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Impact of Non-orthogonal Training on Performance of Downlink Base Station Cooperative Transmission

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Abstract—Base station (BS) cooperative transmission is a promising technique to improve spectral efficiency of cellular systems, using which the channels become asymmetric in average gain. In this paper, we study the impact of the asymmetric channel gains on the performance of coherent cooperative transmission systems, when minimum mean square error (MMSE) and least square (LS) channel estimators are applied for jointly estimating the channel state information (CSI) under non-orthogonal training. We first derive an upper bound of rate loss caused by both channel estimation errors and CSI delay. We then analyze the mean square errors of the MMSE and LS estimators under both orthogonal and non-orthogonal training, which finally reveals the impact of different kinds of training on the precoding performance. It is shown that non-orthogonal training for the users in different cells leads to minor performance degradation for the MMSE channel estimator assisted downlink precoding. The performance degradation induced by channel estimation errors is almost independent of the user’s location. By contrast, the performance loss caused by CSI delay is more severe for users located at the cell center than that for users located at the cell edge. Our analysis is verified via simulation results.

Index Terms—Base station cooperative transmission, channel estimation, channel asymmetry, non-orthogonal training.

I. INTRODUCTION

Coherent base station (BS) cooperative transmission, which is a popular form of coordinated multi-point transmission (CoMP), can provide high spectral efficiency for cellular systems when both data and channel state information (CSI) are available at a central unit (CU) [1–4].

To facilitate downlink (DL) precoding in such CoMP systems, both local and cross channels, i.e., the channels between the cooperative BSs and mobile stations (MSs) which are in the same cell and in different cells, need to be estimated. In time division duplexing (TDD) systems, the required CSI can be estimated through uplink (UL) training by exploiting channel reciprocity. When training signals for all MSs are orthogonal, conventional estimators such as that proposed in [5] can be directly applied with good performance. However, in current cellular systems, such as those complying with the Long Term Evolution (LTE) standard [6], the training sequences of MSs in the same cell are orthogonal but those for the MSs in different cells are not. Moreover, the independent UL frequency scheduling among cells may lead to partially-overlapped training sequences for the MSs in different cells, which results in high cross-correlation of training sequences. To improve channel estimation performance, we can simply apply orthogonal training sequences for the MSs in different cells. However, this not only introduces large overhead which occupies expensive UL resources [7], but also demands inter-cell signalling and protocol to coordinate the training sequences and uplink scheduling among multiple cells [8]. Such a burden will become more noticeable when the cooperative clusters are formed dynamically [4]. From the viewpoint of system compatibility, scalability and complexity, the training sequences of MSs in different cells are highly preferred to be non-orthogonal. Nonetheless, it is not known whether non-orthogonal training can provide acceptable performance or not. In fact, even if we employ orthogonal training, the cross channels which experience severe attenuation seem to be hard to estimate due to the limited transmit power of MS. On the other hand, when the channel is time-varying, the CSI employed by CoMP transmission will become outdated due to the delay between uplink channel estimation and downlink data transmission. Such a channel distortion will inevitably lead to the deterioration of DL transmission performance. Therefore, the extent to which the channel distortion degrades the performance of DL CoMP precoding is of great interest.
In this paper, we study the impact of non-orthogonal training and CSI delay on the performance of DL CoMP transmission. A CoMP channel is an aggregation of multiple single-cell channels and is inherently asymmetric, i.e., the average channel gains from different BSs to one MS and those from MSs in different cells to one BS are different. Such an asymmetric channel feature is fundamental in CoMP systems, since the difference of the average channel gains cannot be compensated by a power control mechanism. Specifically, if the MSs in different cells compensate their average channel gain differences towards one BS by power control, their receive signal energy differences towards other BSs will increase. This is analogous to the interference asynchrony feature, which cannot be dealt with by time-advanced techniques [3].

We consider coherent CoMP multi-user multiple antenna orthogonal frequency-division multiplexing (OFDM) systems. To show the connection of the performance between DL transmission and channel distortion caused by channel estimation and CSI delay, we first derive a lower bound of the achievable per-user rate of CoMP system using zero forcing beamforming (ZFBF). The impact of channel estimation errors on DL CoMP transmission has been studied in [7, 8]. In [7], the authors considered DL channel estimation for feedback, but they did not address the impact of the channel asymmetry and non-orthogonal training. In [8], the authors assume that the cooperative BSs only serve one user on each subcarrier, which is different from our analysis for the multi-user case. The impact of CSI delay on DL CoMP systems has been investigated in [9], in the context of non-coherent CoMP with no data sharing among the BSs.

We then analyze the performance of minimum mean square error (MMSE) and least square (LS) channel estimators, which are widely applied and have been studied extensively in single-cell systems (see [10] and references therein). In multi-cell systems, the authors in [11] showed that when the MSs in different cells use identical sequences for uplink training, the estimation performance of the desired channels will be severely degraded when traditional channel estimator is used, where only the local channel is estimated and the received signals of the cross channels are treated as interference. Considering the channel asymmetry and by jointly estimating the local and cross channels, we will show that the MMSE estimator is robust to the non-orthogonality of training sequences in different cells, whereas the LS estimator is quite sensitive to them. Moreover, our analysis shows that although the cross channels are weak, their estimation errors induce even less rate loss than that of the local channel. In fact, we find that the average per-user rate loss caused by channel estimation errors is nearly independent of the MS’s location when the MMSE estimator is applied under inter-cell non-orthogonal training. By contrast, the performance loss caused by CSI delay is more severe for MSs located at cell center than that for MSs located at cell edge.

