\( \alpha \) decay of high-\( K \) isomers in \( ^{270}\text{Ds} \) and \( ^{266}\text{Hs} \) in a superfluid tunneling model

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We use the superfluid tunneling model (STM) to calculate the half-lives of ground-state \( \alpha \) decays of even-even superheavy nuclei (SHN) with \( Z \geq 100 \). The experimental data are reproduced to accuracies comparable to other contemporary models of \( \alpha \) decay of SHN. We apply the STM to the case of the \( \alpha \) decaying high-\( K \) isomers identified in the decay chains of \( ^{270}\text{Ds} \). By accounting for the \( \alpha \)-decay \( Q \) values, \( Q_{\alpha} \), the angular momentum difference between initial and final states, \( L \), and a reduction in the pairing gap, \( \Delta \), we are able to reproduce the observed \( \alpha \) decay of the isomers, including the unusual competition between \( L \approx 10 \) and \( L \approx 0 \) branches seen for the \( K \) isomer in \( ^{270}\text{Ds} \) (\( Z = 110 \)).

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1. INTRODUCTION

The theory of \( \alpha \) decay was initially formulated in 1928 by Gamow \({\text{[1]}}\), and independently by Gurney and Condon \({\text{[2]}}\), who described the process as a tunneling of the preformed \( \alpha \) particle through a Coulomb barrier. There have been many subsequent approaches to developing a quantitative description of \( \alpha \) decay involving microscopic calculations of both the \( \alpha \) particle formation probability and the barrier penetrability (see, for example, \({\text{[3–5]}}\) and references therein). Many multiparameter empirical relations, starting with the Geiger-Nuttall rule, have also been developed and extensively applied to the description and prediction of \( \alpha \)-decay half-lives \({\text{[6–9]}}\). Generally, these relations are able to reproduce the experimental half-lives across the nuclear chart to within a factor of about 4 \({\text{[10]}}\).

\( \alpha \) decay remains of considerable topical interest, not least because it is a common decay mode of superheavy nuclei (SHN) \({\text{[11]}}\). An accurate description of the \( \alpha \) decay is required in order to understand the suggested proton number \( Z \) and mass number \( A \) assignments of nuclei along particular decay chains. \( \alpha \) decay also provides the very first, and most basic information, on the structure and stability of the heaviest nuclei.

One specific case of \( \alpha \) decay in SHN is the observation of \( \alpha \) decay from high-\( K \) isomers in \( ^{270}\text{Ds} \) (\( Z = 110 \)) and in the daughter \( ^{266}\text{Hs} \) (\( Z = 108 \)) \({\text{[12–14]}}\). Isomers are long-lived (metastable) excited quantum states of a nucleus \({\text{[15]}}\). The isomers found in deformed superheavy nuclei, such as \( ^{270}\text{Ds} \), arise because they involve configurations of nucleon orbitals, which yield high-\( K \) values, where \( K \) is the projection of the total angular momentum onto the axis of symmetry defined by the nuclear shape. Many examples of high-\( K \) isomers are known in the transfermium nuclei \({\text{[16]}}\), but they generally decay via electromagnetic transitions (\( \gamma \)-ray emission and internal conversion). In the case of \( ^{270}\text{Ds} \) chains, not only do the isomers \( \alpha \) decay, including a fine structure that seems to indicate a competition between \( L \approx 10 \) and \( L \approx 0 \) \( \alpha \) transitions from the isomer in \( ^{270}\text{Ds} \), but also the lifetimes of the isomeric states are longer than the corresponding ground states \({\text{[12–14]}}\). This has profound implications for the survival of superheavy nuclei and the possibility of studying these states experimentally \({\text{[17]}}\).

There are three major factors which influence the \( \alpha \) decay of multi-quasiparticle states such as the high-\( K \) isomers discussed above: (i) the energies of the states involved: the larger the \( Q \) value of the \( \alpha \) decay, \( Q_{\alpha} \), the shorter the lifetime will be; (ii) a large difference in angular momentum will give rise to a larger centrifugal barrier resulting in a longer lifetime; (iii) pairing enhances the decay through a barrier and so a reduction in pairing again implies a longer lifetime. Determining the relative influences of these competing effects is important for our understanding of the \( \alpha \) decay of excited states and the survival probability for SHN.

