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Bit Error Rate Performance of Generalized Frequency Division Multiplexing

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***Abstract*—Generalized frequency division multiplexing is a non-orthogonal, digital multicarrier transmission scheme with attractive features that address the requirements of emerging applications of wireless communications systems in areas like cognitive radio and machine-to-machine communication. In this paper, first a linear system description is obtained for the transmitter by ordering data in a time-frequency block structure and representing the processing steps upconversion, pulse shaping and upsampling as matrix operations. Based on the transmitter, three standard ways of detecting the signal are derived and compared in terms of bit error performance in AWGN and Rayleigh multipath fading channels.**

***Index Terms*—flexible physical layer, multicarrier systems, cognitive radio, machine-to-machine communication**

I. Introduction

Many of today's wireless communications systems rely on multicarrier transmission for its proven advantages over traditional singlecarrier (SC) communications in multipath fading channels. Particularly, the orthogonal frequency division multiplexing (OFDM) scheme has found its way into several state-of-the-art wireless standards, including LTE, WiMAX and DVB-T. However, novel applications emerge and impose new requirements to communications systems that cannot be addressed very well by OFDM. For instance in cognitive radio use cases, a communications system needs to exhibit strong frequency localization in order to fit into narrow spectral holes without causing interference to adjacent frequency bands, while at the same time it has to provide the means to aggregate scattered white spaces e.g. across the TV bands [1]. This calls for a frequency agile, scalable wideband system that is capable of shaping the spectrum of the transmit signal. Another use case with growing significance is machine-to-machine communication [2], where important aspects are energy efficiency, the ability to handle an extremely large number of users with varying requirements to traffic, transfer rate, latency, quality of service and mobility. This requires schemes with small communication overhead that are robust to asynchronicity for battery driven devices. When power is not an issue, increased bandwidth efficiency is relevant. To address these aspects, novel multicarrier concepts like generalized frequency division multiplexing (GFDM) [3], [4] and filterbank multicarrier (FBMC) [5], [6] are researched, that generalize the well known OFDM transmission scheme.

GFDM is a digital multicarrier concept that is based on the filter bank approach. The strength of the scheme lies in its high flexibility. The data can be spread across a two-dimensional block structure that spans over time and frequency. The transmit signal exhibits strong frequency localization, which is achieved with adjustable pulse shaping filters. Furthermore, tail biting is applied to prevent rate loss that would otherwise occur from filter tails, and the cyclic prefix technique is used to provide a simple way of equalization when data is transmitted through a multipath channel. However, by introducing variable pulse shaping filters, the orthogonality between the subcarriers is affected. As a result, self-induced intercarrier and intersymbol interferences need to be accounted for in GFDM. In FBMC, a polyphase filter bank structure is used to transmit and receive the signal. The scheme relies on offset-QAM modulation in conjunction with appropriate filters to avoid self-created intersymbol and intercarrier interference. Equalization is performed per-subcarrier without the need for a cyclic prefix.

In previous work, GFDM has been modeled as an arrangement of parallel, independent and partly overlapping subcarriers. For the modulation and detection of the signal, each subcarrier branch has been treated as an individual singlecarrier system with pulse shaping. In this paper, the bit error rate (BER) performance of GFDM is studied and therefore a linear matrix model transmitter model is neccessary. Hence, a system description is presented, in which all subcarriers are jointly processed, i.e. data blocks that span over time as well as frequency resources are modulated and demodulated in one processing step. This allows to derive the three standard methods for receiving the GFDM signal zero forcing (ZF), matched filter (MF) and minimum mean square error (MMSE) and evaluate the performance of the proposed scheme.

The rest of the paper is organized as follows: A generic GFDM system model is presented in Section II and a linear matrix model for the transmitter is found in Section III. In Section IV, three receiver techniques for the GFDM system are derived. In section V, the BER performance of GFDM is discussed and compared with OFDM. Conclusions are drawn in section VI.