The rest of the paper is organized as follows. Section II introduces the system models. Section III analyzes the impact of non-orthogonal training and CSI delay on the performance of DL precoding. Simulation and numerical results are provided in Section IV, and conclusions are given in Section V.

Notations: \((X)^T\), \((X)^+\) and \((X)^H\) denote the transpose, the conjugate, and the conjugate transpose of \(X\). \(\text{vec}(X)\) is the column vector obtained by stacking the columns of \(X\), \(X(i, i)\) and \(X(:, i)\) represent the \((i, i)\)th element and the \(i\)th column of matrix \(X\). \(\|\cdot\|\) represents the two-norm, \(\otimes\) is the Kronecker product, and \(\text{diag}\{\cdot\}\) is a diagonal matrix. \(\mathbb{E}\{\cdot\}\) is the expectation operator. \(I_N\) denotes the identity matrix of size \(N\), and \(0\) is the matrix of zeros.

II. SYSTEM MODELS

Consider a coherent CoMP-OFDM system, where \(B\) BSs each equipped with \(N_t\) antennas cooperatively serve \(M\) single-antenna MSs using multi-cell ZFBF.

We consider TDD systems, where the CSI is estimated through UL training by exploiting channel reciprocity. Time-varying block fading channel is considered, where the channel remains constant for an OFDM symbol duration and changes from symbol to symbol. During the uplink training period, all MSs send training sequences and each BS estimates the CSI between it and all MSs. Then the CU collects the estimated CSI from each BS and computes the precoders based on the estimated CSI. Finally, the CU sends the DL data and precoders to each BS and all BSs serve MSs cooperatively using multi-cell precoding.

We denote the delay between uplink channel estimation and downlink data transmission as \(D\) symbols. Denote the number of subcarriers as \(K\) and the number of resolvable paths of channel impulse response (CIR) as \(L\). To simplify the notations and for the sake of clarity, we assume the number of resolvable paths to be 1, with no loss of generality. Under this assumption, the channel frequency responses (CFR) over all sub-carriers are identical. We will verify in Section IV that the following analysis based on the flat-fading assumption also holds for frequency-selective fading channels. In the following, we will omit the index of subcarrier for brevity.

Denote the composite channel vector between BS\(_n\) and MS\(_m\) at the \(n\)th discrete-time instant as \(h_{m, b}[n] = \alpha_{m, b} g_{m, b}[n]\), where \(\alpha_{m, b}\) is the large scale fading coefficient including both path loss and shadowing, \(g_{m, b}[n] \in \mathbb{C}^{N_t \times 1}\) is the small scale fading channel vector, whose entries are assumed to be independent and identically distributed (i.i.d.) unit variance complex Gaussian variables. The first order Gauss-Markov model is considered to characterize the time-varying property of small-scale
fading channels, by which the current and delayed small scale fading channel vectors are related as
\[ g_{m,b}[n] = \rho g_{m,b}[n - D] + \sqrt{1 - \rho^2} e_{m,b}[n], \]
where \( e_{m,b}[n] \in \mathbb{C}^{N_r \times 1} \) is the channel error vector whose entries are i.i.d. unit variance complex Gaussian variables, \( e_{m,b}[n] \) is uncorrelated with \( g_{m,b}[n - D] \) and \( g_{m,b}[n] \), and \( \rho \in [0, 1] \) is the fading correlation coefficient that characterizes the extent of time variation.

The DL global channel of MS\(_m\) at time \( n \) is the aggregation of \( B \) single-cell channel vectors, which is
\[ h_m[n] = [\alpha_{m,1} g_{m,1}[n], \ldots, \alpha_{m,B} g_{m,B}[n]]^T \]
\[ = \rho [\alpha_{m,1} g_{m,1}[n - D], \ldots, \alpha_{m,B} g_{m,B}[n - D]]^T \]
\[ + \sqrt{1 - \rho^2} [\alpha_{m,1} e_{m,1}[n], \ldots, \alpha_{m,B} e_{m,B}[n]]^T, \]
where \( h_m[n-D] \triangleq [\alpha_{m,1} g_{m,1}[n-D], \ldots, \alpha_{m,B} g_{m,B}[n-D]]^T \) is the DL global channel of MS\(_m\) at time \( (n - D) \), and \( e_{m}[n] \triangleq [\alpha_{m,1} e_{m,1}[n], \ldots, \alpha_{m,B} e_{m,B}[n]]^T \) is the channel error vector of the DL global channel caused by the delay of CSI.

