Estimation of Power Generator Dynamics from Normal Operating Data

Ljung, Lennart; Lindahl, Sture

1973

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Ljung, L., & Lindahl, S. (1973). *Estimation of Power Generator Dynamics from Normal Operating Data*. (Technical Reports 7336(C)). Department of Automatic Control, Lund Institute of Technology (LTH).

Total number of authors:
2

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

ESTIMATION OF POWER GENERATOR DYNAMICS FROM NORMAL OPERATING DATA

STURE LINDAHL
LENNART LJUNG

Report 7336 (C) December 1973
Lund Institute of Technology
Division of Automatic Control

ESTIMATION OF POWER GENERATOR DYNAMICS FROM NORMAL OPERATING DATA

Sture Lindahl and Lennart Ljung
Division of Automatic Control, Lund Institute of Technology
S-220 07 Lund 7, Sweden

Data obtained from a power generator during normal operation is analysed in order to obtain a small-signal dynamic model of a power generator and its environment. A fast fourier transform technique is used to investigate causality between certain pairs of variables. The most decisive input-output pair is found to be from electric torque to angular velocity. Spectral analysis and a maximum likelihood method is applied to determine the transfer function relating angular velocity to electric torque. Since the data has been collected during normal operation all variables arise on a similar footing. To treat them accordingly and to include feedback effects a state space model with all measured variables as outputs and driven by noise only is constructed using some a priori knowledge of the structure. The maximum likelihood method is used to estimate parameters in such a fifth order model describing the generator and its environment.

1. INTRODUCTION

The dynamic analysis of load-frequency control systems requires among other things a reliable model of the power system. Models can be derived from basic physical laws but certain parameters may not be easily obtained. The damping torque coefficient associated with the generator is one such important parameter. In transient stability studies and for the design of voltage regulators there is also a need of a detailed model of the power generator.

To obtain such models various identification methods will be applied to experimental data provided by Dr K.N. Stanton. The data is described in Section 2. A principally important feature of the data is that it has been collected during normal operation. Thus all normal feedback effects between the variables are included. This calls for special attention when using the data for identification. In Section 3 a fast fourier transform technique is used for preliminary investigation of causality relations between the variables. In Section 4 spectral analysis is used to determine the transfer function relating angular velocity to electric torque (the most decisive input-output pair). This transfer function is also investigated by estimating parameters in a general input-output model using the maximum likelihood method.

Since there are no purely causal relationships between the variables, any input-output model will result in a situation where part of the input is caused by the output. Most identification methods do not account for such dependencies, which gives a loss of information. A method to include feedback effects and a priori knowledge of their structure is presented in Section 5. The approach is to treat all measured variables as outputs from a dynamical system driven by noise only. In our case the model describes the generator and its environment and predicts the measured variables: angular velocity (ω), terminal voltage (V), reactive and active component of the armature current (I_q and I_r).

2. AVAILABLE DATA

The available data originates from experiments on a 50 MW, 0.9 PF turboalternator, reported by Stanton (1965). The collection of data from a normally operating, interconnected power system is not straightforward, because the variations in observations are small relative to average operating levels. Since the identification of the power generator dynamics is based on these variations, they must be recorded with adequate resolution and without serious attentuation of rapid changes. The instrumentation has been described by Stanton (1964), and a summary of its performance is included in Table 1. The necessary resolution is achieved by suppressing mean levels, amplifying variations and using digital readout.

Table 1. Details of instrumentation

Variable	Resolution	Errors (10 min time interval)	Frequency response	Sampling rate
I_r	0.01% of I_{r0}	0.05% of I_{r0}	flat to 5c/s	8 per sec
I_q	0.02% of I_{q0}	0.05% of I_{q0}	flat to 5c/s	8 per sec
V	0.01% of V_0	0.05% of V_0	flat to 5c/s	8 per sec
ω	0.001 c/s	0.002 c/s	flat to 1c/s	2 per sec

The machine was not subject to load-frequency control during tests, and, although the voltage regulator was connected, voltage variations were so small that they rarely exceeded the dead-band of the regulator. As we are interested in the dynamics without regulator we will use the data collected with blocked governor. This data was measured during a period of 10 minutes.
To give and idea of the magnitude of the disturbances the normalized standard deviation, σ_x (r.m.s. value of variations/mean value of the variable), normalized maximal variations m_x (maximal variation/mean value of variable),

normalized resolution r_x (resolution/r.m.s. value of the variations) and normalized error e_x (error/ /r.m.s. value of variations) and the operating levels are presented in Table 2.

