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Blue straggler production in globular clusters

Melvyn B. Davies,[1★] Giampaolo Piotto[2] and Francesca De Angeli[2]

[1] *Department of Physics & Astronomy, University of Leicester, Leicester LE1 7RH*

[2] *Dipartimento di Astronomia, Università di Padova, Vicolo dell'Osservatorio, 2, I-35122 Padova, Italy*

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ABSTRACT

Recent *Hubble Space Telescope* observations of a large sample of globular clusters reveal that every cluster contains between 40 and 400 blue stragglers. The population does not correlate with either stellar collision rate (as would be expected if all blue stragglers were formed via collisions) or total mass (as would be expected if all blue stragglers were formed via the unhindered evolution of a subset of the stellar population). In this paper, we support the idea that blue stragglers are made through *both* channels. The number produced via collisions tends to increase with cluster mass. In this paper we show how the current population produced from primordial binaries *decreases* with increasing cluster mass; exchange encounters with third, single stars in the most massive clusters tend to reduce the fraction of binaries containing a primary close to the current turn-off mass. Rather, their primaries tend to be somewhat more massive ($\sim$1–3 $M_\odot$) and have evolved off the main sequence, filling their Roche lobes in the past, often converting their secondaries into blue stragglers (but more than 1 Gyr or so ago and thus they are no longer visible as blue stragglers). We show that this decline in the primordial blue straggler population is likely to be offset by the increase in the number of blue stragglers produced via collisions. The predicted total blue straggler population is therefore relatively independent of cluster mass, thus matching the observed population. This result does not depend on any particular assumed blue straggler lifetime.

Key words: blue stragglers – globular clusters: general.

1 INTRODUCTION

Globular clusters (GCs) are populated by a number of 'exotic' objects that cannot be explained by canonical stellar evolution models. Many of these objects are the results of a number of dynamical interactions that can modify the original structure of stars, particularly in the central regions of the GCs, where the dynamical time-scales are often much shorter than the cluster lifetime.

Among these exotic objects there are the blue straggler stars (BSs). BSs were first identified as an unusual subclass of stars in the cluster M3 (Sandage 1953). Presently, we know that BSs are present in all the GCs (Piotto et al. 2003). The leading explanation for their formation involves mass transfer in and/or the merger of a binary star system, or collisions between stars (whether or not in binary systems). A number of photometric studies showed that probably all the suggested mechanisms of BS formation are at work in different clusters (Fusi Pecci et al. 1992; Piotto et al. 1999; Ferraro et al. 2003), or even within the same cluster (Ferraro et al. 1997).

Very recently, Piotto et al. (2003) have collected a sample of about 3000 BSs, extracted from the colour–magnitude diagrams of 56 GCs, from their *Hubble Space Telescope* (*HST*) snapshot data base (Piotto et al. 2002).

The main result of the Piotto et al. (2003) study is a strong anticorrelation of the BS frequency $F_{\rm bs}$[1] with the cluster total luminosity M_V and with the stellar collision rate. Clusters with higher total luminosity (mass) and clusters with higher collision probability have a much smaller fraction of BSs.

In this paper, we offer explanations for the observations of Piotto et al. (2003). We show that the observed BS frequency can be the result of the combination of the two formation processes, i.e. collisions or mergers in a crowded place (which we call *dynamical blue stragglers*) and evolution of isolated binaries, which we call *primordial blue stragglers*. This result agrees with that of Hurley et al. (2001), who concluded that half of the BSs in M67 have primordial progenitors whilst half have been formed via dynamical interactions.

In Section 2, we discuss the three observational facts that come out from the analysis of Piotto et al. (2003) and which are of relevance to this paper. In Section 3, we consider the relationship between stellar collision rate and total cluster mass, and compute the likely

[1] $F_{\rm bs} = N_{\rm bs}/N_{\rm hb}$, where $N_{\rm bs}$ is the number of BSs and $N_{\rm hb}$ is the number of horizontal branch stars identified in the same field.