In a previous study \({\text{[18]}}\), it was shown that the superfluid tunneling model (STM) \({\text{[19–21]}}\) could be applied to the description of \( \alpha \) decay of ground state and multi-quasiparticle states across different regions of the nuclear chart from the neutron-deficient \( A \sim 150 \) region up through the heavy actinide region. It was also qualitatively shown that it was possible for a \( \alpha \)-decaying isomeric state to be longer lived than the corresponding ground state. In this article we apply the STM to compare against the experimental data on all known even-even SHN with \( 100 \leq Z \leq 118 \) [that is, from isotopes of fermium (\( Z = 100 \)) to oganesson (\( Z = 118 \))]. We find a remarkable quantitative agreement comparable to the fits of other models for the \( \alpha \)-decay of the isomers observed in the \( ^{270}\text{Ds} \) decay chain. We find that we are able to qualitatively reproduce the features of the \( \alpha \) decays including the observation of a strong \( L \approx 10 \) \( \alpha \) transition competing with \( L \approx 0 \) \( \alpha \) transitions from the same isomer in \( ^{270}\text{Ds} \).

After this introduction, we will describe the main features of the STM in Sec. II. In Sec. III, we compare the results of the model with the known experimental data on even-even SHN and with the results of other models for the \( \alpha \) decay of SHN. In Sec. IV, we then present the results on the \( \alpha \) decay from high-\( K \) isomers in \( ^{270}\text{Ds} \) and \( ^{266}\text{Hs} \). This will be followed by a short summary.
II. SUPERFLUID TUNNELING MODEL

In this work we have used the superfluid tunneling model as described in [22], which has been successfully applied previously to calculations of particle emission including α decay and cluster radioactivity [19–21]. The model is intuitively appealing and involves the nucleus evolving to a cluster-like configuration. In the case of α decay, this comprises a touching configuration of the daughter nucleus and α particle. The subsequent decay process is described in terms of standard Gamow theory of tunneling through a barrier. The evolution of the parent nucleus to the cluster-like configuration is dominated by pair-wise rearrangements of nucleons, which occur under the action of the residual nuclear interaction, dominated by pairing.

The Hamiltonian of the model can be written as

$$\hat{H} = \frac{\hbar^2}{2D} \frac{\partial^2}{\partial \xi^2} + V(\xi) \psi(\xi) = E \psi(\xi). \quad (1)$$

\(\xi\) is a generalized deformation variable describing the path of the system in the multidimensional space of deformations. In the case of only quadrupole deformation, this would mean \(\alpha\) such that \(\xi = \eta\). The Bardeen-Cooper-Schrieffer (BCS) model estimated using the Bardeen-Cooper-Schrieffer (BCS) model that \(\Delta_1\xi\) is a generalized deformation variable describing the path of the system in the multidimensional space of deformations. In the case of only quadrupole deformation, this would mean \(\xi\) is proportional to the axial deformation parameter, \(\beta_2\). The parent nucleus evolves from a configuration with a small deformation, \(\xi_0 \approx 0\), to the touching configuration of daughter-plus-α particle at \(\xi = 1\).

Equation (1) can be discretized on a mesh of \(n\) steps such that \(\Delta \xi = 1/n\). One can then derive the expression for the inertial mass parameter as

$$D = -\frac{\hbar^2}{2v}n^2. \quad (2)$$

\(v\) is the transition matrix element between two successive steps. For α decay, \(n = 4\) is assumed [21,22]. The transition matrix element is governed by a pairing operator and is estimated using the Bardeen-Cooper-Schrieffer (BCS) model such that

$$v = -\left(\frac{\Delta_n^2 + \Delta_p^2}{4G}\right). \quad (3)$$

\(G = 25/A\) MeV is the standard pairing strength and \(\Delta_n = \Delta_p = \Delta = 12A^{-1/2}\) MeV are the pair gap parameters.

The decay constant \(\lambda\) can be calculated in terms of the α-particle formation probability \(P\), the assault frequency of the particle against the barrier (also known as the knocking frequency), \(f\), and the transmission coefficient of the α particle through the barrier, \(T\), such that

$$\lambda = PfT. \quad (4)$$

To calculate \(P\) we use the wave function of the ground state of a harmonic oscillator such that \(P = |\psi(\xi = 1)|^2\) with

$$\psi(\xi) = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{1/2} e^{-\frac{1}{2} \alpha^2 \xi^2}, \quad (5)$$

where

$$\alpha^2 = \sqrt{\frac{C}{2|v|}} n. \quad (6)$$

The potential energy parameter \(C = 2V(\xi = 1) = 2(V_N + V_C - Q_\alpha)\), with \(V_N\) and \(V_C\) being the nuclear potential (for which we used the Christensen-Winther potential [23]) and the Coulomb potential, respectively. The details of the potential parameters used can be found in [18]. The assault frequency can then be calculated via the formula \(f = \omega/2\pi\), where \(\omega = \sqrt{C/D}\).

