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Extremal control of Wiener model processes

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Abstract

In this report we describe different approaches to extremal control. Especially, processes of Wiener type are considered. These models consists of a linear part followed by a nonlinearity. In our case we will consider nonlinearities having one extremum point. The purpose of the control is to keep the output of the process as close as possible to the extremum point. Different control schemes are discussed and analyzed. The main problem in the control of this kind of Wiener model processes is the non-uniqueness of the inverse of the nonlinearity. This causes problems, for instance, in the estimation of the states of the process and the identification in the adaptive case.

1. Introduction

There are many application where it is of interest to position the process output at an optimum or extremum point. A typical situation is combustion engines where the emission and efficiency depend on the inputs to the motor such as fuel and air/fuel ratio. Other examples are control of grinding processes, water turbines, and wind mills. It is therefore of interest to study processes having an extremum value in the output and to be able to operate the system as close as possible to the extremum point.

This problem has been studied over a long period of time and there are solutions to some of the problems occurring in extremum control. See, for instance, Draper and Li (1951), Jacobs and Langdon (1970), Keviczsky and Haber (1974), Keviczsky *et al.* (1979), Sternby (1980a), Sternby (1980b), Dumont and Åström (1988), Wellstead and Scotson (1990), Scotson and Wellstead (1990), Wittenmark (1993), Allison (1994), Wittenmark and Urquhart (1995), Navarro and Zarrop (1996), Krstić and Wang (1997), Krstić and Wang (2000), and Gäfvert *et al.* (2000). In many of the earlier references the static optimization problem has mainly been discussed and it is only recently in the work by Krstić and coworkers that the problem with stability in the case of dynamics has been solved for some cases.

The problem of extremum control can be approached in several ways. Among the first approaches was the introduction of perturbation signals. A perturbation signal is then used to get information about the local gradient
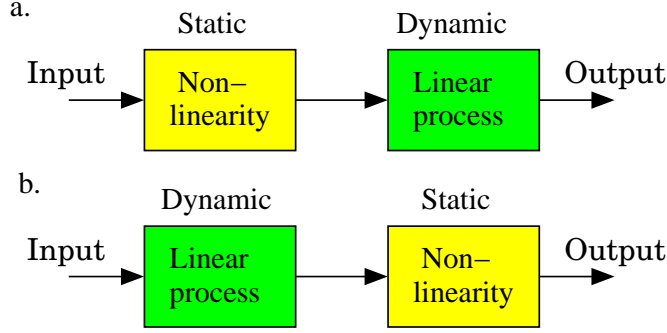
Figure 1 a. Hammerstein model, b. Wiener model

of the nonlinearity. This is done by comparing the phase of the perturbation and its influence in the output. The input signal is then changed using a gradient method to find the extremum point. When a perturbation signal is introduced the dynamics of the process will influence the response of the system and this can corrupt the estimation of the gradient. See Sternby (1980b), Krstić and Wang (1997), and Krstić and Wang (2000). The perturbation signal method is usually only used to find a constant value of the input and/or to be able to follow a varying operating point. The process will then behave as an open loop system around the extremum point. The perturbation signal method has the advantage that it requires very little information about the process. On the other hand the convergence of the system and the steady state performance are not very good, especially in presence of noise.

A second approach is to use more advanced optimization methods. If the nonlinearity is a known function the optimal constant input might be computed directly. This method has the drawback that the static nonlinearity and the open loop gain of the process have to be known. Further, the performance at the extremum point is still as if it were an open loop system. This implies that if the open loop dynamics of the process is slow then the convergence and recovery after a disturbance will be slow.

There are different classifications of nonlinear systems and we will discuss two different classes of systems shown in Figure 1. The first class of models is called *Hammerstein models* where the nonlinearity is at the input of a linear dynamic subsystem. Extremum control of Hammerstein models has been studied in Keviczsky and Haber (1974), Keviczsky *et al.* (1979), Wittenmark (1993), and Wittenmark and Urquhart (1995). In Wittenmark (1993) and Wittenmark and Urquhart (1995) different dynamic controllers together with adaptive schemes are investigated. The Hammerstein models have the advantage that the models are linear in the parameters, which makes it easy to estimate the parameters of the model.

In the second class of models, *Wiener models*, the system has a linear part followed by a nonlinearity. Estimation of the parameters in Wiener models, is discussed in Wigren (1990), Wigren (1993), Hagenblad (1999), and Hagenblad and Ljung (2000), but mainly for the case when the nonlinearity is invertible. In this paper we will discuss extremum control of Wiener models. To start with we will assume that the processes are known and the parameter estimation problem will only be briefly discussed at the end of the paper.

The paper is organized in the following way. The problem is formulated in Section 2 and different controllers are discussed in Section 3. Some of the controllers require that the output or the states of the linear part of the process are known and different ways to make estimators are discussed in Section 4. An example is used in Section 5 to illustrate the behavior of the different control schemes. Section 6 contains a discussion of estimating the unknown parameters of Wiener models. Finally, some conclusions are given in Section 7.

2. Problem formulation

We assume that the process is a Wiener model where the linear part is described by the known discrete-time system

$$
\begin{aligned}
z(k) &+ a_1 z(k-1) + a_2 z(k-2) + \cdots + a_n z(k-n) \\
&= b_0 u(k-d) + b_1 u(k-d-1) + \cdots + b_{n-d+1} u(k-n-1) \quad (1) \\
&\quad + e(k) + c_1 e(k-1) + \cdots + c_n e(k-n)
\end{aligned}
$$

where $u(k)$ is the input signal, $z(k)$ the output of the linear part, and $e(k)$ is Gaussian distributed white noise with zero mean and standard deviation σ.

The model can also be written in polynomial form

$$
A(q)z(k) = B(q)u(k) + C(q)e(k) \quad (2)
$$

where q is the forward shift operator and $\deg A = \deg C = n$ and $\deg B = n - d$. Further, A and C are monic, i.e. the coefficient of the largest power of q is equal to one. The parameter d is the time delay in the system. The model (1) can also be written in state-space form.