### A. Uplink Training

During the uplink training period, each BS needs to estimate the CSI between itself and all MSs. We consider that all the MSs send training sequences at the \( (n - D) \)th discrete-time instant. Assume that the transmit power is the same for all the MSs, and it is denoted as \( p_0 \). Denote the frequency domain training sequence of MS\(_m\) as \( s_m \in \mathbb{C}^{K \times 1} \). Then, the received signal matrix at BS\(_b\) during the uplink training phase can be expressed as
\[ Y_b[n-D] = \sqrt{p_0} H_b[n-D] S^T + N[n-D], \]
where \( Y_b[n-D] = [y_{1,b}[n-D], \ldots, y_{B,b}[n-D]] \in \mathbb{C}^{N_t \times K} \), \( y_{k,b}[n-D] = \sqrt{p_0} s_k[n-D] \) is the received signal vector of BS\(_b\) at the \( k \)th subcarrier, \( H_b[n-D] = [h_{1,b}[n-D], \ldots, h_{M,b}[n-D]] \in \mathbb{C}^{N_t \times M} \) is the channel matrix between BS\(_b\) and all MSs at the \( (n - D) \)th discrete-time instant when channels are estimated, \( S = [s_1, \ldots, s_M] \in \mathbb{C}^{K \times M} \) is the training matrix formed by the training sequences of all MSs, \( N[n-D] \in \mathbb{C}^{N_t \times K} \) is the additive white Gaussian noise (AWGN) matrix, whose elements are random variables with zero mean and covariance \( \sigma_n^2 \).

By vectorizing the received signal in (3) and applying \( \text{vec}(ABC) = (C^T \otimes A)\text{vec}(B) \), we can obtain the vectorization of \( Y_b[n-D] \) as \( \text{vec}(Y_b[n-D]) = \sqrt{p_0} \text{vec}(H_b[n-D]) + \text{vec}(N[n-D]) \), where \( \text{vec}(S) = S \otimes I_{N_t} \). Then \( H_b[n-D] \) can be estimated as
\[ \text{vec}(H_b[n-D]) = \left( S^H S + \mu \frac{\sigma_n^2}{p_0} R_b^{-1} \right)^{-1} \cdot S^H \text{vec}(Y_b[n-D]), \]
where \( R_b = \mathbb{E}\{\text{vec}(H_b[n-D])\text{vec}(H_b[n-D])^H\} \) is the correlation matrix of the vectorized channel matrix \( H_b[n-D] \). We assume that the channels from multiple MSs to BS\(_b\) are uncorrelated, and the separation distance of antennas at each BS are large enough that the channels from multiple antennas of one BS to one MS are spatially uncorrelated. Then we have \( R_b = A_b \otimes I_{N_t} \), where \( A_b = \text{diag}\{\alpha_1^2, \ldots, \alpha_M^2\} \). The estimator in (4) is the MMSE estimator when \( \mu = 1 \) and LS estimator when \( \mu = 0 \).

### B. Downlink Transmission

The CU collects the estimated CSI from each BS to calculate the multi-cell precoders. Denote the estimated downlink channel matrix from all cooperative BSs to all MSs as \( \tilde{H}[n-D] = [\tilde{H}_1^{\text{DL}}[n-D], \ldots, \tilde{H}_M^{\text{DL}}[n-D]] \), then a multicell ZFBF is \( \tilde{V} = \tilde{H}[n-D](\tilde{H}[n-D]^H)^{-1} \). Denote \( d_m[n] \) and \( p_d(m) \) as the data and power allocated for MS\(_m\). For simplicity, we assume that \( \mathbb{E}\{d_m[n]d_m[n]\} = 1 \) and the transmit power is equally allocated to each MS, i.e., \( p_d(m) = p_d \). Then the received signal of MS\(_m\) is
\[ y_m[n] = \sum_{j=1}^M \sqrt{p_d} \text{vec}(H_b[n-D]) v_j[n] d_j[n] + z_m[n], \]
where \( v_j[n] = V[n](::; j)/\|V[n](::, j)\| \) is the precoding vector for MS\(_j\), and \( z_m[n] \) is the AWGN with zero mean and covariance \( \sigma_z^2 \).

For CoMP transmission, the ZFBF precoder should be designed under per-BS power constraint \([1]\), but its performance is hard to analyze. In this paper, we consider a suboptimal but more tractable power constraint, which is the per-user power constraint as in \([3]\). It has been shown that the ZFBF with different power constraints perform closely when the number of users in each cell is large \([12]\).

### III. IMPACT OF DELAYED NON-ORTHOGONAL TRAINING ON COOPERATIVE TRANSMISSION

In this section, we first show the connection between the performance of DL transmission and the distortion of the global composite channel, which is caused by both channel estimation and CSI delay. Then we derive the mean square error (MSE) of the MMSE estimator and LS estimator under both orthogonal and non-orthogonal training, and analyze the impact of the resulting channel estimation errors on the performance of multi-cell ZFBF.

#### A. Relationship between Precoding Performance and Channel Distortion

We analyze the average per-user rate loss of CoMP transmission using ZFBF caused by both channel estimation errors and CSI delay.
The average rate of MS_m achieved by ZFBF with delayed estimation of CSI is obtained as $E\{R_m[n]\}$, where $R_m[n]$ is the data rate of MS_m at the n-th time interval.

