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Energy Optimal Excitation of Radio-Frequency Cavity

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Abstract: We show how to minimize the energy required to build up the electromagnetic field in radio-frequency cavities, which will allow power savings for pulsed particle accelerators. By formulating an optimal control problem for a first-order system we obtain a solution on state-feedback form. We numerically compare the optimal solution to previous approaches.

1. INTRODUCTION

Large particle accelerators require megawatts of power for building up and maintaining the electromagnetic fields in the accelerating radio-frequency (RF) cavities. In pulsed accelerators, the energy required to *fill* the cavities (i.e. build up the electromagnetic fields), is significant, but does not contribute to particle acceleration. For the linear accelerator at the European Spallation Source (ESS), which has 2.86 ms long beam pulses at 14 Hz, it takes 150 µs to fill the superconducting cavities in the high-β section, which amounts to a yearly cost of 100 k€ [1] , Peggs et al. (2013).

Currently, the RF amplifier of choice for high-power accelerators is the klystron, which has constant power consumption regardless of output power. For the accelerator at ESS, the industry has been encouraged to develop an inductive output tube (IOT), for the 84 cavities in the high-β section. IOTs have an almost constant efficiency down to 30% of maximum output power, which will give substantial power savings.

For klystrons, which have constant power consumption, the energy optimal filling strategy is simply to drive it at saturation. However, for IOTs, the energy consumption can be reduced by using a more sophisticated filling strategy.

As remarked in a recent article by Bhattacharyya et al. (2015), little work has been done on energy optimal filling of RF cavities. In their article, they derived analytically how to minimize the energy *reflected* from the cavity during filling, which corresponds to minimization of the wall-plug energy for *ideal amplifiers*. In this contribution we show how to minimize the wall-plug energy consumption for *arbitrary* amplifier characteristics. The components involved in the control problem are illustrated in Figure 1.

⋆ The authors work with development of the RF control system for the linear accelerator at the European Spallation Source, and would like to acknowledge many helpful discussions with the RF group at ESS. They have also received financial support from the ELLIIT Excellence Center and the Swedish Research Council through the LCCC Linnaeus Center.

[1] The filling constitutes $0.15\,\mathrm{ms}/(2.86+0.15)\,\mathrm{ms} = 5\,\%$ of the RF pulse, and the expected average electricity consumption of the RF amplifiers (IOTs) is 6 MW, which gives the following cost estimate: $5\,\% \times 6\,\mathrm{MW} \times 5000\,\mathrm{h/year} \times 0.07$ €/kWh = 100 k€/year.

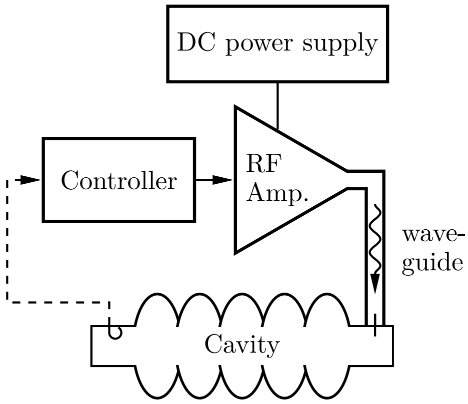


Figure 1. Main components of an RF station. We will not consider feedback in this article (dashed line).

1.1 Problem formulation

The complex-coefficient baseband dynamics for the accelerating cavity mode are, Schilcher (1998),

$$\dot{\mathbf{V}} = (-\omega_{1/2} + i\Delta\omega(t))\mathbf{V} + \omega_{1/2} R_L \mathbf{I_g} + \omega_{1/2} R_L \mathbf{I_b}, \quad (1)$$

where $\mathbf{V}$ is the cavity field voltage (phasor), $\Delta\omega(t)$ is the detuning of the cavity, $\mathbf{I_g}$ is the generator current, which is the controlled output of the RF amplifier, and $\mathbf{I_b}$ is the beam-current, which is equal to 0 during the filling. Throughout, bold letters will denote complex-valued signals. To simplify the exposition we assume, without loss of generality, that $\mathbf{V}$ has nominal value 1, the cavity bandwidth $\omega_{1/2} = 1$, and the loaded shunt impedance $R_L = 1$.

