Generalized Frequency Division Multiplexing: Analysis of An Alternative Multi-Carrier Technique for Next Generation Cellular Systems

Michailow, Nicola; Gaspar, Ivan; Krone, Stefan; Lentmaier, Michael; Fettweis, Gerhard

Published in:
International Symposium on Wireless Communication Systems (ISWCS), 2012

DOI:
10.1109/ISWCS.2012.6328352

2012

Generalized Frequency Division Multiplexing: An Alternative Multi-Carrier Technique for Next Generation Cellular Systems

Nicola Michailow, Ivan Gaspar, Stefan Krone, Michael Lentmaier, Gerhard Fettweis

Vodafone Chair Mobile Communications Systems, Technische Universität Dresden, 01069 Dresden, Germany

{nicola.michailow|ivan.gaspar|stefan.krone|michael.lentmaier|fettweis}@ifn.et.tu-dresden.de

***Abstract*—Generalized frequency division multiplexing (GFDM) is a new concept that can be seen as a generalization of traditional OFDM. The scheme is based on the filtered multi-carrier approach and can offer an increased flexibility, which will play a significant role in future cellular applications. In this paper we present the benefits of the pulse shaped carriers in GFDM. We show that based on the FFT/IFFT algorithm, the scheme can be implemented with reasonable computational effort. Further, to be able to relate the results to the recent LTE standard, we present a suitable set of parameters for GFDM.**

I. Introduction

In the last couple of years, the popularity of smartphones has grown tremendously and as a consequence to growing demand, mobile internet has become an affordable service for many people. Along with increasing data rates and improved coverage, this trend enables novel applications of wireless cellular systems that had not been feasible a few years back. Among those, the Internet of Things (IoT) is particularly prominent. The idea of an IoT is based on the prediction that in a couple of years, the internet will not only be used by people, but it will also constitute an infrastructure for the interaction of all kinds of machines and devices from an extremely broad field of application. Assuming each individual will own several IoT enabled devices, the next generation of cellular systems will be faced with a magnitude of larger number of subscribers. And this will introduce a large variety of new requirements, e.g. regarding mobility, data rates, latency, energy efficiency with respect to low-cost battery driven devices and quality of service. Another approach that tries to satisfy the above requirements in a spectrally flexible way is cognitive radio (CR). One particular goal there is to use dynamic spectrum access to exploit spectrum resources, which although they are assigned to a certain service, remain unused at a given time in a given location. This area of application calls for the ability to transmit narrow-band signals with low out-of-band radiation that can be scattered across a large frequency range.

Many recent wireless standards rely on the orthogonal frequency division multiplexing (OFDM) scheme because of various advantages. Like all multi-carrier systems, OFDM benefits from dividing a high data rate stream into several parallel, low data rate streams that are transmitted on different subcarriers, which allows to exploit frequency diversity. In combination with a cyclic prefix (CP), the scheme enables to consider the individual subcarriers as frequency flat and thus enables an easy single-tap equalization. Further, the orthogonality between the subcarriers enables an efficient, low-complexity transmitter and receiver implementation based on the fast Fourier transform (FFT) algorithm. However, the scheme also exhibits some disadvantageous properties that make it unable to address several of the previously mentioned requirements. With its strong out-of-band radiation, OFDM can present a non-negligible interference in overlay systems applications, which makes additional filters necessary in order to meet a desired spectral mask. OFDM is also very sensitive in terms of carrier frequency offset, which requires sophisticated synchronization mechanisms to guarantee that the orthogonally is not affected. Lastly, the cyclic prefix approach constitutes a necessary overhead that can reduce the overall energy efficiency of the system. Also, depending on the application, the scheme suffers from high peak-to-average power ratio due to the superposition of many subcarriers, which can increase the requirements to amplifiers. Thus, novel transmission schemes are researched. Several concepts that have emerged in this area during the past years are based on the approach of filtered multi-carrer transmission, which has been known even before OFDM gained popularity [1], [2].

