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An Improved Moment Method for Transfer Function Identification

Bo M. Bernhardsson and Per G. Persson
Department of Automatic Control
Lund Institute of Technology, Box 118, 221 00 LUND

Introduction

The interest for simple identification methods has been renewed lately when attempts are made to use expert systems in control loops. The knowledge in expert systems often concerns simple concepts, which are easily interpreted physically, e.g., time delays and time constants. Since the users of these systems can not be expected to understand sophisticated model structures or identification methods simplicity is very important.

A useful model structure, able to approximate many industrial processes and with easily interpreted parameters is

$$\frac{ke^{-sL}}{1+sT} \tag{1}$$

Often, again for simplicity reasons, a step (or impulse) response experiment is used for identification.

One often used identification method for systems of the form (1) was suggested as early as 1928 by Küpfmüller. It uses the point of maximum slope on the step response. A variant of this method is the one by Strejc, which uses two points. A drawback with methods which rely on one or a few points of the step response is the sensitiveness to disturbances and measurement noise.

The method of moments (Ba Hli, 1954), is an attempt to decrease the noise sensitivity. This method deals with integrals of the form $\int_0^\infty t^n h(t)\,dt$, where $h(t)$ is the impulse response of the system. However, due to the weighting, the method will be noise sensitive for large t and the method will rely on a suitable truncation of the integral. Also, most weight will be put on identifying the low frequency part correctly.

This paper describes a generalization of the classical moment method. The simplicity is kept and a large reduction in sensitivity is gained by using a different identification frequency $s = \alpha$, where α is a crude estimate of the interesting time constant of the system.

The Method of Moments

To estimate k, L, and T in (1) using the classical method of moments, define

$$g_n = G^{(n)}(0) = \int_0^\infty (-t)^n h(t)\,dt \quad n = 0, 1, 2 \tag{2}$$

where $h(t)$ is the measured impulse response. Then

$$\begin{aligned} k &= g_0 \\ T &= \sqrt{\frac{g_2}{g_0} - (\frac{g_1}{g_0})^2} \\ L &= -T - \frac{g_1}{g_0}. \end{aligned} \tag{3}$$

When the method of moments is applied in practise two problems arise. First, we must truncate the integrals, secondly noise may destroy the estimation.

If we integrate for a short time we get little information of the process, but the estimation is not corrupted by so much noise. If we integrate for a longer time the noise increases the variance of the estimation.

Analysis will follow.

The New Method

To handle the noise sensitivity we propose to use "moments" of the type $\int_0^\infty t^k e^{-\alpha t} h(t)\, dt$ instead. Less weight will then be put on $h(t)$ for large t were the signal to noise ratio is low. Define

$$g_{n,\alpha} = G^{(n)}(\alpha) = \int_0^\infty (-t)^n e^{-\alpha t} h(t)\, dt \quad n = 0, 1, 2 \tag{4}$$

where $h(t)$ is the measured impulse response (a variant using the step response also exists). Introduce

$$\begin{aligned} k^\star &= \frac{ke^{-\alpha L}}{1+\alpha T} \\ T^\star &= \frac{T}{1+\alpha T} \\ L^\star &= L. \end{aligned} \tag{5}$$

Analogously to (3) we then obtain

$$\begin{aligned} k^\star &= g_{0,\alpha} \\ T^\star &= \sqrt{\frac{g_{2,\alpha}}{g_{0,\alpha}} - (\frac{g_{1,\alpha}}{g_{0,\alpha}})^2} \\ L^\star &= -T^\star - \frac{g_{1,\alpha}}{g_{0,\alpha}}. \end{aligned} \tag{6}$$

These equations can now be solved for k, L and T

$$\begin{aligned} L &= L^\star \\ T &= \frac{T^\star}{1-\alpha T^\star} \\ k &= k^\star(1+\alpha T)e^{\alpha L} = \frac{k^\star e^{\alpha L}}{1-\alpha T^\star}. \end{aligned} \tag{7}$$

Analysis will follow

Simulations show that this method produces estimations with monotonically decreasing variance (in t).

Conclusions

Old method: 1) Choice of T important, 2) Noise sensitive. New method: 1) Choice of α less crucial, 2) Less noise sensitive, 3) highlights 'important' frequencies

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