We want to determine how to fill the cavity, i.e. reach $\mathbf{V}(t_f) = 1$, while minimizing the energy consumption

$$W = \int_0^{t_f} P_{\mathrm{amp}}(|\mathbf{I_g}|)\, dt, \quad (2)$$

where $P_{\mathrm{amp}}(I_g) = I_g^2/\eta(I_g)$ is proportional to the wall-plug power drawn by the amplifier, and $\eta(I_g)$ is the amplifier efficiency as a function of output amplitude. The final time t_f is a free parameter and there is an upper limit $I_g^{\max}$ on $|\mathbf{I_g}|$, see Figure 2 for an illustration.

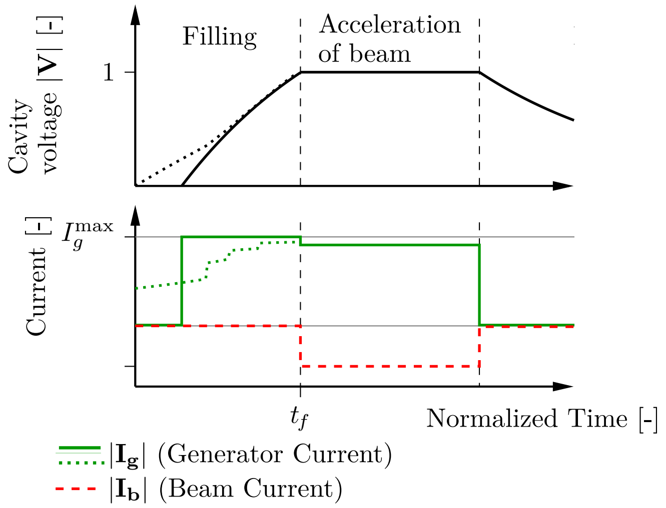


Figure 2. Normalized amplitudes of the cavity field and the generator current during an RF pulse, for the standard approach of minimum time filling, i.e. $|\mathbf{I_g}| = I_g^{\max}$ (solid lines), and energy optimal filling for an IOT (dotted lines).

Formally stated, the problem is

$$\begin{aligned} \underset{I_g, t_f}{\text{minimize}} \quad & \int_0^{t_f} P_{\text{amp}}(|\mathbf{I_g}|)\, dt && \text{(3a)} \\ \text{subject to} \quad & \dot{\mathbf{V}} = (-1 + i\Delta\omega(t))\mathbf{V} + \mathbf{I_g} && \text{(3b)} \\ & |\mathbf{I_g}| \le I_g^{\max} && \text{(3c)} \\ & \mathbf{V}(0) = 0 && \text{(3d)} \\ & \mathbf{V}(t_f) = 1. && \text{(3e)} \end{aligned}$$

In the problem formulation (3), we assume the detuning $\Delta\omega(t)$ for $0 \le t \le t_f$ to be known in advance, that the amplifier has no dynamics, that there are no disturbances and that all relevant parameters are perfectly known. These assumptions are approximations, but we believe that they are sufficiently good for our conclusions to hold.

1.2 *Previous Work*

Bhattacharyya et al. (2015) derived, for $\Delta\omega \equiv 0$ and no limit on $|\mathbf{I_g}|$, how to minimize the *reflected* energy for a fixed final time t_f — a problem similar to minimizing the power consumption for constant amplifier efficiency η. The derived optimal generator current profile was

$$I_g^*(t) = \frac{\exp(t)}{\sinh t_f}, \tag{4}$$

with the corresponding cavity voltage

$$V(t) = \frac{\sinh(t)}{\sinh t_f}.$$

If the amplifier saturation level is too low to implement (4), i.e. $\exp(t_f)/\sinh t_f > I_g^{\max}$, the solution was shown to instead be of the form $I_g^*(t) = \min(K\mathrm{e}^t, I_g^{\max})$, for a suitably chosen K. Bhattacharyya et al. also compared the energy consumption for amplifiers with different efficiency characteristics $\eta(I_g)$, with I_g given by (4), note that I_g^* was computed for $\eta \equiv$ constant, and thus suboptimal for amplifiers whose efficiency depend on the output amplitude.