Finally, the transmission coefficient \(T_L\) for the α particle to tunnel through the Coulomb barrier starting from the daughter-α touching configuration is given by

$$T_L = \frac{\rho}{F_L(\eta, \rho) + G_L(\eta, \rho)}, \quad (7)$$

where \(\rho = R_0k\) with \(k = \sqrt{2\mu Q_\alpha/\hbar}\) (\(\mu\) is the reduced mass) and \(R_0 = 1.2(A_D^{1/3} + A_C^{1/3}) + 0.63\) fm, and \(\eta = 1/ka\) where \(a = \hbar^2/(e^2 \mu Z_D Z_\alpha)\). Here, \(F_L\) and \(G_L\) are the regular and irregular Coulomb functions [24], which take into account the additional centrifugal barrier when the orbital angular momentum \(L\) of the emitted α particle is nonzero.

III. GROUND-STATE α DECAYS OF EVEN-EVEN SHN

Using the model described above, we calculated the ground-state-to-ground-state decays for all known α-decaying even-even nuclei with \(Z \geq 100\) [11,25]. By focusing on the even-even systems, we eliminate ambiguities in \(Q_\alpha\) and \(L\), which might arise due to possible excitations of either the parent or daughter nucleus. The data are presented in Fig. 1, which shows the experimental half-lives of the α decays compared to the results of our calculations. It is remarkable to see that the data are reproduced, to within about a factor of 3, while extending across nine orders of magnitude, with half-lives ranging from about 100 μs for the case of \(^{270}\)Ds to around 10^5 s for \(^{256}\)Fm.