Among those, generalized frequency division multiplexing (GFDM) [3], [4] is a new concept for flexible multi-carrier transmission that introduces additional degrees of freedom when compared to traditional OFDM. In GFDM, the out-of-band radiation of the transmit signal is controlled by an adjustable pulse shaping filter that is applied to the individual subcarriers. Further, a two-dimensional data structure is introduced to group data symbols across several subcarriers and time slots to blocks. The size of the blocks is a variable parameter and allows to implement long filters or to reduce the total number of subcarriers. The processing of these blocks is done based on tail-biting digital filters that preserve circular properties across time and frequency domain. Similar to OFDM, in GFDM a cyclic prefix can be used to combat ISI in a multipath channel.

Filter bank multi-carrier (FBMC) [5], [6] is another technique that can provide strong side lobe suppression of the transmit signal, which is different from GFDM. There, the pulse shaping filter is implemented with the help of a polyphase network. Further, offset QAM modulation is utilized to avoid intercarrier-interference (ICI) between neighboring subcarriers. The scheme discards the concept of cyclic prefix (CP) and relies on a per-subcarrier equalization to combat intersymbol-interference (ISI).

The goal of this paper is on the one hand to extend previous work on GFDM by a low-complexity transmitter model that is suited for a hardware implementation and on the other hand to provide a comparison with the LTE standard.

The rest of this paper is organized as follows: In Section II we discuss the implications of high out of band radiation and recapitulate two ways of looking at the GFDM transmitter, that are known from previous work. In Section III, a new model suited for low complexity implementation is derived. Section IV deals with the comparison of computational expense among the different GFDM models and OFDM and further a set of reference parameters suitable for the comparison of GFDM and OFDM is presented. Finally, conclusions are drawn in Section V.

II. Background

Out-of-band radiation is an important issue for any kind of cellular communication system as spectrum resources are subject to strict government regulations. In OFDM based systems, where each subcarrier is shaped with a rectangular pulse in time, the first side lobes of the corresponding frequency domain $\frac{\sin(f)}{f}$ pulse decay fairly slowy. On the one hand, this makes it necessary to introduce additional filters in order to satisfy a certain spectral mask. On the other hand, it makes it difficult to access vacant resources within a system's bandwidth in an opportunistic fashion without adaptive filtering. This filtering can cause ISI which requires a longer CP, otherwise the ISI will eventually cause ICI when detected with a conventional OFDM receiver and degrade the performance. These two aspects shall serve as the main motivation to introduce additional signal processing efforts to the transmitter and receiver of a wireless system, in order to improve out-of-band radiation properties. In GFDM, each subcarrier individually is shaped with a filter and as can be seen in Fig. 1, depending on the system parameters this allows to significantly improve the spectral properties. And as ISI /ICI are a systematic part of GFDM [4], it is further expected that the requirements towards synchronization can be relaxed. In the rest of this paper, we will introduce a concept for an efficient implementation of GFDM that allows to achieve this strong out-of-band attenuation with reasonable computational complexity and memory requirements.

A. *Transmitter Model*

Consider a system according to [3] that is modeled in baseband. Let a set of complex valued data symbols $d_k[m]$, $k = 0 \dots K-1$, $m = 0 \dots M-1$ be given, which are distributed across K active subcarriers and M active time slots. Each subcarrier on its own is pulse shaped with a transmit filter $\tilde{g}_{\mathrm{Tx}}[n]$ and modulated with a subcarrier center

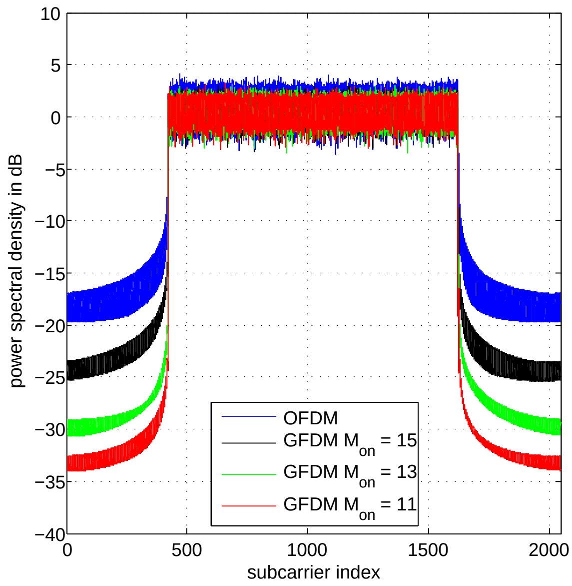


Fig. 1. Power spectral density (PSD) of OFDM and GFDM ($K = 1200$, $N = 2048$ and examplary root raised cosine pulse with roll-off $a = 0.25$)

