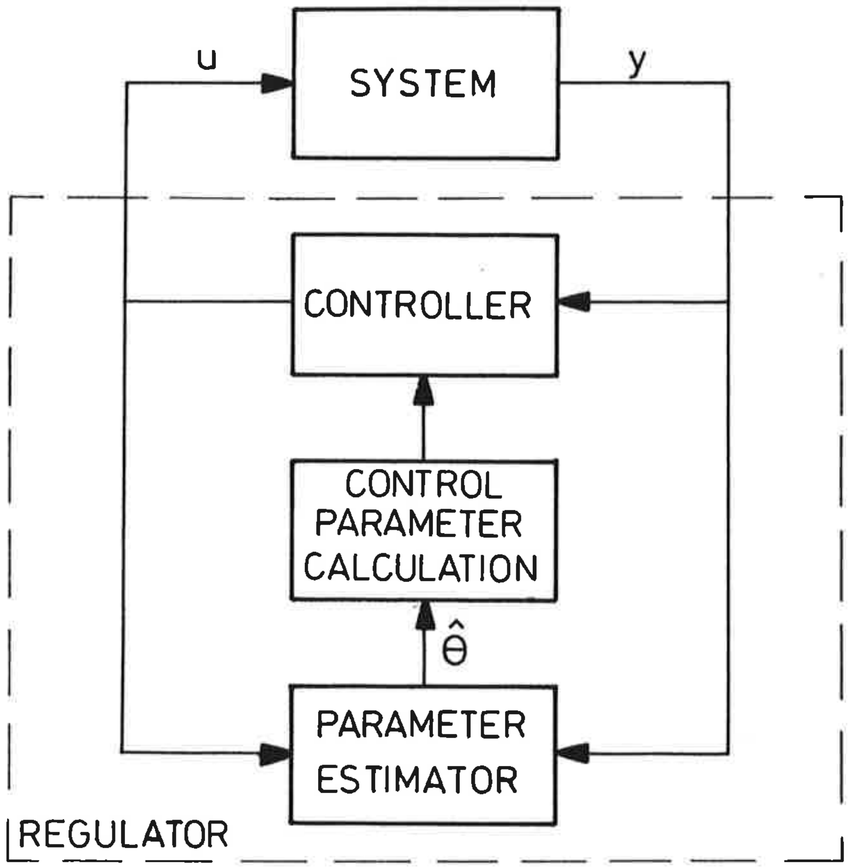

Fig. 1.1. Block diagram of the regulators considered.

The idea behind this program package is that it should provide the user with a simple tool for simulation of some of the possible regulators and to be able to study their behaviour for the quite general class of systems that it is possible to simulate using SIMNON.

This guide describes the program structure, discusses briefly the implemented algorithms and gives some examples of the use of the package. It is assumed throughout this guide that the user is familiar with the interactive program package SIMNON and its use. The basic program structure is described in section 2. In section 3 the start-up of the program is given together with some limitations of the program. The subsystems available in the program package are described in section 4 and in the last section the implemented algorithms are briefly discussed. In section 5 several examples are also shown in order to illustrate the use of the package.

2. BASIC PROGRAM STRUCTURE

In order to simulate the configuration shown in Fig. 1.1 by SIMNON, the process (system) to be controlled and the regulator have to be implemented as two subsystems. The interconnections between these subsystems are then described in a connecting system.

Generally, the user defines the process to be controlled by writing a subsystem in the SIMNON simulation language. He also has to supply the connecting system. The subsystem describing the self-tuning regulator is implemented as a FORTRAN subsystem (REG) in the program package. Three other subsystems are also implemented in the program package, facilitating the simulation of different common problems. All these subsystems are discrete time systems written in FORTRAN. They are described briefly in this section and in more detail in section 4. The user can easily include other subsystems written in the SIMNON simulation language.

Subsystem SYS1 simulates a single input single output linear discrete time system on difference equation form, see section 4.1. This subsystem is sufficient for many self-tuning regulator simulations. If the user wants to control another type of process he has to implement this process as a subsystem written in the SIMNON language. This process may then be of the general type allowed in SIMNON, e.g. continuous and/or nonlinear.

Subsystem REG is the main part of the program package. It includes both the estimator and the controller parts of the self-tuning regulator. Different combinations of estimator and controller can be chosen. For details see section 4.2.

Subsystem NOIS generates a sequence of independent random $N(0,\sigma)$ variables, see section 4.3.

Subsystem INPUT reads data from an IDPAC compatible data file, see Wieslander (1976) and section 4.4. This subsystem can for example be used when performing recursive identification of data from a real process.

Connecting system. The user has to write a connecting system to define the connections between the inputs and the outputs of the used subsystems. A simple example is given below. Further examples can be found in section 5.

Example 2.1. The basic self-tuning regulator STURE1, see section 5.2, is used to control a user system SYS, see Fig. 2.1. The connecting system is given by

```
CONNECTING SYSTEM CON1
TIME T
U1[REG] = Y[SYS]
U[SYS] = UR[REG]
U2[REG] = U[SYS]
END
```

□

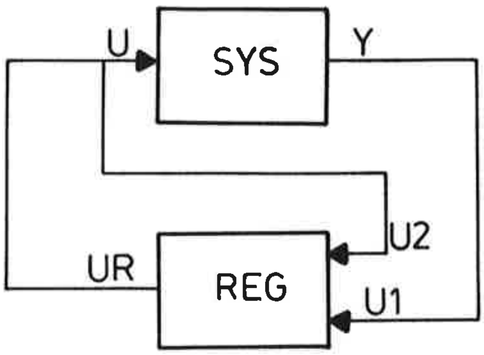

Fig. 2.1. Signal connections for Example 2.1.

Remark 2.1. Normally, the inputs of the regulator REG are the input and the output of the user system SYS, the reference signal or the feedforward signals, if any. It is, however, possible to simulate other configurations. The user may include his own signal processing systems, for filtering for example, and then connect them to the subsystem REG using e.g. the general STURE1 algorithm and its possibilities to handle several feedforward signals. The user can also implement his own control algorithm since the subsystem REG can be used as a pure estimator and it has the estimates available as outputs.

3. PROGRAM START-UP AND LIMITATIONS

3.1. Start-up

The program package for simulation of self-tuning regulators is available on disc No. 9 for the PDP-15 computer. The program is started up by the following sequence of commands+).

```
$PIP
>T RK_RK <AIG> SYSTEM XXX (D)
>T RK_RK <AIG> SYSTEM XCT (B)
>T RK_RK <EXT> SIMNON XXX (D)
>T RK_RK <EXT> SIMNON XCT (B) (escape)
$A RK 3/RK <EXT> 4/NON 5,7,15,16
$BUFFS 6
$E SIMNON
```

3.2. Limitations

Integration of continuous systems. Because of the program size the integration routine HAMPC which is normally used in SIMNON is not implemented. This means that whenever a continuous system is included the user must give the command

>ALGOR RK

before the simulation is started, see also Elmquist (1975). It may also be necessary to decrease the error bound for the integration routine or to choose the maximal time increment in the command SIMU appropriately.

MACRO´s. The program package uses a special version of SIMNON, which is designed to simplify the inclusion of FORTRAN subsystems and to provide more core memory for the user programs. However, as a drawback MACRO´s that include the commands SYST or SIMU cannot be used.

Time delays. If there are time delays in a continuous system a special version of the program has to be used, because the FORTRAN subsystem DELAY must be included in the program package.

+) Before starting up the program make sure that none of the files .ADDR BIN, .BLC BIN, .INTCO BIN, .SIMCO BIN are on the disc.

4. DESCRIPTION OF SUBSYSTEMS

In this section the general structure of the available subsystems, SYS1, REG, NOIS and INPUT, will be described. In the next section the estimation and control algorithms will be described in some detail. This means that in order to be able to use the program package, the user should consult both the description of the subsystems he wants to use (in this section) and the description of the particular algorithms (in section 5).

4.1. Subsystem SYS1

The subsystem SYS1 simulates a single input single output difference equation of the following structure,

$$x(t) + a_1x(t-1) +\ldots+ a_{n_a}x(t-n_a) = b_1u(t-k-1) +\ldots+ b_{n_b}u(t-k-n_b) + \\ + \lambda[e(t) + c_1e(t-1) +\ldots+ c_{n_c}e(t-n_c)] \qquad (4.1.1)$$

$$y(t) = x(t) + d \qquad (4.1.2)$$

where $e(t)$ is a sequence of independent random $N(0,1)$ variables and d is a constant.

4.1.1. Global variable

Before the subsystem SYS1 is used in a SYST command a global variable must be set,

$\boxed{\text{IVS.}}$

This value should be chosen so that $\text{IVS.} \geq \max(n_a, n_b, n_c)$ and $1 \leq \text{IVS.} \leq 5$.

4.1.2. Input

The subsystem SYS1 has one input

$\boxed{\text{U}}$

which is the input signal $u(t)$ in the difference equation (4.1.1). The input U is not used in the output section of SYS1.

4.1.3. Output

The subsystem SYS1 has one output

$\boxed{\text{Y}}$

which is the output signal $y(t)$ from the system (4.1.1)-(4.1.2).

4.1.4. Parameters

To define the system structure the following parameters are used,

$\boxed{\text{NSA}:n_a}$	Order of the A-polynomial. Default value: 0. $(n_a \leqslant 5)$
$\boxed{\text{NSB}:n_b}$	Order of the B-polynomial. Default value: 0. $(n_b \leqslant 5)$
$\boxed{\text{NSC}:n_c}$	Order of the C-polynomial. Default value: 0. $(n_c \leqslant 5)$
$\boxed{\text{KS}:k}$	The number of extra time delays. Default value: 0.

The parameters of the system are defined by

A1: a_1 A2: a_2 . .	The a_i-parameters, $i=1,\ldots,n_a$. Default values: 0.
B1: b_1 B2: b_2 . .	The b_i-parameters, $i=1,\ldots,n_b$. Default values: 0.
C1: c_1 C2: c_2 . .	The c_i-parameters, $i=1,\ldots,n_c$. Default values: 0.
$\boxed{\text{LAMB}:\lambda}$	Standard deviation of the noise e(t). Default value: 1.
$\boxed{\text{YLEV}:d}$	Constant level added to the output of the system. Default value: 0.

The random number generator requires a starting value (an odd number),

$\boxed{\text{NODD}:}$	Default value: 19.

The sampling period is determined by the parameter

$\boxed{\text{DT}:}$	Default value: 1.

4.2. Subsystem REG

The following possibilities of estimators and controllers are available in the subsystem REG,

- o recursive identification: the least squares method, the maximum likelihood method, the extended least squares method, an instrumental variable method, a stochastic approximation method,
- o basic self-tuning regulator, STURE1, with its variants, e.g. STURE0, feedforward signals, reference signal,
- o self-tuning regulators based on linear quadratic gaussian theory solving a Ricatti equation, STURE2 and STUREM,
- o self-tuning algorithms based on pole placement, STURP1 and STURP2,
- o general self-tuning algorithm with feedforward and feedback including MRAS algorithms (all zeroes cancelled).

The detailed information of these algorithms is given in section 5.

4.2.1. Global variables

Before the subsystem REG can be used in the command SYST, three global variables must be set,

IVR.	This value defines the maximum number of estimated parameters, $1 \leq$IVR.≤ 10.
ISA.	This value defines the number of input signals, $1 \leq$ISA.≤ 5. Notice that this value must correspond to the number of inputs of REG that is used in the connecting system. It is possible to set unused inputs equal to zero, which means that ISA. does not necessarily have to be equal to the number of "active" signals.
ISB.	This value defines the maximum number of lagged signal values that are available for the estimator and the controller, $1 \leq$ISB.≤ 20.

There are five more global variables which can be used. In general, however, it is not necessary to set these variables, since they are given certain default values in the program. Four of these variables are related to the print-out, viz.

$\boxed{\text{DATE.}}$

$\boxed{\text{MONTH.}}$

$\boxed{\text{YEAR.}}$

$\boxed{\text{NDOC.}}$

The given date will be written on each documentation page, see the description of the print-out parameter IWR, section 4.2.4. See also Example 5.2. Default values of DATE., MONTH. and YEAR. are 0. The numeration of the documentation pages will be 1,2,... for simulations No. 1,2,..., unless the user sets the variable NDOC. Each time the command SIMU (without -CONT) is used, one is added to the variable NDOC. The user can set NDOC. to a desired value (at most five digits) and the updating then starts from this value.

The global variable

$\boxed{\text{IPL.}}$

determines if the elements of the P- and S1-matrices, see section 4.2.5, are available as variables in the SIMNON sense or not. This feature may be useful if the user has many parameters defined in his own systems. Otherwise there may possibly be a violation of the limit of the number of variables. In that case SIMNON produces the message

NO MORE PLACE FOR NUMBERS

If IPL.=0 the elements are available as variables, if IPL.=1 they are not. The default value of IPL. is 0, unless IVR.≥10. If IVR.≥10 IPL. must be 1.

4.2.2. Inputs

The system REG has ISA. inputs, called

$\boxed{\begin{matrix}\text{U1}\\ \text{U2}\\ \cdot\\ \cdot\end{matrix}}$

4.2.3. Outputs

The following outputs are defined,

$\boxed{\text{UR}}$ The control signal

TH1 TH2 . .	The vector of estimated parameters (≤IVR.)
RES	The residual from the estimation algorithm

4.2.4. Parameters

In this section the parameters that are of more general nature are described. See also Table in Appendix B. The parameters that are specific for a certain estimation or control algorithm will be explained in section 5.

The parameters

ID:	Default value: 1 (means least squares estimation)

and

REG:	Default value: 0 (means no control)

defines the estimator/controller combination. The following possibilities exist,

ID:1	Least squares identification (default)
ID:2	Maximum likelihood identification
ID:3	Extended least squares identification
ID:4	Instrumental variable identification
ID:5	Stochastic approximation identification

REG:0	No control, i.e. pure identification (default)
REG:1	STURE1
REG:2	STURE2
REG:3	STUREM (with STURE2 as a special case)
REG:4	STURP1 (implicit algorithm for pole placement)
REG:5	STURP2 (explicit algorithm for pole placement)
REG:6	General algorithm including MRAS algorithms

Some of the combinations are illegal,see Table 4.1. The general algorithm (i.e. REG:6) can only be combined with least squares estimation (ID:1) or stochastic approximation (ID:5).

The parameters

N1: N2: . .	K1: K2: . .	Default values: 0.

define the model structure used in the estimation algorithm. It thus also defines the regulator structure, directly or indirectly depending on the particular algorithm. There are at most ISA. such values to set. Ni is the number of parameters in polynomial i of the model structure and Ki defines the number of delays for the corresponding signal.

Example 4.1. Let the model structure chosen for an recursive identification application be

$$y(t)= -a_1y(t-k_1-1) -\ldots- a_{n_1}y(t-k_1-n_1) + b_1u(t-k_2-1) +\ldots+ b_{n_2}u(t-k_2-n_2)$$

Then $N1:n_1$, $N2:n_2$, $K1:k_1$ and $K2:k_2$.

□

For details of the model structures for the different algorithms see the descriptions in section 5.

The normal choices of Ni and Ki for the different estimator/controller combinations are given in Table 4.1. For the general algorithm (REG:6) see section 5.7. Notice that N1+N2+...≤IVR.≤10. Of course, there are also restrictions on Ki, e.g. because the control law must be causal. ISB. must also be chosen large enough to allow sufficient space for the required lagged signal values. The required number depends on the parameters Ni, Ki and KDEL.