Using similar techniques as in [13], we can derive the upper bound of the rate loss of MS_m caused by both channel estimation errors and CSI delay, which yields

$$E\{\Delta R_m[n]\} = E\{R^\text{Ideal}_m[n]\} - E\{R_m[n]\} \leq \log_2 \left( 1 + \frac{p_d}{\sigma^2 E[I_m[n]]} \right) = \Delta R_m^{\text{UB}},$$

where $E\{R^\text{Ideal}_m[n]\}$ is the average rate achieved under perfect CSI, $I_m[n] = \sum_{j=1}^M \sum_{j \neq m} |h_{m,1}^H[n]|^2 |v_j[n]|^2$ is the interference power experienced by MS_m caused by channel estimation errors, and $E[I_m[n]]$ is the estimation error as

$$\hat{h}_{m,n}[n-D] \triangleq h_{m,n}[n-D] - \hat{h}_{m,n}[n-D] = [\hat{h}_{m,1,n}[n-D], \ldots, \hat{h}_{m,B,n}[n-D]]^T,$$

where $\hat{h}_{m,b,n}[n-D] = [\hat{h}_{m,b,n,1}[n-D], \ldots, \hat{h}_{m,b,n,M}[n-D]]^T$ is the estimation error vector of the composite channel between MS_m and BS_b. Denote the MSE of the channel estimate between MS_m and the antenna of BS_b as $\varepsilon_{m,b,a}^2 = \mathbb{E}\{[\hat{h}_{m,b,n,a}[n-D]]^2\}$. Since the MSEs of the CSI estimator between different antennas of BS_b and MS_m are equal, we define $\varepsilon_{m,b,a}^2 = \varepsilon_{m,b,1}^2 = \cdots = \varepsilon_{m,b,N_t}^2 = \varepsilon_{m,b}^2$. When the MMSE estimator is used, the channel estimation errors are independent of the channel estimates. Since the precoders are functions of the channel estimates, the channel estimation errors $\hat{h}_{m,n}[n-D]$ and the precoders $v_j[n], j = 1, \ldots, K$, are mutually independent. Then, the average interference power can be derived as follows,

$$E[I_m[n]] = \mathbb{E}\left\{ \sum_{j=1}^M \sum_{j \neq m} |h_{m,j}^H[n]|^2 |v_j[n]|^2 \right\}$$

$$(a) = \sum_{j=1}^M \sum_{j \neq m} \mathbb{E}\left\{ |\rho h_{m,j}^H[n-D]|^2 |v_j[n]|^2 \right\} + \sqrt{1 - \rho^2} E[I_{m, j}[n]]$$

$$(b) = \sum_{b=1}^B \sum_{j=1}^M \mathbb{E}\left\{ |\rho h_{m,b,j}^H[n-D]|^2 |v_{j,b}[n]|^2 \right\} + \sqrt{1 - \rho^2} E[I_{m, b,j}[n]]$$

$$(c) = \left( \sum_{b=1}^B [\rho^2 \varepsilon_{m,b}^2 + (1 - \rho^2) \alpha_{m,b}^2] \right) \cdot \sum_{j=1}^M \mathbb{E}\left\{ |v_{j,b}[n]|^2 \right\}$$

$$(d) = \rho^2 (M-1) \sum_{b=1}^B \varepsilon_{m,b}^2 + (1 - \rho^2) (M-1) \sum_{b=1}^B \alpha_{m,b}^2,$$

where $(a)$ follows because $h_{m,n}[n] = \rho h_{m,n}[n-D] + \sqrt{1 - \rho^2} e_{m,n}[n]$, $e_{m,n}[n] = \rho h_{m,n}[n-D] + \rho h_{m,n}[n-D]$ and $h_{m,n}[n-D]$ is orthogonal to $v_j[n]$ due to ZFBF, $(b)$ comes by the definitions of $e_{m,n}[n]$ and $\hat{h}_{m,n}[n-D]$ in (2) and (7), $(c)$ is obtained by averaging over $h_{m,n}[n-D]$ and $e_{m,b,n}[n]$, in which the statistical property of mutual independence among channel estimation errors $h_{m,n}[n-D]$, the channel error caused by CSI delay $e_{m,n}[n]$, and the precoder vectors $v_j[n], j = 1, \ldots, M$, is used, and $(d)$ comes from the assumption of per-user power constraint, which leads to $|v_{j,b}[n]|^2 \leq \sum_{i=1}^B |v_{j,i}[n]|^2 = 1$.

Then the upper bound of the rate loss is

$$\Delta R_m^{\text{UB}} = \log_2 \left[ 1 + \rho^2 (M-1) \frac{p_d}{\sigma^2} \sum_{b=1}^B \varepsilon_{m,b}^2 \right. + (M-1)(1 - \rho^2) \frac{p_d}{\sigma^2} \sum_{b=1}^B \alpha_{m,b}^2,$$

where the term $\varepsilon_{m,\text{CE}}^2$ reflects the impact of channel estimation errors, and the term $\varepsilon_{m,\text{Delay}}^2$ indicates the performance degradation caused by CSI delay. Then a lower bound of the average rate achieved by MS_m is $E\{R^\text{LB}_m[n]\} = E\{R^\text{Ideal}_m[n]\} - \Delta R_m^{\text{UB}}$.

