Analysis of Linear L1 Adaptive Control Architectures for Aerospace Applications

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Analysis of Linear L1 Adaptive Control Architectures for Aerospace Applications

Anders Pettersson, Karl J. Åström, Anders Robertsson and Rolf Johansson

Abstract— **In some situations the closed-loop system obtained by L1 adaptive control is equivalent to linear systems. The architectures of these systems are investigated and compared with internal model control and the input observer architecture. The analysis is focused on aerospace application. An effort has been made to understand and describe what fundamental control characteristic of flying applications that make L1 adaptive controllers suitable for the task.**

I. Introduction

L1 adaptive control [1] was developed with aerospace control in mind and has been found to be suitable for flying applications [2]. Traditional adaptive schemes such as MRAC [3] can give large transients and slow convergence [4]. In L1 adaptive control fast adaptation is achieved while robust stability to bounded plant parameter changes is claimed. Even though large adaptation gains create large and rapidly varying internal signals, the L1 adaptive controller output is limited in amplitude and frequency, since a low-pass filter directly at the output, is used to make the controller act within the control channel bandwidth [5].

This work is part of a feasibility study of adaptive control for winged aircraft and missiles [6]. The ultimate goal is to address the question whether adaptive control can add value to products that SAAB develops today or in the future.

The study has shown that flight control systems with good performance can indeed be obtained by L1 adaptive control, but also that some L1 adaptive controllers are linear systems with a special architecture. Comparisons with internal model control, input observer control and state feedback give useful insights. One criticism against L1 adaptive control is that it uses high adaptive gains. The analysis gives insight into the choice and implication of gains.

Analysis and evaluation of L1 controllers are performed in the area which the adaptive parameter projection bounds are not active, since then the linear mapping holds. This is the parameter area in which the controller normally operates. In [7], L1 output feedback alternatives are analyzed and mapped to linear time invariant controllers, in this paper similar analysis for piecewise constant types of L1 state feedback controllers is made.

The main contribution of this work is the insight that the L1 adaptive controller of piecewise constant type generates a control signal which can be seen as a modification to a multivariable controller using state feedback with integral action. Comparisons are also made to disturbance observers and it is seen that these types of controllers use the nominal dynamics inverse while L1 controllers use the inverse of the desired dynamics. Doing this analysis and finding a controller that is equivalent to an L1 adaptive controller has a value for industries such as SAAB when online implementation is designed. The vague "sample rate of the available CPU" mentioned in [1] would give problems when prioritizing update rates in real-time software. Knowledge that the resulting inverse is done up to a frequency that is proportional to the sample rate will be helpful when choosing a control algorithm update rate. Industry is also served by the insight that since L1 adaptive controllers of this type are linear time invariant, there are a lot of methods that could end up with the same controller. However, using designs from L1 adaptive methodology points out a suitable set of aerospace controllers with a straightforward and structured method.

The paper is organized as follows. Initially the design of a piecewise constant L1 controller is given. This controller is analyzed and alternative views of the controller are given. Control signal generation is compared for state feedback control with integral action and an L1 controller of piecewise constant type. Transfer functions are analyzed as controllers designed and applied to a fighter jet.

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II. Piecewise constant L1 controller

A piecewise constant L1 controller (detailed in [1]) is defined as in Fig. 1. This controller uses a *State predictor*, an *Adaptation law* and a *Control law* according to:

State predictor:

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + B_m (u(t) + \hat{\sigma}_1(t)) + B_{um} \hat{\sigma}_2(t)$$

Adaptation law:

$$\hat{\sigma}(t) = M\tilde{x}(iT_s) = -B^{-1}\Phi^{-1}(T_s)e^{A_m T_s}\left(\hat{x}(iT_s) - x(iT_s)\right)$$

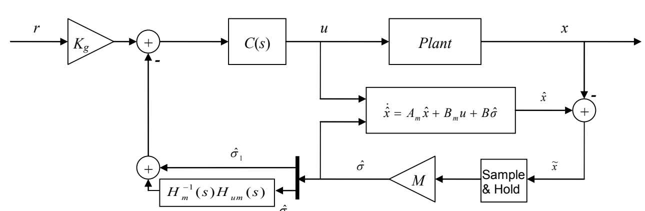

Figure 1. Block diagram of L1-controller, piecewise constant type.

where $B = [B_m \quad B_{um}]$, $\Phi(T_s) = A_m^{-1}(e^{A_m T_s} - I)$ and time argument iT_s effectuates zero order sample and hold at sampling time intervals T_s using index i.