1.3 *Outline of the paper*

In the next section we solve (3) for arbitrary, known, detuning $\Delta\omega(t)$, and arbitrary efficiency characteristics $\eta(I_g)$. In Section 3 we compare the energy optimal filling strategies for different forms of $\eta(I_g)$ and different saturation levels $I_g^{\max}$. We conclude with a remark on cryogenic losses and a discussion of the results.

2. SOLVING THE OPTIMIZATION PROBLEM

2.1 *Optimal phase of* $\mathbf{I_g}(t)$

Transforming the cavity equation (3b) to polar coordinates, gives, Brandt (2007),

$$\begin{aligned} V\dot{\phi} - \Delta\omega V &= I_g \sin(\theta - \phi) && \text{(5a)} \\ \dot{V} + V &= I_g \cos(\theta - \phi), && \text{(5b)} \end{aligned}$$

where $I_g \ge 0$, $V \ge 0$, θ and ϕ are defined via

$$\mathbf{I_g}(t) = I_g(t)\mathrm{e}^{i\theta(t)} \tag{6}$$

$$\mathbf{V}(t) = V(t)\mathrm{e}^{i\phi(t)}. \tag{7}$$

By considering (5b), we see that choosing θ as

$$\theta^*(t) = \phi(t), \tag{8}$$

maximizes $\dot{V}$ for any value of I_g. Since the cost (3a) is independent of ϕ, and we wish to minimize the cost for reaching $V(t_f) = 1$, it is clear that (8) is optimal. With this choice of $\theta(t)$, equation (5a) reduces to $\dot{\phi} = \Delta\omega$, and since we want $\phi(t_f) = 0$, we must have

$$\phi(t) = -\int_t^{t_f} \Delta\omega(t')\, dt'. \tag{9}$$

From (8) it follows that the optimal phase θ^* of the generator current equals the right hand side of (9).

Remark: Actually, for superconducting cavities, $\Delta\omega(t)$ depends on the cavity field V, via Lorenz force detuning. However, since the the optimization of θ and I_g/V is decoupled, the optimal V can be found first.

2.2 *Optimal amplitude of* $\mathbf{I_g}(t)$

The optimal phase of the generator current is given by (8), so finding the optimal amplitude I_g reduces to the following problem,

$$\begin{aligned} \underset{I_g, t_f}{\text{minimize}} \quad & \int_0^{t_f} P_{\text{amp}}(I_g)\, dt && \text{(10a)} \\ \text{subject to} \quad & \dot{V} = -V + I_g && \text{(10b)} \\ & |I_g| \le I_g^{\max} && \text{(10c)} \\ & V(0) = 0 && \text{(10d)} \\ & V(t_f) = 1. && \text{(10e)} \end{aligned}$$

From (10), it seems reasonable that the optimal choice of I_g at each time-instant maximizes the ratio between the increase of the cavity field and the power consumption, i.e.

$$I_g^*(t) = \underset{I_g}{\operatorname{argmax}} \frac{-V(t) + I_g}{P_{\text{amp}}(I_g)}. \tag{11}$$

That (11) indeed is optimal, follows from the following, slightly more general theorem.

Theorem 1. Consider the optimal control problem

$$\begin{aligned}
\underset{u,t_f}{\text{minimize}} \quad & \int_0^{t_f} r(u(t))\,dt && \text{(12a)}\\
\text{subject to} \quad & \dot{x}(t) = f(x(t), u(t)) && \text{(12b)}\\
& x(0) = 0 && \text{(12c)}\\
& x(t_f) = 1 && \text{(12d)}\\
& u(t) \in \mathcal{U}, && \text{(12e)}
\end{aligned}$$

where $\mathcal{U}$ is a compact set, $r(u) > 0$ for all $u \in \mathcal{U}$, $f(x, u)$ and $r(u)$ are continuous functions of u, and

$$\forall x \in [0, 1] \ \exists u \in \mathcal{U} \text{ so that } f(x, u) \geq c > 0. \tag{13}$$

Define

$$u^*(x) := \underset{u \in \mathcal{U}}{\operatorname{argmax}} \frac{f(x, u)}{r(u)}. \tag{14}$$

and assume that u^* is sufficiently well-behaved for $\dot{x} = f(x, u^*(t))$ to have a unique solution $x^*(t)$. Then the optimal control signal is given by $u(t) = u^*(x^*(t))$.