FIG. 1. Decimal logarithm of α-decay half-lives (in seconds) of even-even isotopes with \(Z \geq 100\) as a function of nuclear mass number \(A\). The experimental data are marked with (blue) crosses (errors are typically less than the size of the symbol—see comment in caption of Table I). The results of the calculations from the superfluid tunneling model are shown as filled (red) circles. The data points for each isotope chain, indicated with the corresponding element symbol, are joined by solid lines to guide the eye.
TABLE I. Comparison between the decimal logarithms of the experimental and calculated ground-state-to-ground-state $\alpha$-decay half-lives (in seconds) for all known even-even cases with $Z \geq 100$. The first column gives the nucleus of interest. The second column gives the $Q_\alpha$ value for the $\alpha$ decay, $Q_\alpha$ (in MeV, with uncertainties typically less than 0.1% of the absolute value), either calculated using the AME2016 atomic mass evaluation tables of [26] or taken from [11]. The third column has the decimal logarithm of the experimental half-life taken from the evaluated nuclear data files [25] or from [11]. Generally, the experimental uncertainties in the half-lives are small enough to be ignored for the purposes of plots in Figs. 1 and 2. The fourth, fifth, and sixth columns are the decimal logarithms of the $\alpha$-decay half-lives calculated using the superfluid tunneling model ($T_{1/2,STM}$), the Viola-Seaborg formula [7] using the parameters in [8] ($T_{1/2,VS}$), and the Royer formula [9] ($T_{1/2,Royer}$), respectively.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$Q_\alpha$ (MeV)</th>
<th>$\log_{10}(T_{1/2,exp})(s)$</th>
<th>$\log_{10}(T_{1/2,STM})(s)$</th>
<th>$\log_{10}(T_{1/2,VS})(s)$</th>
<th>$\log_{10}(T_{1/2,Royer})(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{286}$Fm</td>
<td>8.377</td>
<td>0.22</td>
<td>0.62</td>
<td>0.47</td>
<td>0.43</td>
</tr>
<tr>
<td>$^{284}$Fm</td>
<td>7.995</td>
<td>1.56</td>
<td>1.88</td>
<td>1.72</td>
<td>1.69</td>
</tr>
<tr>
<td>$^{282}$Fm</td>
<td>7.557</td>
<td>3.26</td>
<td>3.46</td>
<td>3.26</td>
<td>3.26</td>
</tr>
<tr>
<td>$^{280}$Fm</td>
<td>7.153</td>
<td>4.96</td>
<td>5.05</td>
<td>4.81</td>
<td>4.83</td>
</tr>
<tr>
<td>$^{278}$Fm</td>
<td>7.307</td>
<td>5.07</td>
<td>5.37</td>
<td>4.20</td>
<td>4.16</td>
</tr>
<tr>
<td>$^{276}$Fm</td>
<td>7.027</td>
<td>0.54</td>
<td>0.73</td>
<td>0.67</td>
<td>0.56</td>
</tr>
<tr>
<td>$^{274}$No</td>
<td>8.549</td>
<td>1.75</td>
<td>1.77</td>
<td>1.70</td>
<td>1.60</td>
</tr>
<tr>
<td>$^{272}$No</td>
<td>8.226</td>
<td>0.47</td>
<td>0.54</td>
<td>0.57</td>
<td>0.38</td>
</tr>
<tr>
<td>$^{270}$Fm</td>
<td>8.926</td>
<td>0.32</td>
<td>0.24</td>
<td>0.25</td>
<td>0.08</td>
</tr>
<tr>
<td>$^{268}$Fm</td>
<td>9.193</td>
<td>0.57</td>
<td>0.62</td>
<td>0.63</td>
<td>0.77</td>
</tr>
<tr>
<td>$^{266}$Sg</td>
<td>9.901</td>
<td>2.00</td>
<td>1.92</td>
<td>1.67</td>
<td>1.76</td>
</tr>
<tr>
<td>$^{264}$Sg</td>
<td>9.600</td>
<td>1.50</td>
<td>1.13</td>
<td>0.97</td>
<td>1.28</td>
</tr>
<tr>
<td>$^{262}$Hs</td>
<td>10.591</td>
<td>2.80</td>
<td>3.09</td>
<td>2.82</td>
<td>1.97</td>
</tr>
<tr>
<td>$^{260}$Hs</td>
<td>10.346</td>
<td>2.64</td>
<td>2.51</td>
<td>2.23</td>
<td>2.64</td>
</tr>
<tr>
<td>$^{258}$Hs</td>
<td>9.623</td>
<td>0.40</td>
<td>0.59</td>
<td>0.35</td>
<td>0.71</td>
</tr>
<tr>
<td>$^{256}$Hs</td>
<td>9.070</td>
<td>0.88</td>
<td>1.05</td>
<td>1.23</td>
<td>0.92</td>
</tr>
<tr>
<td>$^{254}$Ds</td>
<td>11.117</td>
<td>4.00</td>
<td>3.80</td>
<td>3.42</td>
<td>3.93</td>
</tr>
<tr>
<td>$^{252}$Fm</td>
<td>10.35</td>
<td>0.40</td>
<td>0.80</td>
<td>0.30</td>
<td>0.90</td>
</tr>
<tr>
<td>$^{250}$Fm</td>
<td>10.07</td>
<td>0.10</td>
<td>0.07</td>
<td>0.44</td>
<td>0.15</td>
</tr>
<tr>
<td>$^{248}$Lv</td>
<td>11.00</td>
<td>1.82</td>
<td>1.87</td>
<td>1.29</td>
<td>1.98</td>
</tr>
<tr>
<td>$^{246}$Lv</td>
<td>10.78</td>
<td>1.74</td>
<td>1.36</td>
<td>0.75</td>
<td>1.45</td>
</tr>
<tr>
<td>$^{244}$Og</td>
<td>11.82</td>
<td>2.74</td>
<td>3.28</td>
<td>2.56</td>
<td>3.36</td>
</tr>
</tbody>
</table>

This indicates that the STM seems to contain all major physical ingredients in order to reproduce the properties of the $\alpha$ decay of even-even SHN.

In Table I, we present the results of our calculations in comparison to the experimental data and also compared to the predictions of two different empirical-fitting approaches. The two empirically fitted formulas are the Viola-Seaborg formula [7] using the parameters of Parkhomenko and Sobiczewski [8] and the Royer formula [9]. To see how well the different approaches reproduce the data, and how they compare to each other, we have plotted the decimal logarithms of the ratios between the experimental and theoretical half-lives in Fig. 2. One sees that data is reproduced rather well and to within a factor of about 3 (corresponding to values of $\pm 0.477$ on the $y$ axis of Fig. 2).