REG / ID	0 or 5 No control STURP2	1 or 4 STURE1 STURP1	2 STURE2	3 STUREM
1,4 or 5 LS,IV or SA	A	B	A	D
2 or 3 ML or ELS	C	Combination illegal	D	C

Table 4.1. Normal model structures

A: Set N1, N2, K2 (K1:0, default value)

B: Set N1, N2, K1=K2

C: Set N1, N2, N3, K2 (K1:0, K3:0, default values)

D: Set N1, N2, K2 (N3:0, K1:0, K3:0, default values)

For the combination REG:6 with ID:1 or ID:5 see section 5.7.

The parameter

$\boxed{\text{KDEL:}}$	Default value: 0.

determines if there is an extra time delay in the regulator or not.

KDEL:0	u(t) = f(y(t),y(t-1),...,u(t-1),...), i.e. it is assumed that y(t) is available when computing u(t).
KDEL:1	u(t) = f(y(t-1),...,u(t-1),...), i.e. it is assumed that y(t) is not available when computing u(t).

The parameter

$\boxed{\text{ULIM:A}}$	Default value: -1 (means no limitation of the control signal)

makes it possible to limit the control signal to ±A (A>0).

There are three parameters controlling the print-out, viz.

$\boxed{\text{IWR:}}$	Default value: 0.
$\boxed{\text{NWR1:}}$	Default value: 10.
$\boxed{\text{NWR2:}}$	Default value: 100.

They are used in the following way.

IWR:0	No print-out.
IWR:1	A documentation page is printed on the line-printer when the command SIMU (without -CONT) is given. It shows all relevant parameter values for the particular simulation. The documentation page is dated by the global variables DATE., MONTH. and YEAR. and is numbered by the global variable NDOC, see section 4.2.1.
IWR:2,3 or 4	A print-out is obtained on the line-printer for the NWR1 first sampling events and then every NWR2nd sampling event. For IWR:2 the parameter estimates (THi), the inputs of the subsystem REG (Ui), the matrix P (Pij), the control signal (UR), the loss functions V and VU and the residual (RES) are printed. For IWR:3 also e.g. the S1-matrix and the vector AL used in STURE2 and STUREM are printed (i.e. if REG:3 or 4). For IWR:4 the matrix S which contains the lagged signal values is also printed.

The parameter

$\boxed{\text{DT:}}$ Default value: 1.

determines the sampling period of the subsystem REG.

4.2.5. Variables.

The following variables are defined

$\boxed{\text{V}}$	Loss function defined as $V = \sum_{1}^{t}[U1(s)]^2$. For the algorithm described in section 5.7 V is defined as $V = \sum_{1}^{t}[U2(s)-YM(s)]^2$.
$\boxed{\text{VU}}$	Loss function defined as $V = \sum_{1}^{t}[UR(s)]^2$
$\boxed{\text{WT}}$	The current weighting factor, see section 5.1.
$\boxed{\text{Pij}}$	The element P(i,j) of the P-matrix. Only available if IPL.=0.
$\boxed{\text{S1ij}}$	The element S1(i,j) of the S1-matrix used in STURE2 and STUREM. Only available if IPL.=0.
$\boxed{\text{ALi}}$	The element i of the control law vector L computed in STURE2 and STUREM.
$\boxed{\text{SAPI}}$	The current value of 1/P(t) for the stochastic approximation algorithm, see eq. (5.1.7).
$\boxed{\text{YM}}$	The output signal of the model (5.7.4).

4.3. Subsystem NOIS

The system NOIS generates a sequence of independent random $N(0,\sigma)$ variables.

4.3.1. Output.

The subsystem NOIS has one output, the random variable generated,

$\boxed{E}$

4.3.2. Parameters.

The system NOIS has three parameters,

$\boxed{\text{NODD:}}$	Starting value for the random number generator. An odd number should be chosen. Default value: 95.
$\boxed{\text{SD:}\sigma}$	Standard deviation of the random variables. Default value: 1.
$\boxed{\text{DT:}}$	The sampling period for the system NOIS. Default value: 1.

4.4. Subsystem INPUT

The system INPUT reads data from an IDPAC compatible data file, see Wieslander (1976). It can read one or two values from the data file at each sampling event.

4.4.1. Global variable.

One global variable defining the name of the data file from which the reading should be done

FIL.

must be set before the command SIMU is given.

Example 4.2. The command

>LET FIL.=FILNM

should be given if the name of the data file is FILNM BIN.

□

Notice that when the system INPUT is used the command SIMU-CONT cannot be used.

4.4.2. Outputs.

There are two outputs

U1
U2

U1 is equal to the value in column NC1 of the data file and U2 is equal to the value in the column NC2 of the data file. If the parameter NC2=0 then the output U2 is meaningless.

4.4.3. Parameters.

The system INPUT has three parameters,

NC1:	NC1 determines from which column the output U1 is read. Default value: 1.
NC2:	NC2 determines from which column the output U2 is read. Default value: 0 (see above).
DT:	The sampling period for the system INPUT. Default value: 1.

5. DESCRIPTION OF ALGORITHMS

The different estimation and controller algorithms available are briefly described in this section.

5.1. Recursive estimation

Five different recursive estimation algorithms are implemented, viz.

- the least squares method, LS,
- the recursive maximum likelihood method, ML,
- the extended least squares method, ELS,
- an instrumental variable method, IV,
- a stochastic approximation algorithm, SA.

For details see Söderström, Ljung and Gustavsson (1974).

Model structures. The parameters of the following model structure can be estimated by the LS, IV and SA method.

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t) + \varepsilon(t) \tag{5.1.1}$$

where

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_a} q^{-n_a}$$

$$B(q^{-1}) = b_1 q^{-1} + \ldots + b_{n_b} q^{-nb}$$

For the ML and ELS methods the model structure

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t) + C(q^{-1})\varepsilon(t) \tag{5.1.2}$$

is used, where

$$C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + c_{n_c} q^{-n_c}$$

Algorithm. The estimates are obtained from

$$\theta(t+1) = \theta(t) + P(t+1)z(t+1)\varepsilon(t+1) \tag{5.1.3}$$

$$P(t+1) = [P(t) - \frac{P(t)z(t+1)\varphi(t+1)^T P(t)}{\lambda(t+1)+\varphi(t+1)^T P(t)z(t+1)}]/\lambda(t+1) \tag{5.1.4}$$

$$\lambda(t+1) = \lambda_o \lambda(t) + (1-\lambda_o) \tag{5.1.5}$$

where $\theta(t)$ is the parameter vector, $P(t)$ the covariance matrix, $\varphi(t)$ and $z(t)$ vectors of old values of $u(t)$, $y(t)$ and $\varepsilon(t)$ (possibly filtered), $\varepsilon(t)$ defined by the model structures given above and $\lambda(t)$ the forgetting profile.

Some remarks concerning this algorithm are given below relating it to the different methods.

Remark 1 - Several inputs. It is possible to have several input signals for the LS, ELS and SA algorithms. The term $q^{-k}B(q^{-1})u(t)$ in the model structures above is then replaced by $\sum_{1}^{m} q^{-k_j}B_j(q^{-1})u_j(t)$, where $m \leq 4$ for LS and SA and $m \leq 3$ for ELS. The output signal of the process should be connected to the input signal U1 of REG. The input signals of the process should be connected to U2, U3, U4 and U5 of REG for LS and SA, and to U2, U4 and U5 for ELS. For ELS the input signal U3 of REG cannot be used and may be put equal to zero in the connecting system if ISA.$\geq$3.

Remark 2 - Forgetting profile. By choosing $\lambda_0=1$ we get $\lambda(t)=\lambda(0)$. A choice of $\lambda(0)<1$ is appropriate in applications with time varying parameters. By choosing $\lambda_0<1$ and $\lambda(0)<1$ $\lambda(t)$ will start in $\lambda(0)$ and tend to 1. This may be used to improve the initial convergence of the algorithm.

Remark 3 - A posteriori estimate of the residual. The a posteriori estimate of $\varepsilon(t)$ can be computed. This may improve the initial convergence of the ELS and ML methods. See parameter IRES.

Remark 4 - Limitation of the residuals. The residuals may be limited by a given value. Again this is of particular importance for the ELS and ML methods. See parameter RLIM.

Remark 5 - Modification of the updating of P(t). The computation of the diagonal elements of the P-matrix can be modified by adding the term

$$(R_{1i} - \delta P_{ii}{}^{2}(t))/\lambda(t+1) \tag{5.1.6}$$

where R_{1i} is the ith element of a vector and δ a constant, both chosen by the user. The modification with R_1 can be derived from the relation between least squares estimation and a Kalman filter. It can be useful in situations with time-varying parameters to secure that the P-matrix does not tend to zero. The second term has been suggested by Ljung (1977) to prevent from difficulties that may occur because of almost singular P-matrices.

For the SA method the corresponding modification is

$$\frac{1}{P(t+1)} = \lambda(t+1)\,\frac{1}{P(t)} + \varphi(t+1)^T\varphi(t+1) + \delta \tag{5.1.7}$$

Remark 6 - The IV algorithm. The implemented IV algorithm uses the old values of the input signal as the instrumental variables. The algorithm can be started up by using the least squares estimate for a number of steps. See parameter ILS.

Remark 7 - The SA algorithm. In the SA algorithm the updating of a matrix P is replaced by the updating of a scalar, i.e. the equation (5.1.4) is replaced by equation (5.1.7).

The inputs of REG are normally (see also Remark 1 above)

U1 - output signal $y(t)$

U2 - input signal $u(t)$

U3 - zero (for the ML and ELS algorithms)

For the LS and SA algorithms the inputs of REG may be

U1 - output signal $y(t)$

U2 - input signal $u_1(t)$

U3 - input signal $u_2(t)$

U4 - input signal $u_3(t)$

U5 - input signal $u_4(t)$

For the ELS algorithm the inputs may be

U1 - output signal $y(t)$

U2 - input signal $u_1(t)$

U3 - zero

U4 - input signal $u_2(t)$

U5 - input signal $u_3(t)$

Notice that U1 is the only input of REG that is used in the output section of REG when performing pure identification (i.e. REG:0). This should be kept in mind when SIMNON gives warnings of the type

U2 IS UNDEFINED IN THE OUTPUT-SECTION OF REG

which the user thus can ignore in this case.

A typical connecting system is given in Example 5.1.

Parameters. The desired estimation algorithm is chosen with the parameter

$\boxed{\text{ID:}}$ Default value: 1 (means least squares estimation)

see section 4.2.4.

The parameter determining control algorithm should be

$\boxed{\text{REG:0}}$ Default value (meaning no control)

The model structure is determined by

N1:n_a
N2:n_b
N3:n_c

Default values: 0.

and

$\boxed{\text{K2:k}}$ Default value: 0.

The parameter vector TH is then defined as $(a_1,\ldots,a_{n_a},b_1,\ldots,b_{n_b},c_1,\ldots,c_{n_c})$.
If e.g. the least squares method is used for the case of four input signals use N1:n_a, N2:n_{b_1}, N3:n_{b_2}, N4:n_{b_3}, N5:n_{b_4}, K2:k_1, K3:k_2, K4:k_3 and K5:k_4.

The initial parameter estimates are determined by

THO1:
THO2:
.
.

Default values: 0.

and the initial value of P(t) is chosen as P(0)=PO·I where the elements of the vector PO are determined by

PO1:
PO2:
.
.

Default values: 100.

The scalar initial value P(0) for the stochastic approximation algorithm, see eq. (5.1.7), is chosen by the parameter

$\boxed{\text{SAPO:}}$ Default value: 100.

The forgetting profile, see (5.1.5), is determined by the parameters

$\boxed{\text{WTI:}\lambda(0)}$ Default values: 1.

$\boxed{\text{WTM:}\lambda_0}$

The parameter

$\boxed{\text{RLIM:A}}$ Default value: -1.

is used to limit the residuals by ±A (A>0). If A<0 there is no limitation.

The estimation of the residual is controlled by the parameter

$\boxed{\text{IRES:}}$

IRES:0 The a priori estimate. Default value.

IRES:1 The a posteriori estimate.

The LS, ML, ELS and IV methods also use the parameters

R11:	Default values:0.
R12:	
.	
.	

which determines the values of the elements of the vector R1, see equation (5.1.6), and

$\boxed{\text{DELTA:}\delta}$ Default value: 0.

which also can be used in the updating of the P-matrix, see (5.1.6). This parameter can also be used by the SA algorithm, see equation (5.1.7).

The IV algorithm has one further parameter

$\boxed{\text{ILS:}}$ Default value: 50.

which determines the number of samples in the start-up of IV for which the LS estimates are computed.

Example 5.1. The intention is to simulate the process

$$y(t) - 0.8y(t-1) = 1.u(t-1) + e(t) + 0.7e(t-1)$$

and then estimate the parameters of the model

$$y(t) + a_1\, y(t-1) = b_1\, u(t-1) + \varepsilon(t) + c_1\varepsilon(t-1)$$

with the ML method. A sequence of independent N(0,1) variables, uncorrelated with e(t), is chosen as the input signal. The standard deviation of e(t) is chosen to one.

The connecting system necessary for this simulation is the following one.

```
CONNECTING SYSTEM CON2
TIME T
U1[REG] = Y[SYS1]
U[SYS1] = E[NOIS]
U2[REG] = E[NOIS]
END
```

The following command sequence was used for the simulation.

```
>LET IVS.=1                          "Global variables
>LET ISA.=2
>LET IVR.=3
>LET ISB.=1
>SYST SYS1 REG NOIS CON2             "Definition of the system
>PAR NSA:1                           "Parameters of SYS1
>PAR NSB:1
>PAR NSC:1
>PAR A1:-0.8
>PAR B1:1.
>PAR C1:0.7
>PAR ID:2                            "Parameters of REG
>PAR N1:1
>PAR N2:1
>PAR N3:1
>PAR WTI:0.95
>PAR WTM:0.99
>PLOT TH1 TH2 TH3 A1 B1 C1           "Variables to be plotted
>AXES H 0 500 V -1.8 1.8             "Definition of axes
>SIMU 0 500                          "Simulation
```

The obtained plot is shown in Fig. 5.1. Notice that default values were used for a great number of variables in the simulation, e.g. the initial values of the parameter estimates, the initialization of the matrix P and the variance of the process disturbances.