★E-mail: mbd@astro.le.ac.uk

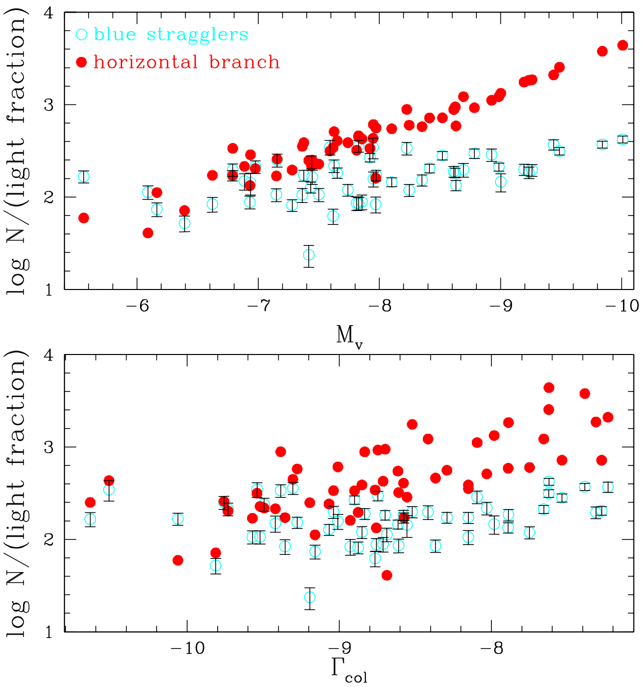

Figure 1. The estimated total number (see text) of BS and HB stars in the sample of 56 GCs as a function of the cluster total magnitude M_V (top panel) and the stellar collision rate, $\Gamma_{\rm coll} \equiv 5 \times 10^{-15}(\Sigma_0^3 r_{\rm c})^{1/2}$ in units of collisions yr^{-1} (bottom panel). See Piotto et al. (2003) for more details.

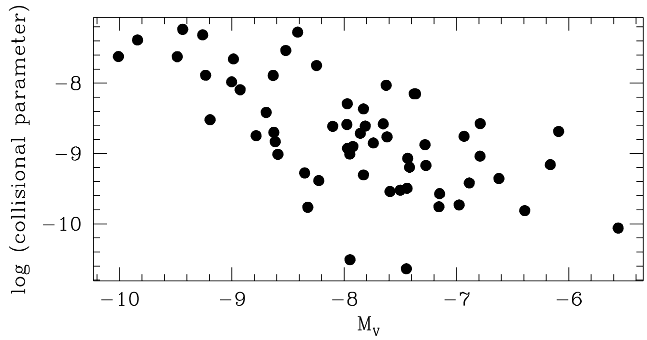

Figure 2. The stellar collision rate $\Gamma_{\rm coll}$ is plotted against the cluster total magnitude M_V.

size of the dynamical BS population. We consider the production of primordial BSs in Section 4, and show why their production rate *decreases* in clusters that are more crowded. We compute the total BS population in Section 5, and show that the decline in the primordial population is at least partially offset by the increase in the formation rate of dynamical BSs in more massive clusters. The combination of the two processes may well reproduce the population seen in Piotto et al. (2003).

In Section 6 we discuss the effect of mass segregation on the likely location of the BS population and show that, at least in some clusters, a large fraction of any primordial BS population is still to be found in the cluster haloes.

2 THE OBSERVATIONAL SCENARIO

Piotto et al. (2003) have shown that there is an anticorrelation between the BS frequency and the GC central collision rate and total mass (cf. their figs 2 and 3). Indeed, this anticorrelation is the consequence of three empirical results that we want to show explicitly here, as they are of relevance to this paper. Figs 1 and 2 show the following.

(i) The total number of horizontal branch (HB) stars in a cluster, $N_{\rm hb}$, scales as expected with M_V such that $N_{\rm hb} \propto M_{\rm tot}$ (Fig. 1). The total number of HB and BS stars of Fig. 1 has been estimated by dividing the number of observed stars by the fraction of cluster total light covered by the WFPC2 images from which the BSs have been extracted.

(ii) There is a correlation between $M_{\rm tot}$ and the current stellar collision rate, $\Gamma_{\rm coll}$ (Fig. 2).[2]

(iii) The total number of BSs in each cluster, $N_{\rm bs}$, is largely independent of both total mass and stellar collision rate (Fig. 1).

The combination of all three of the results listed above produces the anticorrelation seen in the plot of $F_{\rm bs}$ as a function of $\Gamma_{\rm coll}$ shown in fig. 2 of Piotto et al. (2003). Result (i) shows that the number of HB stars scales as the cluster total population, and therefore they must be the result of the ordinary evolution of stars. In this paper we consider the relationship between total cluster mass and current collision rate. We find that the observed correlation between M_V (i.e. total mass) and the current collision rate $\Gamma_{\rm coll}$ [i.e. result (ii) above] is reasonable. Result (iii) is the hardest to explain. We see from Fig. 1 that $N_{\rm bs}$ only varies by a factor of about 10 despite the much larger variations in both $\Gamma_{\rm coll}$ and $M_{\rm tot}$. If all BSs are formed via collisions then we would have expected to find $N_{\rm bs} \propto \Gamma_{\rm coll}$. There is, in fact, a mild correlation between the total number of BSs and the collision rate as shown in Fig. 1, but the correlation is not as strong as would be expected if all BSs were formed via collisions. Alternatively, if all BSs are somehow derived from the initial stellar population (e.g. from original binaries), we would have expected that $N_{\rm bs} \propto M_{\rm tot}$.