For a quantitative comparison between the models, a common approach is to calculate the average of the absolute values of the differences in the decimal logarithms given as

$$\delta = \frac{1}{N} \sum_{k=1}^{N} \log_{10} \left( \frac{T_{1/2,\text{exp},k}}{T_{1/2,\text{theo},k}} \right).$$

(8)

For the different approaches used in this paper, we find that the values of $\delta$ are 0.22, 0.26, and 0.19 for the superfluid tunneling model, the Viola-Seaborg formula, and the Royer formula, respectively. We conclude that the superfluid tunneling model is able to reproduce the experimental data on the $\alpha$ decay of even-even SHN.
decays of even-even SHN to about the same level of accuracy as contemporary empirical formulas.

IV. \(\alpha\) DETAILED STUDY OF THE HIGH-\(K\) ISOMERS IN \(270\)D\(s\) DECAY CHAINS

We now turn to applying the model to the case of the \(\alpha\)-decaying high-\(K\) isomers observed in \(270\)Ds [12–14]. There are only a grand total of 31 \(\alpha\)-decay chains identified for the isotope. This includes decays from both the ground and the isomeric states, which are directly populated with comparable intensity. The salient experimental features can be summarized as follows: there is an isomeric state, presumably of high-\(K\) character, with a suggested configuration of either \(\nu[(613)_{1/2}^{+} \otimes (725)_{11/2}^{-}]_{K^*=9^-}\) or \(\nu[(615)_{5/2}^{+} \otimes (725)_{11/2}^{-}]_{K^*=10^-}\), at an excitation energy of about 1.0 MeV above the ground state. A preliminary half-life of the isomer was reported as 3\(\pm\)0.8 ms, which is considerably longer than the half-life reported for the ground state, which was 0.20\(\pm\)0.02 ms [13]. The isomer decays via three \(\alpha\) transitions: (i) a transition with an energy corresponding to \(Q_\alpha\)\(\approx\) 12.1 MeV, which was assigned as decaying from the isomer to the ground state of \(266\)Hs; (ii) a transition with \(Q_\alpha\)\(\approx\) 11.1 MeV, which decays to a corresponding \(K\) isomer in \(266\)Hs (which is again about 1.0 MeV above the ground state); and (iii) a transition to a state that is about 300 keV below the \(266\)Hs \(K\)-isomer (or, conversely, about 700 keV above the ground state of \(266\)Hs) with \(Q_\alpha\) \(\approx\) 11.4 MeV. Several individual \(\alpha\)-decay events have been associated with each transition, implying that the branching fraction is quite similar for each one. This warrants further investigation, since it implies a strong competition between \(L\) \(\approx\) 10 and \(L\) \(\approx\) 0 or branches.

Figure 3 shows the experimental scheme as described above, based on the information in [13], and we now attempt to reproduce the data using the STM. As can be seen in Table I and Figs. 1 and 2, we are able to reproduce the half-life of the ground-state decay for \(270\)Ds; our calculated value is 0.18 ms, which is to be compared to the experimental value of 0.20\(^{+0.07}_{-0.04}\) ms.

![Figure 3](https://example.com/figure3.png)

**FIG. 3.** Schematic of the decay scenario for \(270\)Ds based on the preliminary data published in [13]. The half-lives of ground and isomeric states and the \(Q_\alpha\) values used in the calculations are indicated.

<table>
<thead>
<tr>
<th>(L)</th>
<th>(T)</th>
<th>(T_{1/2}(\Delta = \Delta_{gs}))</th>
<th>(T_{1/2}(\Delta = 0.6 \times \Delta_{gs}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.27 \times 10^{-13}</td>
<td>1.15 \times 10^{-6}</td>
<td>1.57 \times 10^{-4}</td>
</tr>
<tr>
<td>2</td>
<td>5.02 \times 10^{-13}</td>
<td>1.66 \times 10^{-6}</td>
<td>2.27 \times 10^{-4}</td>
</tr>
<tr>
<td>4</td>
<td>2.12 \times 10^{-13}</td>
<td>3.92 \times 10^{-6}</td>
<td>5.36 \times 10^{-4}</td>
</tr>
<tr>
<td>6</td>
<td>5.57 \times 10^{-14}</td>
<td>1.49 \times 10^{-5}</td>
<td>2.04 \times 10^{-3}</td>
</tr>
<tr>
<td>8</td>
<td>9.22 \times 10^{-15}</td>
<td>9.03 \times 10^{-5}</td>
<td>1.24 \times 10^{-2}</td>
</tr>
<tr>
<td>10</td>
<td>9.79 \times 10^{-16}</td>
<td>8.51 \times 10^{-4}</td>
<td>1.16 \times 10^{-1}</td>
</tr>
<tr>
<td>12</td>
<td>6.81 \times 10^{-17}</td>
<td>1.22 \times 10^{-2}</td>
<td>1.67</td>
</tr>
</tbody>
</table>