□

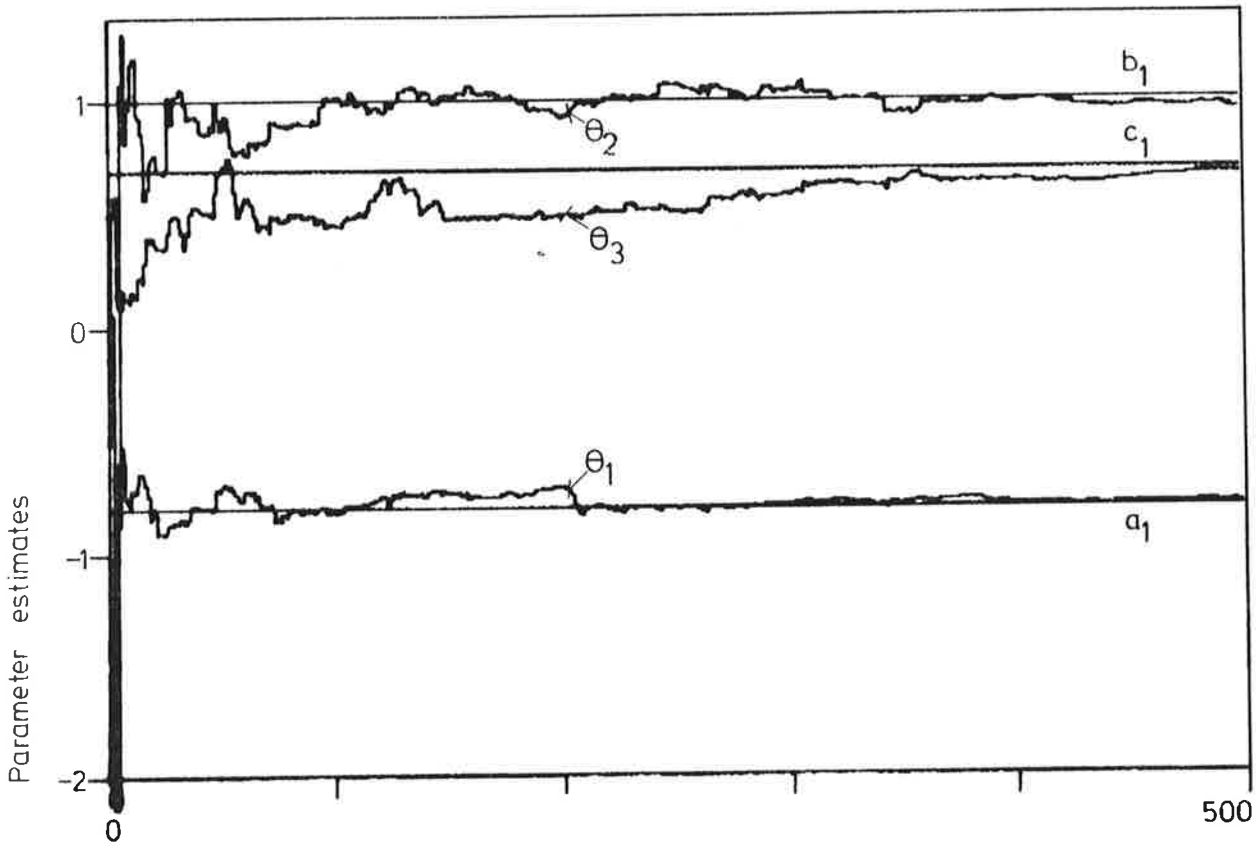

Fig. 5.1 Parameter estimates for Example 5.1.

5.2. Basic self-tuning regulator - STURE1

The basic self-tuning regulator STURE1, Wittenmark (1973), will be described briefly in this section. Different possible variants and extensions of it will also be discussed. Some examples show the use of the program package for simulation of the behaviour of this type of self-tuning regulators.

Method. The algorithm is designed to control a system described by the difference equation

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t) + C(q^{-1})e(t) \tag{5.2.1}$$

where $e(t)$ is a sequence of independent $N(0,\sigma)$ random variables, and

$$A(q^{-1}) = 1 + a_1q^{-1} +\ldots+ a_nq^{-n}$$
$$B(q^{-1}) = b_1q^{-1} +\ldots+ b_nq^{-n}$$
$$C(q^{-1}) = 1 + c_1q^{-1} +\ldots+ c_nq^{-n}$$

It is assumed that all of the parameters of the A-, B- and C-polynomials are unknown but constant. Further it is assumed that the B-polynomial has all its zeroes inside the unit circle, i.e. the system is minimum phase.

If the parameters of the system (5.2.1) are known, the output variance is minimized by using the control law

$$u(t) = -\frac{G(q^{-1})}{B(q^{-1})F(q^{-1})}\, y(t) \tag{5.2.2}$$

where

$$F(q^{-1}) = 1 + f_1q^{-1} +\ldots+ f_kq^{-k}$$
$$G(q^{-1}) = g_o + g_1q^{-1} +\ldots+ g_{n-1}q^{-(n-1)}$$

The polynomials $F(q^{-1})$ and $G(q^{-1})$ are given by the identity

$$C(q^{-1}) = A(q^{-1})F(q^{-1}) + q^{-(k+1)}G(q^{-1}) \tag{5.2.3}$$

If $C(q^{-1})=1$ then the system (5.2.1) can be rewritten as

$$\begin{aligned} y(t) + \alpha_1y(t-k-1) +\ldots+ \alpha_my(t-k-m) = \beta_o[u(t-k-1) + \beta_1u(t-k-2) + \\ +\ldots+ \beta_\ell u(t-k-\ell-1)] + \varepsilon(t) \end{aligned} \tag{5.2.4}$$

where

$$\begin{cases} m = n \\ \ell = n + k - 1 \end{cases} \tag{5.2.5}$$

The disturbance $\varepsilon(t)$ will be a moving average of order k of the noise e(t). The minimum variance regulator for (5.2.4) is simply

$$u(t) = \frac{1}{\beta_0} [\alpha_1 y(t) +\ldots+ \alpha_m y(t-m+1)] - \beta_1 u(t-1) -\ldots- \beta_\ell u(t-\ell) \tag{5.2.6}$$

The basic STURE1 algorithm now consists of two steps which are carried out in every time step,

Step 1: Estimate the parameters $\alpha_1,\ldots, \alpha_m, \beta_1,\ldots,\beta_\ell$ of the model (5.2.4) using the least squares method. The parameter β_0 is assumed to be known.

Step 2: Compute the control signal from (5.2.6) where the parameters α_i and β_i are replaced by the estimates obtained in Step 1.

Remark 1 - The implemented algorithm. The algorithm implemented in the program package is more general than the one described above. The reason is that it should be able also to handle a number of variants of the basic algorithm. The implemented algorithm consists of the following two steps.

Step 1: Estimate the parameters of the model

$$\begin{aligned}
u_1(t) = & -\alpha_1 u_1(t-k_1-k_{del}-1) -\ldots- \alpha_{n_1} u_1(t-k_1-k_{del}-n_1) + \\
& + \beta_0[u_2(t-k_2-1) + \beta_1 u_2(t-k_2-2) +\ldots+ \beta_{n_2} u_2(t-k_2-n_2-1)] + \\
& + \gamma_1 u_3(t-k_3-k_{del}-1) +\ldots+ \gamma_{n_3} u_3(t-k_3-k_{del}-n_3) + \\
& + \delta_1 u_4(t-k_4-k_{del}-1) +\ldots+ \delta_{n_4} u_4(t-k_4-k_{del}-n_4) + \\
& + \eta_1 u_5(t-k_5-k_{del}-1) +\ldots+ \eta_{n_5} u_5(t-k_5-k_{del}-n_5)
\end{aligned}$$

Step 2: Compute the control signal as

$$\begin{aligned}
u(t) = & \frac{1}{\beta_0} [\alpha_1 u_1(t+k_2-k_1-k_{del}) +\ldots+ \alpha_{n_1} u_1(t+k_2-k_1-k_{del}-n_1+1)] - \\
& - \beta_1 u_2(t-1) -\ldots- \beta_{n_2} u_2(t-n_2) - \frac{1}{\beta_0} [\gamma_1 u_3(t+k_2-k_3-k_{del}) +\ldots+ \\
& + \gamma_{n_3} u_3(t+k_2-k_3-k_{del}-n_3+1) + \delta_1 u_4(t+k_2-k_4-k_{del}) +\ldots+ \\
& + \delta_{n_4} u_4(t+k_2-k_4-k_{del}-n_4+1) + \eta_1 u_5(t+k_2-k_5-k_{del}) +\ldots+ \\
& + \eta_{n_5} u_5(t+k_2-k_5-k_{del}-n_5+1)]
\end{aligned}$$

The parameters are stored in the parameter vector as $(\alpha_1,\ldots,\alpha_{n_1}, \beta_1,\ldots, \beta_{n_2}, \gamma_1,\ldots, \gamma_{n_3}, \delta_1,\ldots, \delta_{n_4}, \eta_1,\ldots, \eta_{n_5})$. The notations in this remark, $u_1,\ldots$, $n_1,\ldots$, $k_1,\ldots$ and k_{del}, refer to the corresponding notations used for inputs and parameters in the program.

The inputs of REG are

U1 - process output y(t)

U2 - process input u(t)

U3-U5 - feedforward signals

If a reference signal is included using the structures discussed in Variant 5, (ii) and (iii), page 31, the inputs of REG are

U1 - error signal $y(t)-y_r(t)$

U2 - process input u(t)

U3 - reference signal $y_r(t)$

U4-U5 - feedforward signals

Notice that the inputs U1, U3, U4 and U5 are used in the output section of REG when STURE1 is used (REG:1). This should be kept in mind when SIMNON gives warnings of the type

U3 IS UNDEFINED IN OUTPUT-SECTION OF REG

In such a case a rearranging of the equations in the connecting system may help, see Elmquist (1975).

Typical connecting systems are given in Examples 5.2 and 5.3 and on pages 30 and 32.

<u>Parameters.</u> The parameter determining which control algorithm to use, i.e. in this case STURE1, is

REG:1	Default value: 0.

Normally, the least squares method is used for the estimation, i.e.

ID:1	Default value.

The parameters of the estimation method, i.e.

THO1: THO2: . .	Default values: 0.
PO1: PO2: . .	Default values: 100.

$\boxed{\text{WTI:}}$ Default values: 1.

$\boxed{\text{WTM:}}$

$\boxed{\text{RLIM:}}$ Default value: -1.

$\boxed{\text{IRES:}}$ Default value: 0.

$\boxed{\begin{array}{l}\text{R11:}\\ \text{R12:}\\ \cdot\\ \cdot\end{array}}$ Default values: 0.

$\boxed{\text{DELTA:}}$ Default value: 0.

should be chosen appropriately, see section 5.1 for detailed information.

The model structure parameters, i.e. the orders of the polynomials in the equation (5.2.4) and the estimated time delay k, are chosen by

$\boxed{\begin{array}{l}\text{N1:}m\\ \text{N2:}\ell\\ \text{K1:}k\\ \text{K2:}k\end{array}}$ Default values: 0.

m and ℓ are chosen according to (5.2.5). Notice in particular that <u>both</u> K1 and K2 must be set equal to the estimated system time delay k. The parameter vector TH is then defined as $(\alpha_1,\ldots,\alpha_m,\beta_1,\ldots,\beta_\ell)$.

The control signal may be limited to ±A (A>0) using the parameter

$\boxed{\text{ULIM:A}}$ Default value: -1 (means no limitation)

The value of β_0 ($\neq 0$) should be given by

$\boxed{\text{B0:}\beta_0}$ Default value: 1.

Make sure that β_0 is not estimated, i.e.

$\boxed{\text{IB0:0}}$ Default value

that there is no extra time delay in the regulator, i.e. u(t)=f(y(t), y(t-1),...,u(t-1),...)

$\boxed{\text{KDEL:0}}$ Default value

and that no special structure of the estimation equation (5.2.4) is used as in the presence of a reference signal, i.e.

$\boxed{\text{REF:0}}$ Default value

Example 5.2. The process

$$y(t) - 0.9y(t-1) = 0.25u(t-2) + e(t)$$

will be controlled by the self-tuning regulator STURE1, cf Wittenmark (1973, page 19). The minimum variance controller for this process is

$$u(t) = - \frac{3.24}{1 + 0.9q^{-1}} y(t)$$

The structure that is used by the estimation algorithm is

$$y(t) + \alpha_1 y(t-2) = \beta_0[u(t-2) + \beta_1 u(t-3)] + \varepsilon(t)$$

and the controller will be

$$u(t) = - \frac{\alpha_1/\beta_0}{1 + \beta_1 q^{-1}} y(t)$$

It is assumed that β_0 is 1. A forgetting factor 0.99 is used and the control signal is limited to 5. The result of a simulation is shown in Fig. 5.3. The documentation page is presented in Fig. 5.2.

The connecting system was

```
CONNECTING SYSTEM CON3
TIME T
U1[REG] = Y[SYS1]
U[SYS1] = UR[REG]
U2[REG] = U[SYS1]
END
```

The following command sequence was used for the simulation.

```
>LET IVS.=1                         "Global variables
>LET ISA.=2
>LET IVR.=2
>LET ISB.=3
>LET DATE.=26
>LET MONTH.=5
>LET YEAR.=78
>SYST SYS1 REG  CON3                "Definition of system
>PAR NSA:1                          "Parameters of SYS1
>PAR NSB:1
>PAR A1:-0.9
>PAR B1:0.25
>PAR KS:1
```

```
>PAR REG:1                          "Parameters of REG
>PAR N1:1
>PAR N2:1
>PAR K1:1
>PAR K2:1
>PAR ULIM:5
>PAR WTI:0.99
>PAR IWR:1
>PLOT V                             "Variable to be plotted
>AXES H 0 1000 V 0 2000             "Definition of axes
>SIMU 0 1000                        "Simulation
```

□

```
DOCUMENTATION                 DATE:78 526                 SIMULATION NO:     1
*************


IDENTIFICATION METHOD:          LS
REGULATOR            :        STURE1

N1    1          K1    1
N2    1          K2    1
N3    0          K3    0
N4    0          K4    0
N5    0          K5    0

TH01      0.000000          P01      100.000          R11      0.000000
TH02      0.000000          P02      100.000          R12      0.000000

WTI :     0.990000          RLIM:   -1.00000          DELTA:   0.000000
WTM :      1.00000          IRES:    0

ULIM:     5.00000

KDEL:     0                 Q2  :    0.000000
IB0 :     0                 REF :    0
B0  :     1.00000           INCR:    0

NSA :     1                 KS  :    1                YLEV:    0.000000
NSB :     1                 LAMB:     1.00000
NSC :     0                 NODD:   19

A1      -0.900000         B1      0.250000
```

Fig. 5.2. Documentation page for Example 5.2.

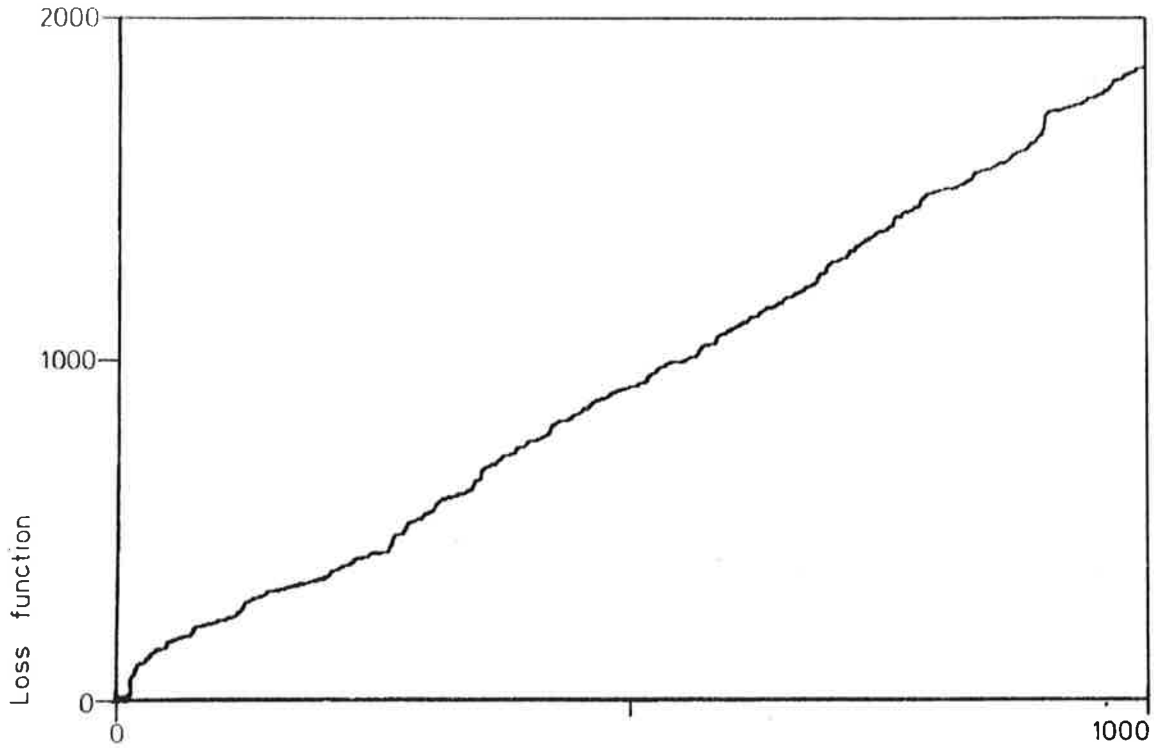

Fig. 5.3. Loss function obtained in Example 5.2.