Control law:

$$u(s) = C(s)\left(K_g r(s) - \hat{\sigma}_1(s) - H_m^{-1}(s) H_{um}(s)\hat{\sigma}_2(s)\right)$$

where $H_m(s) = C(sI - A_m)^{-1} B_m$, $H_{um}(s) = C(sI - A_m)^{-1} B_{um}$ and $C(s) = (I + KD(s))^{-1} KD(s)$.

System state x and state predictor $\hat{x}$ are vectors of equal size (Fig. 1). A_m sets the desired reference dynamics. It could be given by $A_m = A\text{-}B_m L$ where A is the nominal linearized plant dynamics and L corresponds to a linear state feedback. C is a matrix that gives the output vector y, a linear combination of states that will be controlled to follow the equally sized, demand vector r. B_m is given by the nominal plant input, called the matched input matrix, from control signal u which has the same size as y and r. An unmatched input matrix B_{um} is created as the null-space of B_m^T (solving the equation $B_m^T B_{um} = 0$), while keeping the square matrix $[B_m \quad B_{um}]$ of full rank.

For each element in the control input u there is a matched element in $\hat{\sigma}$, called $\hat{\sigma}_1$. Unmatched elements $\hat{\sigma}_2$ are created so that the size of $\hat{\sigma}$ matches the total number of states. $H_m(s)$ is the reference transfer function from the matched input, that is simply how the outputs are affected by the inputs. $H_{um}(s)$ is the reference transfer function from unmatched inputs, which is how the outputs are affected in input directions that are orthogonal to the directions defined by B_m. This design of creating one term in the control signal u by taking the estimated unmatched error and feed it through the inverse of the matched transfer function $H_m^{-1}(s)$ and then the unmatched transfer function $H_{um}(s)$, creates a way to compensate for unmatched disturbances. This is not in any way unique for L1 control; the matched together with unmatched compensation could be used in other types of control designs.

$KD(s)$ is in its simplest form a diagonal matrix K times an integrator so $D(s)=1/s$. In the control law K_g can be chosen as the steady state gain $K_g = H_m^{-1}(0) = -(CA_m^{-1}B_m)^{-1}$. This will, in steady state, couple one reference signal to one output signal by a unity gain.

III. Comparison to Disturbance Observer

In [7] and [8] it is shown that as L1 adaptive gain Γ go to infinity for continuous time controllers and as sampling times T_s go to zero for piecewise constant controllers, there is an equivalent linear time invariant controller. If the adaptive law only uses linear parameter estimates, one example being $\dot{\hat{\sigma}}(t) = -\Gamma B^T P(\hat{x}(t) - x(t))$ of [1], and no projector operators are active, this equivalent controller exists. Since the state predictor and adaptive law in this limit becomes the inverse of a dynamic system, another interpretation of how the controller works can be made. That is to estimate a disturbance at the plant input by inverting the reference dynamics and then compensate for this disturbance by subtracting it from the plant input. Alternative equivalent structures will also make it possible to compare the L1 controller to other linear control design methods.

An L1 equivalent controller in Fig. 2 could be compared to a disturbance observer in Fig 3 ([9], [10] and [11]). The two have many common features. An input disturbance $\hat{\sigma}$ is estimated and a filter $C(s)$ attenuates the high frequency content to the control signal u. There are modifications to these types of controllers; for example the reference r does not have to pass through $C(s)$.

Even if Fig. 2 shows similarities between the L1 controller and the input observer there are several issues that do not appear in Fig. 2 compared with the more detailed block diagram in Fig. 1 which also shows the state error $\tilde{x} = \hat{x} - x$. An important result for L1 adaptive control is that $\tilde{x}$ goes to zero with increasing adaptation gains. The block diagram in Fig. 1 is also useful because the reference model can be augmented with actuator saturation and other nonlinearities. These features are lost by reducing the block diagram to Fig. 2 based on the assumption of linearity.

It is important to note that in the L1 equivalent controller the plant inversion is made based on the *reference system*. In a disturbance observer the *nominal plant dynamics* is inverted.

The Youla parameter $Q(s)$ [12] of a disturbance observer is:

$$Q(s) = C(s)\hat{P}^{-1}(s) \tag{1}$$

where $\hat{P}(s)$ is the nominal plant dynamics.