The nonlinearity is described as a quadratic function of the form

$$
y(k) = h(z(k)) = \gamma_0 + \gamma_1 z(k) + \gamma_2 z(k)^2 \quad (3)
$$

with $\gamma_2 \neq 0$. Other types of nonlinearities can also be assumed. However, we assume, at least close the optimum point, that the nonlinearity can be described by a quadratic function. The nonlinearity has an optimum point, maximum or minimum, depending on the parameter γ_2. For the sake of simplicity we assume that the extremum point is a minimum, i.e. $\gamma_2 > 0$. The minimum of $y(k)$ is obtained for

$$
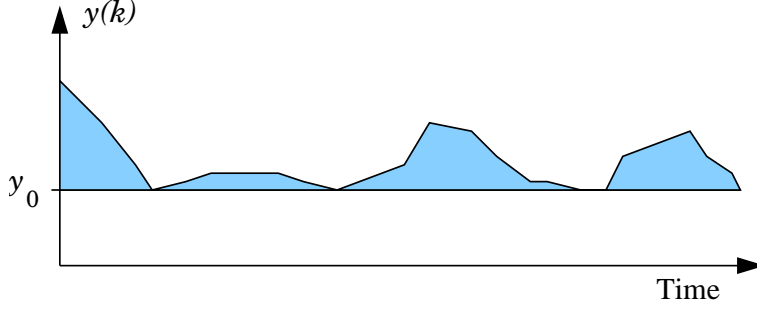z_0 = -\frac{\gamma_1}{2\gamma_2} \quad (4)
$$

The minimum of $y(k)$ is

$$
y_0 = \gamma_0 - \frac{\gamma_1^2}{4\gamma_2} \quad (5)
$$

Independent of the value of $z(k)$ the output can never be below the value y_0.

The control signal, $u(k)$ is allowed to be a function of the process output $y(k)$ and previous inputs and outputs. In the derivation of some of the

Figure 2 The purpose of the control of the Wiener model is to minimize the indicated area.

controllers we will also assume that the control signal may be a function of the outputs of the linear system or its state, i.e. of $z(j)$, $j \leq k$. The estimation of the states is discussed in Section 4.

The purpose of the control is to keep the output $y(k)$ as close as possible to the optimum point y_0. The loss function is formally expressed as

$$\min_{u(k)} (y(k) - y_0) \tag{6}$$

i.e. we want to minimize the area indicated in Figure 2.

3. Control strategies

In this section three different control strategies will be discussed. Firstly, a static controller is derived and then two types of one-step-ahead controllers will be considered.

3.1 Static controller

Assume that there is no noise acting on the system and assume that the input to (2) is constant then $z(k) = z_0$ if

$$u_0 = \frac{A(1)}{B(1)} z_0 = -\frac{\gamma_1 A(1)}{2\gamma_2 B(1)} \tag{7}$$

Using (7) on the system (2) gives

$$z(k) = \frac{A(1)B(q)}{B(1)A(q)} z_0 + \frac{C(q)}{A(q)} e(k) = z_0 + \frac{C(q)}{A(q)} e(k) = z_0 + v(k)$$

The second equality follows since z_0 is constant. This implies that the mean value of z is equal to z_0 but the variation around z_0 is determined by the open loop noise dynamics C/A. The output y will thus deviate from the desired value y_0. The variable z is a Gaussian process but the output y is a non-central χ^2 distribution, see, for instance, Johnson and Kotz (1970). If the open loop system has slow dynamics then the convergence of z will be slow at the startup or after the noise process has driven z away from its desired value.

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The controller (7) can be regarded as a one step minimization of the quadratic nonlinearity opposed to the use of the gradient method when using a perturbation signal. Using (7) gives

$$\begin{aligned} y(k) &= \gamma_0 + \gamma_1(z_0 + v(k)) + \gamma_2(z_0 + v(k))^2 \\ &= y_0 + (\gamma_1 + 2\gamma_2 z_0)v(k) + \gamma_2 v(k)^2 \end{aligned} \tag{8}$$

This implies that

$$\mathrm{E}(y(k) - y_0) = \gamma_2 \sigma_v^2 \tag{9}$$

where σ_v^2 is the variance of the process $v(k)$, which is the same as the open loop variance of the process, i.e. the controller gives the correct mean value, but the stochastic part of the system is not influenced.

3.2 One-step-ahead prediction using the true linear output

We now assume that $z(k)$ is measurable. One way to obtain a good control of the system is to minimize the variance of z around the value z_0. With z available this is essentially the problem of predicting $z(k + d)$ where d is the time delay of the system, i.e. $d = \deg A - \deg B$, see Åström and Wittenmark (1997). To make the prediction we introduce the identity

$$q^d C(q) = A(q)F(q) + G(q) \tag{10}$$

where $\deg F = d$ and $\deg G = n - 1$ and d is the prediction horizon. Further F is monic, i.e. $f_0 = 1$. The linear output at time $k + d$ can then be written as

$$z(k + d) = F(q)e(k) + \frac{1}{C(q)}(B(q)F(q)u(k) + G(q)z(k)) \tag{11}$$

The controller that minimizes the variance of $z(k)$ around z_0 is given by, see Åström and Wittenmark (1997),

$$u(k) = -\frac{G(q)}{B(q)F(q)}z(k) + \frac{C(1)}{B(1)F(1)}z_0 \tag{12}$$

The controller (12) will keep z as close as possible to z_0 and will also make $y(k)$ close to its optimal value y_0. In the case when $d = 1$ as assumed in (2) then $F = 1$ and $G = C - A$ and the controller (12) becomes

$$u(k) = \frac{A(q) - C(q)}{B(q)}z(k) + \frac{C(1)}{B(1)}z_0 = \frac{A(q) - C(q)}{B(q)}z(k) - \frac{\gamma_1 C(1)}{2\gamma_2 B(1)} \tag{13}$$

Using (12) gives

$$\begin{aligned} z(k) &= z_0 + F(q)e(k - d + 1) \\ &= z_0 + e(k) + f_1 e(k - 1) + \cdots + f_{d-1}e(k - d + 1) \end{aligned} \tag{14}$$

which gives the output

$$\begin{aligned} y(k) &= \gamma_0 + \gamma_1 z_0 + \gamma_2 z_0^2 + \gamma_1 F(q)e(k - d + 1) \\ &\quad + 2\gamma_2 z_0 F(q)e(k - d + 1) + \gamma_2(F(q)e(k - d + 1))^2 \end{aligned} \tag{15}$$

Further

$$\mathrm{E}(y(k) - y_0) = \gamma_2(1 + f_1^2 + \cdots + f_{d-1}^2)\sigma^2 \tag{16}$$

Since $F(q)$ is the first $d - 1$ coefficients of the series expansion of $C(q)/A(q)$ it follows that the average loss per step is lower when (12) is used than when (7) is used.