frequency $e^{j2\pi\frac{kn}{N}}$. Each symbol is sampled $N \geq K$ times, leading to a total of MN samples per subcarrier, which is necessary in order to satisfy the Nyquist criterion. The transmit signal

$$x[n] = \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} d_k[m]\, \tilde{g}_{\mathrm{Tx}}[n - mN] e^{j2\pi\frac{kn}{N}}, \qquad (1)$$

is obtained through superposition of the filtered data symbols of all subcarriers and time slots. The filter $\tilde{g}_{\mathrm{Tx}}[n]$ is circular with periodicity $n \bmod MN$, thus the tail-biting technique is applied at the transmitter.

From (1), a linear mapping of a vector $\mathbf{d} = \{d[\ell]\}_{KM}$, $\ell = \ell(k, m)$ containing KM data symbols to a vector $\mathbf{x} = \{x[n]\}_{NM}$ containing NM transmit samples according to

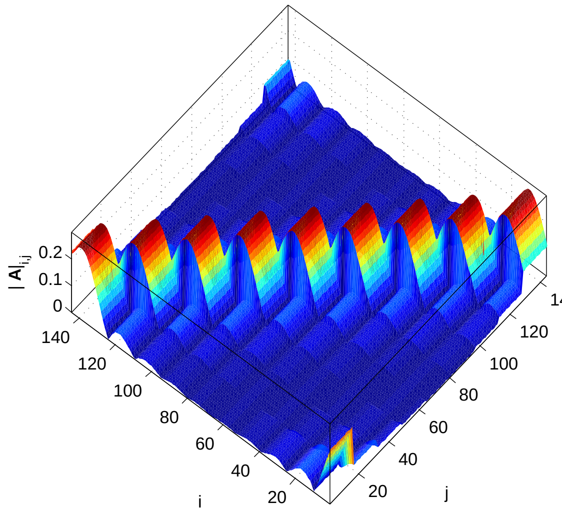
$$\mathbf{x} = \mathbf{A}\mathbf{d} \qquad (2)$$

can be derived, where $\mathbf{A}$ denotes an $NM \times KM$ modulation matrix. This representation allows to easily apply standard receiver methods to the GFDM system [4].

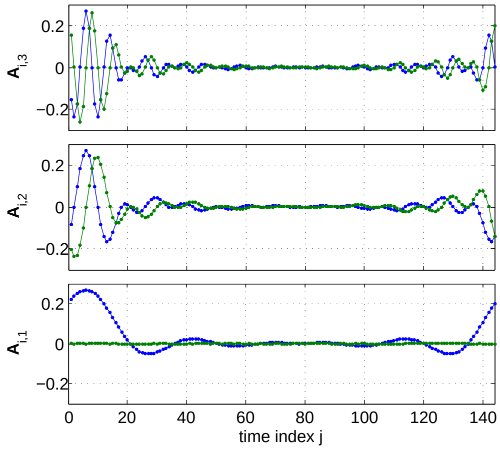
The structure of $\mathbf{A}$ is shown in Fig. 2. From the absolute value of the modulation matrix in Fig. 2(a), it can be seen that there is a repeating pattern that results in a block diagonal structure for all possible phase responses. By closely looking at the individual columns of the first phase response in Fig. 2(d), it becomes evident that the matrix also contains the responses of the pulse shaping filter for all possible subcarriers. This leads to an excellent model for studying the nature of GFDM.

III. A Low Complexity Transmitter Implementation for GFDM

From a hardware perspective, a straightforward implementation of the models (1) and (2) may turn out not very suitable.