II. Generic System Model

In previous work [4], GFDM has been considered as a generic multi-carrier system with pulse shaping. The system is modeled in baseband and it consists of K subcarriers, on

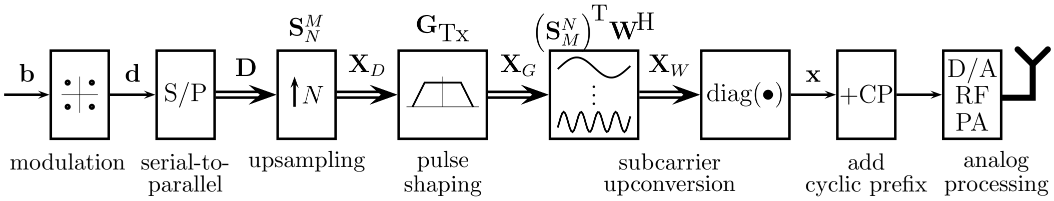


Fig. 1. Baseband transmitter model for GFDM

which a transmit filter $g_{\mathrm{Tx}}[n]$ is applied individually. Blocks with the length of M symbols are processed per subcarrier, where each symbol is sampled N times. The subcarriers are modulated with a respective subcarrier center frequency and the transmit signal

$$x[n] = \sum_{m=0}^{M-1}\sum_{k=0}^{K-1} d_{m,k}\, g_{\mathrm{Tx}}[n-mN] e^{j2\pi\frac{kn}{N}}, \tag{1}$$

is obtained through superposition of all subcarriers, i.e. copies of $g_{\mathrm{Tx}}[n]$, that are weighted with complex valued data symbols $d_{m,k}$, delayed by mN in time and shifted by $k\frac{1}{N}$ in frequency domain, where $\frac{1}{N}$ denotes the subcarrier spacing. The filter $g_{\mathrm{Tx}}[n]$ is circular with periodicity $n \bmod MN$ in order to facilitate tail biting [3] at the transmitter.

Suppose that $y[n]$ are the time samples obtained at the receiver. One way of reconstructing the data is to design the receiver such, that $\hat{d}_{m,k}$ are obtained by reversing the frequency shift, applying the matched filter $g_{\mathrm{Rx}}[n]$ and downsampling the resulting signal at $n = mN$ according to

$$\hat{d}_{m,k} = \left(y[n]e^{-j2\pi\frac{kn}{N}}\right) \circledast g_{\mathrm{Rx}}[n]\Big|_{n=mN}, \tag{2}$$

where $\circledast$ denotes circular convolution with respect to n. Circular convolution is necessary for tail biting at the receiver, which is an essential part of the GFDM concept [4].

From (1), it is already clear that there must be a linear (matrix) model that describes the generation of a GFDM transmit signal, i.e. where blocks of K subcarriers and M time slots are modulated jointly in one step. To obtain this model, in the following section the signal processing steps that are necessary in the GFDM transmitter are first represented in form of matrix operations which, are then rearranged to a very simple matrix expression.

III. Matrix Model for GFDM Transmitters

Consider the model depicted in Fig. 1. The input to the system is a binary sequence $\mathbf{b}$ of length μKM. In the first processing step, the bits are mapped to a 2^{μ}-valued modulation grid with order μ, which yields the complex valued sequence $\mathbf{d} = \left\{\tilde{d}_\ell\right\}_{MK\times 1}$. Next, $\mathbf{d}$ is reshaped by serial-to-parallel conversion to a matrix

$$\mathbf{D} = \left\{d_{m,k}\right\}_{M\times K}, \tag{3}$$

which will be further referred to as a data block. Note that $\left\{d_{m,k}\right\}$ are the data symbols that are used in (1). The elements of $\mathbf{D}$ correspond to a grid such, that the m'th row denotes the data transmitted in the m'th symbol slot and the k'th column contains the data transmitted on the k'th subcarrier, wherein K is the total number of subcarriers and M is the number of symbols in one block. Note that while in OFDM data is transmitted in one-dimensional blocks that occupy one time slot and a number of frequency bins each, in GFDM transmit data is arranged in a two-dimensional block structure that spreads across multiple time slots and frequency bins.

To be able to apply a pulse shaping filter and in order to shift the data to the individual subcarrier frequencies without aliasing, in the subsequent step $\mathbf{D}$ is upsampled by factor N along the columns. Mathematically, zeros can be inserted with a sampling matrix

$$\mathbf{S}_N^M = \left\{s_{n,m}\right\}_{MN\times M}, \quad s_{n,m} = \begin{cases} 1 & n = (m-1)N+1 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

yielding $\mathbf{X}_D = \mathbf{S}_N^M\mathbf{D}$. Note that in (1), the upsampling has inherently been part of the operation $d_{m,k}\, g_{\mathrm{Tx}}[n-mN]$. For the remainder of this paper we consider $N = K$.