Variant 1 - STUREO. It is possible to use a stochastic approximation algorithm instead of the least squares one,

$\boxed{\text{ID:5}}$ Default value: 1.

An instrumental variable method can also be used, ID:4.

Variant 2 - β_0 is estimated. If β_0 should be estimated set

$\boxed{\text{IBO:1}}$ Default value: 0.

The value of the parameter BO is then irrelevant. Also change

$\boxed{\text{N2:}\ell\text{+1}}$ Default value: 0.

The parameter vector TH is then defined as $(\alpha_1,\ldots,\alpha_m,\beta_0,\ldots,\beta_\ell)$.

Variant 3 - Extra delay in the regulator. It is possible to let u(t)= =f(y(t-1),...,u(t-1),...), i.e. y(t) is assumed not to be available when computing u(t). Set

$\boxed{\text{KDEL:1}}$ Default value: 0.

Variant 4 - Feedforward signals. Let the system be

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t) + C(q^{-1})e(t) + q^{-k}D(q^{-1})v(t)$$

where $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ are defined as before and

$$D(q^{-1}) = d_1q^{-1} +\ldots+ d_nq^{-n}$$

Further v(t) is known at time t and independent of e(t). The following algorithm can now be used, Wittenmark (1973),

Step 1: Determine the parameters of the model

$$y(t+k+1) + \alpha_1 y(t) +\ldots+ \alpha_m y(t-m+1) = \beta_0[u(t) + \beta_1 u(t-1) +\ldots+$$
$$+ \beta_\ell u(t-\ell)] + \gamma_1 v(t) +\ldots+ \gamma_p v(t-p+1) + \varepsilon(t+k+1)$$

using the least squares algorithm with m=n, ℓ=n+k-1 and p=n+k. The parameter β_0 is assumed to be known.

Step 2: Determine the control signal as

$$u(t) = \frac{1}{\beta_0}\sum_1^m \alpha_i y(t-i+1) - \sum_1^\ell \beta_i u(t-i) - \frac{1}{\beta_0}\sum_1^p \gamma_i v(t-i+1)$$

The inputs of REG are described on page 25. The connecting system may in this case look like

```
CONNECTING SYSTEM CON4
TIME T
U1[REG] = Y[SYS]
U3[REG] = V[FF]
U[SYS] = UR[REG]
U2[REG] = U[SYS]
END
```

where SYS denotes the system to be controlled and V denotes the feed-forward signal generated by the system FF. The same parameters as above should be used but also set

N3:p K3:k	Default values: 0.

The parameter vector TH is defined as $(\alpha_1,\ldots,\alpha_m,\beta_1,\ldots,\beta_\ell,\gamma_1,\ldots,\gamma_p)$.

Up tp three feedforward signals can be connected. They are all treated analogously. If KDEL:1 there is a delay both in y and v, see Remark above.

Variant 5 - Reference signals. There are three possibilities to take care of the problem with reference signals. The reference signal is denoted by $y_r(t)$ in the sequel.

(i) The first possibility is to substitute $y(t)$ with $y(t)-y_r(t)$ for the basic self-tuning regulator STUREl. To take care of the case with constant reference values it is assumed that the system contains an integrator or that one is introduced by using $\Delta u(t)$ as the control signal. $\Delta u(t)$ can be easily obtained from a simple system written in the SIMNON language. Notice that this structure has clear disadvantages for solving this problem, see Åström and Gustavsson (1978).

(ii) The second possibility is to use the algorithm suggested by Wittenmark (1975). The algorithm consists of the following two steps,

Step 1: Estimate the parameters of the model

$$y(t)-y_r(t) = -\alpha_1[y(t-k-1)-y_r(t-k-1)] - \ldots - \alpha_m[y(t-k-m)-y_r(t-k-m)] + \\ + \beta_0[u(t-k-1) + \beta_1 u(t-k-2) + \ldots + \beta_\ell u(t-k-\ell-1)] - \gamma_0 y_r(t) - \\ - \gamma_1 y_r(t-1) - \ldots - \gamma_p y_r(t-p) + \varepsilon(t)$$

using the least squares algorithm with $m=n$, $\ell=n+k-1$ and $p=n+k$.

Step 2: Compute the control signal from

$$u(t) = \frac{1}{\beta_0}\sum_1^m \alpha_i[y(t-i+1)-y_r(t-i+1)] - \sum_1^\ell \beta_i u(t-i) + \\ + \frac{1}{\beta_0}\sum_0^p \gamma_i y_r(t+k-i+1)$$

The parameter γ_0 is not estimated in the algorithm but fixed to 1. This means that β_0 should normally be estimated and further that β_0 will be identifiable in many situations.

Some precaution must be taken to handle steady state errors. Different possibilities are

a) Control with $\Delta u(t)$ instead of $u(t)$, i.e. force an integrator into the system.

b) There are already integrators in the system.

c) A level can be estimated. Let

N4:1
K4:k

and U4[REG]=1 in the connecting system. This has been used by Clarke and Gawthrop (1975).

d) Increase the order of the polynomials in the model so that it is possible to get an integrator in the estimated model.

It is of course possible to combine this algorithm with feedforward signals. In this case, however, no more than two feedforward signals can be connected, to U4[REG] and U5[REG] respectively.

The inputs of REG are defined on page 25. An example of a connecting system is the following one.

```
CONNECTING SYSTEM CON5
TIME T
U1[REG] = Y[SYS] - Y[REF]
U3[REG] = Y[REF]
U[SYS] = UR[REG]
U2[REG] = U[SYS]
END
```

where SYS is the system to be controlled and REF generates the command (reference) signal.

The parameters are defined as for the basic algorithm but use

N1:m	Default values: 0.
N2:ℓ	
N3:p	
K1:k	
K2:k	
K3:k+1	

and

REF:1	Default value: 0.

The parameter vector TH is defined as $(\alpha_1,\ldots,\alpha_m,\beta_1,\ldots,\beta_\ell,\gamma_1,\ldots,\gamma_p)$.

(iii) The third possibility is to use the algorithm suggested by Clarke and Gawthrop (1975), which consists of the two steps,

Step 1: Estimate the parameters of the model

$$y(t)-y_r(t) = -\alpha_1 y(t-k-1) - \ldots - \alpha_m y(t-k-m) + \beta_0[u(t-k-1) + \beta_1 u(t-k-2) + \ldots + \beta_\ell u(t-k-\ell-1)] - \gamma_0 y_r(t) - \gamma_1 y_r(t-1) - \ldots - \gamma_p y_r(t-p) + \delta + \varepsilon(t)$$

using the least squares algoritm with $m=n$, $\ell=n+k-1$ and $p=n_c$ and δ=the level to be estimated, see page 32.

Step 2: Compute the control signal from

$$u(t) = \frac{1}{\beta_0}\sum_1^m \alpha_i y(t-i+1) - \sum_1^\ell \beta_i u(t-i) + \frac{1}{\beta_0}\sum_0^p \gamma_i y_r(t+k-i+1) - \frac{\delta}{\beta_0}$$

Exactly the same comments apply to this algorithm as to algorithm (ii) above. In this case, however, the parameter

REF:2	Default value: 0.

should be chosen.

This algorithm can be modified in the way Clarke and Gawthrop (1975) suggest. They claim that it is possible to have other cost functions than $I=E[y(t)-y_r(t)]^2$, viz.

$$I_1 = E\{[y(t)-y_r(t)]^2 + \lambda u^2(t-k-1)\}$$

$$I_2 = E\{[y(t)-y_r(t)]^2 + \lambda[u(t-k-1)-u(t-k-2)]^2\}$$

These modifications can be obtained by choosing the parameters

Q2:λ	Default value: 0.

and

INCR:	Default value: 0.
INCR:0	Cost function I_1.
INCR:1	Cost function I_2.

Notice that cost function I_1 is identical to I for $\lambda=0$.

Example 5.3. Consider the process

$$y(t) - 0.75y(t-1) = u(t-1)$$

Let the reference signal be a square wave with period 40 time steps. The system is controlled by a self-tuning regulator using the structure given in (iii) above. The parameter β_0 is estimated as well as a level.

In Fig.5.4 the results are shown. At time t=90 a disturbance has been introduced. The parameter ULEV in the connecting system CON6 was changed from zero to 0.3.

The connecting system was

```
CONNECTING SYSTEM CON6
TIME T
U1[REG] = Y[SYS1] - Y[REF]
U3[REG] = Y[REF]
U4[REG] = 1
U[SYS1] = UR[REG] + ULEV
U2[REG] = UR[REG]
ULEV:0
END
```

The system REF is listed in Appendix C.

The following command sequence was used for the simulation.

```
>LET IVS.=1                        "Global variables
>LET ISA.=4
>LET IVR.=3
>LET ISB.=3
>SYST SYS1 REG REF CON6            "Definition of system
>PAR NSA:1                         "Parameters of SYS1
>PAR NSB:1
>PAR A1:-0.75
>PAR B1:1
>PAR LAMB:0
>PAR REG:1                         "Parameters of REG
>PAR REF:2
>PAR N1:1
>PAR N2:1
>PAR N4:1
>PAR K3:1
>PAR IB0:1
>PAR TH02:1
>PAR WTI:0.95
>PLOT Y[REF] Y[SYS1]               "Variables to be plotted
>AXES H 0 250 V -3.6 3.6           "Definition of axes
```

```
>SIMU 0 90                            "Simulation
>PAR ULEV:0.3                         "Change the parameter ULEV of CON6
>SIMU 0 160 - CONT                    "Continuation of simulation
```

□

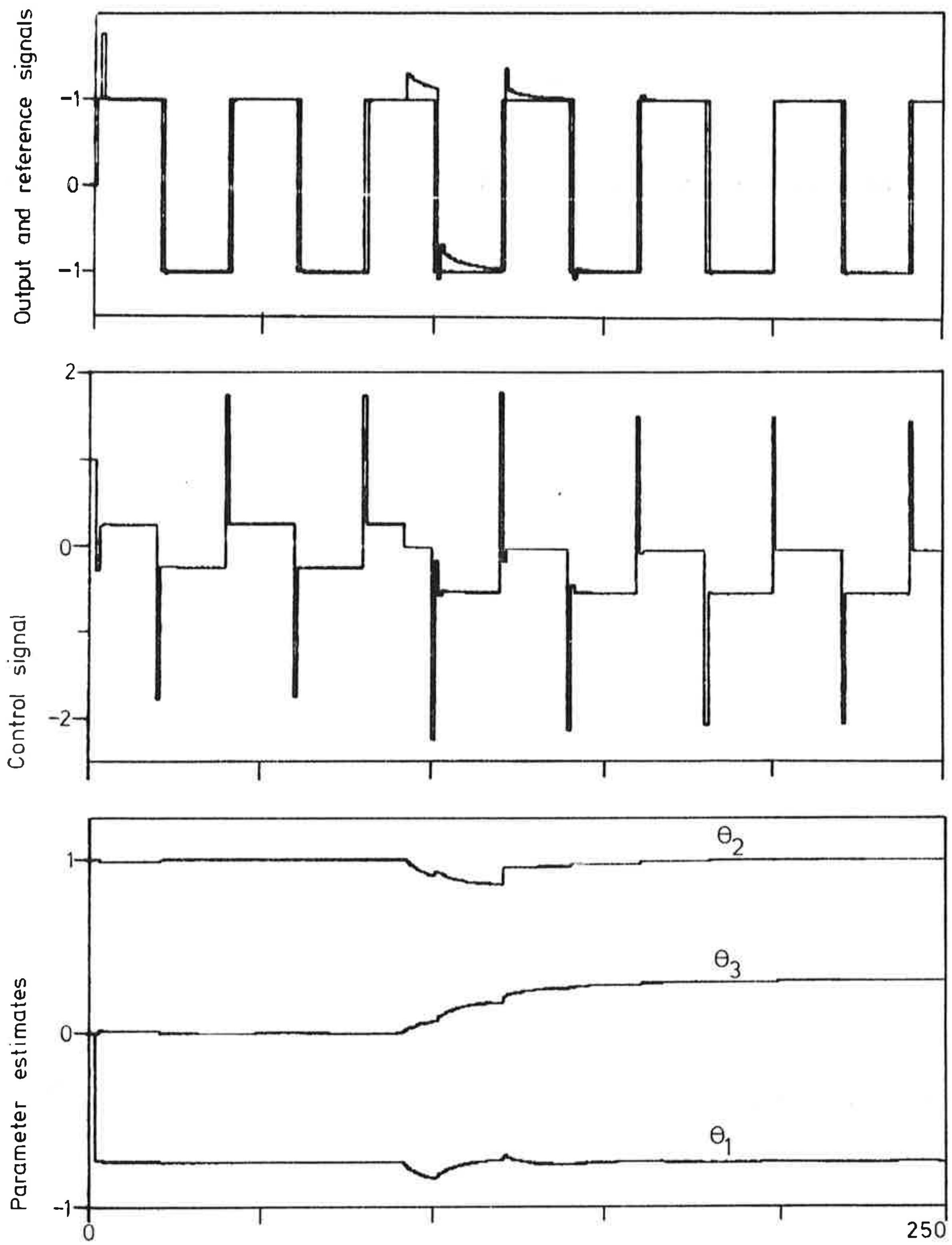

Fig. 5.4 Output and reference signals, control signal and parameter estimates for Example 5.3.

5.3. The self-tuning regulator STURE2

Instead of estimating the parameters of the control law as in STURE1 the parameters of the system can be estimated and then used to compute the control signal. In this section an algorithm is described which uses this idea, see Åström and Wittenmark (1974).

Method. The algorithm STURE2 is designed for the control of the system

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t) + e(t) \qquad (5.3.1)$$

$A(q^{-1})$ and $B(q^{-1})$ are defined as in (5.2.1). The parameters of these polynomials are estimated by e.g. the least squares method. The control law is then computed using the linear quadratic gaussian theory for the problem of minimizing the output variance. The implemented algorithm can also put a penalty on the control signal.