3 DYNAMICAL BLUE STRAGGLERS

In this section we compute the size of the BS population today formed via collisions using the current collision rate, $\Gamma_{\rm coll}$. In the case of a cluster very close to core collapse, using $\Gamma_{\rm coll}$ may overestimate the number of BSs that should be observable today. The collision rate may change considerably over the lifetime of currently observed BSs. This effect is only important in the very few clusters in the process of core collapse today, and does not affect the BS population seen in the GC population taken as a whole. Indeed, Piotto et al. (2003) show that the post-core-collapse clusters have BS frequencies comparable to those of the normal King model clusters.

The stellar collision rate within the cluster core is given by $\Gamma_{\rm coll} \propto \rho^2 r_{\rm c}^3/\sigma$, where ρ is the mass density of stars within the cluster core, $r_{\rm c}$ is the core radius, and σ is the velocity dispersion of the stars which is $\propto \sqrt{M_{\rm tot}/r_{\rm h}}$, where $M_{\rm tot}$ is the total mass and $r_{\rm h}$ is the half-mass radius. Also $M_{\rm c} \propto \rho r_{\rm c}^3$. Hence we have

$$\Gamma_{\rm coll} \propto \frac{\rho^2 r_{\rm c}^3}{\sigma} \propto \frac{\rho^2 r_{\rm c}^3}{\sqrt{M_{\rm tot}/r_{\rm h}}} \propto \frac{M_{\rm c}^2 r_{\rm c}^{-3}}{\sqrt{M_{\rm tot}/r_{\rm h}}} \propto \frac{f_{\rm c}^2 r_{\rm h}^{1/2}}{r_{\rm c}^3} M_{\rm tot}^{3/2}, \qquad (1)$$

where $f_{\rm c} = M_{\rm c}/M_{\rm tot}$. Assuming for simplicity that $f_{\rm c}$, $r_{\rm c}$ and $r_{\rm h}$ are the same for all clusters, we see that $\Gamma_{\rm coll} \propto M_{\rm tot}^{1.5}$. Clearly $f_{\rm c}$, $r_{\rm c}$ and $r_{\rm h}$ all vary between clusters, although this simply produces a spread around the relationship. Hence we can understand why $\Gamma_{\rm coll}$ is seen to be strongly correlated with $M_{\rm tot}$ in the observed properties of the GCs.

[2] $\Gamma_{\rm coll} = 5 \times 10^{-15}(\Sigma^3{}_0 r_{\rm c})^{1/2}$, where Σ_0 is the central surface brightness in units of $L_{\odot V}$ pc^{-2} (equivalent to $\mu_v = 26.41$), and $r_{\rm c}$ is the core radius in pc.

We now calculate the number of BSs, formed via collisions, that are likely to be visible today. The current collision rate is given by

$$\Gamma_{\rm coll} = N_{\rm c} n_{\rm c} \Sigma_{\rm coll} v_\infty, \tag{2}$$

where $n_{\rm c}$ is the number density of stars in the core, and $N_{\rm c}$ is the total number of stars in the core. The collision cross-section, $\Sigma_{\rm coll}$, is given by

$$\Sigma_{\rm coll} = \pi r_{\rm coll}^2 \left(1 + \frac{v^2}{v_\infty^2}\right), \tag{3}$$

where $v^2 = 4Gm_\star/r_{\rm coll}$ (where we are assuming that all stars have the same mass, $m_\star$, and $r_{\rm coll}$ is the minimum separation of the two stars). The number of BSs produced via collisions during the previous 10^9 yr is given by

$$N_{\rm coll,BS} = 0.03225 \left(\frac{f_{\rm mms}^2 N_{\rm c} n_{\rm c,5} r_{\rm coll} m_{\rm bs}}{v_\infty}\right), \tag{4}$$

where $r_{\rm coll}$ and $m_{\rm bs}$ are in solar units, $n_{\rm c,5}$ is the stellar number density in units of 10^5 stars pc^{-3}, and $f_{\rm mms}$ is the fraction of stars in the core that are massive main-sequence stars (such that when two collide, they form a BS). The value of $f_{\rm mms}$ is uncertain, but, adopting a value of $f_{\rm mms} = 0.25$ (see e.g. Davies & Benz 1995), we plot $N_{\rm coll,BS}$ as a function of cluster luminosity in Fig. 3. From this figure we can see that collisions seem likely to produce ∼10–100 BSs in a number of clusters. It should be noted that encounters between binaries and other binaries/single stars also lead to collisions, but the numbers will be of the same order. Also included in this figure is a dashed line following $N_{\rm coll,BS} \propto M_{\rm tot}^{1.5}$: the relationship derived in equation (1). $f_{\rm mms}$ is likely to vary by at least a factor of 2–3 between clusters, with more concentrated clusters tending to have larger values of $f_{\rm mms}$ as the stars have undergone a greater degree of mass segregation (see e.g. Davies & Benz 1995). This has a significant effect on $N_{\rm coll,BS}$. There is also some uncertainty concerning the lifetime of BSs. In equation (4) we have in effect assumed a BS lifetime of 1 Gyr. The recent literature predicts quite a broad range of lifetimes, the spread being somewhat connected with the treatment of any excess angular momentum obtained via off-axis collisions (Sills et al. 2001; Lombardi et al. 2002). Even with all these uncertainties, we are able to conclude that *in clusters having the highest collision rates, BS formation via collisions seems likely to produce a current population of* ∼10–100*BSs*.