Next, the pulse shaping filter is applied. As stated in the previous section, a circular filter is used in GFDM to create tail biting. For this purpose, a pulse shaping filter with length of M symbols is sampled N times per symbol, which yields a vector $\mathbf{g}_{\mathrm{Tx}} = \left\{g_n\right\}_{MN\times 1}$, where $g_n = g_{\mathrm{Tx}}[n]$. It is used to construct a matrix

$$\mathbf{G}_{\mathrm{Tx}} = \begin{pmatrix} g_1 & g_{MN} & \cdots & g_2 \\ g_2 & g_1 & & g_3 \\ \vdots & & \ddots & \vdots \\ g_{MN} & g_{MN-1} & \cdots & g_1 \end{pmatrix} \tag{5}$$

which is applied according to $\mathbf{X}_G = \mathbf{G}_{\mathrm{Tx}}\mathbf{X}_D$.

Similarly to OFDM, the upconversion of the subcarriers can be done with an inverse Fourier transform (IFFT). This operation corresponds to a multiplication with a Fourier matrix

$$\mathbf{W} = \frac{1}{\sqrt{MN}}\left\{w^{k,n}\right\}_{MN\times MN}, \quad w^{k,n} = e^{-j2\pi\frac{(k-1)(n-1)}{N}}. \tag{6}$$

Since there are $K = N$ subcarriers in the system, in order to maintain a subcarrier spacing of $\frac{1}{N}$, only every M'th column of $\mathbf{W}$ is selected by using a sampling matrix according to $\mathbf{X}_W = \mathbf{X}_G\left(\mathbf{S}_M^N\right)^{\mathrm{T}}\mathbf{W}^{\mathrm{H}}$. The transmit signal of the system $\mathbf{x} = \left\{x_n = x[n]\right\}_{MN\times 1}$ is obtained by taking the elements from the diagonal of $\mathbf{X}_W$. This additional step is necessary due to the two-dimensional nature of the transmit data block. The off-diagonal elements of $\mathbf{X}_W$ contain cross-mixing terms and are not relevant for the transmitted signal.

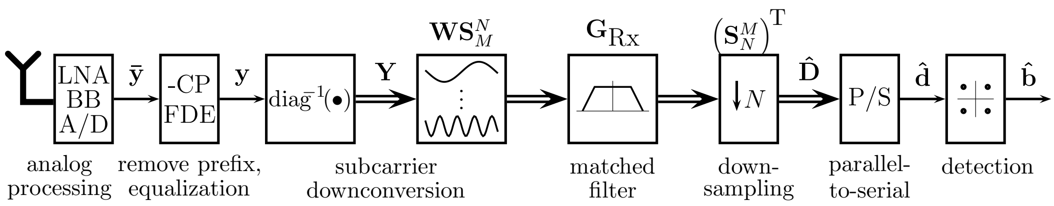


Fig. 2. Baseband receiver model for GFDM

Putting all processing steps together yields the expression

$$\mathbf{x} = \mathrm{diag}\Big(\mathbf{G}_{\mathrm{Tx}}\mathbf{S}_N^M\mathbf{D}\left(\mathbf{S}_M^N\right)^{\mathrm{T}}\mathbf{W}^{\mathrm{H}}\Big), \tag{7}$$

which denotes the samples of the transmit signal that corresponds to a block of data $\mathbf{D}$, which is obtained from a sequence $\mathbf{b}$. A cyclic prefix is added to preserve the circular structure of the transmit signal and to make frequency domain equalization possible at the receiver after multi-path effects apply in the channel. Subsequently, the signal is converted to the analog domain, mixed up to radio frequency and amplified before transmission.