The problem is solved in the following way, see Bengtsson and Egardt (1974). A state space model is introduced with the states

$$x_1(t) = y(t)$$

$$x_2(t) = -a_2y(t-1) -\ldots-a_ny(t-n+1) + b_1u(t-k) +\ldots+ b_nu(t-n-k+1)$$

$$\ldots$$

$$x_{k+1}(t) = -a_{k+1}y(t-1) -\ldots- a_ny(t-n+k) + b_1u(t-1) +\ldots+ b_nu(t-n)$$

$$\ldots$$

$$x_n(t) = -a_ny(t-1) + b_{n-k}u(t-1) +\ldots+b_nu(t-k-1)$$

$$x_{n+1}(t) = b_{n-k+1}u(t-1) +\ldots+b_nu(t-k)$$

$$\ldots$$

$$x_{n+k}(t) = b_nu(t-1)$$

The equation (5.3.1) can then be written as

$$x(t+1) = \Phi x(t) + \Gamma u(t) + Ke(t)$$

where (Φ is of dimension $(n+k)\mathrm{x}(n+k)$)

$$\Phi = \begin{bmatrix} -a_1 & 1 & & & & \\ -a_2 & & 1 & & 0 & \\ \cdot & & & \cdot & & \\ \cdot & & & & \cdot & \\ -a_n & & & & \cdot & \\ 0 & & 0 & & 1 & \\ \cdot & & & & & 1 \\ 0 & & & & & 0 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} 0 \\ \cdot \\ 0 \\ b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{bmatrix} \qquad K = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

The control law which minimizes the loss function

$$J = E\{\Sigma(x^T Q_1 x + u^T Q_2 u)\}$$

is given by

$$u(t) = -L\hat{x}(t|t) \tag{5.3.2}$$

where

$$L = \Gamma^T S\Phi(Q_2 + \Gamma^T S\Gamma)^{-1}$$

$$S = \Phi^T S(\Phi - \Gamma L) + Q_1$$

The output variance is minimized if $Q_1 = \mathrm{diag}(1,0,\ldots,0)$ and $Q_2=0$. It is also possible to use the control law

$$u(t) = -L\hat{x}(t|t-1) \tag{5.3.3}$$

if y(t) is not available when computing u(t).

The inputs of REG are

U1 - process output y(t)

U2 - process input u(t)

Notice that U1 is used in the output section of REG. An example of a connecting system is given in Example 5.4.

Parameters. The parameter determining the control algorithm to use, i.e. STURE2 in this case, is

REG:2	Default value: 0.

Normally the least squares method is used for the estimation, i.e.

ID:1	Default value.

The parameters which control the estimation algorithm are used as described in section 5.1.

The parameters determining the model structure are

N1:n_a N2:n_b K1:0 K2:k	Default values: 0.

The parameter vector TH is defined as $(a_1,\ldots,a_{n_a},b_1,\ldots,b_{n_b})$.

The control signal may be limited by ±A using the parameter

$\boxed{\text{ULIM:A}}$ Default value: -1 (means no limitation)

The control signal may depend on the last output y(t) or not, see (5.3.2) and (5.3.3). The choice between the alternatives is made with the parameter

$\boxed{\text{KDEL:}}$ Default value: 0.

KDEL:0 The control law (5.3.2) is used.

KDEL:1 The control law (5.3.3) is used.

The Ricatti equation is solved iteratively. The maximum number of iterations in each step is determined by the parameter

$\boxed{\text{ITER:}}$ Default value: 10.

It is possible also to penalize the control signal by putting $Q2 \neq 0$. This is done with the parameter

$\boxed{\text{Q2:}}$ Default value: 0.

It is possible to force all poles of the closed loop system to be inside a circle with the radius $r \leq 1$. This is done with the parameter

$\boxed{\text{RO:r}}$ Default value: 1.

<u>Example 5.4.</u> Consider the system

$$y(t) - 0.95y(t-1) = u(t-1) + e(t) - 0.5e(t-1)$$

which should be controlled by the self-tuning regulator STURE2. The example is also used in Bengtsson and Egardt (1974). The weighting factor is chosen to 0.99. It is assumed that y(t) is not available when computing u(t) and the control signal is limited by ±10. The parameter estimates are shown in Fig. 5.5.

The connecting system was

```
CONNECTING SYSTEM CON7
TIME T
U1[REG] = Y[SYS1]
U[SYS1] = UR[REG]
U2[REG] = U[SYS1]
END
```

5.4. The self-tuning regulator STUREM

In this section an algorithm is described which principally is similar to STURE2 but which requires an extended least squares or maximum likelihood estimation algorithm since the system is allowed to have coloured disturbances.

Method. The algorithm STUREM can control the system

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t) + C(q^{-1})e(t) \tag{5.4.1}$$

where $C(q^{-1}) \neq 1$. A, B and C are defined as in (5.2.1). If the system is described by such a model the STURE2 algorithm may converge to a non-optimal regulator. To get an improved behaviour the parameters of the C-polynomial are also estimated and used when calculating the control law using the linear quadratic gaussian theory. This algorithm is called STUREM and includes of course STURE2 as a special case.

The state space model used in STUREM, see Bengtsson and Egardt (1974), is

$$x(t+1) = \Phi x(t) + \Gamma u(t) + Ke(t)$$

$$y(t) = Cx(t) + e(t)$$

where Φ and Γ are defined as in section 5.3. and $K=(c_1-a_1, c_2-a_2,\ldots, c_n-a_n,0,\ldots,0)^T$ and $C=(1,0,\ldots,0)$. The states are defined as

$$x_1(t) = y(t) - e(t)$$

$$x_2(t) = -a_2y(t-1) -\ldots- a_ny(t-n+1) + b_1u(t-k) +\ldots+ b_nu(t-n-k+1) + c_2e(t-1) +\ldots+ c_ne(t-n+1)$$

...

$$x_{k+1}(t) = -a_{k+1}y(t-1) -\ldots- a_ny(t-n+k) + b_1u(t-1) +\ldots+ b_nu(t-n) + c_{k+1}e(t-1) +\ldots+ c_ne(t-n+k)$$

...

$$x_n(t) = -a_ny(t-1) +b_{n-k}u(t-1) +\ldots+ b_nu(t-k-1) + c_ne(t-1)$$

$$x_{n+1}(t) = b_{n-k+1}u(t-1) +\ldots+ b_nu(t-k)$$

...

$$x_{n+k}(t) = b_nu(t-1)$$

The following command sequence was used for the simulation.

```
>LET IVS.=1                          "Global variables
>LET ISA.=2
>LET IVR.=2
>LET ISB.=2
>SYST SYS1 REG CON7                  "Definition of system
>PAR NSA:1                           "Parameters of SYS1
>PAR NSB:1
>PAR NSC:1
>PAR A1:-0.95
>PAR B1:1
>PAR C1:-0.5
>PAR REG:2                           "Parameters of REG
>PAR N1:1
>PAR N2:1
>PAR KDEL:1
>PAR P01:10
>PAR P02:10
>PAR WTI:0.99
>PAR ULIM:10
>PLOT TH1 TH2                        "Variables to be plotted
>AXES H 0 1000 V -1.8 1.8            "Definition of axes
>SIMU 0 1000                         "Simulation
```

□

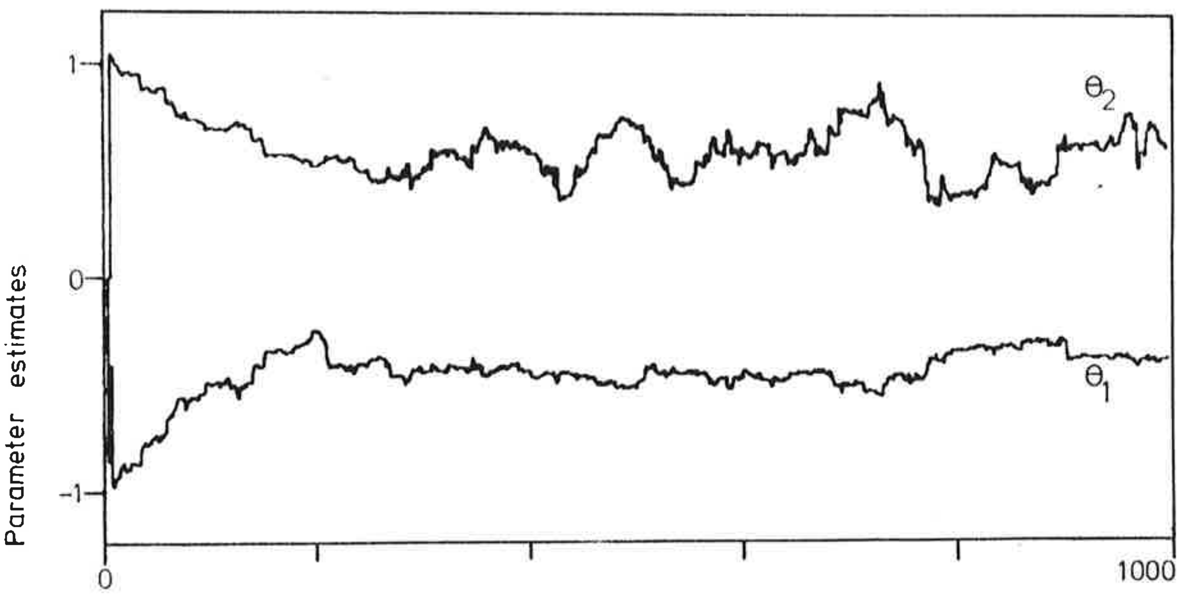

Fig. 5.5 Parameter estimates for Example 5.4

The predicted state vector $\hat{x}(t|t-1)$ is updated using the model, see Bengtsson and Egardt (1974),

$$
\hat{x}(t+1|t) = \begin{bmatrix}
-\hat{a}_1 & \cdot & \cdot & \cdot & -\hat{a}_n & 0 & \cdot & 0 \\
-\hat{a}_2 & & & \cdot & & & & \\
\cdot & & \cdot & & & & & \\
\cdot & \cdot & & & & & & \\
-\hat{a}_n & & & & 0 & & & \\
0 & & & & & & & \\
\cdot & & & & & & & \\
0 & & & & & & &
\end{bmatrix}
\begin{bmatrix}
y(t) \\ y(t-1) \\ \cdot \\ \cdot \\ y(t-n+1) \\ 0 \\ \cdot \\ 0
\end{bmatrix} +
$$

$$
+ \begin{bmatrix}
0 & & \hat{b}_1 & \cdot & \cdot & \cdot & \hat{b}_n \\
& \cdot & & & & \cdot & \\
& \cdot & & & \cdot & & \\
\hat{b}_1 & \cdot & \cdot & \cdot & \hat{b}_n & & \\
\hat{b}_2 & \cdot & \cdot & \hat{b}_n & & & \\
\cdot & & \cdot & & & 0 & \\
\cdot & \cdot & & & & & \\
\hat{b}_n & & & & & &
\end{bmatrix}
\begin{bmatrix}
u(t) \\ \cdot \\ \cdot \\ u(t-k) \\ \cdot \\ \cdot \\ \cdot \\ u(t-n-k+1)
\end{bmatrix} +
$$

$$
+ \begin{bmatrix}
\hat{c}_1 & \cdot & \cdot & \cdot & \hat{c}_n & 0 & \cdot & 0 \\
\hat{c}_2 & & & \cdot & & & & \\
\cdot & & \cdot & & & & & \\
\cdot & \cdot & & & & & & \\
\hat{c}_n & & & & & & & \\
0 & & & & 0 & & & \\
\cdot & & & & & & & \\
0 & & & & & & &
\end{bmatrix}
\begin{bmatrix}
\hat{e}(t) \\ \hat{e}(t-1) \\ \cdot \\ \cdot \\ \hat{e}(t-n+1) \\ 0 \\ \cdot \\ 0
\end{bmatrix}
$$

and the control law would be

$$u(t) = -L\hat{x}(t|t-1)$$

The control law

$$u(t) = -L\hat{x}(t|t)$$

can be used if the estimate $\hat{x}(t|t)$ is computed from $\hat{x}(t|t-1)$. This can be done observing that

$$\hat{x}(t|t) = \hat{x}(t|t-1) + K_1[y(t) - C\hat{x}(t|t-1)] = \hat{x}(t|t-1) + K_1\varepsilon(t) \qquad (5.4.2)$$

and thus

$$\hat{x}(t+1|t) = \Phi\hat{x}(t|t-1) + \Phi K_1\varepsilon(t)$$

Because of equation (5.4.1) we then have

$$\Phi K_1 = (c_1-a_1,\ c_2-a_2,\dots,c_n-a_n,0,\dots,0)^T$$

From this system of equations the elements of the vector K_1 in equation (5.4.2) can be solved, provided $a_n \neq 0$.

The inputs of REG are

U1 - process output y(t)

U2 - process input u(t)

U3 - zero

Notice that U1 is used in the output section of REG. An example of a connecting system is given in Example 5.5.

Parameters. The parameter determining the control algorithm to use, in this case STUREM, is

REG:3	Default value: 0.

Normally, the maximum likelihood or the extended least squares method is used, i.e.

ID:2 or 3	Default value: 1.

The parameters which control the estimation algorithms are used as described in section 5.1.

The parameters determining the model structure are

N1:n_a	Default values: 0.
N2:n_b	
N3:n_c	
K1:0	
K2:k	
K3:0	

The parameter vector TH is defined as $(a_1,\dots,a_{n_a},b_1,\dots,b_{n_b},c_1,\dots,c_{n_c})$.

The other parameters controlling the algorithm STUREM are

ULIM:	Default value: -1.
KDEL:	Default value: 0.

$\boxed{\text{ITER:}}$ Default value: 10.

$\boxed{\text{Q2:}}$ Default value: 0.

$\boxed{\text{R0:}}$ Default value: 1.

and they have the same interpretation as described in section 5.3.

Example 5.5. Consider the system

$$y(t) - 0.95y(t-1) = u(t-1) + e(t) - 0.5e(t-1)$$

which should be controlled with the self-tuning regulator STUREM using the ML estimation algorithm. The example is also used in Bengtsson and Egardt (1974). The forgetting profile is determined by $\lambda(0)=0.99$ and $\lambda_0=0.99$. It is assumed that y(t) is not available when computing u(t). The control signal is limited by ±10 and the residuals are limited by ±10. The parameter estimates are shown in Fig. 5.6.