Proof. See the Appendix for a proof based on the Hamilton-Jacobi-Bellman technique.

Remark 1: The assumption (13) guarantees finite-time feasibility.

Remark 2: The maximum in (14) exists since a continuous function is optimized over a compact set. If several u maximize the expression, any can be chosen.

Remark 3: It is clear that the actual functions $r(u)$ and $f(x, u)$ considered in (10), give a well-behaved u^*.

Remark 4: A constraint $t_f < t_{\max}$ on the final time, can be handled by adding a constant term to $r(u)$, and doing a binary search over that constant.

Solution for constant efficiency For a constant efficiency $\eta \equiv \eta_0$ we have $P_{\text{amp}}(I_g) = I_g^2/\eta_0$, and (11) becomes,

$$I_g^* = \underset{I_g}{\operatorname{argmax}} \frac{-V + I_g}{I_g^2/\eta_0} = 2V.$$

Note that this corresponds to V not reaching 1 in finite time, which is possible since the cost is not strictly greater than 0. If a fixed final time is imposed, the solution follows from the maximum principle, or as in Bhattacharyya et al. (2015).

3. RESULTS

In this section we compare the energy consumption for three filling strategies:

- *Minimum time*, i.e. $I_g(t) = I_g^{\max}$.
- *Minimum reflection*, (Bhattacharyya et al. (2015)), i.e. $I_g(t) = \min(e^t/\sinh\hat{\tau}_i,\, I_g^{\max})$, $\hat{\tau}_i = 2$.
- *Energy optimal*, according to (11),

considering four amplifier characteristics (Figure 3):

- Tetrode
- Doherty architecture solid state amplifier (SSA)
- Inductive output tube (IOT)
- Ideal (constant efficiency) amplifier,

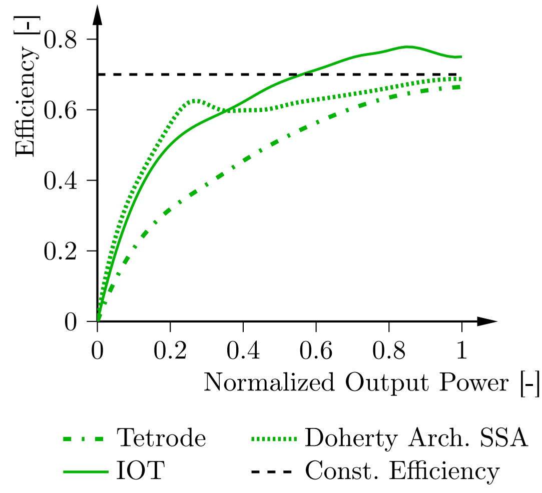


Figure 3. Efficiency as a function of normalized output power $(I_g/I_g^{\max})^2$, for the considered amplifiers types. The data is from (Bhattacharyya et al., 2015, Fig. 9), and has been slightly smoothed.

Table 1. Energy consumption of minimum time (MT) and minimum reflection (MR) filling relative energy optimal (EO) filling.

Amplifier	$I_g^{\max} = 1.5$		$I_g^{\max} = 2.25$	
	$\frac{W_{\text{EO}}}{W_{\text{MT}}}$	$\frac{W_{\text{EO}}}{W_{\text{MR}}}$	$\frac{W_{\text{EO}}}{W_{\text{MT}}}$	$\frac{W_{\text{EO}}}{W_{\text{MR}}}$
Tetrode	98 %	94 %	97 %	82 %
Doherty Arch. SSA	90 %	98 %	82 %	94 %
IOT	92 %	98 %	87 %	92 %
$\eta = 1$	83 %	96 %	68 %	96 %

and two saturation levels, given in normalized units by:

- $I_g^{\max} = 1.5$
- $I_g^{\max} = 2.25$.