Turning to the decay from the isomeric state, we first consider the decay of the isomer to the ground state of \(266\)Hs via the transition with \(Q_\alpha\) \(\approx\) 12.1 MeV. The variation of the lifetime as a function of angular momentum of the transition is shown in Table II. The formation probability \(P\) and assault frequency \(f\) are unaffected by the change in \(L\) and have values of \(P = 1.37 \times 10^{-6}(1.66 \times 10^{-5})\) and \(f = 6.09 \times 10^{20}(3.65 \times 10^{20})\) s\(^{-1}\) for \(\Delta = \Delta_{gs}\) (or \(\Delta = 0.6 \times \Delta_{gs}\)).

The reduction of the pairing parameter dramatically reduces the transition matrix elements. As discussed in [18], much of the additional hindrance can be traced to the multi-quasiparticle character of the isomer, which results in a significant reduction in the pairing. It was found that by reducing the pairing gap \(\Delta\) to 60\%, the value used for the calculation of ground-state properties, \(\Delta = 0.6 \times \Delta_{gs}\), one is able to reproduce the data on known \(\alpha\)-decaying two-quasiparticle isomers. Note that even though we call this the \(\alpha\)-pairing, and we expect pairing to be a dominant component of the residual interaction, there may be additional nuclear structure effects that are being included: the overall effect is to reduce the transition matrix element \(\nu\) of Eq. (2). Assuming the same reduction factor of \(\Delta = 0.6 \times \Delta_{gs}\), applies in the case of the high-\(K\) isomer in \(270\)Ds, we can repeat the calculations, again varying the angular momentum of the emitted \(\alpha\) particle. The results are shown in the fourth column of Table II.

The reduction of the pairing parameter dramatically reduces the formation probability \(P\) of the \(\alpha\) particle, lowering it by about two orders of magnitude. There is also a reduction in the assault frequency \(f\), which is lowered by a factor of 2. The effects on \(P\) and \(f\) reflect the increased inertia of the system as the pairing is reduced. The transmission
coefficients remain unchanged by the reduction of the pairing. The overall effect is an increase in the half-lives extracted. Assuming different values of the angular momentum for the emitted \( \alpha \) particle, we find that the half-life (partial decay constant) of the transition is \( T_{1/2} = 3.6 \times 10^{-2} \) s (\( \lambda_1 = 19 \) s \(^{-1} \)) or \( T_{1/2} = 1.2 \times 10^{-1} \) s (\( \lambda_1 = 6 \) s \(^{-1} \)), for \( L = 9 \) and \( L = 10 \) \( \alpha \) transitions, respectively.

Now we look at the decays from the isomer in \( ^{270}\text{Ds} \) to excited states in \( ^{266}\text{Hs} \). Since the structures of the two isomers in \( ^{270}\text{Ds} \) and \( ^{266}\text{Hs} \) are likely to be very similar, we assume that the angular momentum of the \( \alpha \) decay between them, with \( Q_\alpha \approx 11.1 \) MeV, is \( L \approx 0 \). Using the reduced pairing parameter, \( \Delta = 0.6 \times \Delta_{ms} \), the calculated value for the half-life (partial decay constant) of this transition is \( T_{1/2} = 3.1 \times 10^{-2} \) s (\( \lambda_2 = 22 \) s \(^{-1} \)). For the transition, with \( Q_\alpha \approx 11.4 \) MeV, to the state lying \( \approx \) 300 keV below the isomer in \( ^{266}\text{Hs} \), we again assume that \( L \approx 0 \) and that \( \Delta = 0.6 \times \Delta_{ms} \). The calculated half-life (partial decay constant) is then \( T_{1/2} = 5.9 \times 10^{-3} \) s (\( \lambda_3 = 117 \) s \(^{-1} \)).