The connecting system was

```
CONNECTING SYSTEM CON8
TIME T
U1[REG] = Y[SYS1]
U3[REG] = 0
U[SYS1] = UR[REG]
U2[REG] = U[SYS1]
END
```

The following command sequence was used for the simulation.

```
>LET IVS.=1                         "Global variables
>LET ISA.=3
>LET IVR.=3
>LET ISB.=2
>SYST SYS1 REG CON8                 "Definition of system
>PAR NSA:1                          "Parameters of SYS1
>PAR NSB:1
>PAR NSC:1
>PAR A1:-0.95
>PAR B1:1
>PAR C1:-0.5
>PAR REG:3                          "Parameters of REG
>PAR ID:2
```

```
>PAR N1:1
>PAR N2:1
>PAR N3:1
>PAR KDEL:1
>PAR P01:10
>PAR P02:10
>PAR P03:10
>PAR WTI:0.99
>PAR WTM:0.99
>PAR ULIM:10
>PAR RLIM:10
>PLOT TH1 TH2 TH3                    "Variables to be plotted
>AXES H 0 1000 V -1.8 1.8            "Definition of axes
>SIMU 0 1000                         "Simulation
```

□

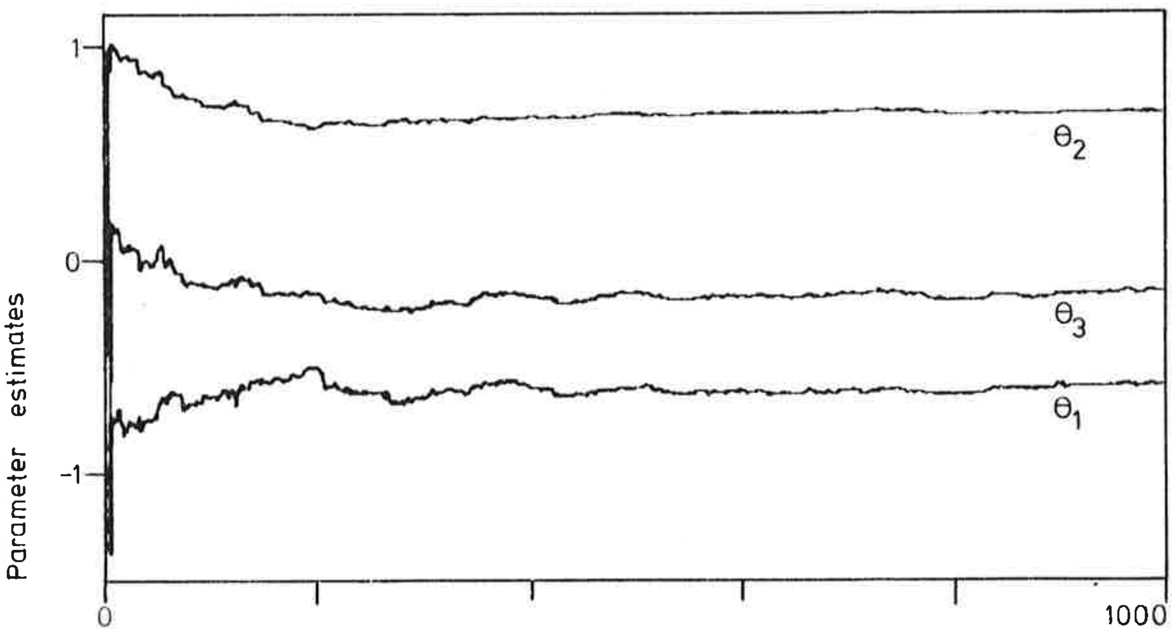

Fig. 5.6 Parameter estimates for Example 5.5.

5.5. Implicit algorithm based on pole placement - STURP1

A self-tuning algorithm based on pole placement will be described briefly in this section. It is principally similar to the STURE1 algorithm in that the parameters of the control law are estimated directly.

Method. This algorithm is designed to control a system described by the equation

$$A(q^{-1})y(t) = q^{-(k+1)}B(q^{-1})u(t) + e(t) \tag{5.5.1}$$

where k is the pure time delay of the system, and

$$A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_{n_a}q^{-n_a}$$
$$B(q^{-1}) = b_o + b_1q^{-1} + \ldots + b_{n_b}q^{-n_b}$$

It is assumed that the polynomial $B(q^{-1})$ has all its zeroes inside the unit circle, i.e. the system is minimum phase.

The control strategy has the structure

$$u(t) = -\frac{S(q^{-1})}{R(q^{-1})}y(t) + \frac{T(q^{-1})}{R(q^{-1})}Gy_r(t) \tag{5.5.2}$$

where G is a gain factor (see below), and

$$T(q^{-1}) = 1 + t_1q^{-1} + \ldots + t_{n_t}q^{-n_t}$$
$$R(q^{-1}) = r_o + r_1q^{-1} + \ldots + r_{n_r}q^{-n_r}$$
$$S(q^{-1}) = s_o + s_1q^{-1} + \ldots + s_{n_s}q^{-n_s}$$

This structure can be obtained if an observer is combined with a linear feedback from the reconstructed states. The polynomial $T(q^{-1})$ is then the characteristic polynomial of the observer. The pole placement is not influenced by the choice of the polynomial $T(q^{-1})$, only the transient behaviour is changed. The polynomial $T(q^{-1})$ is chosen by the user.

The closed loop system will have the desired poles, i.e. will have the characteristic polynomial equal to $A_m(q^{-1})$ given by the user, if the polynomials $R(q^{-1})$ and $S(q^{-1})$ satisfy the identity

$$A_m(q^{-1})T(q^{-1})B(q^{-1}) = A(q^{-1})R(q^{-1}) + q^{-(k+1)}B(q^{-1})S(q^{-1}) \tag{5.5.3}$$

where

$$A_m(q^{-1}) = 1 + a_1^mq^{-1} + \ldots + a_{n_{am}}^m q^{-n_{am}}$$

Straightforward calculations give the closed loop system

$$y(t) = q^{-(k+1)} \frac{G}{A_m(q^{-1})} y_r(t) + \frac{R(q^{-1})}{A_m(q^{-1})T(q^{-1})B(q^{-1})} e(t)$$

The gain factor G is chosen equal to $A_m(1)$, so that the steady state gain from y_r to y is one.

In this so called implicit algorithm the parameters of the polynomials $R(q^{-1})$ and $S(q^{-1})$ are estimated directly in the model structure

$$A_m(q^{-1})T(q^{-1})y(t) = q^{-(k+1)}R(q^{-1})u(t) + q^{-(k+1)}S(q^{-1})y(t) + \bar{R}(q^{-1})e(t) \tag{5.5.4}$$

where

$$R(q^{-1}) = \bar{R}(q^{-1})B(q^{-1})$$

$$\bar{R}(q^{-1}) = \bar{r}_o + \bar{r}_1 q^{-1} + \ldots + \bar{r}_{n_{\bar{r}}} q^{-n_{\bar{r}}}$$

This model can be derived from the identity (5.5.3) and the system equation (5.5.1). Notice that $A_m(q^{-1})T(q^{-1})$ is a filter chosen by the user and thus known at the time of the execution. The left hand side of the estimation equation (5.5.4) is thus just a filtered signal of $y(t)$.

The degrees of the polynomials $R(q^{-1})$ and $S(q^{-1})$ should be chosen as

$$\begin{cases} n_r = n_b + k \\ n_s = \max(\deg A_m + \deg T - k - 1, n_a - 1) \end{cases} \text{ or } \begin{cases} n_r = \max(\deg A_m + \deg T + n_b - n_a, n_b + k) \\ n_s = n_a - 1 \end{cases} \tag{5.5.5}$$

which gives $n_{\bar{r}} = k$. This means that a least squares algorithm can be used for the estimation, since $\bar{R}(q^{-1})e(t)$ will be independent of the other two terms of the right hand side of the estimation equation (5.5.4)

The algorithm is then,

Step 1: Estimate the parameters of the polynomials $R(q^{-1})$ and $S(q^{-1})$ from the equation (5.5.4).

Step 2: Compute the control signal from the equation (5.5.2) with the estimated parameters inserted.

For details see Westerberg (1977) and Åström, Westerberg and Wittenmark (1978).

The inputs of REG are in this case

U1 - error signal $y(t)-y_r(t)$

U2 - process input $u(t)$

U3 - reference signal $y_r(t)$

Notice that the inputs U1 and U3 are used in the output section of REG.

A typical connecting system is given i Example 5.6.

Parameters. Choose the parameter determining the control algorithm to use, i.e. STURP1,

REG:4	Default value: 0.

and the parameter determining the estimation algorithm, normally the least squares method,

ID:1	Default value.

The instrumental variables method (ID:4) and the stochastic approximation algorithm (ID:5) may also be used.

The parameters which control the estimation algorithm are described in section 5.1. Notice that the initial value of r_0 should be non-zero.

The orders of the polynomials $S(q^{-1})$ and $R(q^{-1})$ and the time delay of the system are chosen by

N1:n_s+1	Default values: 0.
N2:n_r+1	
K1:k	
K2:k	

n_s and n_r are chosen according to (5.5.5).

The parameter vector TH is defined as $(s_0,\ldots,s_{n_s},r_0,\ldots,r_{n_r})$.

The control signal may be limited by ±A using

ULIM:A	Default value: -1 (means no limitation).

The observer polynomial $T(q^{-1})$ is defined in the program as

$$T(q^{-1}) = t_1 + t_2q^{-1} + \ldots + t_{n_t}q^{-(n_t-1)} \qquad (n_t \leq 5)$$

which is determined by the parameters

NT:n_t	Default value: 1.
T1:t_1	Default value: 1.
T2:t_2	Default values: 0.
.	
.	

The desired characteristic polynomial of the closed loop system, $A_m(q^{-1})$, is defined in the program as

$$A_m(q^{-1}) = a_1^m + a_2^m q^{-1} + \ldots + a_{n_{a_m}}^m q^{-(n_{a_m}-1)} \qquad (n_{am} \leq 5)$$

which is determined by the parameters

NAM: n_{am}	Default value: 1.
AM1: a_1^m	Default value: 1.
AM2: a_2^m	Default values: 0.
.	
.	

Example 5.6. Consider the system

$y(t) - 0.9y(t-1) = u(t-1)$

It is assumed that the desired behaviour of the closed loop system is characterized by the transfer function $1/(q-0.7)$. Further a first order observer with a pole in 0.5 is used. Hence $T=1-0.5q^{-1}$ and $A_m=1-0.7q^{-1}$. The behaviour of the closed loop system for the algorithm STURP1 is shown in Fig. 5.7, see also Åström, Westerberg and Wittenmark (1978).

The connecting system was

```
CONNECTING SYSTEM CON9
TIME T
U1[REG] = Y[SYS1] - Y[REF]
U3[REG] = Y[REF]
U[SYS1] = UR[REG]
U2[REG] = U[SYS1]
END
```

The following command sequence was used.

```
>LET IVS.=1                      "Global variables
>LET ISA.=3
>LET IVR.=3
>LET ISB.=2
>SYST SYS1 REG REF CON9          "Definition of system
>PAR NSA:1                       "Parameters of SYS1
>PAR NSB:1
>PAR A1:-0.9
>PAR B1:1
>PAR LAMB:0
```

```
>PAR REG:4                            "Parameters of REG
>PAR N1:2
>PAR N2:1
>PAR TH03:2
>PAR P01:10
>PAR P02:10
>PAR P03:10
>PAR WTI:0.99
>PAR NAM:2
>PAR AM2:-0.7
>PAR NT:2
>PAR T2:-0.5
>PAR PER:200                          "Parameters of REF
>PAR NIV1:2
>PAR NIV2:-2
>PLOT Y[SYS1] Y[REF]                  "Variables to be plotted
>AXES H 0 500 V -3.6 3.6              "Definition of axes
>SIMU 0 500                           "Simulation
```

□

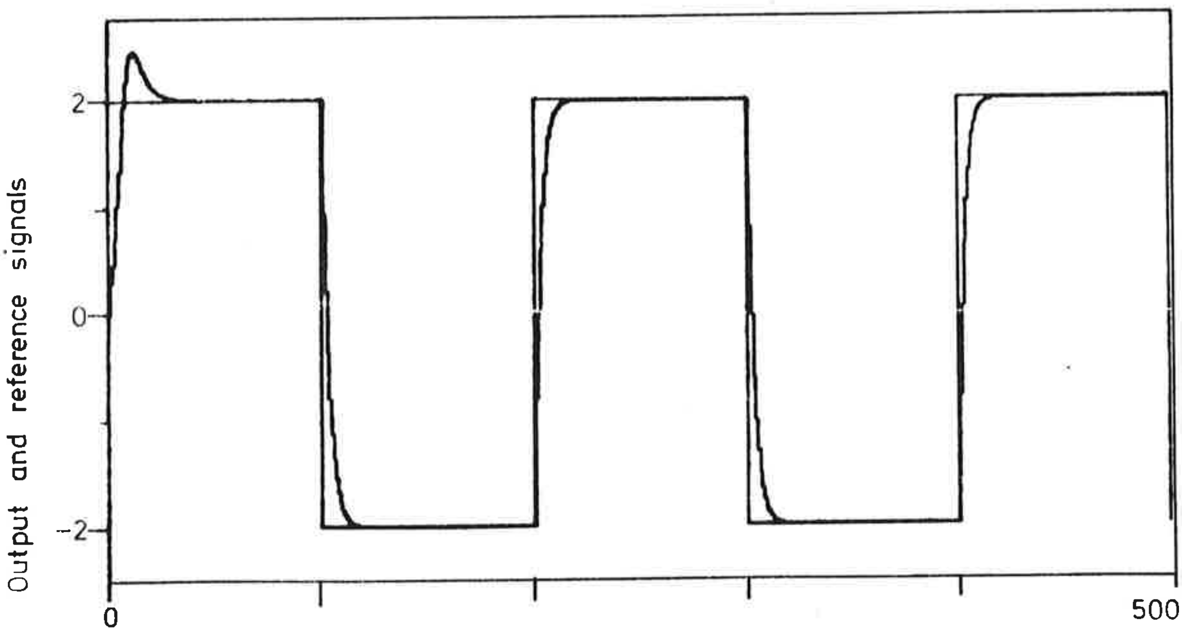

Fig. 5.7 Output and reference signals for Example 5.6.

5.6. Explicit algorithm based on pole placement - STURP2

A second self-tuning regulator based on pole placement is described in this section. It differs from STURP1 in that in this algorithm the parameters of the process are estimated and the controller parameters are then computed from them.

Method. The algorithm is designed for the control of the system described by the equation (5.5.1). The system may in this case be non minimum phase, however. The control strategy has the structure (5.5.2). In order to get the polynomials $R(q^{-1})$ and $S(q^{-1})$ of (5.5.2) the identity

$$A_m(q^{-1})T(q^{-1}) = A(q^{-1})R(q^{-1}) + q^{-(k+1)}B(q^{-1})S(q^{-1}) \tag{5.6.1}$$

is solved, where as in section 5.5 the polynomial $A_m(q^{-1})$ is the desired characteristic polynomial of the closed loop system, and the polynomial $T(q^{-1})$ is the characteristic polynomial of the observer.

Inserting the control strategy (5.5.2) into the system equation (5.5.1) and using the identity (5.6.1) gives the closed loop system

$$y(t) = q^{-(k+1)} \frac{B(q^{-1})}{A_m(q^{-1})} Gy_r(t) + \frac{R(q^{-1})}{A_m(q^{-1})T(q^{-1})} e(t)$$

Thus with the control strategy (5.5.2) the closed loop system can have arbitrary poles without cancellation of the zeroes.

In this algorithm the parameters of the polynomials $A(q^{-1})$ and $B(q^{-1})$ are estimated using a least squares method. The polynomials $R(q^{-1})$ and $S(q^{-1})$ are then solved from the identity (5.6.1) using the estimated polynomials $A(q^{-1})$ and $B(q^{-1})$. The identity has a solution provided that

$$\begin{cases} n_r = n_b + k \\ n_s = \max(\deg A_m + \deg T - n_b - k - 1,\ n_a - 1) \end{cases} \tag{5.6.2}$$

or that

$$\begin{cases} n_r = \max(\deg A_m + \deg T - n_a,\ n_b + k) \\ n_s = n_a - 1 \end{cases} \tag{5.6.3}$$

The estimated steady state gain is adjusted to one in the algorithm with the gain factor G, which is chosen as

$$G = A_m(1)/B(1) \tag{5.6.4}$$

where $B(q^{-1})$ is the estimated polynomial.