For L1 adaptive control of piecewise constant type, the Youla parameter $Q(s)$ is:

$$Q(s) = \left(I + \hat{P}(s)(I - C(s))C(s)H_m^{-1}(s)\right)^{-1}(I - C(s))C(s)H_m^{-1}(s) \tag{2}$$

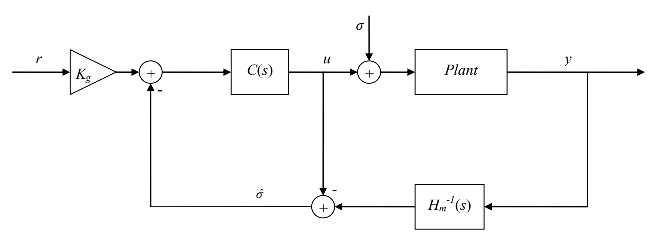

Figure 2. L1 controller with replaced state predictor.

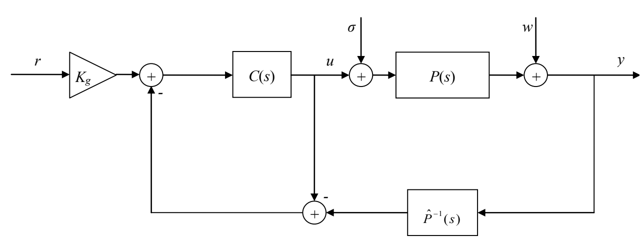

Figure 3. Disturbance observer acting on plant *P(s)*.

or expressed in $KD(s)$:

$$Q(s) = \left(I + \hat{P}(s)KD(s)H_m^{-1}(s)\right)^{-1} KD(s)H_m^{-1}(s) \tag{3}$$

where $\hat{P}(s)$ is the plant nominal dynamics and $H_m^{-1}(s)$ is the reference system inverse.

The two methods become equal if the reference system inverse of the L1 equivalent controller is set to the nominal plant inverse.

IV. Comparison of Feedback Laws

Feedback laws for three different system architectures will be discussed.

The control law for state feedback with integral action is:

$$u = K_g r - Lx - L_i K_g \frac{1}{s}(r - y) \tag{4}$$

essentially a PID control in aerospace applications. The state x contains a proportional P-part and a derivative D-part. Proportional parts in aerospace are angle of attack and sideslip as being the control objective and approximate derivatives are pitch and yaw rates. The integral I-part is created as the control error [13] and can be transformed to corresponding input directions by the steady state gain K_g.

As an alternative to (4) the reference signal can be filtered through a reference system $H_m(s) = C(sI - A_m)^{-1} B_m$ to reduce overshoot due to integral windup:

$$u = K_g r - Lx - L_i K_g \frac{1}{s}\left(H_m(s)K_g r - y\right) \tag{5}$$

An L1 controller of piecewise constant type augmented to a state feedback (Fig. 4) corresponds to the control law:

$$u = K_g r - Lx + KD_0(s)\frac{1}{s}\left(K_g r - H_m^{-1}(s)y\right) \tag{6}$$

Instead of integrating smoothed reference signals, the L1 controller takes the raw reference signal and feeds the output of the plant through an inverse approximation of the reference system. The start of the high frequency roll of to the $H_m^{-1}(s)$ approximation increases with increasing adaptive gains Γ and decreasing sampling periods T_s [1]. It should be noted that the reference signal is used both outside and inside the integral expression. The standard L1 controller procedure is to add it inside only, although the (6) alternative has been used as well in flying applications [2]. $D_0(s) = sD(s)$ in (6) is in its simplest form unity. It can be noted that (5) and (6) are identical if $D_0(s) = H_m(s)$. This could guide tuning of the low-pass filter $C(s)$ which is an important controller design variable.

$$C(s) = \left(I + KD(s)\right)^{-1} KD(s) = \left(sI + KD_0(s)\right)^{-1} KD_0(s) \tag{7}$$

The gain K in (7), which sets bandwidth of the low-pass filter $C(s)$, is usually a diagonal matrix. There will be three design parameters for a three channel aerospace roll-pitch-yaw controller (if $D_0(s)$ is unity). Nominal settings for the diagonal elements in K are available bandwidth values in the control channels (roll, pitch, yaw, respectively).

The L1 controller structure aids the design of a control law by pointing out gain directions. It focuses on the error at the input of the plant instead of the commonly used output error. The input error is integrated over time to generate the control signal.