It is worth to note that the analysis in [13] is for single-cell systems and is based on the assumption of i.i.d. channel. By contrast, we analyze the rate loss for CoMP transmission. Since the global composite channel is not i.i.d. except for the case when the MSs are located at cell edge, and the cell-edge MSs perform the worst among all the MSs, (9) can serve as a pessimistic rate loss upper bound. Nonetheless, this does not affect our latter results. **Remark 1:** The rate loss caused by CSI delay depends on the sum of average channel gains of the local and cross channels. When a MS moves from the cell center to the cell edge, the path loss of its local channel increase exponentially. Though the path losses of its cross channels reduce exponentially, the sum of average channel gains of a cell-center MS exceeds that of a cell-edge MS. This implies that the impact of CSI delay on CoMP transmission is larger for the MSs located at the cell center than for the MSs located at cell edge.

It is shown from (9) that the impact of channel estimation on the upper bound of rate loss depends on the sum of MSEs of the composite local and cross channels, which is independent of the CSI delay. In the following, we will analyze the MSE of the composite CSI estimates under both orthogonal and non-orthogonal training.

**B. Impact of Non-orthogonal Training on DL CoMP Transmission**

Here, we strive to derive an explicit and unified expression of the MSE for both kinds of training. Then, we
can understand how non-orthogonal training performs in CoMP system by comparing with orthogonal training.

1) MSEs of the MMSE and LS estimators: The covariance matrix of channel estimation error vector, vec\( (\mathbf{H}_b[n-D]) \) = vec\( (\mathbf{H}_b[n-D] - \mathbf{H}_b[n-D]) \), can be derived from (4) as
\[
C_b \triangleq \mathbb{E}\{\text{vec}(\mathbf{H}_b[n-D])\text{vec}(\mathbf{H}_b[n-D])^H\} = \left( \mu \mathbf{R}_b + \frac{p_u}{\sigma_n^2} \mathbf{S}^H \mathbf{S} \right)^{-1} = \left[ \mu \mathbf{A}_b \otimes \mathbf{I}_{N_t} + \frac{p_u}{\sigma_n^2} (\mathbf{S} \otimes \mathbf{I}_{N_t})^H (\mathbf{S} \otimes \mathbf{I}_{N_t}) \right]^{-1} = \left[ \mu \mathbf{A}_b^* \otimes \mathbf{I}_{N_t} + \frac{p_u}{\sigma_n^2} (\mathbf{S}^H \mathbf{S}) \otimes \mathbf{I}_{N_t} \right]^{-1} = \left( \mu \mathbf{A}_b^* + \frac{p_u}{\sigma_n^2} \mathbf{S}^H \mathbf{S} \right)^{-1} \otimes \mathbf{I}_{N_t}.
\]
Then, the MSE of the estimated CSI between MS\(_m\) and the \(a\)th antenna of BS\(_b\), i.e., \(\varepsilon_{m,b,a}\), is the \((m-1)N_t + a\)th diagonal element of \(C_b\). Define \(B_{ba} \triangleq (\mu \mathbf{A}_b^* + \frac{p_u}{\sigma_n^2} \mathbf{S}^H \mathbf{S})^{-1}\). The MSE of the estimated channel coefficient between any antenna of BS\(_b\) and MS\(_m\) is the same, which is the \((m,m)\)th element of \(B_{ba}\), i.e., \(\varepsilon_{m,b,1} = \ldots = \varepsilon_{m,b,N_t} = \varepsilon_{m,b} = B_{ba}(m,m)\). Due to the matrix inverse operation in \(B_{ba}\), it is nontrivial to derive an explicit general expression of \(\varepsilon_{m,b}\). For mathematical tractability, we consider a simple but fundamental scenario, where \(B\) BSs cooperatively serve two MSs.

Then, it is not hard to respectively derive the MSES of the MMSE and LS estimators, as follows,
\[
\varepsilon_{m,b}^{\text{MMSE}} = \eta_{m,b}^{\text{MMSE}} \frac{\sigma_n^2}{p_u} \frac{1}{K} \frac{1}{1 - \beta_b \lambda}, \quad m = 1, 2, \tag{11}
\]
\[
\varepsilon_{m,b}^{\text{LS}} = \frac{\sigma_n^2}{p_u} \frac{1}{K} \frac{1}{1 - \lambda}, \quad m = 1, 2, \tag{12}
\]
where \(\eta_{m,b}^{\text{MMSE}} = 1/(1 + \frac{\sigma_n^2}{\alpha_s^2 p_u K}), \beta_b = 1/\prod_{j=1}^2 (1 + \frac{\sigma_n^2}{\alpha_s^2 p_u K})\) and \(\lambda = \frac{|s[k][s][s]^H|}{|s[k][s][s]^H|^2} = \frac{|s[k][s][s]^H|^2}{\lambda^2}\) represents the cross-correlation between the training sequences of MS\(_1\) and MS\(_2\).

For the MMSE estimator, it is shown that the MSE of the composite CSI depends on \(\alpha_{m,b}^2\), which is the large scale fading gain of H\(_{m,b}\). If MS\(_m\) is in the same cell with BS\(_c\), then H\(_{m,c}\) is the local composite channel for MS\(_m\) and H\(_{m,b}\) for \(b \neq c\) are cross composite channels. Since in general \(\alpha_{m,c}^2 \geq \alpha_{m,b}^2\), \(b \neq c\), we can observe that the MSES of the weak cross channels are even less than that of the strong local channel. For the LS estimator, the MSES of local and cross composite channels are identical.