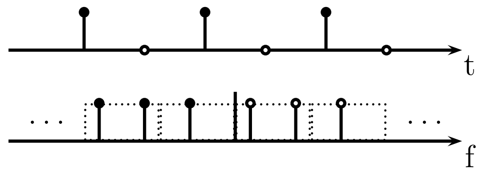


(a) Absolute value of $\{\mathbf{A}\}_{i,j}$

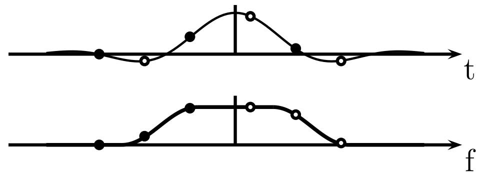


(d) Real and imaginary part of $\{\mathbf{A}\}_{i,1}$, $\{\mathbf{A}\}_{i,2}$ and $\{\mathbf{A}\}_{i,3}$

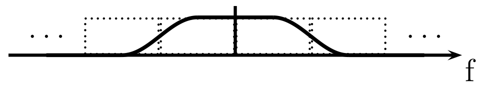
Fig. 2. Structure of the modulation matrix $\mathbf{A}$ for a system with $K = 16$ subcarriers and $M = 9$ time slots.



(a) Upsampling in time and frequency domain



(b) Pulse shaping filter in time and frequency domain



(c) Filtering in frequency domain

Fig. 3. Subcarrier processing in time and frequency domain

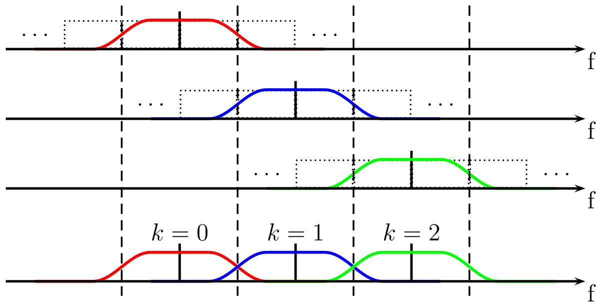


Fig. 4. Subcarrier superposition in frequency domain

By assessing just the number of complex valued multiplications that are necessary to produce $x[n]$, the two approaches result in a number $C_{\text{GFDM},\Sigma} = C_{\text{GFDM},\mathbf{A}} = NKM^2$.

But there is a big potential for savings, when reformulating the GFDM transmitter in a fashion that is similar to the well known IFFT/FFT approach that is used in OFDM. To be able to do that, the transmit signal from (1) shall be rewritten as $x[n] = \sum_k x_k[n]$, where

$$x_k[n] = [(d_k[m]\delta[n - mN]) \circledast g_{\text{Tx}}[n]]\, e^{j2\pi \frac{k}{N} n} \tag{3}$$

is the transmit signal of the kth subcarrier. Note that here $g_{\text{Tx}}[n]$ constitutes one complete period of $\tilde{g}_{\text{Tx}}[n]$ and thus the circular convolution denoted by $\circledast$ is performed with respect to n and with periodicity NM. So the modulation of an individual subcarrier in (3) can be broken down to the convolution of a Dirac pulse train $d_k[m]\delta[n - mN]$ with a filter response $g_{\text{Tx}}[n]$ and a subsequent multiplication with a complex valued oscillation $e^{j2\pi \frac{k}{N} n}$. Carrying over this operation to frequency domain, it can be equally written as

$$\begin{aligned} x_k[n] = \text{IDFT}_{NM}\,(&\text{DFT}_{NM}(\, d_k[m]\delta[n - mN]\,) \cdot \\ &\text{DFT}_{NM}(\, g_{\text{Tx}}[n]\,) \circledast \text{DFT}_{NM}\Big(e^{j2\pi \frac{k}{N} n}\Big)\Big)\,, \end{aligned} \tag{4}$$

where $\text{DFT}_{NM}(\,\bullet\,)$ is the NM-point discrete Fourier transform and $\text{IDFT}_{NM}(\,\bullet\,)$ denotes the corresponding inverse operation. Now the left side of the product, $\text{DFT}_{NM}(\, d_k[m]\delta[n - mN]\,)$ can be interpreted as capturing N periods of the M points periodic sequence $\text{DFT}_M(\, d_k[m]\,)$, which contains all necessary information. Thus the result can be equally produced by copying the values of the M point DFT instead of actually performing arithmetic operations necessary for an NM point DFT. This concept is illustrated in Fig. 3(a) with $N = 2$, where three data symbols in time domain, represented by the black dots, produce the same number of points in frequency domain. And adding zero samples between the data symbols then results in a repetition of the sequence in frequency domain.