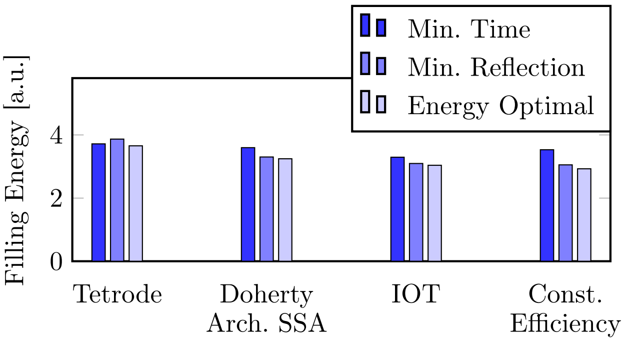
The levels correspond to a normal conducting cavity and a heavily beam-loaded superconducting cavity respectively.

The energy consumption for the different parameter combinations are shown in Figure 4. The energy required by the optimal filling strategy relative to minimum time filling and minimum reflection filling are compared in Table 1. The corresponding profiles for the cavity voltage and generator current are shown in Figure 5.

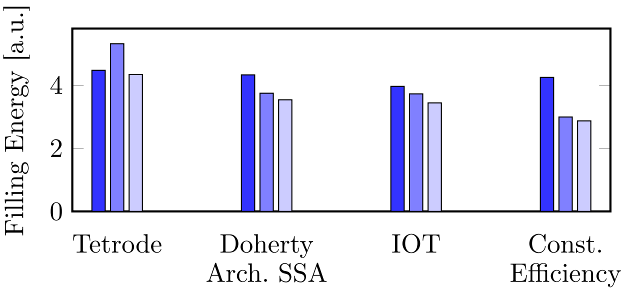
For the high-β section at ESS, where IOTs will be used, and $I_g^{\max} = 2.25$, the energy reduction of using energy optimal filling, relative to minimum time filling, is 13%, corresponding to 13 k€/year.

3.1 Remark on cryogenic losses

As seen in Figure 5, the energy optimal filling profiles take about 50-100% longer time than minimum time filling. This implies increased RF heating of the cavities, and a higher load on the cryogenic system, corresponding to a yearly



(a) $I_g^{\max} = 1.5$.



(b) $I_g^{\max} = 2.25$.

Figure 4. Comparison of the energy consumption for minimum time filling (standard approach), minimum reflection filling according to Bhattacharyya et al. (2015), and energy optimal filling according to (11). Four different amplifier types and two different saturation levels are considered.

cost of 1–2 k€.[2] This cost is clearly outweighed by the savings from using energy optimal filling.

If desired, it is easy to incorporate cryogenic losses into the optimization problem by adding a term $\alpha_{\text{cryo}} V^2$ to the integrand in (10a). We performed some experiments with this, and found that the resulting energy optimal trajectories were similar, but slightly faster, than the original ones.

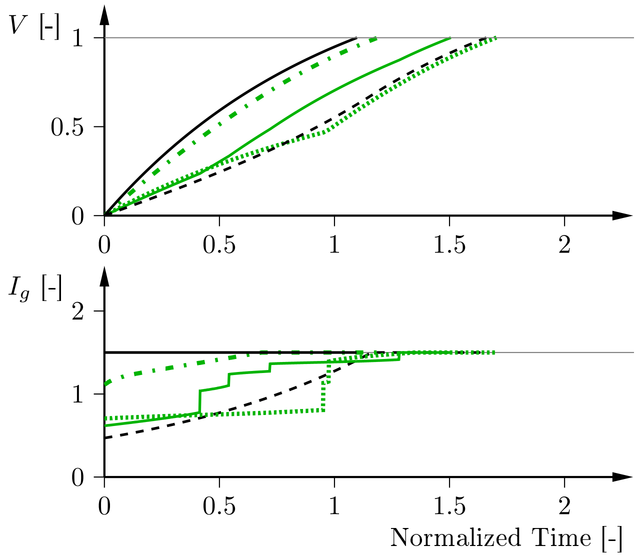
4. CONCLUSIONS

We have shown how to reduce the energy required to build up the fields in an RF cavity, compared to the standard approach of driving the amplifier at saturation. We proved that the energy optimal amplitude and phase of the generator current, for a normalized cavity with $\omega_{1/2} = 1$ and $R_L = 1$, are given by

