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Extension of Pozharitsky Theorem for Partial Stabilization of a System with Several First Integrals[1]

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Abstract. The paper is devoted to an extension of one particular fact within theory of partial stability, the so-called *Pozharitsky Theorem*, to the case of partial stabilization of nonlinear controlled system. It is shown that under appropriate assumptions *partial stabilization* of the system based on usage some Lyapunov function constructed from first integrals of the unforced system, implies that the Lyapunov function of a simplified form also leads to a controller that partially stabilizes the system. The theoretical results are illustrated by the problem of partial stabilization of the downward equilibrium of the *Inertia Wheel Pendulum.*

1 Introduction and Problem Statement

Stability of a dynamical system with respect to part of variables is a natural extension of the classical concept of stability in the sense of Lyapunov. Partial stability is intensively studied within last 50 years, see the books [5, 8]. This paper is devoted to an extension of one particular fact within the partial stability theory, the so-called *Pozharitsky Theorem.* Let us formulate this statement precisely [4, 5]:

Theorem 1 (Pozharitsky, 1957) *Consider a nonlinear system*

$$\frac{d}{dt}x(t) = f(x,t), \tag{1}$$

where $x \in R^n$, $f(t,x)$ is vector function with $f(t,0) \equiv 0$. Suppose that

1. *the state vector x has a partition*

$$x = \Big(x_1,\, x_2,\, \dots,\, x_n\Big)^T = \begin{pmatrix} y \\ z \end{pmatrix}$$

such that

(a) *The right-hand side of (1) is continuous and provides the uniqueness of the solution of (1) in the domain*

$$t \geq 0, \quad \|y\| \leq H, \quad \|z\| < +\infty \quad (H = const > 0)$$

(b) *Any solution $x(t)$ of the system (1) is well defined for all $t \geq 0$, provided that its part $y(t)$ satisfies to the constraint*

$$\|y(t)\| \leq H$$

2. *The system (1) has k first integrals*

$$I_p(t,x) = const, \quad p = 1,\dots,k,$$

such that $I_p(t,0) \equiv 0$, $p = 1,\dots,k$.

Given any C^1-smooth function

$$V(t,x) := F\Big(I_1(t,x), I_2(t,x), \dots, I_k(t,x)\Big) \tag{2}$$

with $F(0,0,\dots,0) = 0$, and the function

$$\begin{aligned} V_0(t,x) \;&:=\; F_0\Big(I_1(t,x), I_2(t,x), \dots, I_k(t,x)\Big) \\ &= \frac{1}{2}I_1^2(t,x) + \frac{1}{2}I_2^2(t,x) + \dots + \frac{1}{2}I_k^2(t,x), \end{aligned} \tag{3}$$

consider both functions $V(t,x)$ and $V_0(t,x)$ as a Lyapunov function candidates. If there exists any y-positive definite function $V(t,x)$ of the form (2), i. e.

$$V(t,x) \geq \alpha(\|y\|), \quad \alpha \in \mathcal{K},$$

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then the function $V_0(t,x)$ defined in (3) is also y-positive definite, i.e. there exists $\beta \in \mathcal{K}$ such that

$$V_0(t,x) \geq \beta(\|y\|).$$

In other words, if there exists a Lyapunov function of the form (2) that provides y-stability of the system (1), then the function $V_0(t,x)$ will be by necessity the Lyapunov function for the system (1) that guarantees the partial stability of the system (1). ■

As the reader can see this fact gives the simple and efficient algorithm for checking partial stability based on a Lyapunov function constructed from first integrals of the system. To check the partial stability following this way is then sufficient to consider **the simplest Lyapunov function candidate (3), and if it does not guarantee the partial stability, any other candidate (2) will not guarantee it.**

It is worth also mentioning the origin for a such type consideration. Prof. N.G. Chetaev suggested in [1] to consider Lyapunov function candidates of the form

$$V(t,x) = \sum_{p=1}^{k} \lambda_p I_p(t,x) + \sum_{p=1}^{k} \mu_p I_p^2(t,x)$$

with objective to find parameters λ_p, μ_p, that ensure stability of the system (1). At present this form of a Lyapunov function is known as *Chetaev's Method of Bundles of First Integrals.* As seen, *Pozharitsky Theorem* shows that for the partial stability one should try only the simplest case: $\lambda_p = 0$, $\mu_p = 1$, $p = 1, \ldots, k$.