3.3 One-step-ahead prediction using the estimated linear output

The controller (12) is an idealized controller since the output (or the state) of the linear part cannot be measured. An obvious modification of the controller is then to assume that the separation principle holds and then replace $z(k)$ with the estimated value $\hat{z}(k)$, i.e. to use a certainty equivalence controller, see Wittenmark (1995). The estimation of $z(k)$ is assumed to be based on measurements of $y(k)$ and previous values of the measured outputs and inputs. The control law based on the estimated linear output is

$$u(k) = -\frac{G(q)}{B(q)F(q)}\hat{z}(k) + \frac{C(1)}{B(1)F(1)}z_0 \qquad (17)$$

There are obvious difficulties in the estimation of z, especially since the nonlinear part of the system has a non-unique inverse. The estimation problem is discussed in Section 4.

3.4 An example

A simple example is used to illustrate the problem formulation and the different controllers. Assume that the process is described by

$$z(k) + az(k-1) = bu(k-1) + e(k) \qquad (18)$$

The output nonlinearity is given by

$$y(k) = \gamma_0 + \gamma_1 z(k) + \gamma_2 z(k)^2 \qquad (19)$$

The output of the system at time $k+1$ can be written as

$$
\begin{aligned}
y(k+1) =& \gamma_0 + \gamma_1 z(k+1) + \gamma_2 z(k+1)^2 \\
=& \gamma_0 - \gamma_1 az(k) + \gamma_1 bu(k) + \gamma_2[a^2 z(k)^2 + b^2 u(k)^2 - 2abz(k)u(k)] \\
& + \gamma_1 e(k+1) + \gamma_2[e(k+1)^2 - 2az(k)e(k+1) + 2bu(k)e(k+1)]
\end{aligned}
$$
$$(20)$$

Since $e(k+1)$ is independent of $z(k)$ and $u(k)$ it follows that the expected mean of $y(k+1)$ is given by

$$
\begin{aligned}
\mathrm{E}y(k+1) =& \gamma_0 - \gamma_1 az(k) + \gamma_2 a^2 z(k)^2 + \gamma_2 \sigma^2 \\
& + (\gamma_1 b - 2\gamma_2 abz(k))u(k) + \gamma_2 b^2 u(k)^2
\end{aligned}
\qquad (21)
$$

which is quadratic in $u(k)$. The mean value is minimized using the control law

$$u(k) = \frac{2\gamma_2 az(k) - \gamma_1}{2\gamma_2 b}$$

which obviously is the same as the controller (12). This controller gives the expected loss per step equal to

$$V_{pred} = \gamma_2 \sigma^2 \qquad (22)$$

The constant controller corresponding to (7) is in this case

$$u_0 = -\frac{\gamma_1(1+a)}{2\gamma_2 b}$$

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Assuming that the linear part is stable, i.e. $|a| < 1$, then the expected loss per step using this controller is

$$V_{const} = \frac{\gamma_2 \sigma^2}{(1 - a^2)} \tag{23}$$

which is larger than (22).

4. The state estimation problem

The controller (17) requires an estimate of the output of the linear part of the process based on the output from the nonlinearity. This constitutes a nonlinear estimation problem. There are different ways of approaching this problem. One approach is to use the extended Kalman filter. Another way is to try to utilize the structure of the process and approach the estimation of $z(k)$ in some other ways. For simplicity, it is assumed that there is no extra time delay in the system (1) in this section. This is not any restriction and will simplify the notation and the discussion in the sequel.

4.1 Extended Kalman filter

The process (1) can be written in the state space form

$$
\begin{aligned}
x(k+1) &= \Phi x(k) + \Gamma u(k) + \Gamma_e e(k+1) \\
z(k) &= C x(k) \\
y(k) &= h(z(k)) + \varepsilon(k)
\end{aligned} \tag{24}
$$

In the measurement of $y(k)$ a noise term is also included and it is assumed that the variance of ε is r_2. From (1) the state representation is not unique and it is only the output $z(k)$ that we need to estimate for the controller (17). We use the notation in (24) to comply with the normal formulation of state estimation. The extended Kalman filter, see e.g. Gelb (1974), is based on a linearization of the output nonlinearity when computing the gain in the filter. The estimator can now be written as

$$
\begin{aligned}
\hat{x}(k) &= \hat{x}(k|k-1) + K(k)[y(k) - h(\hat{x}(k|k-1))] \\
\hat{x}(k|k-1) &= \Phi \hat{x}(k-1) + \Gamma u(k-1) \\
P(k|k-1) &= \Phi P(k-1)\Phi^T + \Gamma_e \Gamma_e^T \sigma^2 \\
P(k) &= [I - K(k)H_k(\hat{x}(k|k-1))]P(k|k-1) \\
K(k) &= P(k|k-1)H_k^T(\hat{x}(k|k-1)) \\
&\quad \cdot [H_k(\hat{x}(k|k-1))P(k|k-1)H_k^T(\hat{x}(k|k-1)) + r_2]^{-1} \\
H_k(\hat{x}(k|k-1)) &= \gamma_1 C + 2\gamma_2 \hat{x}^T(k|k-1)C^T C
\end{aligned} \tag{25}
$$