For the \( \alpha \) transitions from the isomeric state, it can be seen from the above results that the partial decay constants of \( \lambda_1 = 19 \) s \(^{-1} \) (assuming the \( L = 9 \) transition), \( \lambda_2 = 22 \) s \(^{-1} \), and \( \lambda_3 = 117 \) s \(^{-1} \) suggest expected \( \alpha \) branching fractions of \( \approx 0.15, \approx 0.15, \) and \( \approx 0.70 \) for the \( Q_\alpha \approx 12.1, 11.1, \) and 11.4 MeV, transitions, respectively. The total decay constant \( \lambda_{T_{TOT}} = \lambda_1 + \lambda_2 + \lambda_3 = 158 \) s \(^{-1} \) gives a half-life of the isomeric state of \( T_{1/2} = 6.4 \) ms, which is in agreement with the experimental value of 3.9\(^{+1.3}_{-0.8} \) ms. Given the uncertainties in the model calculations (exemplified by the comparison to data on the ground-state \( \alpha \) decays in Fig. 2), and the sparse experimental information, the agreement with experiment is remarkable. The STM can qualitatively account for the observed fine structure in the \( \alpha \) decay of the isomer in \( ^{270}\text{Ds} \), including the competition between the \( L \approx 10 \) and \( L \approx 0 \) transitions.

Finally, we consider the \( \alpha \) decays of the ground-state and \( K \)-isomer in \( ^{264}\text{Hs} \). As shown in Fig. 3, the ground-state decays with a half-life of 2.9/0.51 ms. With \( Q_\alpha = 10.35 \) MeV, the calculated half-life is 3.1 ms. For the isomer, only a single \( \alpha \) branch to an excited state in \( ^{262}\text{Sg} \) is observed. The experimental half-life is measured to be 74\(^{+4}_{-3} \) ms [13]. Using the estimated value of \( Q_\alpha = 10.6 \) MeV [13], assuming an \( L = 0 \) \( \alpha \) decay, and taking account of the reduction in the pairing gap parameter, we estimate that the half-life of the isomer is \( T_{1/2} = 129 \) ms. Once more, the agreement between experiment and the STM calculations is very good.

V. SUMMARY AND CONCLUSIONS

In this article we have applied the superfluid tunneling model to compare against the experimental data on all known \( \alpha \) decays of even-even SHN with \( 100 \leq Z \leq 118 \), i.e., from isotopes of fermium (\( Z = 100 \)) to oganesson (\( Z = 118 \)). We have found a remarkable quantitative agreement between the data and the results of our calculations. The agreement is at a level that is comparable to empirical parametrizations exemplified by comparison with the Viola-Seaborg formula and the Royer formula.

We have used the model to examine the decay of the high-\( K \) isomers observed in the \( ^{270}\text{Ds} \) decay chain, which cannot be described using empirical formulas like the Viola-Seaborg or Royer prescriptions. It is recognized that in order to understand the \( \alpha \) decay of isomers in the SHN it is important to include the changes in the angular momentum, which have a strong effect on the barrier through which the \( \alpha \) particle must tunnel. However, the effect of the centrifugal barrier alone is insufficient to account for the observations of isomers in \( ^{266}\text{Hs} \) and \( ^{270}\text{Ds} \) which have half-lives longer than their corresponding ground states. There is also the observed fine structure in the \( \alpha \) decay from the isomer in \( ^{270}\text{Ds} \) in which an \( L \approx 10 \) transition competes with \( L \approx 0 \) transitions. We find that the effects of the nuclear structure of the multi-quasiparticle isomer, which is accounted for in the STM model by a reduction in the pairing gap parameter, must be included. We are then able to reproduce the observed features of the decays including the lifetimes of ground states and isomers, and the fine structure of the isomer decay.

As noted earlier, the observation of isomers that are longer lived than the ground states has important implications for experiments that are searching for new elements. One consequence is that we may simply be able to reach further in the nuclear chart using techniques that are more sensitive to finding the longer-lived isomeric states. Understanding the properties and decay of these isomers will be an important topic for both experiment and theory. It is also interesting to note that we are seeing essentially parallel \( \alpha \)-decay chains from excited- and ground-state decays in the same nuclei. In the case we have studied here there are parallel \( \alpha \)-decay chains for \( ^{270}\text{Ds} \rightarrow ^{266}\text{Hs} \rightarrow ^{262}\text{Sg} \). It will be important to distinguish the decay chains and assign them accurately since this effect could easily mislead isotopic assignments. Calculations such as those presented in this article may help with such issues.

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