Table 2. Details of the data

Vari-able	σ_x %	m_x %	r_x %	e_x %	Operating level
I_r	0.32	2	4	16	0.90 p.u.
I_q	1.5	8	2	4	0.24 p.u.
V	0.11	0.8	10	46	1.04 p.u.
ω	0.036	0.3	6	12	314 rad/sec.

The electrical variables are expressed in per unit with a base power of 50 MW and a base voltage of 33 kV. Although sophisticated instrumentation was used the signal to noise ratio is low. We observe that the resolution of the terminal voltage is as low as ten per cent of the r.m.s. value of the variations and we expect difficulties when using this data for identification.

3. PRELIMINARY ANALYSIS OF THE DATA

To make a preliminary investigation of causality relationships between the variables we will estimate the coefficients h_r in a model

$$y_k = \sum_{-\infty}^{\infty} h_r u_{k-r} \qquad (1)$$

that does not assume a purely causal influence from time series (u_k) to (y_k). For a white noise input (u_k), h_r coincides with the cross correlation function between (y_k) and (u_k). The estimates of h_r are formed by a straightforward application of fourier transform technique. The fast fourier transform (FFT), which gives very short computing time, is used. The accuracy of the method motivates its use for preliminary analysis. The information wanted could also be obtained by determining the cross correlation function for pre-whitened data. A comparison shows that the FFT method is preferable in the present case.

The method was applied to several time series pairs, formed from the measured variables. The most decisive input-output pair among those considered turned out to be electric torque $(V \cdot I_r/\omega)$ to angular velocity (ω). Fig. 1 shows the estimated impulse response for this pair. The estimates also yield a basis for general theoretical considerations. For example, the direct coupling between reactive component of armature current and terminal voltage is readily seen from Fig. 2.

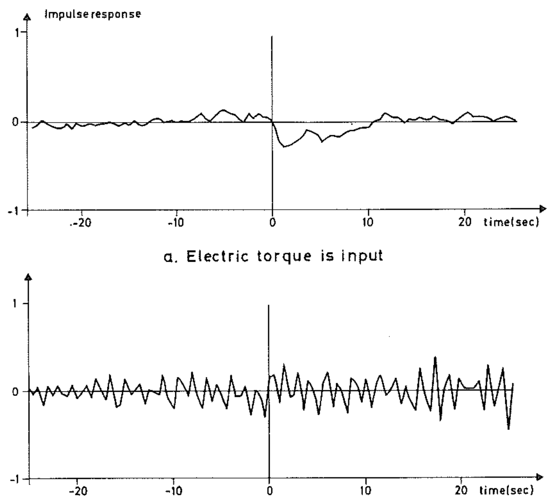


Fig. 1. Causality between electric torque and angular velocity. The figure shows h_r as in Eqn. (1). The data has been normalized so that $h_0 = 1$ corresponds to a direct coupling.

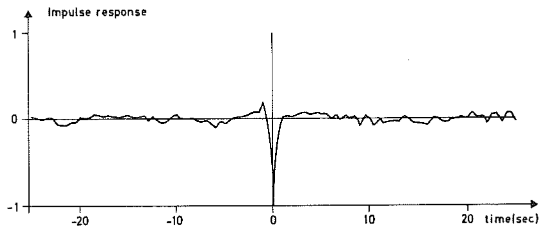


Fig. 2. Causality between reactive component of armature current and terminal voltage, when the reactive current is input. The figure shows h_k as in Eqn. (1). The data has been normalized so that $h_0 = 1$ corresponds to a direct coupling.

4. IDENTIFICATION OF THE DYNAMICS FROM ELECTRIC TORQUE TO ANGULAR VELOCITY

The electric torque to angular velocity dynamics can approximately be described, Lindahl (1972), by the following differential equation

$$J \frac{d\omega}{dt} + D\omega + I_r V/\omega = 0 \qquad (2)$$

where ω is the angular velocity of the rotor, J is the combined turbine-rotor inertia, D is the damping torque coefficient, V is the terminal voltage of the generator and I_r the active component of the armature current. Stanton (1965) has estimated D by curve fitting in a Bode diagram obtained by spectral analysis. Eqn. (2) corresponds to an exponential impulse response. The response in Fig. 1a is approximately of this type.