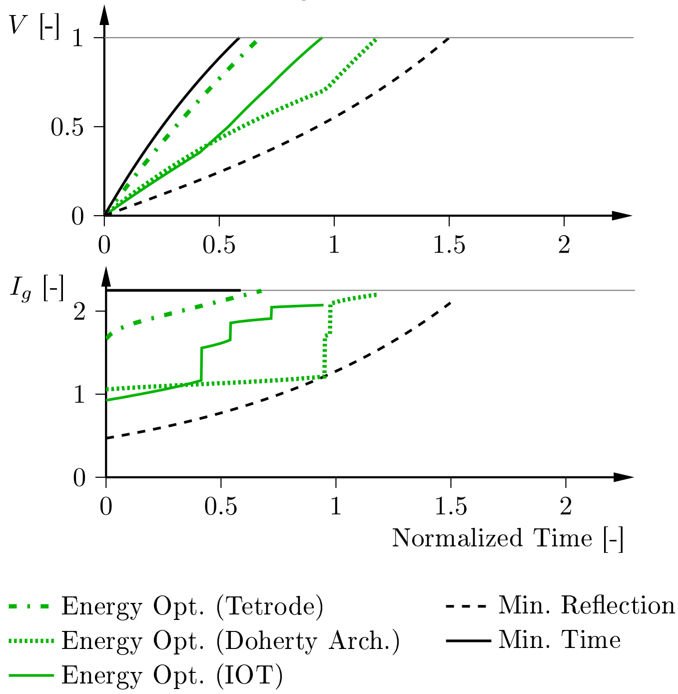
$$I_g^*(t) = \underset{I_g}{\operatorname{argmax}} \frac{-V(t) + I_g}{P_{\text{amp}}(I_g)}$$

$$\theta^*(t) = \int_t^{t_f} \Delta\omega(t)\, dt.$$

We compared the energy savings for different amplifier characteristics and found that the energy consumption could be reduced by up to 30%. For the high-β section at the European Spallation Source we estimated the yearly savings to be about 10 k€. As the proposed filling strategy, or at least an approximate version thereof, is relatively easy to implement, it provides an straight-forward way to reduce the operating costs and environmental footprint of pulsed accelerators.



(a) $I_g^{\max} = 1.5$.



(b) $I_g^{\max} = 2.25$.

Figure 5. Cavity voltage and generator current for different filling strategies: minimum time, minimum reflection and optimal with respect to the efficiency characteristics in Figure 3 (ideal amplifier ↔ minimum reflection filling). Note that minimum time and minimum reflection filling are independent of the amplifier characteristics. Two saturation levels are considered.

[2] The cryogenic load due to RF heating in the high-β section is 1.6 kW@4.5K, cooling efficiency of 250 W/W, the required wall-plug power is 400 kW, Peggs et al. (2013). Assuming a linear increase of V during the filling, the average heating is proportional to $\int_0^T (V/T)^2 dV = T/3$, on average a third relative when $V = 1$: 5 % × 1/3 × 400 kW × 5000 h/year × 0.07 €/kWh = 2 k€/year.

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Appendix A. PROOF OF THEOREM 1

Proof. Define the optimal cost-to-go function

$$V(x) := \int_x^1 \frac{r(u^*(x'))}{f(x', u^*(x'))}\, dx'.$$

Let u be an arbitrary control signal with $u(t) \in \mathcal{U}$, such that the corresponding state trajectory x satisfies (12b)–(12d). It then holds that

$$\begin{aligned} r(u) + \frac{d}{dt}V(x(t)) &= r(u) + \frac{dV}{dx} f(x,u) \\ &= r(u) - \frac{r(u^*(x))}{f(x,u^*(x))} f(x,u) = \\ &= \frac{r(u)r(u^*(x))}{f(x,u^*(x))}\left(\frac{f(x,u^*(x))}{r(u^*(x))} - \frac{f(x,u)}{r(u)}\right) \geq 0, \quad \text{(A.1)} \end{aligned}$$

where the inequality follows from $f(x, u^*(x)) > 0$, $r(u) > 0$ and the definition (14) of u^*. Equality holds for $u = u^*$.

Integration of (A.1) gives, since $x(0) = 0$ and $x(t_f) = 1$,

$$\int_0^{t_f} r(u(t))\, dt \geq V(x(0)) - V(x(t_f)) = V(0),$$

with equality for $u = u^*$. This proves optimality of u^*.