The expression in (7) can be carried over to the more convenient form

$$\mathbf{x} = \mathbf{Ad}, \tag{8}$$

where $\mathbf{A}$ is an $MN \times MN$ complex valued modulation matrix. Let $\mathbf{G}'_{\mathrm{Tx}} = \mathbf{G}_{\mathrm{Tx}}\mathbf{S}_N^M$ and $\mathbf{W}'_{\mathrm{Tx}} = \left(\mathbf{S}_M^N\right)^{\mathrm{T}}\mathbf{W}^{\mathrm{H}}$. Then (7) can be written as $\mathbf{x} = \mathrm{diag}(\mathbf{G}'_{\mathrm{Tx}}\mathbf{D}\mathbf{W}'_{\mathrm{Tx}})$. Since only the elements on the diagonal of the matrix product are transmitted, the n'th element $x_n = [\mathbf{X}_W]_{n,n}$ depends only on the n'th row of $\mathbf{G}'_{\mathrm{Tx}}$ given as $\mathbf{g}'_{\mathrm{Tx},n}$, the data matrix $\mathbf{D}$ and the n'th column of $\mathbf{W}'_{\mathrm{Tx}}$ denoted by $\mathbf{w}'_{\mathrm{Tx},n}$. Consequently, $[\mathbf{X}_W]_{n,n} = \mathbf{g}'_{\mathrm{Tx},n}\mathbf{D}\mathbf{w}'_{\mathrm{Tx},n}$, which can be rewritten with a Kronecker product [7] to

$$[\mathbf{X}_W]_{n,n} = \Big(\left(\mathbf{w}'_{\mathrm{Tx},n}\right)^{\mathrm{T}} \otimes \mathbf{g}'_{\mathrm{Tx},n}\Big)\,\mathrm{vec}(\mathbf{D}) = \mathbf{a}_n\mathbf{d}. \tag{9}$$

Here, $\mathrm{vec}(\mathbf{D})$ denotes the operation of stacking all columns of $\mathbf{D}$ into a vector. If the elements $\tilde{d}_\ell$ of $\mathbf{d}$ are mapped to the elements $d_{m,k}$ of $\mathbf{D}$ according to

$$d_{m=((\ell-1)\bmod M)+1,\,k=\lfloor\frac{\ell-1}{M}\rfloor+1} = \tilde{d}_\ell, \tag{10}$$

then $\mathrm{vec}(\mathbf{D}) = \mathbf{d}$. Computing $\mathbf{a}_n$ for all $n = 1,\ldots,MN$ and storing them in the rows of a matrix finally gives $\mathbf{A}$ and leads to (8).

This expression for generating the transmit signal now allows to apply standard methods for receiving it. Also, with respect to an implementation, the transmitter is just a matrix multiplication. Hence, a benefit of GFDM is that scaling the matrix or using different precomputed matrices is an easy way to adjust the transmit signal to different frequency bands.

IV. Three Receiver Models for GFDM

A. Channel Model

Let $\mathbf{y}$ be the vector that contains the time samples $y[n]$, that are obtained at the receiver after low-noise amplification, downmixing to baseband and analog-to-digital conversion. Further let $\mathbf{n} \sim \mathcal{N}\left(\mathbf{0},\sigma_{\mathbf{n}}^2\right)$ denote a noise vector containing AWGN samples of variance $\sigma_{\mathbf{n}}^2$. Assuming the analog processing is ideal, the received signal can be expressed as

$$\bar{\mathbf{y}} = \mathbf{Hx} + \mathbf{n}, \tag{11}$$

where $\mathbf{H}$ the channel filter. In additive white Gaussian noise (AWGN) channels $\mathbf{H} = \mathbf{I}$, hence $\mathbf{y} = \mathbf{x} + \mathbf{n}$. Also, for that case the cyclic prefix is omitted.

For Rayleigh multipath channels, $\mathbf{H}$ has been constructed from channel responses with exponential power delay profile. By inserting a CP to the transmit signal $\mathbf{x}$ and removing it from the received vector $\bar{\mathbf{y}}$, single tap frequency domain equalization (FDE) is facilitated. When $\mathbf{H}$ is known perfectly at the receiver $\mathbf{y} = \mathbf{x} + \mathbf{n}^+$ is obtained, where $\mathbf{n}^+$ is now colored noise.

B. Matched Filter Receiver

One way to receive the GFDM signal is to apply a matched filter (MF) on each subcarrier separately, which corresponds to (2). Let $\mathbf{Y}$ be a matrix that contains only zeros except on the main diagonal, thus $[\mathbf{Y}]_{n,n} = y[n]$. Then, according to Fig. 2 and in analogy with the steps described in the previous section,

$$\hat{\mathbf{D}} = \left(\mathbf{S}_N^M\right)^{\mathrm{T}}\mathbf{G}_{\mathrm{Rx}}\mathbf{YWS}_M^N \tag{12}$$

can be found. Therein $\mathbf{G}_{\mathrm{Rx}} = \mathbf{G}_{\mathrm{Tx}}^{\mathrm{H}}$ denotes the receiver matched filter. With $\mathrm{vec}\big(\hat{\mathbf{D}}\big)$ the received data is arranged in a vector and the matched filter receiver follows as