The disagreement between the actual response and a first order one indicates that Eqn. (2) does not hold strictly.

Spectral Analysis.
Straightforward spectral analysis, see Jenkins (1968), was applied to this input-output pair. For the spectral estimates a Tukey window with 100 lags as the width of the lag window was found to be suitable. Prefiltering of data gave no significant improvement and the result shown here is for the original data. Confidence intervals have been computed from the coherency spectrum, Jenkins (1968). In Fig. 3 the result of the identification is shown as a Bode diagram of the transfer function.

Estimation of Parameters in a General Linear Input-Output Model.
As it is not known a priori that a model like Eqn. (2) is compatible with the data a general input-output model with the canonical structure

$$y(t) + a_1 y(t-1) + \dots + a_n y(t-n) = b_1 u(t-1) + \dots + b_n u(t-n) + \lambda[e(t) + c_1 e(t-1) + \dots + c_n e(t-n)] \quad (3)$$

has been fitted to the data. The parameters have been estimated using the maximum likelihood method, see Åström (1966) and Gustavsson (1969). The loss functions obtained from models of different orders are shown in Table 3. The computations are based on N = 1000 input-output pairs. Since the sampling interval of (3) is 0.5 sec and 0.125 sec for the other variables a compatible data set is created by interpolating the angular velocity.

Table 3. Minimal values of the loss function V_n (sum of squared residuals) for maximum likelihood identification of models of different orders (n).

n	V_n	$t_{n,n+1}$
1	0.014421	126
2	0.009562	26
3	0.008560	73
4	0.006656	17
5	0.006230	2.0
6	0.006180	

The test quantities

$$t_{n,n+1} = \frac{V_n - V_{n+1}}{V_{n+1}} \cdot \frac{N - 4(n+1)}{4}$$

for testing the order of the model are given in Table 3. A straightforward application of the order tests indicates that a fifth order model is appropriate. The coefficients and the accuracy estimates of the fifth order model are shown in Table 4.

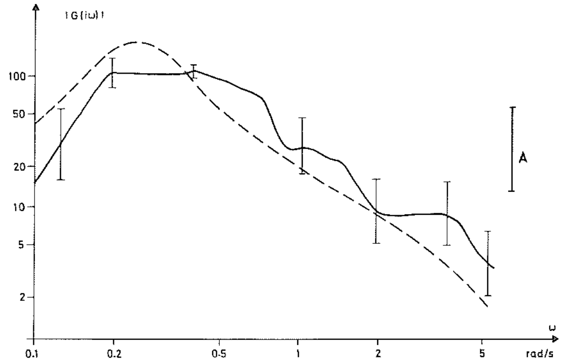
Table 4. Coefficients and accuracy estimates.

i	$\hat{a}_i \pm \sigma(\hat{a}_i)$	$\hat{b}_i \pm \sigma(\hat{b}_i)$	$\hat{c}_i \pm \sigma(\hat{c}_i)$
1	-2.628±0.031	-0.835±0.092	-0.899±0.036
2	2.290±0.074	1.120±0.244	-0.005±0.039
3	-0.715±0.082	-0.805±0.308	-0.013±0.036
4	0.078±0.067	0.599±0.250	-0.624±0.034
5	-0.025±0.026	-0.080±0.100	0.579±0.029

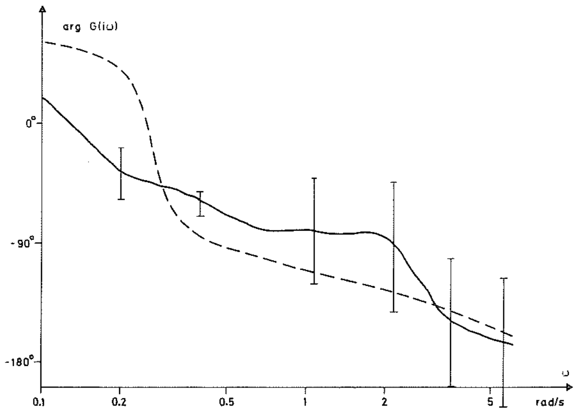
The standard deviation of the residuals is $0.25 \cdot 10^{-2}$ rad/sec and the r.m.s. value of the variations in angular velocity is $11.3 \cdot 10^{-2}$ rad/sec.