This paper is aimed at deriving a version of *Pozharitsky Theorem* for partial *stabilization* of the time invariant nonlinear controlled system of the form

$$\frac{d}{dt}x(t) = f(x) + g(x)u. \tag{4}$$

Here $u \in R^m$ is a controlled variable, f and $g(x)$ is a smooth vector field of appropriate dimensions. As in *Pozharitsky Theorem*, we will assume that

I). The state space vector x has a partition

$$x = \Big(x_1,\, x_2,\, \ldots,\, x_n\Big)^T = \begin{pmatrix} y \\ z \end{pmatrix}$$

and any solution $x(t)$ of the system (4) with $u = 0$, is well defined and unique for all $t \geq 0$, provided that its part $y(t)$ satisfies to the constraint

$$\|y(t)\| \leq H$$

II). The system (4) with $u = 0$ has k first integrals

$$I_p(x) = \text{const}, \quad p = 1, \ldots, k$$

that satisfy to the conditions $I_p(0) \equiv 0$, $p = 1, \ldots, k$.

To motivate further development, let us briefly mention two examples. The first one is the stabilization of the downward position of *a spherical pendulum* where the bob consists of a closed reservoir filled with a liquid. This system is described partly by the pair second order differential equations, and partly by the partial differential equation covering the motion of the liquid (the Navier-Stocks equations).

The exact movement of liquid corresponds to an element of the infinite dimensional state space, it is no really of interest in the problem, while, some averaged characteristics like, the vector of the total momentum of the liquid, its total energy, are of interest. So the problem of stabilization of the spherical pendulum at its downward position without taking into account the exact behavior of the liquid after transition falls into the area of partial stabilization.

Another motivating example is the stabilization of the downward equilibrium of the novel pendular system, the so-called *Inertia Wheel Pendulum,* introduced in [6]. Below we suggest a stabilizing controller, when only a position and velocity of one of two independent variables of the system are available for measurements.

The paper is organized as follows. The main result is stated in Section 2. As an example of its application, the problem of partial stabilization of the downward equilibrium of the novel *Inertia Wheel Pendulum* is considered in Section 3, while some computer simulations of the results are drawn in Section 4. Some conclusions then are collected in Section 5.

2 The Main Result

Theorem 2 *Suppose that the assumptions* **I)** *and* **II)** *are valid and that there exists a C^1-smooth function $F : R^k \to R_{\geq 0}$ such that a Lyapunov function candidate*

$$V(x) = F\Big(I_1(x), I_2(x), \ldots, I_k(x)\Big) \tag{5}$$

satisfies to the following conditions:

1. *The function $V(t)$ is y-positive definite, that is the inequality*

$$V(x) \geq \alpha(\|y\|), \quad \alpha \in \mathcal{K} \tag{6}$$

holds;

2. *The feedback control defined by*

$$u(x) = -Pg(x)^T \left[\frac{\partial V(x)}{\partial x}\right]^T \tag{7}$$

ensures the y-asymptotic stability of the closed loop system (4), (7) with P being some $m \times m$ positive definite matrix.

Suppose, in addition, that

III). *There is no solution $x(t)$ of unforced system (4) such that the rank of the $m \times k$ matrix function*

$$g(x(t))^T \begin{bmatrix} \frac{\partial I_1(x(t))}{\partial x_1} & \cdots & \frac{\partial I_k(x(t))}{\partial x_1} \\ \vdots & \vdots & \vdots \\ \frac{\partial I_1(x(t))}{\partial x_n} & \cdots & \frac{\partial I_k(x(t))}{\partial x_n} \end{bmatrix} \tag{8}$$

is less than k for all $t \geq 0$, and such that $x(t)$ belongs to any neighborhood $\mathcal{O}$ of the set

$$\mathcal{I} := \Big\{x : I_1(x) = \ldots = I_k(x) = 0\Big\} \subset \Big\{x : y = 0\Big\} \tag{9}$$

IV). *Any solution $x(t)$ of the closed loop system (4) with the feedback controller*

$$\begin{aligned} u &= -g(x)^T \left[\frac{\partial V_0(x)}{\partial x}\right]^T \\ &= -g(x)^T \begin{bmatrix} \frac{\partial I_1(x)}{\partial x_1} & \cdots & \frac{\partial I_k(x)}{\partial x_1} \\ \vdots & \vdots & \vdots \\ \frac{\partial I_1(x)}{\partial x_n} & \cdots & \frac{\partial I_k(x)}{\partial x_n} \end{bmatrix} \begin{bmatrix} I_1(x) \\ \vdots \\ I_k(x) \end{bmatrix} \end{aligned} \tag{10}$$

has a non-empty compact ω-limit set. Here the function $V_0(x)$ is defined in (3).