4.2 Analysis of the extended Kalman filter

Equation (25) looks very reasonable, but there is an intricate problem due to the non-uniqueness of the output non-linearity or equivalently the

existence of a minimum. This is easily illustrated using the example in Section 3.4. For the example we get

$$
\begin{aligned}
\hat{z}(k) &= -a\hat{z}(k-1) + bu(k-1) + K(k)[y(k) - h(-a\hat{z}(k-1) + bu(k-1))] \\
P(k) &= [I - K(k)H_k](a^2 P(k-1) + \sigma^2) \\
K(k) &= (a^2 P(k-1) + \sigma^2)H_k^T[H_k(a^2 P(k-1) + \sigma^2)H_k^T + r_2]^{-1} \\
H_k &= \gamma_1 + 2\gamma_2(-a\hat{z}(k-1) + bu(k-1))
\end{aligned}
$$

$$(26)$$

Some properties of the extended Kalman filter can now be seen. Firstly, there will be problems when using the controller with feedback from the estimated linear output. Using the controller (17) gives

$$
\begin{aligned}
H_k &= \gamma_1 + 2\gamma_2(-a\hat{z}(k-1) + bu(k-1)) \\
&= \gamma_1 + 2\gamma_2\left(-a\hat{z}(k-1) + b\frac{2\gamma_2 a\hat{z}(k-1) - \gamma_1}{2\gamma_2 b}\right) \equiv 0
\end{aligned}
$$

$$(27)$$

I.e. the gain in the filter will be equal to zero, which eliminates the updating of the estimate.

Secondly, consider the noise-free case, i.e. $\sigma = 0$ and $r_2 = 0$. It then follows that $K(k) = 1/H_k$. This implies that even if we are not using (17) the variable H_k can be small and the gain in the filter will be large and the filter may become unstable. Away from the optimum point the extended Kalman filter is able to make good estimation of the the full state of the linear part of the process.

Another way of analyzing the instability of the filter is by assuming that $\gamma_0 = \gamma_1 = b = 0$ in the example in Section 3.4. The estimator equations can then be written as

$$
\begin{aligned}
\hat{z}(k) &= -a\hat{z}(k-1) + K(k)(y(k) - h(-a\hat{z}(k-1)) \\
&= -a\hat{z}(k-1) + K(k)(a^2 z(k-1) - a^2 \hat{z}(k-1))
\end{aligned}
$$

$$(28)$$

Using $z(k) = -az(k-1)$ and introducing $\tilde{z}(k) = z(k) - \hat{z}(k)$ we get

$$
\begin{aligned}
\tilde{z}(k) &= -a\tilde{z}(k-1) + K(k)(a^2 z(k-1) - a^2 \hat{z}(k-1)) \\
&= -a\tilde{z}(k-1) + K(k)a^2(z(k-1) + \hat{z}(k-1))(z(k-1) - \hat{z}(k-1)) \\
&= [-a + K(k)a^2(z(k-1) + \hat{z}(k-1))]\tilde{z}(k-1) = \alpha\tilde{z}(k-1)
\end{aligned}
$$

$$(29)$$

This is a time-varying system and it is clear that the magnitude of α can be larger than 1 and the estimation error will grow exponentially if $K(k)$ is sufficiently large.

The analysis indicates two possibilities for eliminating the drawback of the extended Kalman filter. Either the control law has to be changed. This can be done by changing the goal for the controller by trying to reach a different mean value for the process output. Another way to change the control law is to introduce a perturbation signal, which also will increase the loss. A second possibility is to look at different ways of making the estimator. The extended Kalman filter uses the estimate of the derivative of the nonlinearity giving H_k, but other types of filters may use other approximations.

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4.3 Second order nonlinear filter

It is possible to make further extensions of the extended Kalman filter. In Gelb (1974), pp 191–192, second order nonlinear filters are also described. These filters take up to second order terms in the series expansions into account. Since the output nonlinearity is quadratic this should be sufficient. The second order filter has the same structure as (25) with some slight modifications. The main problem with the extended Kalman filter that $H_k \equiv 0$ when the controller (17) is used remains. The equations for the second order extended Kalman filter will thus not be given.

4.4 Probabilistic estimator

A new estimator will now be derived by fully utilizing the structure of the problem. Since there is not any measurement noise in $y(k)$ in (1) and the nonlinearity is quadratic we can solve for the value of $z(k)$ using $y(k)$. The two possible solutions for the output of the linear system is

$$z_{1,2}(k) = \frac{1}{2\gamma_2} \left(-\gamma_1 \pm \sqrt{\gamma_1^2 + 4\gamma_2(y(k) - \gamma_0)} \right) \tag{30}$$

Using (11) we introduce the following one-step-ahead predictor for the output of the linear system

$$\hat{z}(k|k-1) = \frac{B(q)}{C(q)} u(k-1) + \frac{G(q)}{C(q)} \hat{z}(k-1) \tag{31}$$

This can be interpreted as the second equation in (25). We now assume that we have an estimate of the probabilities $p_1(k-1)$ and $p_2(k-1)$ that the linear output at time $k-1$ are $z_1(k-1)$ and $z_2(k-1)$, respectively. An algorithm to obtain the estimate at time k is the following

1. Use $y(k-1)$ to compute the two possible values of z at time $k-1$ giving $z_1(k-1)$ and $z_2(k-1)$.

2. Predict one step ahead using (31) with $\hat{z}(k-1) = \hat{z}_i(k-1)$. (For previous values of $\hat{z}(k-1)$ the previous estimates are used.) This results in $\hat{z}_1(k)$ and $\hat{z}_2(k)$, respectively. The prediction error has the normal distributed frequency function $f_n(z)$

3. The probabilities for being in the two states can now be updated using the following equations

$$\begin{aligned}
p_1(k) &= \alpha_{norm}[p_1(k-1)f_n(z_1(k) - \hat{z}_1(k|k-1)) \\
&\quad + p_2(k-1)f_n(z_1(k) - \hat{z}_2(k|k-1))] \\
p_2(k) &= \alpha_{norm}[p_1(k-1)f_n(z_2(k) - \hat{z}_2(k|k-1)) \\
&\quad + p_2(k-1)f_n(z_2(k) - \hat{z}_1(k|k-1))]
\end{aligned} \tag{32}$$

where α_{norm} is a normalization factor making the sum of $p_1(k)$ and $p_2(k)$ equal to one.