At the first glance, the results for both the LS and MMSE estimators may appear inconsistent with intuition, where the MSES of the estimates of the cross channels should exceed that of the local channel. However, this is true only for estimating small scale fading channels with unit average energy. To see this, we normalize the MSE of H\(_{m,b}\) by \(\alpha_{m,b}^2\) to obtain a normalized MSE (NMSE), which is actually the MSE for estimating the small scale fading channel \(\mathbf{g}_{m,b}\). The NMSE for LS and MMSE estimators can be expressed as
\[
\text{NMSE}_{m,b}^{\text{MMSE}} = \frac{\varepsilon_{m,b}^{2\text{MMSE}}}{\alpha_{m,b}^2} = \frac{1}{1 + \frac{1}{\gamma_{m,b} K}} \frac{1}{1 - \beta_b \lambda}, \quad m = 1, 2, \tag{13}
\]
where \(\gamma_{m,b} = \frac{\sigma_s^2}{\alpha_{m,b}^2 p_u K}\) is in fact the average UL receive SNR when estimating the small scale fading channel vector \(\mathbf{g}_{m,b}\). Thus, the NMSE for estimating a small scale fading channel with low SNR exceeds that with high SNR.

2) Impact of the Training Sequences: When the training sequences for the two MSs are orthogonal, we have \(\lambda = 0\). Then both MMSE and LS estimators achieve their minimal MSES, which are \(\varepsilon_{m,b}^{2\text{MMSE}} = \frac{1}{1 + \frac{1}{\gamma_{m,b} K}} \frac{\sigma_s^2}{p_u K}\) and \(\varepsilon_{m,b}^{2\text{LS}} = \frac{\alpha_{m,b}^2}{\gamma_{m,b} K}\), respectively. In typical cellular OFDM systems, the receive SNR of the link from MS\(_m\) to BS\(_b\) \(\gamma_{m,b}\) is larger than 0 dB and the value of \(1/K\) is usually small. Consequently, \(\varepsilon_{m,b}^{2\text{MMSE}} \approx \varepsilon_{m,b}^{2\text{LS}}\).

When the training sequences of MSs in different cells are not orthogonal, we have \(\lambda \neq 0\). For the LS estimator, if \(\lambda\) is close to 1, the value of \(\varepsilon_{m,b}^{2\text{LS}}\) will be extremely large, which means that the estimation performance will severely degrade. Therefore, the LS estimator is quite sensitive to the non-orthogonality of training sequences no matter where the MSs are located.

For the MMSE estimator, \(\lambda\) is weighted by \(\beta_b\) in the expression of MSE, where \(\beta_b < 1\) and its value is related to the large scale fading gains between the two MSs and the BS\(_b\). This indicates that the MSE of MMSE estimator depends on the location of the MSs.

If the two MSs are within the same cell with BS\(_b\), both \(\alpha_{1,b}^2\) and \(\alpha_{2,b}^2\) are large, then \(\beta_b \approx 1\) and \(\eta_{m,b}^{\text{MMSE}} \approx 1\). As a result, \(\varepsilon_{m,b}^{2\text{LS}} \approx \varepsilon_{m,b}^{2\text{MMSE}}\). This implies that if the MSs in the same cell use non-orthogonal training sequences, MMSE estimator is also quite sensitive to the non-orthogonality of training sequences.

If the two MSs are in different cells, on the other hand, no matter where both MSs are located, \(\beta_b\) will be small hence \(\varepsilon_{m,b}^{2\text{MMSE}} \approx \varepsilon_{m,b}^{2\text{LS}}\). This is because if both MSs are at the cell edge, their average channel energies are small thereby both values of \(1/\gamma_{m,b} K\), \(m = 1, 2\), are large. If at least one MS is at the cell center, thanks to the severe energy attenuation of the channels from this
MS to its non-serving BSs, at least one of the $\frac{1}{\gamma_m} \frac{1}{K}$ is large. This indicates that the MMSE estimator is robust to the inter-cell non-orthogonal training.

Substituting the expression of $\frac{\Delta R_{m,k}}{\gamma_m}$ in (11) into (9), the upper bound of the rate loss becomes

$$\Delta R_{m}^{UB} = \log_2 \left[ 1 + (M - 1) \frac{\rho_d}{\sigma_z^2} \sum_{b=1}^{B} (1 - \rho^2) \alpha_{m,b}^{2} + \rho^2 (M - 1) \frac{\rho_d}{\sigma_z^2} \sum_{b=1}^{B} \frac{1}{1 + \frac{1}{\gamma_m,k} \frac{1}{K} \frac{1}{p_u} \frac{1}{1 - \beta_k \lambda}} \right]$$

$$\approx \log_2 \left[ 1 + (1 - \rho^2) (M - 1) \frac{\rho_d}{\sigma_z^2} \sum_{b=1}^{B} \alpha_{m,b}^{2} \right]$$

$$+ \rho^2 B (M - 1) \frac{\rho_d}{\sigma_z^2} \frac{\sigma_n^2}{p_u} \frac{1}{K},$$ (15)

where the approximation of (15) is obtained when the values of both $\frac{1}{\gamma_m,k}$ and $\beta_k \lambda$ approach 0. When the number of subcarriers $K$ is large, the value of $\frac{1}{\gamma_m,k} \frac{1}{K}$ will be close to 0. Regarding to $\beta_k \lambda$, as we have analyzed, its value approaches 0 when the training signals are orthogonal, or when the training signals are non-orthogonal but the MSs are in different cells.