Then the feedback controller (10) makes the closed loop system (4), (10) y-asymptotically stable. ■

Some comments to Theorem 2

1. As the reader can see the contribution of Theorem is that an existence of y-stabilizing controller (7) derived by usage of some appropriate Lyapunov function $V(x)$ implies that the controller (10) of simplified form is also y-stabilizing. This fact can be seen as a direct extension for *Pozharitsky Theorem*: a presence of some Lyapunov function $V(t, x)$ *implies* an existence of Lyapunov function $V_0(t, x)$ of the simplified form (3).
2. The assumption **III)** restricts the consideration to the case when the number of independent first integrals $I_1(x), \ldots, I_k(x)$ of the unforced system (4) is less or equal to the number m of control inputs, i. e.
$$k \leq m$$
3. The matrix (8) may loose the rank on the set $\mathcal{I}$, see (9), while it is important that its rank remains equal to k in some vicinity of $\mathcal{I}$.
4. It seems that the assumption **IV)** is the main difficulty in checking Theorem conditions. We specially avoid mentioning particular sufficient conditions guaranteeing its validity. In the examples, such a property may be obtained due to topology of the phase space (if, for example, some of coordinates are periodic or bounded for sure) or it comes from properties of first integrals $I_j(x)$ (if, for example, the function $V_0(x)$ is proper on the state space of the system). Another ways to checking this assumption are presented in Section 3, where the partial stabilization of the downward equilibrium of the *Inertia Wheel Pendulum* is treated in detail.

3 Example: Partial Stabilization of Downward Position of Inertia Wheel Pendulum

The *Inertia Wheel Pendulum* is a physical pendulum with a symmetric disk attached to the end. The disc is controlled by a DC-motor that can change the angular acceleration of the disc, while the physical pendulum itselfi is freely rotating. The details and the description of hardware can be found in [6, 7].

The mathematical model of this system derived in [6, 7], is

$$d_{11}\ddot{\theta}_1 + d_{12}\ddot{\theta}_2 = -\bar{m}g\sin(\theta_1) \tag{11}$$

$$d_{21}\ddot{\theta}_1 + d_{22}\ddot{\theta}_2 = \tau \tag{12}$$

where θ_1 is the angle that the pendulum makes with the vertical line; θ_2 is the angle that describes the position of the symmetric disc; and

$$d_{11} = m_1 l_{c1}^2 + m_2 l_1^2 + J_1 + J_2 \tag{13}$$

$$d_{12} = d_{21} = d_{22} = J_2 \tag{14}$$

$$\bar{m} = m_1 l_{c1} + m_2 l_1 \tag{15}$$

Here l_1 is the length of the pendulum; l_{c1} is the position of the center of mass of the pendulum; m_1 is the mass of the pendulum; m_2 is the mass of the disc; J_1, J_2 are inertia of the pendulum and the disc around their centers of masses.

Changing the variables θ_1, θ_2 to new ones, as done in [3, 7],

$$q_1 = \theta_1, \quad q_2 = \theta_1 + \theta_2$$

brings the system (11), (12) into simplified decoupled form

$$d_{11}\,\ddot{q}_1 = -\bar{m}g\sin(q_1) - \tau \tag{16}$$

$$d_{22}\,\ddot{q}_2 = \tau \tag{17}$$

The problem is: *to stabilize the downward equilibrium of the pendulum, that is $q_1 = 0$, $\dot{q}_1 = 0$, while the available measurements are only the current values of* *the angle q_1, and its velocity $\dot{q}_1$.*

As seen the problem exactly falls into the subject of partial stabilization with respect variables q_1, $\dot{q}_1$, while the rest of variables q_2, $\dot{q}_2$ are not of interest.