As the DFT is an operation with periodic inputs and periodic outputs, further computational savings can be harvested when the periodicity of the time domain signal is maintained during the filtering operation, i.e. tail-biting circular filters are used

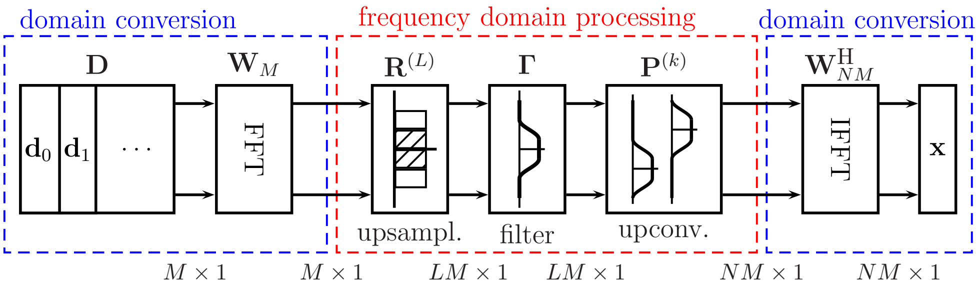


Fig. 5. Low complexity GFDM implementation model

in the process [7]. In that case the (circular) convolution of the data sequence and pulse shaping filter from (3) turns into a regular multiplication in frequency domain in (4). Also, since the aim of the pulse shaping is to keep out-of-band radiation minimal, the utilized pulse may turn out to be sparse in frequency domain, i.e. many of the coefficients can be zero and thus multiplications do not need to be carried out. Consequently, in general the filter pulse spans over $1 \leq L \leq N$ subcarriers in frequency domain. For the root-raised cosine (RRC) typically $L = 2$, which again saves operations as outlined in Fig. 3(c).

Lastly, the DFT of a sinusoid $\mathrm{DFT}_{NM}\left(e^{j2\pi \frac{k}{N} n}\right)$ corresponds to $\delta\left(f - \frac{k}{N}\right)$ in frequency domain and convolution with a Dirac results in a shift. Consequently, the subcarrier upconversion can be implemented by shifting the samples in frequency domain according to Fig. 4.

The modifications listed above lead to a GFDM transmitter model as depicted in Fig. 5.

A. Matrix model

Consider a data matrix $\mathbf{D}$ that contains $M \times K$ complex valued data symbols $d_k[m]$, where $\mathbf{d}_k$ is the kth column of $\mathbf{D}$ and denotes the data transmitted on the kth subcarrier. First, an M point DFT is performed on each vector $\mathbf{d}_k$, which can be expressed mathematically with a Fourier matrix

$$\mathbf{W}_M = \frac{1}{\sqrt{M}} \left\{w^{k,n}\right\}_{M \times M}, \quad w^{k,n} = e^{-j2\pi \frac{(k-1)(n-1)}{N}}. \quad (5)$$

Sequentially, each of the transformed vectors $\mathbf{W}_M \mathbf{d}_k$ undergoes three stages of processing in frequency domain. First, the samples of the vector are reproduced L times according to $\mathbf{R}^{(L)} \mathbf{W}_M \mathbf{d}_k$, by multiplying with a matrix $\mathbf{R}^{(L)} = \{\mathbf{I}_M, \mathbf{I}_M, \ldots, \mathbf{I}_M\}^{\mathrm{T}}$, which is a concatenation of L identity matrices $\mathbf{I}_M$ of size $M \times M$. Next, the pulse shaping filter $\mathbf{\Gamma}$ is applied through multiplication according to $\mathbf{\Gamma} \mathbf{R}^{(L)} \mathbf{W}_M \mathbf{d}_k$, where $\mathbf{\Gamma}$ is a matrix that contains $\mathbf{W}_{LM}\mathbf{g}$ on its diagonal and zeros otherwise and $\mathbf{g} = \{g[\ell]\}_{LM}$ contains the time samples of the filter pulse. In the last stage, the kth subcarrier's signal $\mathbf{X}_k$ is created by moving the vector to the position of the corresponding subcarrier with a permutation matrix $\mathbf{P}^{(k)}$, such that $\mathbf{X}_k = \mathbf{P}^{(k)} \mathbf{\Gamma} \mathbf{R}^{(L)} \mathbf{W}_M \mathbf{d}_k$. Therein $\mathbf{P}^{(1)} = \{\mathbf{I}_{LM}\, \mathbf{0}_{LM}\, \mathbf{0}_{LM}\, \ldots\}^{\mathrm{T}}$, $\mathbf{P}^{(2)} = \{\mathbf{0}_{LM}\, \mathbf{I}_{LM}\, \mathbf{0}_{LM}\, \ldots\}^{\mathrm{T}}$ and so on, with $\mathbf{0}_{LM}$ being an $LM \times LM$ all zero matrix. Finally, all subcarrier signals are superpositioned. The transmit signal is then produced with an NM point IDFT according to