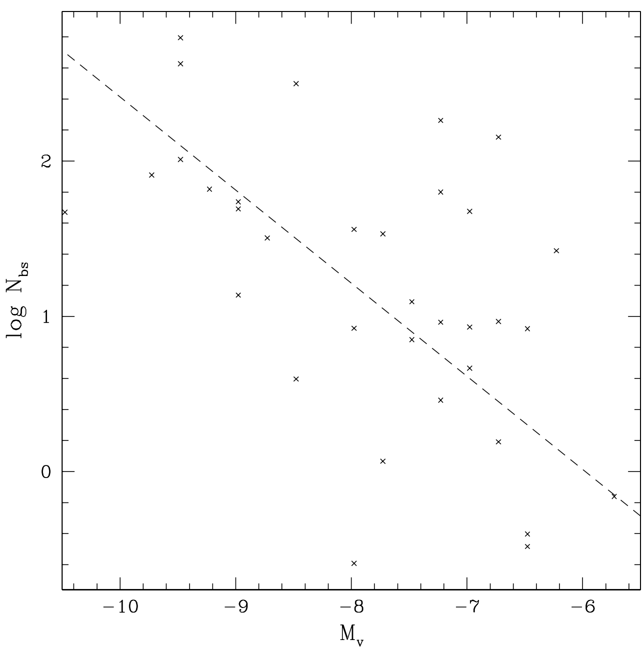

Figure 3. The number of BSs produced via collisions during the last 10^9 yr as a function of absolute cluster luminosity, M_V, assuming $M/L_V = 3$ for all clusters and $f_{\rm bs} = 0.25$ (see Section 3 for details).

4 PRIMORDIAL BLUE STRAGGLERS

In this section we consider the formation of BSs through the isolated evolution of binaries as discussed by Preston & Sneden (2000) and Carney et al. (2001). This must be the mechanism to produce the BSs observed in the field: we will call them *primordial BSs* in this paper. Preston & Sneden (2000) and Carney et al. (2001) consider that such primordial BSs are formed in relatively *wide* binaries. Rather than having the two stars merge via angular momentum loss, in these wider systems a BS is formed when the primary evolves off the main sequence and fills its Roche lobe. Mass transfer on to the secondary (which is still a main-sequence star) may then produce a BS. Certainly very tight binaries will merge, but Preston & Sneden argue that this represents a much smaller population.

How would the cluster habitat affect the evolution of primordial BSs? One might wonder whether the binaries would simply be broken up by encounters. This is unlikely, as most are hard (i.e. binding energy larger than the kinetic energy of any incoming star). One might also wonder whether exchange encounters could alter the system. For example, if a neutron star exchanged into the system, we would have a low-mass X-ray binary on mass transfer. This certainly occurs, but probably not in sufficient numbers to be the whole story. Being in a cluster produces one very important effect, though, which is shown in Fig. 4. The idea is based on a result obtained to explain millisecond pulsar production in globulars (Davies & Hansen 1998). In clusters having high collision rates, exchange encounters produce binaries containing more-massive main-sequence primaries. Once they evolve, mass transfer on to the secondaries produce BSs, but they will have been formed earlier than in clusters having low collision rates (where the primary masses are lower). By today, most of the BSs formed in this way in high collision rate clusters have evolved, hence we see fewer BSs in them compared with less crowded clusters.

To quantify the effect described above, we consider the following simple prescription. We place two stars, drawn from an initial mass function (IMF, see below), in some initial binary. We then consider encounters between this binary and a third star, again drawn from the same IMF. In such encounters involving relatively wide binaries, by far the most common outcome is the ejection of the least massive of the three stars, with the remaining two forming a new binary (see e.g. Davies & Benz 1995). Considering a population of binaries, we calculate the fraction of systems for which the primary masses are in the range $0.8 < M_1 < 0.816\,{\rm M}_\odot$, this being the range of masses that have evolved off the main sequence within the last 1 Gyr [assuming a current turn-off mass of $0.8\,{\rm M}_\odot$ and using the single-star evolution code of Hurley, Pols & Tout (2000) with a metallicity $Z = 0.001$].