3.1 The Controller I

The unforced system (16), (17) has two independent first integrals

$$I_1 = \frac{d_{11}}{2}\dot{q}_1^2 + \bar{m}g\left(1 - \cos(q_1)\right) \qquad (18)$$

$$I_2 = \frac{d_{22}}{2}\dot{q}_2^2 \qquad (19)$$

Both integrals are non-negative. Theorem 2 suggests to choose the controller τ as follows, see (10),

$$\tau = -g(q)^T \left[\frac{\partial V_0(q)}{\partial q}\right]^T = \dot{q}_1 \cdot I_1 \qquad (20)$$

where the Lyapunov function candidate $V_0(q)$ is chosen as $V_0(q) = \frac{1}{2}I_1^2(q)$, and $q = \left(q_1,\, \dot{q}_1,\, q_2,\, \dot{q}_2\right)$.

To conclude partial stability of the closed loop system (16), (17), (20) let us check the assumptions of Theorem 2. Some of them, like the given by the next statement, are obvious.

Lemma 1 *The function $V_0(q)$ is q_1, $\dot{q}_1$-positive definite around the downward equilibrium $q_1 = \dot{q}_1 = 0$.* ■

Lemma 2 (Assumption III) *There is no solution $q(t)$ of the unforced system (16), (17) around the downward equilibrium $q_1 = \dot{q}_1 = 0$ such that the matrix (8) looses the rank for all $t \geq 0$.* ■

Proof. In this case the matrix (8) is just a scalar equals to $\left(-\dot{q}_1\right)$ and it is obviously not identically equal to zero for any solution of the unforced system (16), (17) except the downward $q_1 = \dot{q}_1 = 0$ and upright $q_1 = \pi$, $\dot{q}_1 = 0$ equilibria. ■

Lemma 3 (Assumption IV) *Any solution of the closed loop system (16), (17), (20) has non-empty compact ω-limit set belonging to the cylindrical phase space.* ■

Proof. Let us choose any point $\left(q_1^0,\, \dot{q}_1^0,\, q_2^0,\, \dot{q}_2^0,\right)$ that is closely located to the downward equilibrium, and consider the solution

$$q(t) = \left(q_1(t),\, \dot{q}_1(t),\, q_2(t),\, \dot{q}_2(t)\right)$$

of the closed loop system (16), (17), (20) with the origin in this point. The equations (16), (17) are decoupled, so we can consider and analyze separately the dynamics of the variable q_1, see (16), and, then, the dynamics of the variable q_2, see (17).

The dynamics in $q_1(t)$ is covered by (16), (20), and satisfies to the differential inequality

$$\begin{aligned}\frac{d}{dt}V_0\left(q(t),\dot{q}(t)\right) &= \frac{1}{2}\frac{d}{dt}I_1^2\left(q_1(t),\dot{q}_1(t)\right)\\ &= -\dot{q}_1^2(t)\, I_1^2\left(q_1(t),\dot{q}_1(t)\right) \leq 0 \quad (21)\end{aligned}$$

The function $V_0(q(t))$ is proper of the sub-space $(q_1, \dot{q}_1) \in S^1 \times R^1$ of the cylindrical phase space of the system (16), (17). This fact, the inequality (21) and Barbalat lemma imply that the solution $q_1(t)$, $\dot{q}_1(t)$ has non-empty ω-limit set, remains in the neighborhood of the downward equilibrium, and

$$\dot{q}_1(t) \cdot I_1\left(q_1(t),\dot{q}_1(t)\right) \to 0, \quad \text{as} \quad t \to +\infty.$$

This limit relation, in turn, implies that the solution $q_1(t)$, $\dot{q}_1(t)$ converges to the downward equilibrium, i.e.

$$q_1(t) \to 0, \quad \dot{q}_1(t) \to 0, \quad \text{as} \quad t \to +\infty \qquad (22)$$

On the cylindrical phase space the variable $q_2(t) \in S^1$ is always bounded, therefore to prove a existence and compactness of ω-limit set for $q(t)$, we has to show that the function $\dot{q}_2(t)$ is bounded.