The Bode diagram of the corresponding transfer function is shown in Fig. 3. It should be noted when comparing the results of spectral analysis and maximum likelihood identification that the two methods show different features for systems with feedback effects. Feedback effects may cause the impulse response to be nonzero for $t<0$. In spectral analysis the impulse response for $t<0$ is taken account of in the model while it is neglected for the maximum likelihood method. The results are also illustrated in Fig. 4, which shows the measured input and output, the model output, the difference between the measured output and the model output, and the residuals. Fig. 4 shows that only about half of the observed output can be related to the input.

The results obtained indicate that the approximate first order model (2) is not compatible with data. An attempt was also made to determine the maximum likelihood estimates of D and J in (2) but only a minor part of the output could be explained by the model (2) with these estimates $\hat{D}$ and $\hat{J}$. This could be explained by the feedback from angular velocity to electric torque. Assuming that the generator is connected to an infinite bus via a lossless line the electric torque is given by $V \cdot E \cdot \sin\theta/(X_\ell)$ where E is the voltage of the bus and X_ℓ is the reactance of the line. The rotor angle θ is related to ω by $d\theta/dt = \omega$. This illustrates that the input (electric torque) depends on previous values of the output (angular velocity). The fact that all changes in electric torque result in limited changes in rotor angle explains the low gain at low frequencies in the Bode diagram.



a) Amplitude diagram.



b) Phase diagram.

Fig. 3. Bode diagram of the transfer function relating angular velocity (ω) to electric torque (VI_r/ω). Broken line: Fifth order model from maximum likelihood identification. Solid line: Model resulting from spectral analysis. 95% confidence intervals are marked out. Estimates whose confidence interval is larger than distance A or 180° resp. have been omitted and replaced by interpolated values.

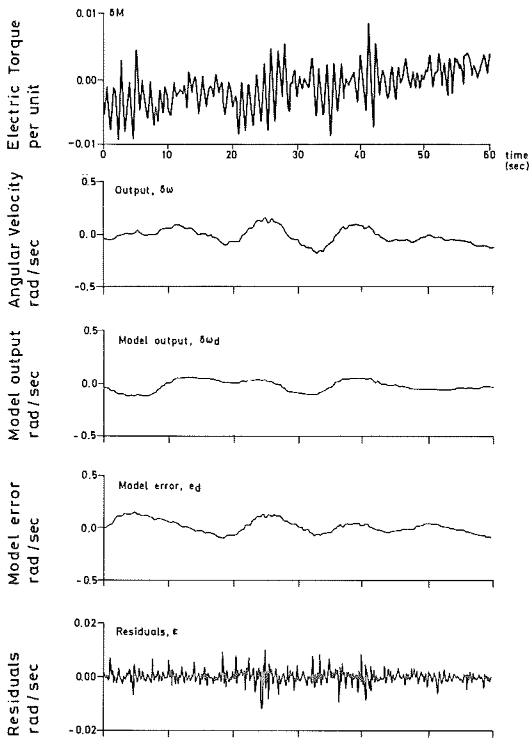


Fig. 4. Results of maximum likelihood identification of electric torque to angular velocity dynamics. The uppermost curve shows the input (electric torque, I_rV/ω, in per unit) and the measured output (angular velocity in rad/sec). The model output computed from the fifth order model with coefficients given in Table 4, the model error and the residuals are also shown in the diagram.

5. ESTIMATION OF PARAMETERS IN A FIFTH ORDER STATE SPACE MODEL

If attempt is made to treat any measured variable as input to a dynamic system there will be a feedback from the output to the input. The idea is to treat all measured variables as outputs from a dynamical system driven by noise only and to exploit basic physical laws governing the system. The following simple example may clarify the first point.