We assume a population of stars with an IMF given by Eggleton, Fitchett & Tout (1989). The mass of one star belonging to such a population may be generated using the formula below:

$$M = \frac{0.19x}{(1-x)^{0.75} + 0.032(1-x)^{0.25}}\,{\rm M}_\odot, \tag{5}$$

where x is a random number between 0 and 1. We are thus able to generate the masses of all stars within a population by repeated use of the above equation and a series of random numbers. The fraction

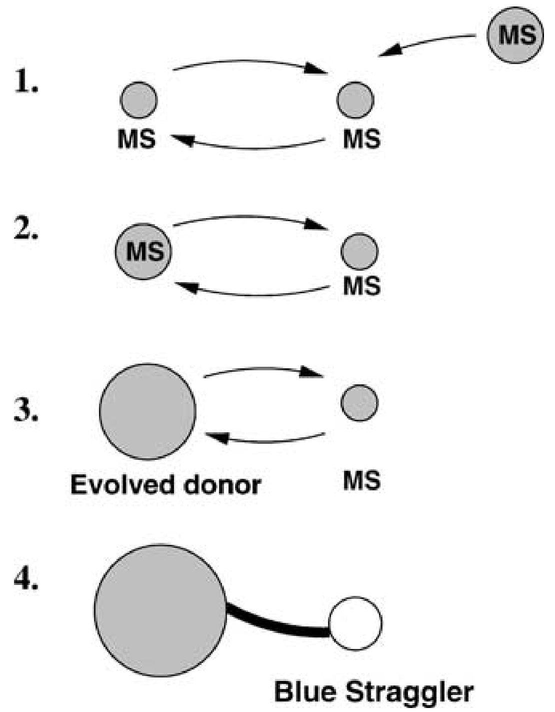

Figure 4. The evolutionary pathway to produce BSs through mass transfer in wide binaries in globular clusters. A more-massive main-sequence star exchanges into a binary containing two main-sequence stars (phase 1). The typical primary mass after encounters in a sufficiently crowded cluster is $M_1 \simeq 1.5$–$3\ \mathrm{M}_\odot$ (Davies & Hansen 1998). This primary evolves off the main sequence and fills its Roche lobe (phase 3). The secondary gains mass from the primary becoming a BS (phase 4) at a time roughly equal to the main sequence lifetime of the donor star. Hence BSs have formed earlier in binaries containing more-massive primaries (i.e. in high collision rate clusters). Given the finite lifetime of BSs, the BS population in the most crowded clusters today could be lower than in very sparse clusters.

of binaries containing a primary in the required range, $f_{\rm pbs}$, is shown in Fig. 5 as a function of the number of encounters, $N_{\rm enc}$. From this figure we clearly see that $f_{\rm pbs}$ decreases significantly when $N_{\rm enc}$ is greater than 10–20. This is a reasonable number of encounters for the most crowded globular clusters. Clearly, there are a number of uncertainties in such a calculation. The cluster density and stellar mass distribution both vary as a function of time. The numbers of encounters are also a function of the distribution of binary separations.

There is also considerable uncertainty concerning the lifetime of BSs. Here we have considered a value of 1 Gyr. Stellar evolution modelling predicts lifetimes between 100 Myr and 5 Gyr (see for example Sills et al. 2001; Lombardi et al. 2002). However, we have found that the exact lifetime considered has no effect on the shape of the curve in Fig. 5, or on the results in later sections of this paper. The general effect described here is robust, namely that more-massive stars tend to exchange into binaries, and, given enough encounters, the number of systems containing primary stars close to the turn-off mass is decreased.

The most crowded clusters could indeed have contained a much larger BS population in the past because the primaries in binaries get replaced by systematically more-massive stars. It could also be that the primary masses are sufficiently large in binaries in the most crowded clusters, and that the evolution once the donor fills its Roche lobe is quite different: a more extreme mass ratio may well