The dynamics in q_2 is covered by (17), (20), that is

$$d_{22}\, \ddot{q}_2(t) = \tau = \dot{q}_1(t) \cdot I_1\left(q_1(t),\dot{q}_1(t)\right)$$

Therefore

$$\dot{q}_2(t) = \frac{1}{d_{22}}\int_0^t \left\{\dot{q}_1(s) \cdot I_1\left(q_1(s),\dot{q}_1(s)\right)\right\} ds \qquad (23)$$

To show that the integral in the right hand side of (23) has a limit as $t \to +\infty$, we use the Abel criterion which states: *For any scalar smooth converging to zero function $f(t)$ the boundedness of the integral*

$$\int_0^{+\infty} f(s)ds$$

implies the boundedness of the integral

$$\int_0^{+\infty} f(s)\mu(s)ds$$

provided the function $\mu(t)$ is nonnegative and monotonically decreasing.

We can directly apply the Abel criterion to our case. Indeed, put

$$f(t) = \dot{q}_1(t), \quad \mu(t) = I_1\left(q_1(t),\dot{q}_1(t)\right),$$

then the improper integral has the value

$$\int_0^\infty \dot{q}_1(s)ds = \lim_{t\to+\infty}\int_0^t \dot{q}_1(s)ds = \lim_{t\to+\infty}\Big[q_1(t)-q_1(0)\Big] = -q_1(0).$$

Here we use the already proven limit relation (22) for $q_1(t)$. At the same time, the function $I_1\Big(q_1(t),\dot{q}_1(t)\Big)$ is nonnegative, and due to (21) is monotonically decreasing. As a result the Abel criterion allows us to conclude that the integral in the right hand side of (23) has a limit when $t\to+\infty$. Therefore $\dot{q}_2(t)$ has also limit as $t\to+\infty$ and, hence, it is bounded. This finishes the proof. ∎

Remark 1 As the reader can see from the proof of Lemma 3, an existence and compactness of ω-limit set for the closed loop system (16), (17), (20) was derived by a particular property of the dynamics of the closed loop system, but not by a properness of the storage function $V_0(q)$ that is commonly in use in literature. In our case the function $V_0(q)$ is, in fact, not proper. ∎

3.2 The Controller II

The partial stabilization of the downward equilibrium of the *Inertia Wheel Pendulum* shown in Section 3.1 was made by usage only one first integral of the system, $I_1(q)$. A simple question: *What is original (simplest) form of the first integral I_1?* leads to some ambiguity in the choice of $V_0(q)$. Indeed, for any smooth scalar function F, the function $F(I_1(q))$ is again first integral, and only some personal preferences defines the expression for $I_1(q)$.

Let us show that another form of I_1 results again in the partial stabilization, while a transient performance is proven to be exponential. Suppose that

$$I_{1new}(q) = \sqrt{2}\sqrt{\frac{d_{11}}{2}\dot{q}_1^2 + \bar{m}g\Big(1-\cos(q_1)\Big)}$$

Then the Lyapunov function candidate is

$$V_{0new}(q) = \frac{1}{2}I_{1new}^2(q) = I_1(q)$$

and the controller suggested in Theorem 2 is then

$$\tau = -g(q)^T\left[\frac{\partial V_0(q)}{\partial q}\right]^T = \dot{q}_1 \qquad (24)$$

To use Theorem 2 we need to check again assumptions **III)** and **IV)**.

Lemma 4 (Assumption III) *There is no solution $q(t)$ of the unforced system (16), (17) around the downward equilibrium $q_1 = \dot{q}_1 = 0$ such that the matrix (8) looses the rank for all $t \geq 0$.* ∎

Proof. In our case the matrix (8) is just a scalar equals to $\left(-\frac{\dot{q}_1}{I_{1new}(q)}\right)$ and it is obviously not identically equal to zero for any solution of the unforced system (16), (17) except the upright $q_1 = \pi$, $\dot{q}_1 = 0$ equilibrium, while in the downward equilibrium $q_1 = \dot{q}_1 = 0$ it has an uncertainty of the type $0/0$. ∎

Lemma 5 (Assumption IV) *Any solution of the closed loop system (16), (17), (24) has non-empty compact ω-limit set belonging to the cylindrical phase space. Furthermore for any solution $q(t)$ of the closed loop system the variable $q_1(t)$ exponentially converges to zero.* ∎

Proof. We can proceed in the same way as in the proof of Lemma 3, but let us show another arguments. Considering only the dynamics of the closed loop system (16), (17), (24) we have

$$d_{11}\ddot{q}_1 = -\bar{m}g\sin(q_1) - \dot{q}_1$$

As seen the linearization of this differential equation around the downward equilibrium $q_1 = \dot{q}_1 = 0$ is asymptotically stable, that is $q_1(t)$ will exponentially converge to the downward equilibrium provided that the initial conditions are chosen close to this equilibrium.