$$\mathbf{x} = \mathbf{W}_{NM}^{\mathrm{H}} \sum_k \mathbf{P}^{(k)} \mathbf{\Gamma} \mathbf{R}^{(L)} \mathbf{W}_M \mathbf{d}_k. \quad (6)$$

Note that the processing chain in Fig. 5 can be divided into three general parts. Initially, the data matrix $\mathbf{D}$, in which each row corresponds to a time slot and each column corresponds to a subcarrier, is given in time-frequency domain. By applying the M point DFT along each column, the data is converted to the frequency-frequency domain, where all the processing takes place. Finally, the signal is transformed back to time-time domain by the NM point IDFT, which is the domain necessary for transmission.

IV. Results

A. Complexity Analysis

Assuming that an M point DFT can implemented with the FFT algorithm at the expense of $M \log_2 M$ complex valued multiplications, the processing of (6) requires

- K times $M \log_2 M$ multiplications for the M point FFTs of K subcarriers,
- K times LM multiplications for the filtering of K subcarriers,
- $NM \log_2 NM$ multiplications for the NM point IFFT.

The operations related to $\mathbf{R}^{(L)}$ and $\mathbf{P}^{(k)}$ can be realized by means of pointer/memory operations and are thus not counted. This leads to an implementation effort of

$$\begin{aligned} C_{\text{GFDM,FFT}} &= KM \log_2 M + KLM + MN \log_2 MN \\ &= \underbrace{MN \log_2 N}_{\text{OFDM complexity}} + \underbrace{(K+N)M \log_2 M + KLM}_{\text{GFDM overhead}} \end{aligned} \quad (7)$$

for GFDM, while generating the OFDM transmit signal of the same amount of data is at the cost of $C_{\text{OFDM}} = MN \log_2 N$. Altough not analyzed in detail here, an implementation according to (6) also gives savings in the memory consumption, because the processing is performed on vectors and does not require storing the $NM \times KM$ modulation matrix $\mathbf{A}$ from (2).

A comparison of the implementation complexity of the different transmitter approaches in terms of complex valued multiplications is given in Fig. 6. An OFDM signal can be generated with the lowest computational effort. For certain parametrization, i.e. $L = 2$, with the new model the benefits of pulse shaped subcarriers in GFDM can be exploited at the cost of an increase in complexity by a factor as low as roughly 2. In the impractical case that the pulse shaping filter spans the complete signal bandwidth, the number of multiplications increases by an order of two magnitudes compared to OFDM. Implementations according to (1) and (2) suffer the highest computational load, because they do not benefit from the $\log_2$ savings of the FFT/IFFT.

B. A Case Study for an LTE-like GFDM System

The power spectral density (PSD) of OFDM and GFDM is compared in Fig. 1. Therein, the parameters of the OFDM system are chosen such, that they match the specifications of the Long Term Evolution (LTE) standard [8], [9]. For the GFDM system, a comparable set of parameters has been derived in Table I such, that they can serve as a reference for comparing both concepts in terms of equal sampling time, channel bandwidth and subcarrier bandwidth. Thus the FFT size N and the number of active subcarriers K is also the same for both systems. In GFDM, a block has the duration of $M = 15$ time slots, which is comparable to a transmission time interval (TTI) of LTE. Further, the parameter M_{on} is introduced, which denotes how many of the time slots are actually filled with data. In total, $M - M_{\text{on}}$ time slots remain as a guard time to phase the GFDM block in and out. Consequently, the GFDM employs a block structure as depicted in Fig. 5. In the context of this comparison, a root-raised cosine (RRC) filter with roll-off factor $\alpha = 0.25$ is chosen because of the narrow spectrum that it can produce. Note that the GFDM scheme is not restricted to this exemplary pulse.