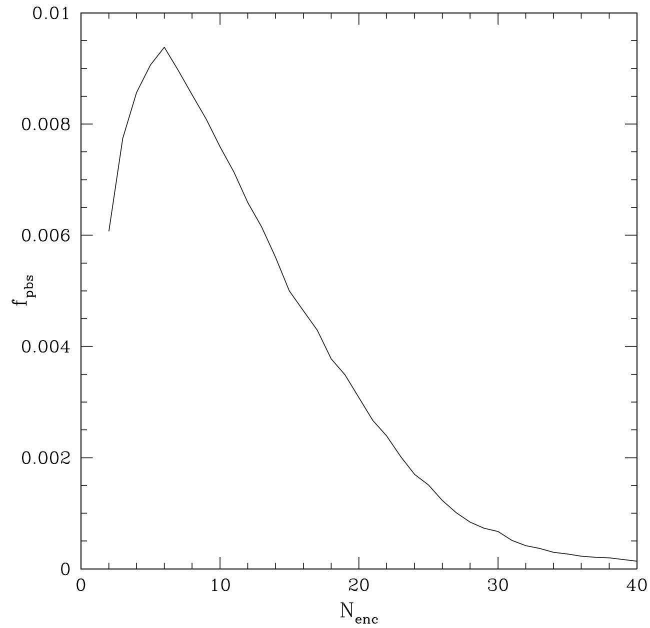

Figure 5. The fraction of binaries containing a primary of mass between 0.8 and 0.816 $\mathrm{M}_\odot$ as a function of the number of encounters that the binary undergoes with single stars, $N_{\rm enc}$. Assuming a current turn-off mass of 0.8 $\mathrm{M}_\odot$, these are the stars that have evolved off the main sequence within the last 1 Gyr.

produce a common-envelope phase rather than conservative mass transfer on to the envelope of the secondary. In both cases, the BS population will be small in such clusters today.

5 THE TOTAL BLUE STRAGGLER POPULATION

In this section we calculate the total number of BSs visible today within GCs combining the contributions from both dynamical BSs (as described in Section 3) and primordial BSs (as described in Section 4).

Following equation (1), we assume that the number of *dynamical* BSs visible today in a particular GC is given by

$$N_{\rm dbs} = k_{\rm dbs} M_{\rm tot}^{3/2}, \qquad (6)$$

where $k_{\rm dbs}$ is a suitably chosen constant. Here we assume that $N_{\rm dbs} = 60$ for $M_{\rm tot} = 10^6\ \mathrm{M}_\odot$ (see, for comparison, Fig. 3, where $M_{\rm tot} = 10^6\ \mathrm{M}_\odot$ is equivalent to $M_V = -9.0$).

The number of *primordial* BSs visible today in a particular GC is given by

$$N_{\rm pbs} = f_{\rm pbs} N_{\rm bin}, \qquad (7)$$

where $f_{\rm pbs}$ is the fraction of binaries containing a primary of the required mass (as plotted in Fig. 5), and $N_{\rm bin}$ is the number of binaries in the given cluster. Here we assume that $N_{\rm bin} = 8 \times 10^4$ for $M_{\rm tot} = 10^6\ \mathrm{M}_\odot$. This may, at first, seem to be rather on the low side, but one should recall that the primordial BSs are believed to form from binaries having a somewhat restricted range of separations (see Preston & Sneden 2000), and this reduces the number of potentially relevant binaries by a factor of a few. The binary fraction tends to be relatively higher for lower mass clusters as more binaries are, on average, resilient to break-up. The separation of binaries lying on the hard–soft boundary $a_{\rm hs} \propto \sigma^2$. For clusters having similar radii, the velocity dispersion within the cluster $\sigma \propto \sqrt{M_{\rm tot}}$. If the binaries

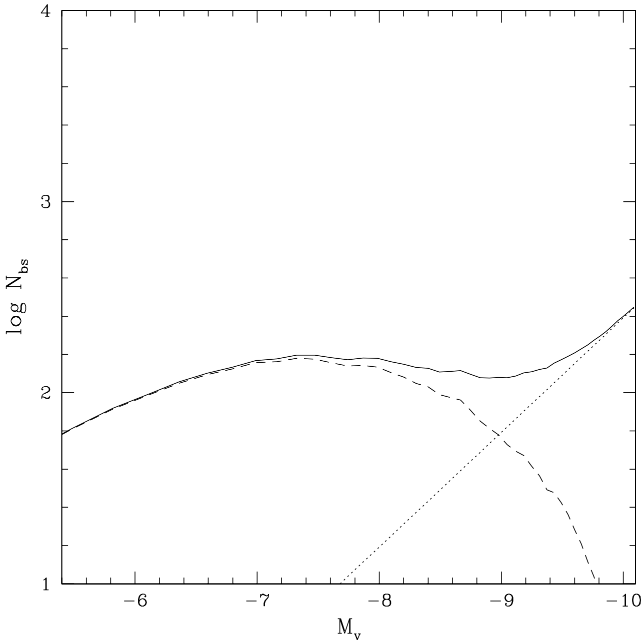

Figure 6. The number of BSs produced over the last 1 Gyr as a function of absolute cluster luminosity, M_V, assuming $M/L_V = 3$ for all clusters. The contribution from primordial systems is shown with a dashed line, whilst those produced via collisions (involving either two single stars or binaries) is shown as a dotted line. The total is given as a solid line.

are distributed uniformly in log(separation) (i.e. equal numbers of binaries per decade), then the binary fraction will scale linearly with $\log(M_{\rm tot})$. We make this assumption here with $N_{\rm bin} = 15 \times 10^3$ for $M_{\rm tot} = 10^5\ {\rm M}_{\odot}$.