Along any solution the variable $q_2 \in S^1$ is bounded, and to prove compactness of ω-limit set we have to check boundedness of $\dot{q}_2(t)$. It is defined by (16), (24), i.e.

$$d_{22}\,\ddot{q}_2(t) = \dot{q}_1.$$

Therefore

$$\dot{q}_2(t) = \frac{1}{d_{22}}\int_0^t \dot{q}_1(s)ds = \frac{1}{d_{22}}\Big(q_1(t) - q_0(0)\Big)$$

Due to the fact that $q_1(t)\to 0$ as $t\to+\infty$, we can conclude that $\dot{q}_2(t)$ is bounded and the limit relation

$$\lim_{t\to+\infty}\dot{q}_2(t) = -\frac{q_1(0)}{d_{22}}$$

is valid. ∎

3.3 Computer Simulations

To check theoretical results of Section 3.1 and 3.2 we simulate the closed loop system with both controllers (20) and (24). The parameters of the system (16), (17) were chosen as

$$d_{11} = 0.004571, \quad d_{22} = 2.495\times10^{-5}, \quad \bar{m}_3 = 0.35481$$

that are physical parameters of the system located at the Automatic Control Dept., Lund Institute of Technology. The initial conditions are

$$q_1^0 = 1, \quad \dot{q}_1^0 = 0.1, \quad q_2^0 = \dot{q}_2^0 = 0.$$

Figure 1 presents the behavior of $q_1(t)$ in the closed loop system with the controller (20) and the controller (24). As seen, the transient performance of the system with the controller (20) is oscillatory (and may be not acceptable to real-time implementation, while the friction in the system will certainly help to damp these slowly decreasing oscillations and this fact has been verified by experiment). At the same time, the closed loop system with the controller (24) shows quite fast convergence $q_1(t)$ to zero.

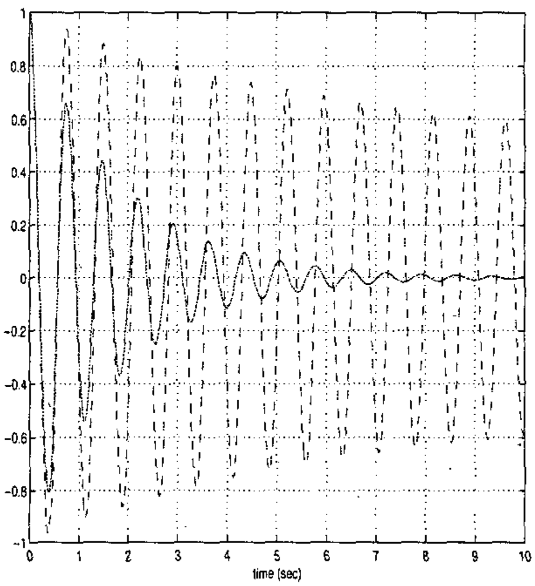

Figure 1: The values of $q_1(t)$ for the controller (20), the blue line (--), and the controller (24), the green line (—)

4 Conclusions

This paper is aimed at revealing an interesting extension of one particular (classical) fact within the partial stability theory to the case of *partial stabilization* of nonlinear control systems. This fact, the so-called *Pozharitsky Theorem*, states: If partial stability is derived based on the Lyapunov function constructed from first integrals of the system then partial stability can be proven by using new Lyapunov function of a simple form.

This fact substantially simplifies the search of an appropriate Lyapunov function leading to partial stability of the system. Indeed, if the simplest choice of Lyapunov function candidate constructed from the first integrals of the system, does not result in partial stability, then *no other choice of Lyapunov function candidate constructed from first integrals of the system* will result in partial stability.

Omitting some technical assumptions, the main result of the present paper states that: an existence of a partially stabilizing controller with the associated storage function constructed from the first integrals of the unforced system, implies that *the controller derived by speed-gradient algorithm [2] from new storage function of a simplified form is also stabilizing provided that any solution of the closed loop system with this feedback controller has non-empty compact ω-limit set.*

To illustrate this basic result the problem of partial stabilization of the downward equilibrium of the novel *Inertia Wheel Pendulum* is considered and solved.

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