4. The estimate $\hat{z}(k)$ is chosen as the $z_i(k)$ which has the largest probability.

The probability based estimator uses the previous measurement of y to compute two possible values of z. These two outputs of the linear system are then predicted into the future using a slightly modified predictor. The two predicted values represent two possible outcomes for z at time k and these are compared with the two possibilities that are obtained from the present measurement. The probabilities for the two outcomes are calculated and the one of the possible present values of $z_i(k)$ that has the highest probability is chosen as the estimate of z at time k. The estimator has the advantage that it can't be unstable. It is only selecting one of two possible values. There is, however, a possibility that the estimator may sometimes choose the wrong value. An estimator of the form (32) was discussed in Jacobs and Langdon (1970).

It is also straightforward to show that $p_1(k) = p_2(k) = 0.5$ independent of $p_1(k-1)$ and $p_2(k-1)$ when the constant controller (7) is used. This implies that the constant controller doesn't give any information about which of the values $z_i(k)$ to choose. The maximum probability rule then doesn't give any possibility to decided which value to use. Another possibility to determine the estimate is to use the mean value, i.e.

$$\hat{z}(k) = p_1(k)z_1(k) + p_2(k)z_2(k) \tag{33}$$

A controller based on this estimate will move the control signal towards u_0, which implies that there will be less excitation of the process compared when the largest probability is used for the decision.

5. An example

The example in Section 3.4 will be used to illustrate the properties of the different controllers. The following numerical values will be used in the simulations: $a = -0.99$, $b = 0.1$, $\sigma^2 = 0.1$, $\gamma_0 = 12$, $\gamma_1 = -4$, and $\gamma_2 = 0.5$. The system is simulated using the the constant controller (7), one-step-ahead prediction controller using the true linear output (12) and the one-step-ahead prediction controller using the estimated linear output (17) when the estimation is carried out using the probabilistic estimator in Section 4.4.

Figure 3 shows the input u and the output y when using the three controllers. It is clearly seen that output y deviates much more from the optimum y_0 when the constant controller is used compared to the other two controllers. This is also seen in Figure 4 showing the accumulated loss function

$$V(k) = \sum_{i=1}^{k}(y(k) - y_0)$$

when the different controllers are used. The loss when using (12) corresponds well with the predicted loss per step given by (22), while the loss when using the constant controller is less in the simulation than what is predicted by (23). Longer simulations are required to obtain a better agreement with (23) since the process in this case has a very low frequency behavior. (The pole of the open loop system is close to the unit circle.) The use of the estimated linear output gives a loss that is 2.3 times larger, while the constant controller gives a loss that is 28 times larger
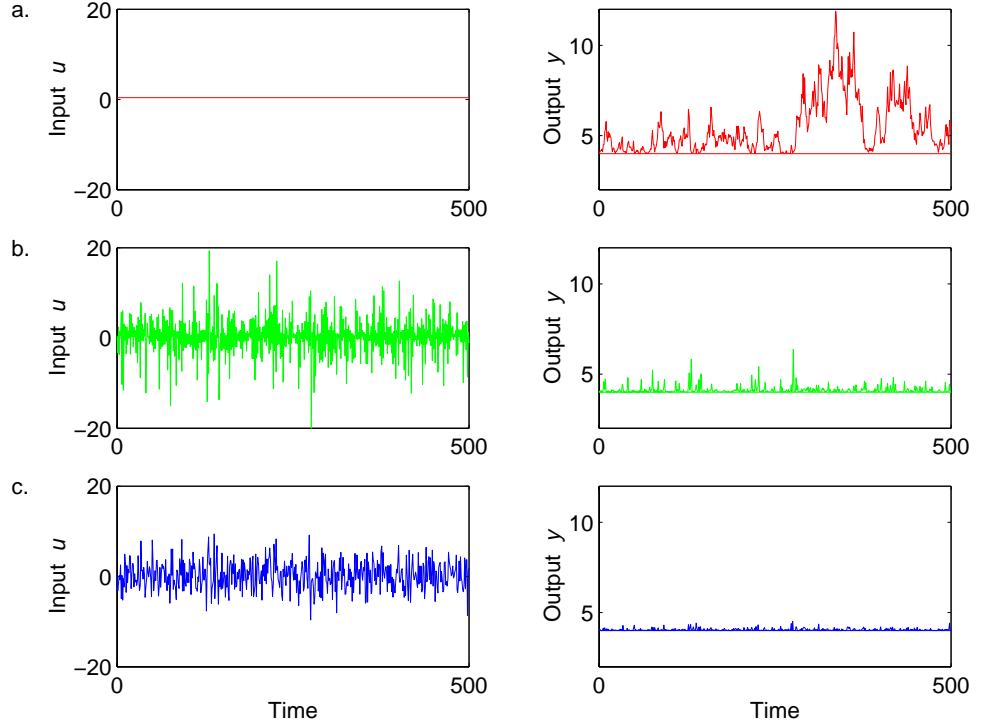
Figure 3 The left diagrams show the input u and the right diagrams the output y together with the minimum value y_0 when using a. (7); b. (17); and c. (12).

than the loss when using feedback from the true linear output. Despite the much improved result with the one-step-ahead controller using the estimated linear output there are still improvements that can be achieved by using a better estimator or an improved controller. Simulations also verify that using the mean value estimator (33) gives a much worse performance than when using the most probable value. The accumulated loss is 12.5 times the loss when using the feedback from the true linear output.

The histogram of the distribution of the output for 20000 steps of time is shown in Figure 5. When using (17) the distribution gets a "heavier" tail then when (12) is used. The properties of the estimator is seen in Figure 6. The curves show the estimated and true linear output when the controller (17) is used. There is a quite good estimation of the true linear output but the estimator picks the wrong solution in about 35% of the cases. This is due to the noise and that the process is operating close to the optimum where it is difficult for the estimator to distinguish between the influence of the noise and the influence of the control signal.