V. Transfer Function Analysis

Relevant transfer functions from reference, input disturbance and output measurement noise are generated from Fig. 4 assuming that the plant aided by the state feedback nominally has dynamics similar to the reference system. To get simple but yet indicative expressions in (8) and (9) it is also assumed that the gain K is a scalar k times the unity matrix, $K=kI$. This assumption is only made in (8) and (9) for illustration.

The transfer functions to output y and to control signal u_a with these assumptions are:

$$y = H_m(s)K_g r + \left(I - C(s)\right)H_m(s)\sigma - \left(I - C(s)\right)w \tag{8}$$

and

$$u_a = K_g r - C(s)\sigma + C(s)H_m^{-1}(s)w \tag{9}$$

In (8) the transfer function from input disturbance σ to y will be reduced for high frequencies by $H_m(s)$ and for low frequencies by I-$C(s)$. The low-pass filter $C(s)$ will have similar or slightly higher bandwidth than $H_m(s)$, the recommendation is to design $C(s)$ to the available bandwidth of the control channel, by doing so a small amount of the disturbance σ will be passed to y. Output disturbance attenuation will be I-$C(s)$, low frequency disturbances in w will be compensated for up to a bandwidth corresponding to $C(s)$.

In (9) input disturbance σ is fed to the L1 control signal u_a through $C(s)$. The output disturbance w and equivalently measurement noise goes to u_a through the reference system inverse which could be problematic so analysis of this case will be provided.

To give examples of transfer function magnitudes a fighter jet realization is provided. Singular values for a roll-pitch-yaw controller as in Fig. 4 are presented. Reference elements in r and corresponding control objective y is roll

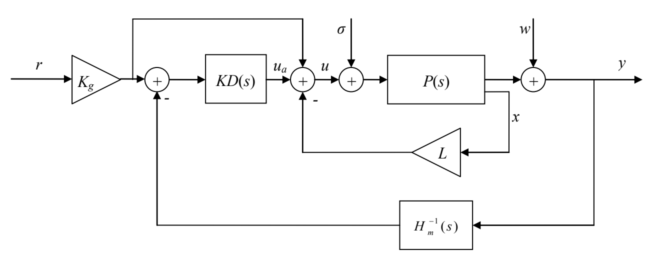

Figure 4. Low pass filter $C(s)$ replaced by $KD(s)$ and linear state feedback added to the plant.

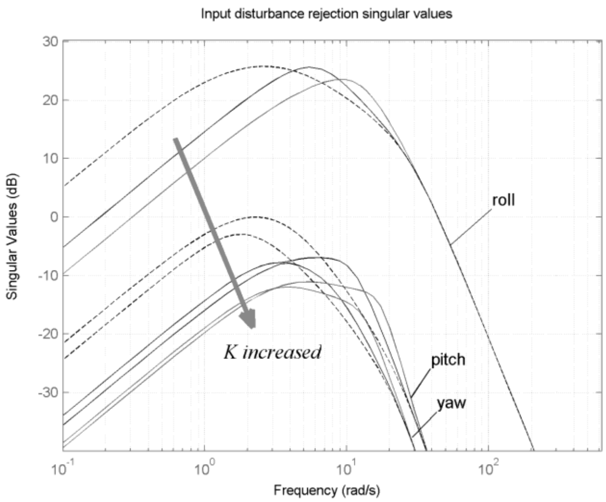

Figure 5. Input disturbance attenuation from σ to control objective y, for (5) dashed and (6) solid, as low pass filter bandwidth in (6) is K and $2K$.

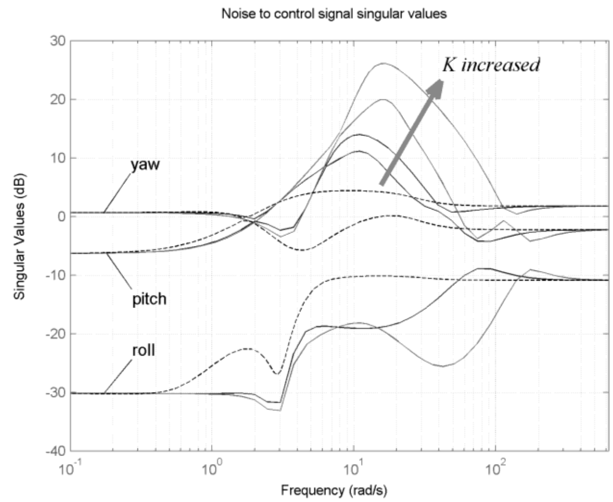

Figure 6. Disturbance and noise feedthrough from w to control signal u, for (5) dashed and (6) solid, as low pass filter bandwidth in (6) is K and $2K$.