The algorithm consists of the following steps,

Step 1: Estimate the parameters of the polynomials $A(q^{-1})$ and $B(q^{-1})$ from the equation (5.5.1).

Step 2: Solve the identity (5.6.1) for the polynomials $R(q^{-1})$ and $S(q^{-1})$.

Step 3: Compute the gain factor G from (5.6.4).

Step 4: Compute the control signal from the equation (5.5.2).

For details see Westerberg (1977) and Åström, Westerberg and Wittenmark (1978).

The inputs of REG are in this case

U1 - error signal $y(t)-y_r(t)$

U2 - process input u(t)

U3 - reference signal $y_r(t)$

Notice that the inputs U1 and U3 are used in the output section of REG.

A typical connecting system is given in Example 5.6.

Parameters. Choose the parameter determining which control algorithm to use, i.e. STURP2,

REG:5	Default value: 0.

and the parameter determining the identification method, which normally is chosen as the least squares method,

ID:1	Default value

The instrumental variable method (ID:4), the stochastic approximation method (ID:5) as well as the extended least squares method (ID:3) and the maximum likelihood method (ID:2) may be used.

The parameters which control the estimation algorithm are described in section 5.1. Notice that the initial value of b_{nb} should be non-zero.

The orders of the polynomials $A(q^{-1})$ and $B(q^{-1})$ and the time delay of the system are chosen by

N1:n_a N2:n_b K1:0 K2:k	Default values: 0.

The parameter vector TH is defined as $(a_1,\dots,a_{n_a},b_1,\dots,b_{n_b})$.

If the extended least squares method or the maximum likelihood method is used, then

N3:n_c K3:0	Default values: 0.

The control signal may be limited by ±A using

ULIM:A	Default value: -1 (means no limitation)

The observer polynomial $T(q^{-1})$ is defined in the program as

$$T(q^{-1}) = t_1 + t_2 q^{-1} + \ldots + t_{n_t} q^{-(n_t-1)} \qquad (n_t \leq 5)$$

which is determined by the parameters

NT:n_t	Default value: 1.
T1:t_1	Default value: 1.
T2:t_2	Default value: 0.
.	
.	

The desired characteristic polynomial of the closed loop system, $A_m(q^{-1})$, is defined in the program as

$$A_m(q^{-1}) = a_1^m + a_2^m q^{-1} + \ldots + a_{n_{am}}^m q^{-(n_{am}-1)} \qquad (n_{am} \leq 5)$$

which is determined by the parameters

NAM:n_{am}	Default value: 1.
AM1:a_1^m	Default value: 1.
AM2:a_2^m	Default values: 0.
.	
.	

The user can choose either the solution (5.6.2) or the solution (5.6.3) of the identity (5.6.1) by the parameter

IOP:	Default value: 0.
IOP:0	Solution (5.6.2).
IOP:1	Solution (5.6.3).

The identity is solved using the subroutines DECOM and SOLVE, see program library. The parameter

EPS:	Default value: 0.

controls the accuracy used in DECOM. If EPS:0 the a computer dependent default value is used.

5.7. General self-tuning regulator including MRAS

A brief description of the general self-tuning regulator implemented will be given. For details, see Egardt (1978). The algorithm is implicit meaning that the parameters of the regulator are estimated directly. Different special cases will be discussed. In particular, the structure of the MRAS included in the formulation will be described.

Method. The algorithm is designed to control a system described by

$$A(q^{-1})y(t) = q^{-(k+1)}b_o B(q^{-1})u(t) + w(t) \tag{5.7.1}$$

where k is the pure time delay of the system, w is a non-measurable disturbance and

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \ldots + a_{n_a} q^{-n_a} \\ B(q^{-1}) &= 1 + b_1 q^{-1} + \ldots + b_{n_b} q^{-n_b} \end{aligned} \tag{5.7.2}$$

It is assumed that b_o is positive and that the system is minimum phase.

The objective of the controller is to make the error

$$e(t) = y(t) - y_m(t) \tag{5.7.3}$$

as small as possible. The reference model output $y_m(t)$ is given by

$$A^m(q^{-1})y_m(t) = q^{-(k+1)}B^m(q^{-1})u_m(t) \tag{5.7.4}$$

where

$$\begin{aligned} A^m(q^{-1}) &= 1 + a_1^m q^{-1} + \ldots + a_{n_{am}}^m q^{-n_{am}} \\ B^m(q^{-1}) &= b_o^m + b_1^m q^{-1} + \ldots + b_{n_{bm}}^m q^{-n_{bm}} \end{aligned} \tag{5.7.5}$$

The control law has the structure

$$b_o B(q^{-1})S(q^{-1})u(t) = -R(q^{-1})y(t) + T(q^{-1})B^m(q^{-1})u_m(t) \tag{5.7.6}$$

where

$$\begin{cases} R(q^{-1}) = r_o + r_1 q^{-1} + \ldots + r_{n_r} q^{-n_r} \\ S(q^{-1}) = 1 + s_1 q^{-1} + \ldots + s_{n_s} q^{-n_s} \\ T(q^{-1}) = 1 + t_1 q^{-1} + \ldots + t_{n_t} q^{-n_t} \end{cases} \tag{5.7.7}$$

This structure can be obtained if an observer is combined with a linear feedback from the state estimates. Furthermore the zeroes are cancelled and new zeroes are added by $B^m(q^{-1})$. The polynomial $T(q^{-1})$ is then the characteristic polynomial of the observer. The closed loop transfer function is not influenced by the choice of the polynomial $T(q^{-1})$, only the transient behaviour and the effect of disturbances are affected. In particular, assume that $w(t)$ in (5.7.1) is given by

$$w(t) = C(q^{-1})v(t) \tag{5.7.8}$$

where $\{v(t)\}$ is a sequence of independent, zero-mean random variables and

$$C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + c_{n_c} q^{-n_c} \tag{5.7.9}$$

Then the optimal (in the sense of minimum variance) choice of the observer polynomial is $T(q^{-1})=C(q^{-1})$.

The error $e(t)$ in (5.7.3) is minimized by the control law (5.7.6) if the polynomials $R(q^{-1})$ and $S(q^{-1})$ satisfy the identity

$$T(q^{-1})A^m(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-(k+1)}R(q^{-1}) \tag{5.7.10}$$

where $T(q^{-1})$ is arbitrary in the deterministic case and $T(q^{-1})=C(q^{-1})$ if the disturbance is given by (5.7.8).

Define the filtered error

$$e_f(t) = \frac{Q(q^{-1})}{P(q^{-1})}\, e(t) \tag{5.7.11}$$

where

$$\left\{\begin{aligned} Q(q^{-1}) &= q_1 + q_2 q^{-1} + \ldots + q_{n_q} q^{-(n_q-1)} \\ P(q^{-1}) &= P_1(q^{-1})P_2(q^{-1}) \\ P_1(q^{-1}) &= p_{11} + p_{12} q^{-1} + \ldots + p_{1n_{p1}} q^{-(n_{p1}-1)} \\ P_2(q^{-1}) &= p_{21} + p_{22} q^{-1} + \ldots + p_{2n_{p2}} q^{-(n_{p2}-1)} \end{aligned}\right. \tag{5.7.12}$$

These polynomials are chosen by the user. Using the polynomial identity (5.7.10), the system equation (5.7.1) and the reference model equation (5.7.4), the following expression is obtained for the filtered error,

$$e_f(t) = \frac{Q}{TA^m}\, b_o q^{-(k+1)}\left[\frac{u(t)}{P_1} + (BS-P_2)\,\frac{u(t)}{P} + \frac{R}{b_o}\,\frac{y(t)}{P} - \frac{1}{b_o}\,\frac{TB^m}{P}\, u_m(t)\right] \tag{5.7.13}$$

This model is used in the algorithm and the parameters of $(BS-P_2)$, R/b_o and the parameter $1/b_o$ are estimated. If the choice $T(q^{-1})=C(q^{-1})$ is desired, the parameters of $T(q^{-1})/b_o$ are estimated instead of just $1/b_o$. The T-polynomial in the transfer function Q/TA^m is then replaced by unity. The degrees of the polynomials $R(q^{-1})$ and $S(q^{-1})$ should be chosen as

$$\begin{cases} n_r = \max(n_a-1,\ \deg A_m + \deg T - k - 1) \\ n_s = k \end{cases} \qquad (5.7.14)$$

In summary the algorithm consists of two steps,

Step 1: Estimate the parameters of the model (5.7.13).

Step 2: Compute the control signal from the equation (5.7.6).

The inputs of REG are

U1 - control signal $u(t)$

U2 - plant output $y(t)$

U3 - reference model input $u_m(t)$

U4 - zero

Notice that the inputs U2 and U3 are used in the output section of REG. A typical connecting system is given in example 5.7.

Parameters. Choose the parameter determining the control algorithm to use,

REG:6 Default value: 0.

and the parameter determining the estimation algorithm, e.g. the least squares method,

ID:1 Default value.

The stochastic approximation algorithm (ID:5) may also be used.

In addition to the parameters described in section 5.1, there are two parameters which control the estimation algorithm. Ordinary LS or SA is obtained with

FILT:1 Default value.

This means that the signals in (5.7.13) are filtered with Q/TA^m and the positive realness condition on Q/TA^m is eliminated.

The MRAS structure of the identification (see Egardt(1978)) is obtained using

MRAS:1 Default value.

The a priori estimate of the residual is always used in this algorithm, i.e. the parameter IRES is irrelevant.

The number of estimated parameters and the time delay are chosen as

N1:1	Default values: 0.
N2:n_b+n_s	
N3:n_r+1	
N4:1	
K1:k	

n_s and n_r are chosen according to (5.7.14).

The control signal may be limited by ±A using

ULIM:A	Default value: -1 (means no limitation)

An estimate β_o ($\neq$ 0) of b_o is given by

B0:β_o	Default value: 1.

The parameter b_o is not estimated if

IB0:0	Default value.

The observer polynomial $T(q^{-1})$ is chosen beforehand if

OBS:0	Default value.

The parameter vector TH is then defined as $(\alpha_1,\ldots,\alpha_{n_b+n_s},r_o/b_o,\ldots,r_{n_r}/b_o,1./b_o)$ where $\alpha_1,\ldots,\alpha_{n_b+n_s}$ are the parameters of the polynomial $BS\text{-}P_2$.

The observer polynomial $T(q^{-1})$ is defined in the program as

$$T(q^{-1}) = t_1 + t_2q^{-1} +\ldots+ t_{n_t}q^{-(n_t-1)}$$

which is determined by the parameters

NT:n_t	Default value: 1.	($n_t \leq 5$)
T1:t_1	Default value: 1.	
T2:t_2	Default values: 0.	
.		
.		

The desired characteristic polynomial of the closed loop system, $A^m(q^{-1})$, is defined in the program as

$$A^m(q^{-1}) = a_1^m + a_2^mq^{-1} +\ldots+ a_{n_{am}}^m q^{-(n_{am}-1)}$$

which is determined by the parameters

NAM: n_{a_m}	Default value: 1.	($n_{a_m} \leq 5$)
AM1: a_1^m	Default value: 1.	
AM2: a_2^m	Default values: 0.	
⋮		

Analogously the closed loop numerator polynomial

$$B^m(q^{-1}) = b_1^m + b_2^m q^{-1} + \ldots + b_{n_{b_m}}^m q^{-(n_{b_m}-1)}$$

is determined by the parameters

NBM: n_{b_m}	Default value: 1.	($n_{b_m} \leq 5$)
BM1: b_1^m	Default value: 1.	
BM2: b_2^m	Default values: 0.	
⋮		

The polynomials $Q(q^{-1})$, $P_1(q^{-1})$ and $P_2(q^{-1})$ are defined in the program as in (5.7.12) and are determined by the parameters

NQ: n_q	Default value: 1.	($n_q \leq 5$)
QP1: q_1	Default value: 1.	
QP2: q_2	Default values: 0.	
⋮		

NP1: n_{p_1}	Default value: 1.	($n_{p_1} \leq 5$)
PP11: p_{11}	Default value: 1.	
PP12: p_{12}	Default values: 0.	
⋮		

NP2: n_{p_2}	Default value: 1.	($n_{p_2} \leq 5$)
PP21: p_{21}	Default value: 1.	
PP22: p_{22}	Default values: 0.	
⋮		

Variant 1 - b_o is estimated. The parameter b_o can be estimated using

IBO:1	Default value: 0.

Notice that BO is used anyway, see Egardt (1978). The parameter vector TH is in this case defined as $(b_o,\alpha_1,\ldots,\alpha_{n_b+n_s},r_o/b_o,\ldots,r_{n_r}/b_o,1./b_o)$ where $\alpha_1,\ldots,\alpha_{n_b+n_s}$ are the parameters of the polynomial BS-P_2.

Variant 2 - Ordinary estimation structure. By choosing

MRAS:0	Default value: 1.

b_o is included in the bracket in (5.7.13) and the model becomes linear in the unknown parameters. The fact that the first parameter in $T(q^{-1})$ is known to be unity, makes it necessary to change the parameter N4,

N4:0	Default value.

The parameter b_o is always estimated and thus IBO and BO are irrelevant. The parameter vector TH is in this case defined as $(b_o,\alpha_1,\ldots,\alpha_{n_b+n_s}, r_o,\ldots,r_{n_r})$ where $\alpha_1,\ldots,\alpha_{n_b+n_s}$ are the parameters of the polynomial b_o(BS-P_2).

Variant 3 - MRAS. The model (5.7.13) with the transfer function Q/TA^m outside the brackets is used by model reference adaptive regulators. These algorithms can thus be simulated using

FILT:0	Default value: 1.

Variant 4 - Observer polynomial estimated. Instead of fixing the observer polynomial $T(q^{-1})$, it can be estimated. Compare the discussion above for the stochastic case. Choose

OBS:1	Default value: 0.

and change N4 to

N4:n_t if MRAS:0 n_t+1 if MRAS:1	Default value: 0.

where n_t is defined by (5.7.7). The parameter vector TH is then defined as $(\alpha_1,\ldots,\alpha_{n_b+n_s},r_o/b_o,\ldots,r_{n_r}/b_o,1./b_o,t_1/b_o,\ldots,t_{n_t}/b_o)$ where $\alpha_1,\ldots,\alpha_{n_b+n_s}$ are the parameters of the polynomial BS-P_2 if MRAS is 1, and as $(b_o,\alpha_1,\ldots,\alpha_{n_b+n_s},r_o,\ldots,r_{n_r},t_1,\ldots,t_{n_t})$ where $\alpha_1,\ldots,\alpha_{n_b+n_s}$ are the parameters of the polynomial b_o(BS-P_2) if MRAS is 0.

Example 5.7. Consider the system

$y(t) - 0.9y(t-1) = u(t-1)$

It is assumed that the desired behaviour of the closed loop system is characterized by the transfer function $0.3/(q-0.7)$. Thus $A_m=1-0.7q^{-1}$ and $B_m=0.3$. The stochastic approximation method is used to identify the parameters. The influence of filtering the signals in (5.7.13) with Q/TA_m is illustrated in Fig. 5.8. In both cases the MRAS structure is used and the observer polynomial is not estimated. The documentation page is shown in Fig. 5.9.