Figure 7 shows the probability p_1 when using different controllers. Curve a. shows that the estimator gives equal probabilities of the two possible values of the linear output when the control signal is constant. This is due to the poor excitation of the process. For the other two controllers the probabilities are moving rapidly around the value 0.5.

The performance of the estimators when the input signal is not generated by (17) will now be investigated. Figure 8 shows the input u, output y, and the linear output z. The input is

$$u(k) = 0.2 + 2\sin(0.1k) \tag{34}$$

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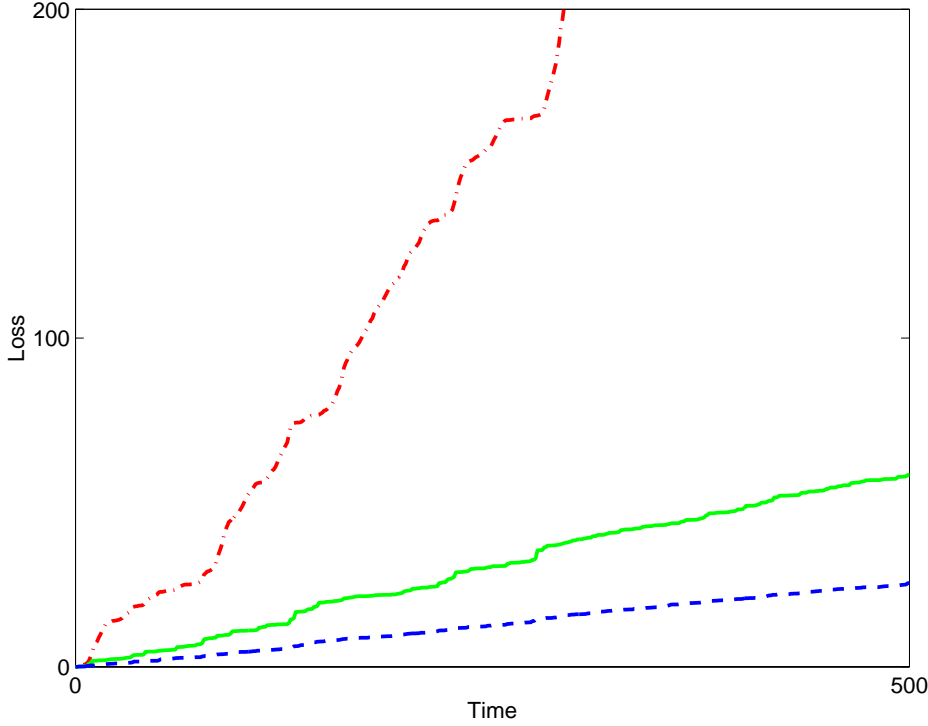
Figure 4 The accumulated loss function when using the three controllers (7) (dash-dotted), (12) (dashed), and (17) (full).

and is chosen such that z passes the value z_0 several times during the simulation. Figure 9 shows the true linear output together with the estimated output using three different estimators. Curve a. shows the case when the extended Kalman filter is used. Curve b. is when the second order filter is used and curve c. when the probabilistic estimator is used. The extended Kalman filter has difficulties to choose the branch of the nonlinearity while the second order filter most of the time is making a good estimate. The probabilistic estimator is the best of the three investigated estimators. The performance of the estimators can be measured in how large percentage of the estimates that are within 10% of the true value. For the extended Kalman filter it is 54%, for the second order filter 87%, and for the probabilistic estimator 98%. Within 1% accuracy the numbers are 23%, 35%, and 96%, respectively.

The special case $a = -1$, $b = 1$, $\gamma_0 = \gamma_1 = 0$, and $\gamma_2 = 1$ is discussed in Jacobs and Langdon (1970) and Sternby (1980a). The optimal controller is numerically derived in Jacobs and Langdon (1970) by using dynamic programming. The average loss per step is shown to be $2.2\sigma^2$. A suboptimal controller controller based on an approximate least squares estimation of the linear output and a two-step loss function is proposed in Sternby (1980b) and gives an average loss $2.64\sigma^2$ per step. The proposed probabilistic estimator and the certainty equivalence controller suggested in this report gives a loss of $2.42\sigma^2$ per step, which is 10% larger than the optimal loss, but better than earlier proposed suboptimal controllers. The improvement is probably due to the perturbation introduced by choosing the most probable value of $z_i(k)$.

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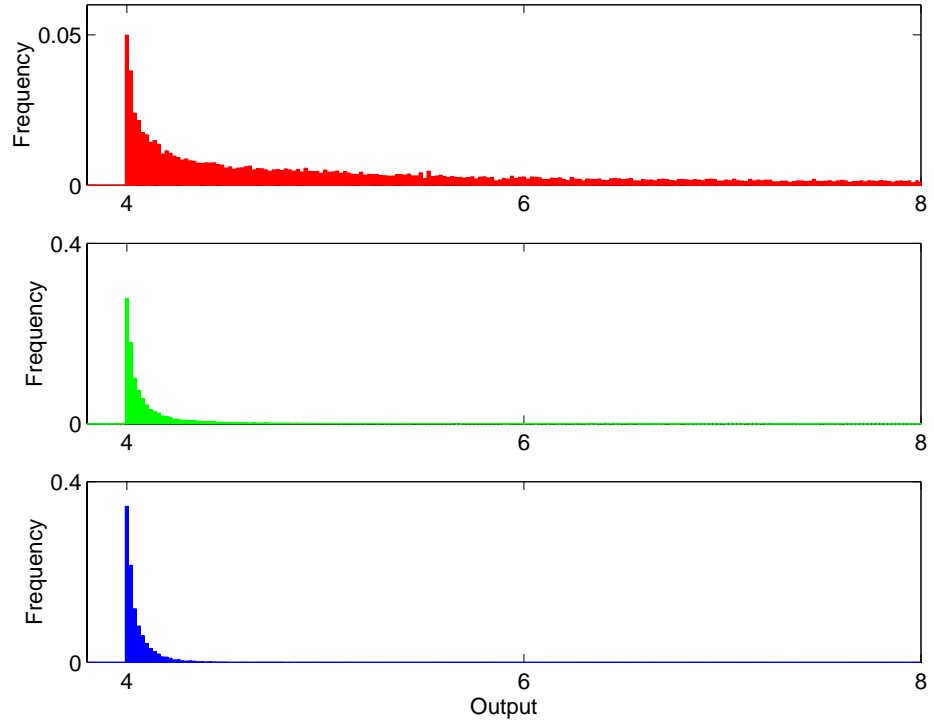
Figure 5 The histogram of the output y when using the three controllers (7) (upper), (17) (middle), and (12) (lower). Notice the difference in the vertical axis of the upper curve.