$$\hat{\mathbf{d}} = \mathbf{A}^{\mathrm{H}}\mathbf{y}. \tag{13}$$

C. Zero Forcing Receiver

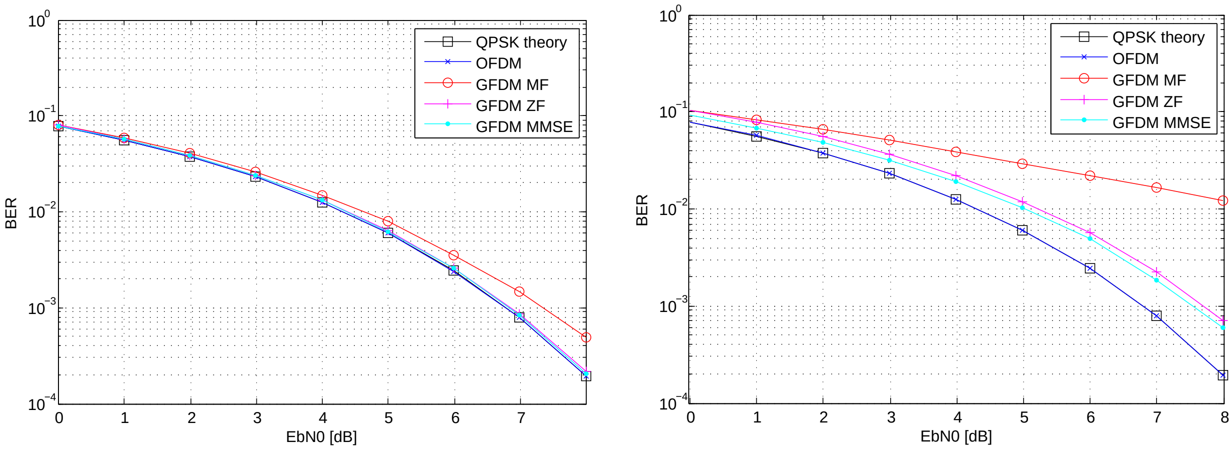
Another receiver method can be obtained directly from (8) by finding the pseudo-inverse $\mathbf{A}^+$ of $\mathbf{A}$ such, that $\mathbf{A}^+\mathbf{A} = \mathbf{I}$. Depending on the row and column rank of $\mathbf{A}$, it can be computed as $\mathbf{A}^+ = \left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{H}}$ or $\mathbf{A}^+ = \mathbf{A}^{\mathrm{H}}\left(\mathbf{A}\mathbf{A}^{\mathrm{H}}\right)^{-1}$. Then with $\hat{\mathbf{d}}$ denoting the received data symbols

$$\hat{\mathbf{d}} = \mathbf{A}^+\mathbf{y} \tag{14}$$

will be further referred to as the zero forcing (ZF) receiver.

D. Minimum Mean Square Error Receiver

A major drawback of the zero forcing receiver is its inherent property of potential noise amplification, which strongly depends on $\mathbf{A}^+$. This weakness is addressed by the minimum



(a) AWGN, $K=128$, $M=5$, $\alpha=0.1$

(b) AWGN, $K=128$, $M=5$, $\alpha=0.5$

Fig. 3. OFDM and GFDM BER performance for uncoded QPSK transmission in AWGN channels.

mean square error (MMSE) receiver [8]

$$\hat{\mathbf{d}} = A^{\dagger}\,\mathbf{y} \quad \text{with} \quad \mathbf{A}^{\dagger} = \left(\frac{\sigma_{\mathbf{n}}^2}{\sigma_{\mathbf{d}}^2}\mathbf{I}+\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{H}} \tag{15}$$

by balancing the variance of the noise samples $\sigma_{\mathbf{n}}^2$ and the data symbols $\sigma_{\mathbf{d}}^2$.

V. Performance Comparison

A. Simulation Setup

With (8), (14), (13) and (15), the bit error rates (BER) of GFDM can be studied. The subsequent results are obtained through simulation of a GFDM and an OFDM system with the parameters listed in Table I. Two setups of uncoded transmission through AWGN and Rayleigh multipath fading channels are considered.

description	parameter	value
number of subcarriers	K	128
number of time slots	M	5
pulse shaping filter	g	RRC
roll-off factor	α	$\{0.1, 0.5\}$
modulation order	μ	2 (QPSK)
length of cyclic prefix	N_{CP}	32
exponent of power delay profile	γ	0.1

TABLE I
Simulation parameters for the GFDM system.