Remark 2: Since the MSEs of the weak cross composite channels are usually smaller than that of the local composite channel, it is shown from (14) that the channel estimation errors of cross channels have less contribution to the rate loss than that of local channel. Moreover, if the training signals for MSs within each cell are orthogonal whereas those for the MSs in different cells are non-orthogonal, the average interference part $I_{m,c}^{CEE}$ caused by channel estimation errors in (15) will be a constant no matter where the MSs are located.

IV. SIMULATION AND NUMERICAL RESULTS

In this section, we verify our previous analysis via simulations. We consider a CoMP system of $B$ BSs, each with four omnidirectional antennas. Considering that the DL transmit power is usually larger than the UL transmit power, we set $p_d = p_u + 5$ dB. $K = 128$. Recall that in the derivations we assume flat-fading channels mainly for simplicity of notations and clarity of presentation. Similar derivations can likewise be made for frequency-selective channels, which are appropriate for new generation of broadband cellular systems. Hence, the channels used in the simulations are frequency-selective, the CIR of which is a tapped-delay line with independent Rayleigh fading coefficients and an exponential power delay profile with attenuation factor $1.4$, $L = 20$. These parameters are the same as those in SCME channels [6]. Though a permissive time-varying channel model is employed in the previous derivation of rate loss for mathematical tractability, in simulations we consider a more realistic channel mode, Jakes’ Model, which is a widely accepted channel model by various standardization organizations. Its temporal correlation function is $R_e(t) = J_0(2\pi f_d DT_s)$, $J_0(\cdot)$ is the zeroth order Bessel function of the first kind, $f_d$ is the Doppler spread and $T_s$ is the symbol duration. The carrier frequency is 2 GHz, the OFDM symbol duration is 1 ms and the speed of the MSs is 3 km/h.

The training sequences are constructed from the Constant Amplitude Zero Autocorrelation Code (CAZAC) as $t(k) = e^{-j2\pi n_k/N_{ZC}}$, $k = 0, \ldots, K - 1$ [6], where $N_{ZC} = 127$, $n_k = \text{mod}(k, N_{ZC})$. The training sequences for MSs may be orthogonal or non-orthogonal. For orthogonal training, the training sequences for all MSs are the cyclic shift version of one CAZAC. For non-orthogonal training, the CAZACs used by MSs are of different $c$’s.

A. Impact of the Training Sequence Orthogonality on the NMSEs of Different Estimators

To observe the robustness of different estimators to the non-orthogonal training, we consider a cooperative cluster of two BSs. Though our analytical results are derived for the case of two MSs, we consider four MSs in the simulation to demonstrate that the analysis is also valid for more general cases. We assume that the four MSs are symmetrically located as shown in Fig. 1. Then, all MSs achieve the same performance and we only need to show the performance of one MS.

To be consistent with the traditional understanding, we show the NMS of the composite CIR, which is defined in (13) and is actually the MSE for estimating the small scale fading channel. In Fig. 2, the NMSEs of the MMSE and LS estimators under orthogonal and non-orthogonal training versus the UL local SNR for both the local and cross channels are shown. The UL local SNR is the UL average receive SNR of the local link, which is $\gamma_{m,c,m} = \frac{\alpha_{m,c,m} p_u}{\sigma_n^2}$ for $MS_m$, where $MS_m$ and $BS_c$ are in the same cell. As
we have analyzed, only when both the training sequences for MSs within a cell and for MSs in different cells are orthogonal, the LS estimator achieves good performance, which is only slightly inferior to that of the MMSE estimator. On the other hand, when the MMSE estimator is used and when the training sequences for the MSs within a cell are orthogonal but those for MSs in different cells are non-orthogonal, the performance of both cross and local channel degrades slightly. By contrast, when the training sequences for the MSs within a cell and for the MSs in different cells are both non-orthogonal, the performance of the local channel degrades severely especially when the MSs are located at cell center, whereas the performance of the cross channels only degrades a little since their channel estimation is noise-limited. This agrees well with our previous analysis, i.e., the MMSE estimator is robust to the non-orthogonality of the training signals for the MSs in different cells owing to the channel asymmetry.

**B. Impact of Training Sequence Orthogonality and CSI delay on the DL Average Per-User Rate**

To observe the impact of different estimates and CSI delay on the performance of CoMP transmission, we first consider a case where the settings are the same as that in the previous subsection.