A simple example. Consider a linear time-invariant first order stochastic system:

$$x(t+1) = ax(t) + bu(t) + v(t)$$

$$y(t) = cx(t) + e(t) \qquad (4)$$

with the output feedback

$$u(t+1) = -\ell y(t) + w(t) \quad (5)$$

Substitution of (4) into (5) yields

$$\begin{bmatrix} x(t+1) \\ u(t+1) \end{bmatrix} = \begin{bmatrix} a & b \\ -\ell c & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} v(t) \\ -\ell e(t) + w(t) \end{bmatrix}$$

$$y(t) = [c \quad 0] \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} + e(t) \quad (6)$$

Instead of trying to estimate parameters in the first order system (4) with output feedback (5) we can try to estimate the parameters in the second order system (6) which is driven by noise only.

Construction of a Model Including Feedback.
The theory of electric machinery and electrical circuits provides some knowledge, which can be used to construct a model of the generator and its environment including feedback effects. Using this knowledge we will construct a model with angular velocity (ω), terminal voltage (V), reactive and active current (I_q and I_r) as outputs. To include a simple model of the environment it is assumed that the generator (generator 1) is connected to the network via a lossless transmission line. Furthermore it is assumed that the network can be represented by another generator (generator 2) with constant terminal voltage. To describe one generator we need at least two state variables, rotor angle and angular velocity. We decided to include also the field flux linkage into the state vector, which gives us a fifth order model. This is consistent with the results obtained in Section 4. The time derivatives of the states ($\dot{x} = dx/dt$) can be computed from a nonlinear function $f(\dot{x},x,z,\alpha)$, given in Appendix, where x is the state vector $(\theta_1,\omega_1,\psi_f,\theta_2,\omega_2)$, z is a vector of intermediate variables and α is a vector of parameters having physical interpretation. The output vector (y) is defined by a nonlinear equation $g(y,x,z,\alpha)=0$ given in Appendix. Another nonlinear equation, $h(x,z,\alpha) = 0$, defining the intermediate variable z, is also given in the Appendix.

Construction of a Linear Stochastic Difference Equation.
The objective of the identification is in the first place to determine the physical parameters α. From these parameters and the functions $f(\dot{x},x,z,\alpha)$, $g(y,x,z,\alpha)$ and $h(x,z,\alpha)$ various models can then be derived. We are particularly interested in a linear stochastic discrete-time model:

$$x(t+T) = \Phi(\alpha)x(t) + Ke(t)$$
$$y(t) = C(\alpha)x(t) + e(t) \quad (7)$$

The deterministic part of (7) $\left(\Phi(\alpha) \text{ and } C(\alpha)\right)$ is derived by linearization of f, g and h. The first step is to compute the operating point $(\bar{x},\bar{y},\bar{z})$ by solving the equations $f(0,\bar{x},\bar{z},\alpha) = 0$, $g(\bar{y},\bar{x},\bar{z},\alpha) = 0$ and $h(\bar{x},\bar{z},\bar{\alpha}) = 0$. After linearization around the operating point we have

$$f_{\dot{x}} \frac{d}{dt}(x-\bar{x}) + f_x(x-\bar{x}) + f_z(z-\bar{z}) = 0$$

$$g_y(y-\bar{y}) + g_x(x-\bar{x}) + g_z(z-\bar{z}) = 0$$

$$h_x(x-\bar{x}) + h_z(z-\bar{z}) = 0$$

where the partial derivatives are evaluated at $dx/dt = 0$, $x = \bar{x}$, $y = \bar{y}$ and $z = \bar{z}$. Now a model building program, described by Eklund (1970) is used to eliminate the intermediate variables z, and the state space model

$$\frac{dx}{dt} = A(\alpha)x \qquad y = C(\alpha)x$$

is obtained.

After transformation to discrete time the resulting model is

$$x(t+T) = \Phi(\alpha)x(t) \qquad y(t) = C(\alpha)x(t)$$

where $\Phi(\alpha) = \exp[A(\alpha)T]$ and T is the sampling interval. The linearization must be repeated for each current value of the vector α, suggested by the iterative identification algorithm. The stochastic part of the model (7) (K and the covariance matrix of e) is estimated together with the parameter vector α as described below.