B. AWGN Performance

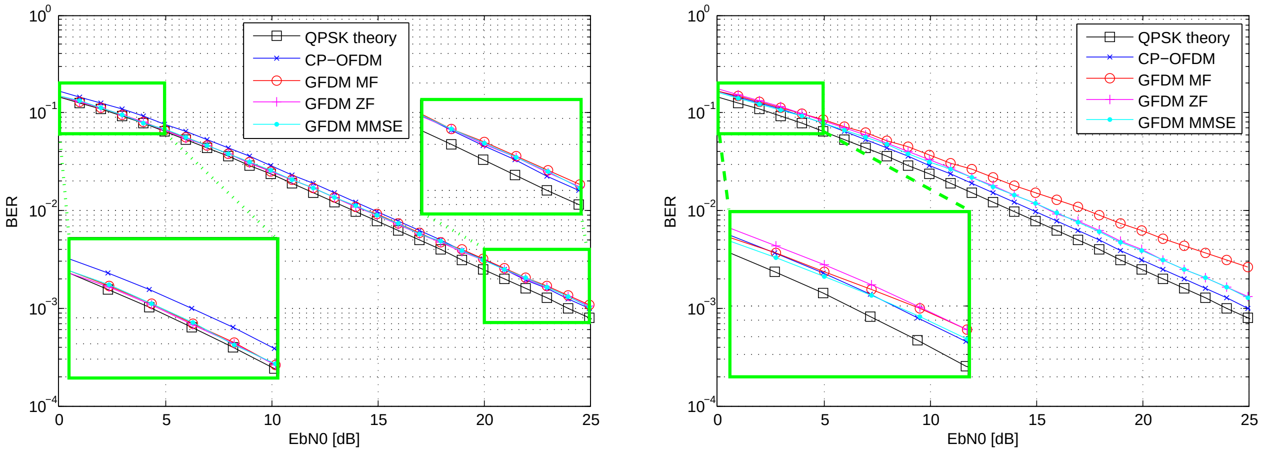
Looking at the performance in AWGN channels is appropriate to study the self induced interference. It occurs because non-orthogonal subcarriers are tolerated in GFDM, in order to improve the spectral properties of the transmitted signal by applying a pulse shaping filter. In Fig. 3(a), the BER curve that corresponds to the matched filter receiver exhibits a behavior that is equivalent to previous work [4], [9]. At low signal-to-noise ratio (SNR) the noise is dominant and the MF performance is close to the theoretical BER, that can be achieved with QPSK transmission without self-interference. However, when the SNR increases and the relative noise power decreases, the self-induced interference remains and as a result the BER deviates from the ideal QPSK curve. Ultimately an error floor is to be expected. How much the MF performance degrades, strongly depends on the choice of the pulse shaping filter. In this simulation root-raised cosine filters with roll-off factors $\alpha=0.1$ (Fig. 3(a)) and $\alpha=0.5$ (Fig. 3(b)) are used. Increasing the roll-off factor increases the SNR gap.

The behavior of the ZF receiver is different. When no noise is present, it can reverse the effect of the self-interference, since $\mathbf{A}^{+}\mathbf{A}=\mathbf{I}$. When AWGN is considered, the data symbols can be adequately reconstructed even in the high SNR region, however a constant SNR shift can be observed, which is due to the the noise enhancement that is well known for this approach. How much the ZF curve deviates from the theoretical performance depends on the properties of $\mathbf{A}^{+}$. Particularly the roll-off factor of the pulse shaping filter has been found to have a strong impact. While there is a signficant deviation for $\alpha=0.5$ in Fig. 3(b), the offset is nearly not present in Fig. 3(a) where $\alpha=0.1$.

Further, the MMSE receiver provides a balance between MF and ZF, yielding the best performance. This is achieved at the cost of higher computational effort, because $\mathbf{A}^{\dagger}$ needs to be computed every time $\sigma_{\mathbf{n}}^2$ changes, while $\mathbf{A}^{+}$ and $\mathbf{A}^{\mathrm{H}}$ are independent of the channel.