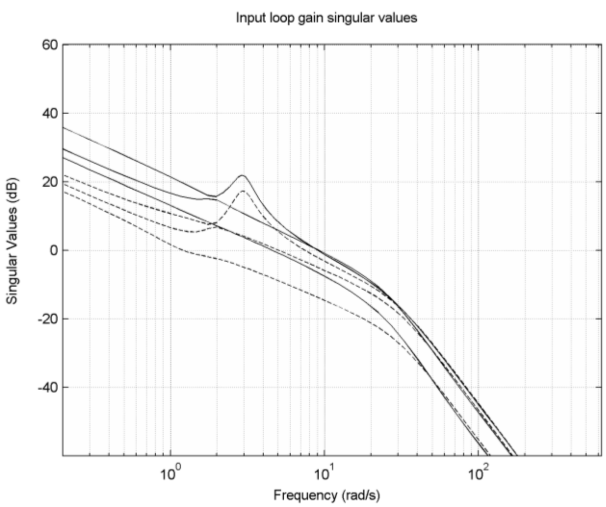

Figure 7. Input loop-gain singular values for (5) dashed and (6) solid. Roll is of highest magnitude, followed by pitch and yaw in descending order.

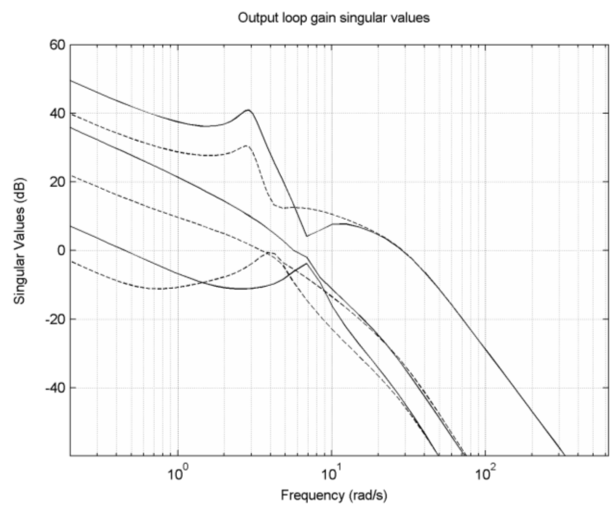

Figure 8. Output loop-gain singular values for (5) dashed and (6) solid. Roll is of highest magnitude, followed by pitch and yaw in descending order.

rate p, angle of attack α and angle of sideslip β (see appendix). The control signal u has roll-pitch-yaw control surface deflection elements δ_a, δ_e and δ_r. The state feedback gain L is such that the nominal linearized dynamics aided by the state feedback has dynamics similar to the reference system $H_m(s)$, ($A_m = A\text{-}B_mL$). Dashed lines in Figs. 5-8 correspond to a linear state feedback controller with integral action as in (5), solid lines correspond to an augmented L1 controller with a high adaptive gain as in (6).

Fig. 5 shows how input load disturbances σ are attenuated to the output y. In Fig. 5 one K value that results in a second order $C(s)$ with a bandwidth corresponding to the control channel bandwidth and another K that corresponds to twice that value. Input load attenuation is significantly higher for the L1 controller than the state feedback and that load attenuation increases with K.

Fig. 6 shows how output disturbance or measurement noise w propagates to the control signal u for the same variations as in Fig. 5. The L1 controller feeds more noise through $C(s)$ to the control signal than the state feedback.

The choice of bandwidth K of the low-pass filter is a tradeoff between load disturbance attenuation and injection of measurement noise. A low value gives less disturbance attenuation with low noise injection. Increasing the bandwidth improves load disturbance attenuation but more measurement noise is injected causing large actuator demands. This noise injection will be damped by actuator and plant dynamics but could cause actuator wear and undesired excitation of the plant dynamics.

Open loop-gain singular values from Fig. 4 at the input u are presented in Fig. 7 and loop-gain singular values at the output y are presented in Fig. 8. Singular values of the loop-gain (solid) are clustered at the input (Fig. 7) when designing an L1 controller (magnitude of singular values are made equal). The unity gain crossover frequency is higher than for the linear state feedback controller and is increased even further if K is increased beyond the control channel bandwidth. Fig. 8 shows an increase in crossover frequency for the L1 controller compared to the state feedback but there is no cluster of singular values.