We also need to relate the number of encounters that a binary is likely to have to the total cluster mass. As the total number of encounters within the *entire* cluster $\propto M^{3/2}{}_{\rm tot}$, the number of encounters that one binary is likely to have is $N_{\rm enc} \propto M_{\rm tot}^{3/2}/N_{\rm bin} \propto M_{\rm tot}^{1/2}$. The time-scale for a given binary in a GC to undergo an encounter with a third star may be approximated as (Binney & Tremaine 1987)

$$\tau_{\rm enc} \simeq 7 \times 10^{10}\ {\rm yr} \left(\frac{10^5 {\rm pc}^{-3}}{n}\right) \left(\frac{V_\infty}{10 {\rm kms}^{-1}}\right) \left(\frac{{\rm R}_{\odot}}{R_{\rm min}}\right) \left(\frac{{\rm M}_{\odot}}{M}\right), \tag{8}$$

where n is the number density of single stars and M is the combined mass of the binary and a typical single star. Taking $n = 3 \times 10^4$ stars pc^{-3}, $V_\infty = 10$ km s^{-1}, $M = 2.5\ {\rm M}_{\odot}$ and $R_{\rm min} = 200\ {\rm R}_{\odot}$, we find $\tau_{\rm enc} \simeq 420$ Myr, or in other words $N_{\rm enc} \simeq 30$, assuming a cluster age of 14 Gyr. We assume here that these are values typical of the most-massive clusters, i.e. for $M_{\rm tot} = 10^6\ {\rm M}_{\odot}$.

The combined BS populations are shown in Fig. 6. To aid direct comparison with Fig. 1, we have converted total cluster mass to absolute visual magnitude, assuming $M/L_V = 3$ for all clusters. The agreement between the population predicted by the theory and the actual observations shown in Fig. 1 is very encouraging. More important than the absolute numbers is the shape of the solid curve plotted in Fig. 6: the declining primordial population combines with the increasing dynamical population to produce a population whose *size is relatively independent of cluster absolute magnitude (and thus mass)*. This is the main result of this paper. This result is robust to changes in the assumed BS lifetimes. It also changes very little if we change our assumptions concerning the binary fraction. If we assume that the binary fraction $f_{\rm bin} = 0.08$ for all clusters, then $N_{\rm bs}$ will decrease by a factor of 2 [i.e. $\delta\log(N_{\rm bs}) \simeq 0.3$] for the faintest clusters plotted in Fig. 6 whilst the population in the brightest clusters will be largely unchanged.

Piotto et al. (2003) split their observed GCs into two groups: those with $M_V > -8.8$ and those with $M_V \leqslant -8.8$. From Fig. 6, we would predict that the BSs in the more massive (and thus brighter) clusters would be produced predominantly via collisions. Such BSs are expected to be brighter than those formed from primordial binaries (Bailyn & Pinsonneault 1995, but see also fig. 8 in Piotto et al. 1999). Indeed Piotto et al. (2003, their fig. 4) find that the luminosity functions for more-massive clusters do favour brighter BSs, providing further support to the picture suggested here.

6 MASS SEGREGATION

Heavy stars sink within the potential well of a cluster on a time-scale given by the local relaxation time. Stars very far out in the halo of a cluster therefore take much longer to sink into the core. In some clusters this may prove to be a good observational test of the formation mechanism of BSs: in clusters having extremely long relaxation time-scales, any BSs formed from primordial binaries in the halo of a cluster should still be in the halo, and contained within wide binaries.

How does the degree of mass segregation scale with M_V and thus $M_{\rm tot}$? A good indication of what happens can be obtained by considering the relaxation time-scale. The local relaxation time-scale (in years) is given by (see Binney & Tremaine 1987)

$$\begin{aligned} t_{\rm r} &= 0.34 \frac{\sigma^3}{G^2 m \rho \ln \Lambda} \\ &= \frac{1.8 \times 10^{10}}{\ln \Lambda} \left(\frac{\sigma}{10\ {\rm km\ s}^{-1}}\right)^3 \left(\frac{{\rm M}_{\odot}}{m}\right) \left(\frac{10^3\ {\rm M}_{\odot}\ {\rm pc}^{-3}}{\rho}\right). \end{aligned} \tag{9}$$