This indicates that further improvements are possible.

6. Unknown process parameters

In the analysis and simulations so far we have assumed that the process and its parameters are fully known. In practical applications this is not the case. In many situations the parameters of the process will change depending on the environment in which the process is working. This is the case, for instance, for combustion engines.

One way to circumvent the lack of knowledge about the process is to use a method that does not depend on the process parameters. The use of a perturbation signal only relies on the assumption that the nonlinearity is a concave or convex function. The input signal is changed based on the estimate of the gradient. The dynamics of the process and the noise, however, have a heavy influence on the estimation of the gradient. The phase lag introduced by the process dynamics can be compensated for as suggested in Sternby (1980b), Krstić and Wang (1997), and Krstić and Wang (2000).

An alternative way is to estimate the parameters of the process on-line or off-line. The estimation of the parameters of the Wiener models is discussed, for instance, in Wigren (1990), Wigren (1993), Hagenblad (1999), and Hagenblad and Ljung (2000). The method suggested in Wigren (1990), Wigren (1993) is essentially based on the idea of the extended Kalman filter and the references are only briefly considering the more difficult case
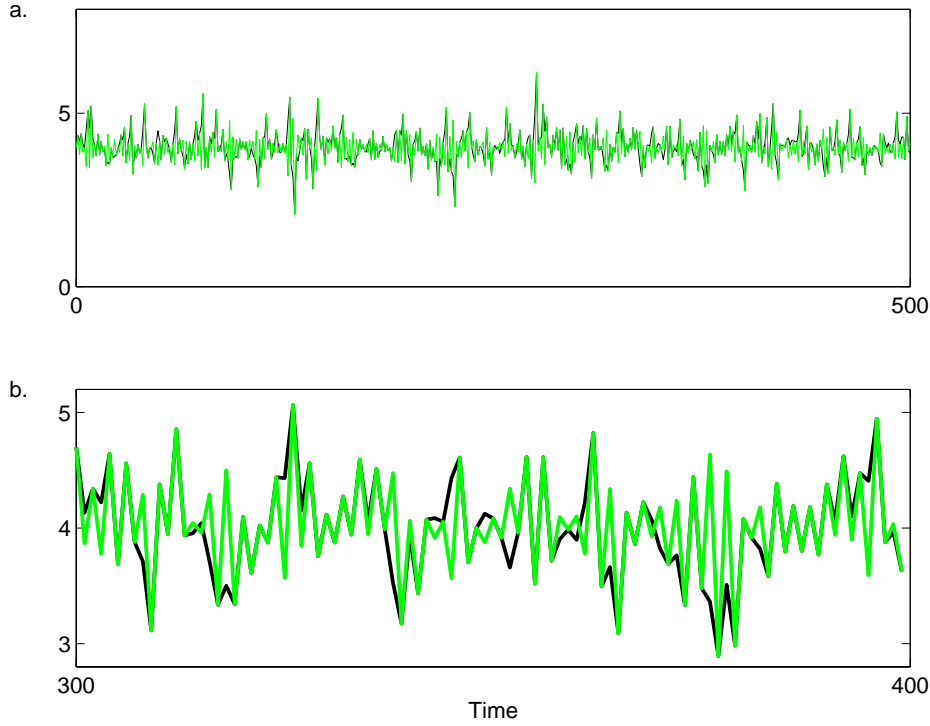
Figure 6 The true (black) and estimated state z (grey) when using the controller (17). Curve b. show an enlarged part of curve a.

with non-unique inverse of the nonlinearity that is discussed in this report. This topic requires more research.

7. Summary

Extremum control of Wiener model processes has been discussed. For known processes there are several possibilities to obtain good control of the process. A crucial part of the controller is the estimation of the output of the linear part of the process. Several types of estimators have been discussed and most of the estimators have the drawback that they has a singular point at the optimum point of the process and this is where we want to keep the process. The method proposed to avoid this problem is to use a probabilistic based estimator that selects between two possible values of the output based on previous measurements and input signals. The combination of this estimator and a prediction controller has the advantage that especially close to the optimum it is insensitive to the accuracy of the estimates.

The controllers discussed have been based on the assumption that the separation principle is valid, which implies that the true linear output can be replaced by its estimate. The behavior when using the constant controller also indicates that perturbation signals should be introduced to improve the performance of the closed loop system. This implies that the controller should have a dual property, which ensures that that the control action is a compromise between making good control and obtaining
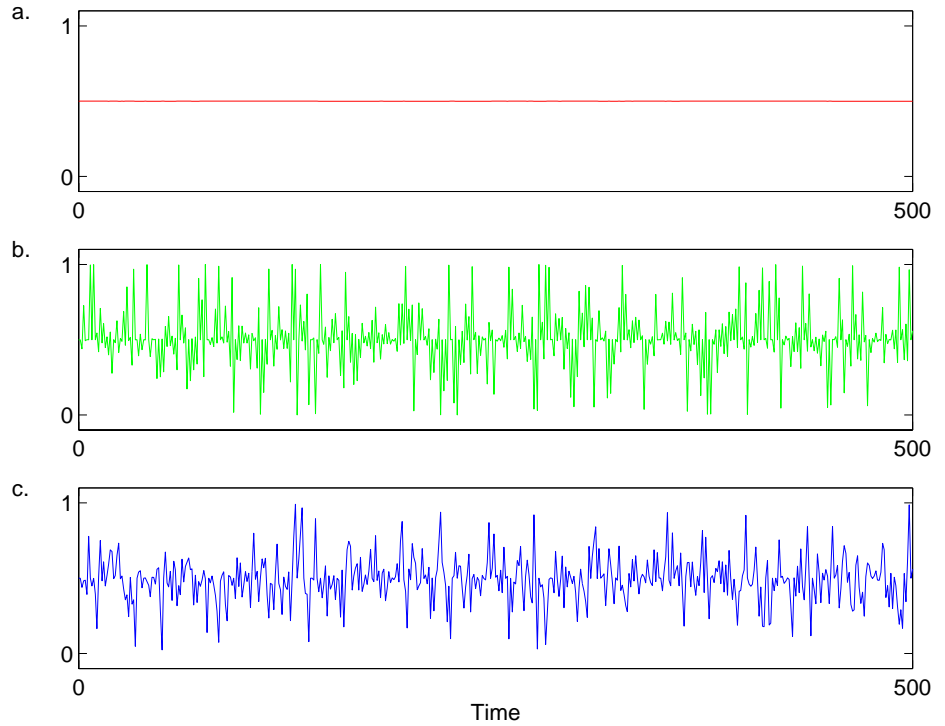
Figure 7 The probability p_1 as function of time when using the controllers a. (7); b. (17); and c. (12).