VI. Conclusion

L1 adaptive control methodology can be used for flying vehicle controllers. The idea of having a reference system, creating plant deviations from this reference system and rejecting the difference is both intuitive and straightforward.

Some L1 adaptive controllers are linear time invariant as long as projection operators are inactive. Comparisons of linear L1 adaptive controllers have been made to the type of internal model controller which is known as a disturbance observer. They share a lot of characteristics such that they can be seen as estimating and compensating for disturbances at the plant input by using inverse dynamics. L1 controllers focus on the desired reference dynamics while disturbance observers use the nominal plant dynamics.

By using the desired dynamics reference system inverse, L1 controllers accomplish both reference following and disturbance attenuation, without caring about if deviation comes from model error or an external disturbance from outside the plant. However, if the L1 controller is augmented to a feedback controller that make the plant dynamics nominally behave like the reference system dynamics (such as a linear state feedback), better reference following is achieved since then only truly unknown factors will have to be compensated. The high gain in L1 controllers can be seen as a measure to approximate a reference system inverse up to a certain frequency.

The augmentation of an L1 controller to the plant makes input disturbance attenuation significantly better than what a typical linear state feedback controller accomplishes in aerospace applications. The L1 controller input disturbance estimation and compensation in aerospace applications focus on keeping the angular velocity of the vehicle correct. Since angle of attack/sideslip over a short period of time is integrated angular velocity, the control objective will be close to demands.

For real-time implementation it is important to have understanding of the fundamental parts that are needed for exploiting L1 adaptive control benefits. Analysis of alternative equivalent structures gives options in how to implement the controller in a real-time application where it has to fit into a larger software structure.

Mapping of control methods to each other will also make it possible to use benefits from L1 adaptive control gradually. It will be possible to blend in design features such as a reference system inverse as a modification to standard aerospace feedback laws. It is also possible to gradually add non-linearities in the state predictor. Such options could be important when compromises are needed to get clearance in use of new control designs in live flying products.

Appendix

Linear vehicle dynamics used in Figs. 5-8: In pitch motion there is one input from elevator deflection δ_e that affects two states: angle of attack α and pitch nose up angular rate q.

$$\begin{pmatrix} \dot{\alpha} \\ \dot{q} \end{pmatrix} = A_p \begin{pmatrix} \alpha \\ q \end{pmatrix} + B_p \delta_e$$

$$A_p = \begin{pmatrix} -\dfrac{q_d S}{mV} C_{N_\alpha} & 1 - \dfrac{q_d Sc}{2mV^2} C_{N_q} \\ \dfrac{q_d Sc}{I_y} C_{m_\alpha} & \dfrac{q_d Sc^2}{2 I_y V} C_{m_q} \end{pmatrix} \quad B_p = \begin{pmatrix} -\dfrac{q_d S}{mV} C_{N\delta_e} \\ \dfrac{q_d Sc}{I_y} C_{m\delta_e} \end{pmatrix}$$

where $q_d = \rho V^2 / 2$ is dynamic air pressure, ρ air density, V airspeed, S vehicle aerodynamic reference area, c a reference length (wing cord), m mass and I_y mass inertia around y-axis. Aerodynamic coefficients 'C_{xx}' are non-dimensional linear approximations of how aerodynamic forces and moments depend on states and inputs [13].

In roll-yaw motion there are two inputs, aileron δ_a and rudder δ_r, that affect three states, roll rate p, angle of sideslip β and yaw rate r:

$$\begin{pmatrix} \dot{p} \\ \dot{\beta} \\ \dot{r} \end{pmatrix} = A_y \begin{pmatrix} p \\ \beta \\ r \end{pmatrix} + B_y \begin{pmatrix} \delta_a \\ \delta_r \end{pmatrix}$$

where A_y and B_y are matrices with elements similar to those of the pitch matrices, that is, elements dependent on vehicle mass and aerodynamic quantities together with dynamic air pressure and airspeed.

There are no linear couplings between the pitch and roll-yaw motion so the two are combined to a five state model. Second order actuator dynamics are added and model parameters are set to values of a fighter jet with airspeed corresponding to Mach 0.6 at an altitude of 1000m.

Acknowledgment

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