The connecting system was

```
CONNECTING SYSTEM CON10
TIME T
U2[REG] = Y[SYS1]
U3[REG] = Y[REF]
URR = UR[REG]
URP = PL1*URR + PL2
PL1:1
PL2:0
U[SYS1] = URR
U1[REG] = U[SYS1]
U4[REG] = 0.
END
```

The following command sequence was used

```
>LET IVS.=1                         "Global variables
>LET ISA.=4
>LET IVR.=3
>LET ISB.=3
>SYST SYS1 REG REF CON10            "Definition of system
>PAR NSA:1                          "Parameters of SYS1
>PAR NSB:1
>PAR A1:-0.9
>PAR B1:1
>PAR LAMB:0
>PAR ID:5                           "Parameters of REG
>PAR REG:6
>PAR N1:1
```

```
>PAR N3:1
>PAR N4:1
>PAR IB0:1
>PAR TH01:1
>PAR SAP0:10
>PAR WTI:0.1
>PAR NAM:2
>PAR AM2:-0.7
>PAR BM1:0.3
>PAR FILT:0
>PAR IWR:1
>PAR PER:200                         "Parameter of REF
>PAR PL2:-2                          "Parameter of CON10
>PLOT Y[REF] Y[SYS1] URP             "Variables to be plotted
>AXES H 0 500 V -4 1.8               "Definition of axes
>SIMU 0 500                          "Simulation
>AXES                                "New axes
>PAR FILT:1                          "Parameter of REG
>SIMU 0 500                          "Simulation
```

□

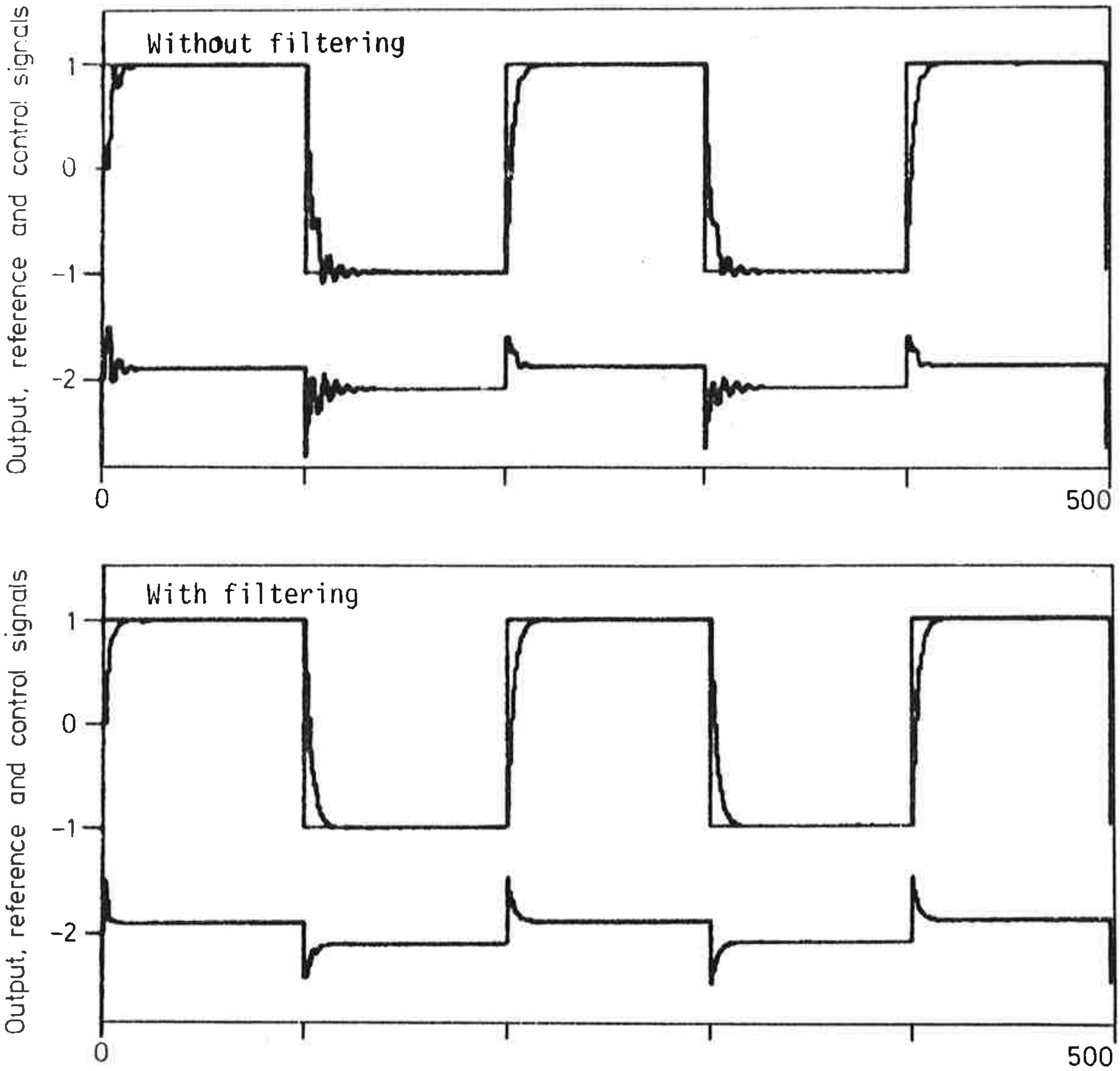

Fig. 5.8 Output, reference and control signals for example 5.7. Notice that the control signal has been shifted 2 units downwards.

```
DOCUMENTATION                 DATE:78- 8-21              SIMULATION NO:     7
*************
IDENTIFICATION METHOD:          SA
REGULATOR            :         MRAS

N1    1          K1    0
N2    0          K2    0
N3    1          K3    0
N4    1          K4    0
N5    0          K5    0

TH01      1.00000
TH02     0.000000
TH03     0.000000
SAP0:     10.0000

WTI :    0.100000            RLIM:  -1.00000             DELTA: 0.000000
WTM :     1.00000            IRES:    0

ULIM:   -1.00000

IB0 :    1                   B0   :   1.00000
MRAS:    1                   OBS:     0                  FILT:   1

NAM:     2       AM:        1.00000       -0.700000
NT:      1       T:         1.00000
NBM:     1       BM:       0.300000
NQ:      1       QP:        1.00000
NP1:     1       PP1:       1.00000
NP2:     1       PP2:       1.00000

NPOL1:   1       POLY1:     1.00000
NPOL2:   2       POLY2:     1.00000       -0.700000
NPOL3:   2       POLY3:     1.00000       -0.700000
NPOL4:   1       POLY4:    0.300000
NPOL5:   2       POLY5:     1.00000       -0.700000

NSA  :   1                   KS   :   0                  YLEV:   0.000000
NSB  :   1                   LAMB:    0.000000
NSC  :   0                   NODD:   19

A1     -0.900000            B1       1.00000
```

Fig. 5.9 Documentation page for example 5.7.

6. ACKNOWLEDGEMENTS

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APPENDIX A - Parameters of the subsystem REG

Parameter	Type	Dimension	Default value	Value of the parameter REG						
				0	1	2	3	4	5	6
ID			1	x	x	x	x	x	x	x
REG			0	x	x	x	x	x	x	x
REF			0		x					
N	Vector	5	(0,0,...)	x	x	x	x	x	x	x
KDEL			0		x	x	x			
K	Vector	5	(0,0,...)	x	x	x	x	x	x	x
ULIM			-1		x	x	x	x	x	x
IBO			0		x					x
BO			1		x					x
DT			1	x	x	x	x	x	x	x
WTI			1	x	x	x	x	x	x	x
WTM			1	x	x	x	x	x	x	x
RLIM			-1	x	x	x	x	x	x	x
IRES			0	x	x	x	x	x	x	
ILS			50	x	x	x	x	x	x	
DELTA			0	x	x	x	x	x	x	x
INCR			0		x					
Q2			0		x	x	x			
ITER			10			x	x			
RO			1			x	x			
IOP			0						x	
EPS			0						x	
FILT			1							x
MRAS			1							x
OBS			0							x
IWR			0	x	x	x	x	x	x	x
NWR1			10	x	x	x	x	x	x	x
NWR2			100	x	x	x	x	x	x	x
NT			1					x	x	x
T	Vector	5	(1,0,...)					x	x	x
NAM			1					x	x	x
AM	Vector	5	(1,0,...)					x	x	x

Parameter	Type	Dimen-sion	Default value	Value of the parameter REG						
				0	1	2	3	4	5	6
NBM			1							x
BM	Vector	5	(1,0,...)							x
NQ			1							x
QP	Vector	5	(1,0,...)							x
NP1			1							x
PP1	Vector	5	(1,0,...)							x
NP2			1							x
PP2	Vector	5	(1,0,...)							x
IPP	Vector	5	(0,0,...)							
RPP	Vector	5	(1,1,...)							
THO	Vector	10	(0,0,...)	x	x	x	x	x	x	x
PO	Vector	10	(100,100,...)	x	x	x	x	x	x	x
SAPO			100	x	x	x	x	x	x	x
R1	Vector	10	(0,0,...)	x	x	x	x	x	x	x

Table giving the parameters of the subsystem REG, their default values and telling in which algorithms they are used. x denotes that the parameter is used in the corresponding algorithm.

APPENDIX B - External systems and MACRO´s for the simulated examples

The necessary commands for the examples in section 5 are listed in this appendix where sequences of commands have been gathered into MACRO´s in order to simplify a reproduction of the results.

EXAMPLE 5.1

External system required: CON2

MACRO´s required: GLOBL, EX51

```
GLOBL 2 3 1 1
SYST SYS1 REG NOIS CON2
EX51
```

EXAMPLE 5.2

External system required: CON3

MACRO´s required: GLOBL, EX52

```
GLOBL 2 2 3 1
SYST SYS1 REG CON3
EX52
```

EXAMPLE 5.3

External systems required: REF, CON6

MACRO´s required: GLOBL, EX53

```
GLOBL 4 3 3 1
SYST SYS1 REG REF CON6
EX53
```

EXAMPLE 5.4

External system required: CON7

MACRO´s required: GLOBL, EX54

```
GLOBL 2 2 2 1
SYST SYS1 REG CON7
EX54
```

EXAMPLE 5.5

External system required: CON8

MACRO´s required: GLOBL, CON8

```
GLOBL 3 3 2 1
SYST SYS1 REG CON8
EX55
```

EXAMPLE 5.6

External systems required: REF, CON9

MACRO´s required: GLOBL, EX56

GLOBL 3 3 2 1

SYST SYS1 REG REF CON9

EX56

EXAMPLE 5.7

External systems required: REF, CON10

MACRO´s required: GLOBL, EX57

GLOBL 4 3 3 1

SYST SYS1 REG REF CON10

EX57

External systems

```
DISCRETE SYSTEM REF

TIME T
OUTPUT Y
TSAMP TS

OUTPUT
Y=IF MOD(T,PER)<(0.5*PER-EPS) THEN NIV1 ELSE NIV2

DYNAMICS
TS=T+DT

PER:40
NIV1:1
NIV2:-1
EPS:0.000001
DT:1

END


CONNECTING SYSTEM CON2
TIME T
U1[REG] = Y[SYS1]
U[SYS1] = E[NOIS]
U2[REG] = E[NOIS]
END


CONNECTING SYSTEM CON3
TIME T
U1[REG] = Y[SYS1]
U[SYS1] = UR[REG]
U2[REG] = U[SYS1]
END


CONNECTING SYSTEM CON6
TIME T
U1[REG] = Y[SYS1] - Y[REF]
U3[REG] = Y[REF]
U4[REG] = 1
U[SYS1] = UR[REG] + ULEV
U2[REG] = UR[REG]
ULEV:0
END
```

```
CONNECTING SYSTEM CON7
TIME T
U1[REG] = Y[SYS1]
U[SYS1] = UR[REG]
U2[REG] = U[SYS1]
END
```

```
CONNECTING SYSTEM CON8
TIME T
U1[REG] = Y[SYS1]
U3[REG] = 0
U[SYS1] = UR[REG]
U2[REG] = U[SYS1]
END
```

```
CONNECTING SYSTEM CON9
TIME T
U1[REG] = Y[SYS1] - Y[REF]
U3[REG] = Y[REF]
U[SYS1] = UR[REG]
U2[REG] = U[SYS1]
END
```

```
CONNECTING SYSTEM CON10
TIME T
U2[REG] = Y[SYS1]
U3[REG] = Y[REF]
URR = UR[REG]
URP = PL1*URR + PL2
PL1:1
PL2:0
U[SYS1] = URR
U1[REG] = U[SYS1]
U4[REG] = 0
END
```

MACRO´s

```
MACRO GLOBL ISA IVR ISB IVS
LET ISA.=ISA
LET IVR.=IVR
LET ISB.=ISB
LET IVS.=IVS
END

MACRO EX51
PAR NSA:1
PAR NSB:1
PAR NSC:1
PAR A1:-0.8
PAR B1:1
PAR C1:0.7
PAR ID:2
PAR N1:1
PAR N2:1
PAR N3:1
PAR WTI:0.95
PAR WTM:0.99
END

MACRO EX52
PAR NSA:1
PAR NSB:1
PAR A1:-0.9
PAR B1:0.25
PAR KS:1
PAR REG:1
PAR N1:1
PAR N2:1
PAR K1:1
PAR K2:1
PAR ULIM:5
PAR WTI:0.99
PAR IWR:1
END

MACRO EX53
PAR NSA:1
PAR NSB:1
PAR A1:-0.75
PAR B1:1
PAR LAMB:0
PAR REG:1
PAR REF:2
PAR N1:1
PAR N2:1
PAR N4:1
PAR K3:1
PAR IBO:1
PAR THO2:1
PAR WTI:0.95
END

MACRO EX54
PAR NSA:1
PAR NSB:1
PAR NSC:1
PAR A1:-0.95
PAR B1:1
PAR C1:-0.5
PAR REG:2
PAR N1:1
PAR N2:1
PAR KDEL:1
PAR PO1:10
PAR PO2:10
PAR WTI:0.99
PAR ULIM:10
END

MACRO EX55
PAR NSA:1
PAR NSB:1
PAR NSC:1
PAR A1:-0.95
PAR B1:1
PAR C1:-0.5
PAR REG:3
PAR ID:2
PAR N1:1
PAR N2:1
PAR N3:1
PAR KDEL:1
PAR PO1:10
PAR PO2:10
PAR PO3:10
PAR WTI:0.99
PAR WTM:0.99
PAR ULIM:10
PAR RLIM:10
END
```

```
MACRO EX56
PAR NSA:1
PAR NSB:1
PAR A1:-0.9
PAR B1:1
PAR LAMB:0
PAR REG:4
PAR N1:2
PAR N2:1
PAR THO3:2
PAR PO1:10
PAR PO2:10
PAR PO3:10
PAR WTI:0.99
PAR NAM:2
PAR AM2:-0.7
PAR NT:2
PAR T2:-0.5
PAR PER:200
PAR NIV1:2
PAR NIV2:-2
END

MACRO EX57
PAR NSA:1
PAR NSB:1
PAR A1:-0.9
PAR B1:1
PAR LAMB:0
PAR ID:5
PAR REG:6
PAR N1:1
PAR N3:1
PAR N4:1
PAR IBO:1
PAR THO1:1
PAR SAPO:10
PAR WTI:0.1
PAR NAM:2
PAR AM2:-0.7
PAR BM1:0.3
PAR FILT:0
PAR IWR:1
PAR PER:200
PAR PL2:-2
END
```