Note that $t_{\rm r} \propto 1/\rho$: the relaxation time-scale is much longer in the lower density haloes of clusters. We can get some idea of the overall evolution of the cluster by considering the half-mass relaxation time-scale, which is given by

$$\begin{aligned} t_{\rm rh} &= \frac{0.14 N}{\ln(0.4N)} \sqrt{\frac{r_{\rm h}^3}{G M_{\rm tot}}} \\ &= \frac{6.5 \times 10^8 {\rm yr}}{\ln(0.4N)} \left(\frac{M_{\rm tot}}{10^5\ {\rm M}_{\odot}}\right)^{1/2} \left(\frac{{\rm M}_{\odot}}{m}\right) \left(\frac{r_{\rm h}}{1\ {\rm pc}}\right)^{3/2}, \end{aligned} \tag{10}$$

where m is the mass of the particular, heavy star which sinks in time. Heavier stars tend to take longer to sink in more-massive clusters (the extreme example being Omega Centauri). Assuming that the BSs are formed from the isolated evolution of primordial binaries, roughly half of them have sunk to the core in one half-mass relaxation time. For the globular clusters considered here, $t_{\rm rh} \ll 10^{10}$ yr in virtually all cases. However, the relaxation time-scale further out than the half-mass radius is much longer. In some clusters at least, there are some stars observed where the local relaxation time-scale is longer than the cluster age. Indeed, in a number of clusters the observations go further out than two half-mass radii.

If BSs are seen out in the haloes, either they are evolved from isolated primordial binaries, or they have been kicked out from the core [e.g. in M3 (Sigurdsson, Davies & Bolte 1994)]. The latter works more often in low-dispersion clusters. Information about the radial distribution of the BSs in each cluster could prove to be extremely useful. In clusters having relatively high velocity dispersions, any blue stragglers found in the haloes would have to be derived from primordial binaries.

As an example of the likely population of halo BSs, we consider an analytic approximation to a King model as given below (see Hut, McMillan & Romani 1992):

$$\begin{aligned} \rho &= \rho_c (r < r_c), \\ &= \rho_c (r/r_c)^{-2} (r_c < r < r_h), \\ &= \rho_h (r/r_c)^{-4} (r_h < r < r_t), \end{aligned} \tag{11}$$

where $\rho_h = \rho_c (r_h/r_c)^{-2}$ and $\rho_c = M_{tot}/[8\pi r_c^2 (r_h - \frac{2}{3} r_c)]$. Noting that $t_{rh} \simeq 2 \times 10^9$ yr, for a number of clusters, we need to consider the mass of the cluster external to 2 r_h (where $t_r \simeq 10^{10}$ yr, as $t_r \propto 1/\rho$). Taking $r_c = 0.1$ pc, $r_h = 5$ pc and $r_t = 50$ pc, which corresponds to a concentration parameter $c = \log_{10}(r_t/r_c) = 2.7$, and integrating equation (10), yields

$$M_{external} = \int_{2r_h}^{r_t} 4\pi r^2 \rho \, dr \simeq M_{tot}/4. \tag{12}$$

In other words, with the above cluster model, 1/4 of any BSs formed from the isolated evolution of binaries should still be located in the outer halo.

7 CONCLUSIONS

The observations of the BS population within GCs by Piotto et al. (2004) reveal that the number of BSs within each cluster is restricted to a relatively narrow range of values (between 40 and 400 BSs per cluster). If the observed BS population is derived entirely from the unhindered evolution of primordial binaries (as presumedly must occur in the field), then we would have expected the number of BSs to scale linearly with total cluster mass (and thus the BS population per cluster would vary by a factor of about 100). Alternatively, if all BSs are formed via stellar collisions, then we would expect to see a strong correlation between the size of the BS population and the collision rate within a given cluster. This is not observed.

In relatively wide binaries, a BS may be produced when the primary evolves off the main sequence, transferring material on to the secondary which is still on the main sequence. Observations suggest that this is what produces the BSs seen in the halo (Carney et al. 2001; Sneden, Preston & Cowan 2003). We have dubbed them primordial BSs in this paper. Such binaries are vulnerable to exchange encounters in the crowded environments of stellar clusters. In exchange encounters, low-mass components of the binary are replaced by more-massive single stars. The effect is to increase the average mass of the primary with increasing numbers of encounters. The BS population observed today is derived from systems where mass transfer has occurred in the last 1 Gyr or so (i.e. where the primary has evolved off the main sequence within the last 1 Gyr). Encounters between binaries and single stars tend to *reduce* the number of binaries containing primaries with masses close to the present turn-off mass when the number of encounters exceeds 10–20. Thus the population of primordial BSs is reduced in more crowded (and, on average, more massive) clusters.

The stellar collision rate is correlated with cluster mass. Hence the number of dynamically formed BSs increases with cluster mass. This increase offsets the decline in the population of primordial BSs. The predicted total BS population is relatively uniform for all cluster masses, reproducing the observed BS populations.

We have also considered the effects of mass segregation on the current BS population. In some clusters, any surviving primordial BSs may still be found in the outer parts of the halo.

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