**Fig. 2.** NMSEs of MMSE and LS estimators for both local and cross channels versus the UL average receive SNR of the local link, $\gamma_{m,c_m} = \sigma^2_{m,c_m} P_M / \sigma^2_n, m = 1, \ldots, 4$. When the MSs are located at the exact cell edge, $\gamma_{m,c_m} = 5$ dB. For non-orthogonal training, the ZACACs used by MSs in two cells are constructed from $c_2 = 1$ and $c_2 = 7$ respectively, which leads to moderate cross correlation. The NMSEs are obtained by averaging over 1000 realizations of small scale fading channels. When $\gamma_{m,c_m}$ increases, the large scale fading gains of the cross channels decrease, which leads to large NMSE of the cross channels. For the local channels, the NMSE of the estimators are overlapped under orthogonal training. The legend 'Intra-O & Inter-O' means that the training sequences both for MSs within a cell and for MSs in different cells are orthogonal, and vise versa. The legend 'Intra-O & Inter-NO' means that the training sequences for MSs within a cell are orthogonal while those for MSs in different cells are non-orthogonal, and vise versa.

**Fig. 3.** The average rate achieved by each MS obtained via simulation and its lower bound calculated from (15) when the MMSE estimator and orthogonal training are considered for all the four MSs. The per-MS rate of CoMP transmission with perfect CSI is also provided for reference.

Figure 3 shows the tightness of the per-user rate lower bound derived in Section III-A. The simulated per-user rate under perfect ZFBF and that with MMSE estimator under orthogonal training and different delays are plotted, together with the lower bound of the achievable rate derived from (15). It shows that the derived lower bound based on i.i.d. channel assumption is indeed a lower bound and is not very tight, which is due to the overestimated interference induced by the imperfect CSI for the CoMP channel. When no delay is introduced, i.e., $D = 0$, the performance degradation caused by only channel estimation errors are almost independent of the location of MS. By contrast, the per-user average rate loss of MS caused by the CSI delay is more severe for MS with high local SNR than that for MS with low local SNR. When the delay is large, the per-user average rate will arrive at a ceiling at high local SNR. These results agree well with our previous analysis.

The simulation results for the DL average rate of each MS with different estimators are shown in Fig. 4, where the average rate under non-CoMP transmission is also presented as a reference. To highlight the impact of different estimators and different training sequences, no CSI delay is considered. It is shown that under orthogonal training, the per-user rates with different estimators are close. Moreover, for both the LS and MMSE estimators, the performance gaps from the perfect CSI-based ZFBF are almost independent of the location of MS. When the training sequences of MSs in different cells are not orthogonal, the performance degradation is minor when using the MMSE estimator. On the contrary, the performance degradation is severe when the LS estimator is used, and the rate is even lower than the non-CoMP transmission. We can also observe that the performance of the CoMP transmission reduces more by
imperfect CSI than that of non-CoMP transmission. This is no surprise because the rate loss of the CoMP transmission is induced by the sum of the estimation errors of multiple single-cell channel components, i.e., the channel estimation errors are larger in the CoMP system, given the same single-cell training resources with the non-CoMP system. Furthermore, the non-CoMP transmission is interference-limited, hence imperfect CSI leads to relatively less performance loss.

Finally, we simulate a more realistic setting, where $B = 3$ and each cell contains two MSs. The 3 BSs cooperatively serve all the 6 MSs without scheduling. With better scheduling algorithms such as those considering sum rate maximizing or fairness among MSs, both CoMP and Non-CoMP systems will perform better. All the MSs are randomly distributed in a 'cell-edge region'. In particular, the ratio of the large scale fading gain of the local channel to the sum of those of the cross channels of the MSs in this region, e.g., $\alpha_{m,c_m}^2 / \sum_{b=1, b\neq c_m}^B \alpha_{m,b}^2$ for MS$_m$, is less than a pre-defined value. The CSI delay is set as 4 symbols. In Fig. 5, we present the average cell-edge region throughput, which is the average sum rate of two MSs in the cell-edge region of each cell. We can see that the same conclusion can be drawn as before no matter how small the 'cell-edge region’ is, where the channels are not very asymmetric. When the MSs are randomly located in the whole cell region, CoMP only outperforms Non-CoMP slightly. This result seems to be pessimistic. However, it is worth to note that only channel estimation is considered here. In practical systems, various channel prediction methods can be applied to alleviate the impact of CSI delay and enhance the performance of CoMP, which is out of the scope of this paper. We also simulate the throughputs when regularized ZFBF [14] is used for the DL CoMP transmission, but the results overlapped with those of ZFBF, which are not shown to make the figures clearer. This is because the regularized ZFBF outperforms ZFBF at low SNR, but in CoMP systems, the SNR is high. Furthermore, the analysis based on the simulated channels is successfully verified using recently measured CoMP channels with three BS sites [15].

V. CONCLUSION

In this paper, we analyzed the performance of LS and MMSE channel estimators under non-orthogonal training and the impact of both channel estimation errors and CSI delay on the CoMP transmission. Our analysis showed that the LS estimator is quite sensitive to the non-orthogonal training, but the MMSE estimator is robust to the inter-cell non-orthogonal training owing to the unique feature of CoMP channels. When using the MMSE channel estimator, the estimation errors of the weak cross channels contribute even less to the average per-user rate loss for multi-cell precoding than those of local channels. The performance loss caused by channel estimation errors almost does not depend on the MS's location, whereas the impact of CSI delay on CoMP transmission is larger for the MSs located at the cell center than for the MSs located at the cell edge. The simulation results show that
CoMP transmission performs fairly well without inter-cell orthogonal training. This means that the cumbersome inter-cell signalling for coordinating the training resources may not be necessary.

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