C. Multipath Fading Performance

When OFDM and GFDM are used for transmission through Rayleigh multipath fading channels with exponential power delay profile, a cyclic prefix is added to prevent ISI. Both systems differ in the amount of CP that is inserted. While in OFDM every symbol is prefixed, in GFDM one CP is added for every block of M symbols. As a consequence, in Fig. 4(a)



(a) Multipath, $K = 128$, $M = 5$, $\alpha = 0.1$

(b) Multipath, $K = 128$, $M = 5$, $\alpha = 0.5$

Fig. 4. OFDM and GFDM BER performance for uncoded QPSK transmission in Rayleigh multipath channels.

the CP-OFDM curve deviates more from the theoretical QPSK error rate than all three GFDM techniques, when the low SNR region is considered. For high SNR the GFDM performance deviates stronger from the ideal curve, which hints that in time dispersive channels, neither of the receiver methods can efficiently cope with the self-created interference from non-orthogonal subcarriers.

However, the performance among the three GFDM receivers still differs. Considering Fig. 4(b), becomes evident, that the MF performs well at low SNR, while it is being outperformed by the ZF at high SNR. The MMSE provides best performance and converges as to be expected towards the MF curve for low SNR and towards the ZF curve for high SNR.

VI. Conclusions

The research of novel physical layer concepts is a relevant topic, because future wireless communication systems will be facing novel requirements regarding their flexibility, spectral efficiency, energy efficiency, quality-of-service, etc. Among various competing techniques, GFDM is one approach towards addressing these aspects.

In this paper a linear matrix description for the GFDM transmitter has been derived. This now allows to apply standard receiver techniques, i.e. the MF, ZF and MMSE. The BER performance in AWGN channels is studied and insight on how the receiver methods cope with the non-orthogonal subcarriers in GFDM is gained. It is found that in the absence of multipath propagation, the ZF receiver can eliminate the self-created interference, yielding nearly the theoretical BER performance. However this property strongly depends on the pulse shaping filter that is used.

When Rayleigh multipath fading channels are considered, one main gain in the performance of GFDM over OFDM is the reduced amount of CP that is inserted. Further, the ZF receiver removes large portions of the self-created interference in GFDM, however it also exhibits some residual performance degradation in the high SNR region. While the MF receiver yields best results at low SNR, it is outperformed by the ZF receiver at higher SNR and the MMSE receiver performs best. In summary, using sharp pulse shaping filters in GFDM not only yields good spectral properties of the transmitted signal, but also reduces the self-created interference. Then, particularly in a multipath fading environment there is only marginal difference in the BER performance of the different receiver methods. When the self-interference is more severe, the MMSE yields the lowest error rates at the cost of higher complexity. Hence, that receiver method is favorable in an uplink scenario, where computational complexity is not an issue for the receiving base station.

Acknowledgement

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References

[1] Electronic Communications Committee, "CEPT Report 25: Technical Roadmap proposing relevant technical options and scenarios to optimise the Digital Dividend," Tech. Rep., 2008.

[2] EXALTED FP7 258512, "D2.1: Description of baseline reference systems, scenarios, technical requirements & evaluation methodology," Tech. Rep., 2011.

[3] G. Fettweis, M. Krondorf, and S. Bittner, "GFDM - Generalized Frequency Division Multiplexing," in *Proc. IEEE 69th Vehicular Technology Conf. VTC Spring 2009*.

[4] N. Michailow, M. Lentmaier, P. Rost, and G. Fettweis, "Integration of a GFDM Secondary System in an OFDM Primary System," in *Future Network & Mobile Summit 2011*.

[5] T. Ihalainen, A. Viholainen, and M. Renfors, "On spectrally efficient multiplexing in cognitive radio systems," in *Proc. 3rd Int. Symp. Wireless Pervasive Computing ISWPC 2008*.

[6] T. Ihalainen, A. Viholainen, T. H. Stitz, and M. Renfors, "Spectrum monitoring scheme for filter bank based cognitive radios," in *Proc. Future Network & Mobile Summit 2010*.

[7] Anil K. Jain, *Fundamentals of Digital Image Processing*, chapter 2.8, Prentice Hall, 1988.

[8] Steven M. Kay, *Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory (v. 1)*, pp. 389–392, Prentice Hall, 1993.

[9] R. Datta, N. Michailow, M. Lentmaier, and G. Fettweis, "GFDM Interference Cancellation for Flexible Cognitive Radio PHY Design," in *Submitted to 76th IEEE Vehicular Technology Conference, VTC Fall 2012*.