good estimates of the linear output. Dual control is discussed in, for instance, Wittenmark (1995), Åström and Wittenmark (1995), and Filatov and Unbehauen (2000).

More research is required for the case when the process is unknown.

Acknowledgments

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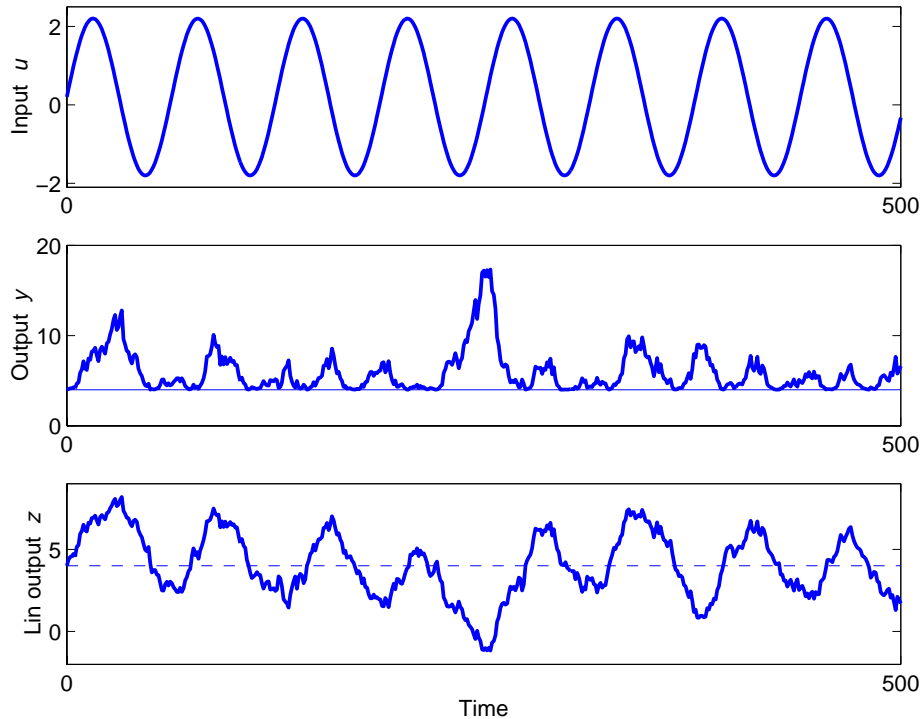
Figure 8 The input u, the output y together with the minimum value y_0, and the linear output z of the example when the input (34) is used.

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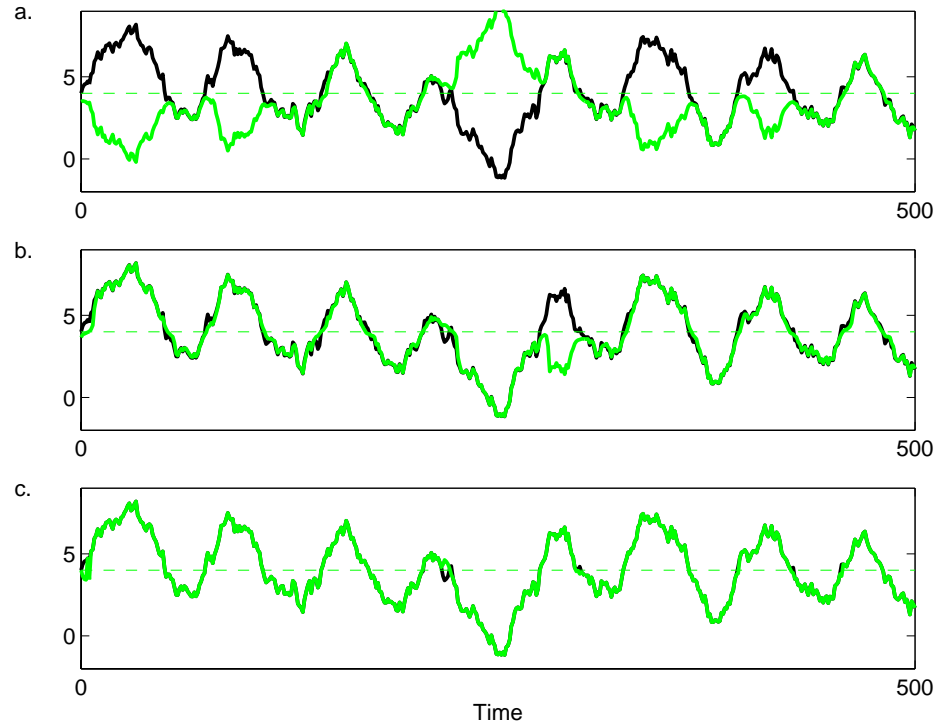
Figure 9 The estimates (grey) and the true linear output (black) for the data in Figure 8 when using a. The extended Kalman filter; b. The second order filter; and c. The probabilistic estimator.

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