Parameter	Value	Description
T_s	0.66 μs	sampling time
B	20 MHz	channel bandwidth
B_{SC}	15 kHz	subcarrier bandwidth
N	2048	FFT size
K	1201	active subcarriers
M	15	block size
M_{on}	$\{15, 13, 11\}$	active time slots
filter	RRC	pulse shaping filter shape
α	0.25	filter roll-off
L	2	filter size in freq. domain

TABLE I
LTE-LIKE PARAMETERS FOR A GFDM SYSTEM

The curves in Fig. 1 show that a GFDM signal with significantly stronger out-of-band suppression can be produced. The benefit over OFDM increases with larger guard times at the edges of the GFDM block. A further improvement is expected from optimizing the filter pulse.

V. CONCLUSIONS

In this paper we motivate the need for a flexible multi-carrier communication system that is able to address the expected needs of future cellular networks. We show that pulse shaped subcarriers can be achieved in GFDM at reasonable computational cost, which is approximately in the same order of magnitude as traditional OFDM. But at the same time, in terms of out of band radiation, GFDM can outperform OFDM by several orders of magnitude. In order to be able to draw a comparison between both systems, we introduce a set of suitable parameters for GFDM, which relate to the recent LTE standard.

The investigation of an optimal filter pulse shape, that is matched to the properties of GFDM, as well as the design of a low complexity receiver remain interesting topics for further research.

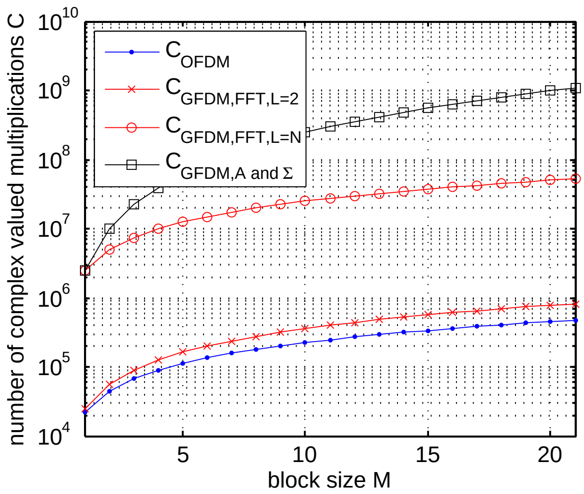


Fig. 6. Comparison of implementation complexity

ACKNOWLEDGMENT

This work has been performed in the framework of the ICT project ICT-5-258512 EXALTED, which is partly funded by the European Union.

REFERENCES

[1] B. Saltzberg, "Performance of an Efficient Parallel Data Transmission System," *IEEE Transactions on Communication Technology*, vol. 15, no. 6, pp. 805–811, December 1967.

[2] R. W. Chang, "High-Speed Multichannel Data Transmission with Bandlimited Orthogonal Signals," *Bell Syst. Tech. J.*, vol. 45, pp. 1775–1796, December 1966.

[3] G. Fettweis, M. Krondorf, and S. Bittner, "GFDM - Generalized Frequency Division Multiplexing," in *Proc. 69th IEEE Vehicular Technology Conference, VTC Spring 2009*.

[4] N. Michailow, S. Krone, M. Lentmaier, and G. Fettweis, "Bit Error Rate Performance of Generalized Frequency Division Multiplexing," in *Proc. 76th IEEE Vehicular Technology Conference, VTC Fall 2012*.

[5] B. Farhang-Boroujeny, "OFDM versus Filter Bank Multicarrier," *IEEE Signal Processing Magazine*, vol. 28, no. 3, pp. 92–112, May 2011.

[6] T. Ihalainen, A. Viholainen, and M. Renfors, "On Spectrally Efficient Multiplexing in Cognitive Radio Systems," in *Proc. 3rd Int. Symp. Wireless Pervasive Computing ISWPC 2008*.

[7] R. Datta, N. Michailow, M. Lentmaier, and G. Fettweis, "GFDM Interference Cancellation for Flexible Cognitive Radio PHY Design," in *Proc. 76th IEEE Vehicular Technology Conference, VTC Fall 2012*.

[8] 3GPP, "TS 36.211: Physical Channels and Modulation," Tech. Rep.

[9] 3GPP, "TS 36.104: Base Station Radio Transmission and Reception," Tech. Rep.