Method of Identification.
To estimate the parameter vector α in (7) we use the maximum likelihood method. The likelihood function is given by

$$-L(\alpha,K,R) = \frac{1}{2}\sum_{t=1}^{N} \varepsilon(t)R^{-1}\varepsilon(t) + \frac{N}{2}\ln \det R + \frac{Nr}{2}\ln 2\pi$$

where r is the dimension of the e-vector, N is the record length, and R the covariance of e(t). The maximization with respect to R can be done analytically, and Eaton (1967) has shown that the maximum of $L(\alpha,K,R)$ is obtained by finding α, K which minimizes

$$V(\alpha,K) = \det\left[\sum_{t=1}^{N} \varepsilon(t)\varepsilon^T(t)\right] \quad (8)$$

These values are found by numerical minimization. The maximization with respect to R yields

$$\hat{R} = \frac{1}{N}\sum_{t=1}^{N} \varepsilon(t)\varepsilon^T(t)$$

where $\{\varepsilon(t)\}$ are the residuals defined by

$$\varepsilon(t) = y(t) - C(\alpha)\hat{x}(t)$$

$$\hat{x}(t+1) = \Phi(\alpha)\hat{x}(t) + K\varepsilon(t)$$

and

$$\hat{x}(1) = x_0$$

Number of Parameters and Identifiability.
The deterministic part of the model (7) is defined by ten parameters, (α). Since nothing is known about K, there are 30 parameters in the stochastic part (K and R) of the model. A canonical representation of the model will, in this case, contain 50 unknown parameters, which gives an upper limit for the number of identifiable parameters, Åström (1971). The problem is now to find a set of parameters which gives an identifiable model and for which the fixed parameters do not essentially restrict the dynamics defined by the model. An attempt was made to estimate all 40 parameters, but the values of r_a and r_f became unrealistic without changing the loss function $V(\alpha,K)$ essentially. We then decided not to include these parameters in the model and the number of parameters became 38. The estimation of parameters in a general linear model relating angular velocity to electric torque indicated that the sampling interval (0.125 sec) was short compared to the dominant time constants. To avoid interpolation in the ω-measurements the sampling interval was increased to 0.5 sec.

Result of Identification.
The result is based on 500 sampling events and the numerical values of the estimated parameters defining the deterministic part of the model are given below and initial values are given within brackets.

$\hat{J}_1$ = 0.00872 (0.22) $\quad$ $\hat{J}_2$ = 0.121 (5.0)
$\hat{D}$ = 0.0051 (0.01) $\quad$ $\hat{X}_\ell$ = 1.112 (0.1)
$\hat{X}_{ff}$ = 2.36 (2.0) $\quad$ $\hat{X}_{af}$ = 1.341 (1.2)
$\hat{X}_d$ = 0.846 (1.0) $\quad$ $\hat{X}_q$ = 0.888 (0.8)

The estimated K-matrix, $\hat{K}$ equals

$$\hat{K} = \begin{bmatrix} 0.00530 & -1.147 & -0.222 & -0.0633 \\ 1.201 & 0.388 & 0.0159 & -0.0162 \\ -0.000012 & 0.0081 & 0.00177 & 0.000527 \\ 0.00521 & 1.417 & 0.160 & 0.0320 \\ 1.205 & -0.0576 & 0.0161 & 0.00939 \end{bmatrix}$$

The estimated covariance matrix, $\hat{R}$ equals

$$\hat{R} = 10^{-6}\begin{bmatrix} 602.0 & 1.21 & 9.37 & -24.1 \\ 1.21 & 0.231 & -0.843 & -0.233 \\ 9.37 & -0.843 & 4.41 & -1.45 \\ -24.1 & -0.233 & -1.45 & 5.90 \end{bmatrix}$$

The standard deviations of the one-step ahead prediction error then are:

Angular velocity: $2.46\cdot10^{-2}$ $(11.3\cdot10^{-2})$ rad/sec
Terminal voltage: $4.88\cdot10^{-4}$ $(11.4\cdot10^{-4})$ p.u.
Reactive current: $2.13\cdot10^{-3}$ $(3.6\cdot10^{-3})$ p.u.
Active current: $2.44\cdot10^{-3}$ $(3.2\cdot10^{-3})$ p.u.

The r.m.s. values of the corresponding variations are given within brackets.

The value of the loss function (8) has been decreased from $8.0\cdot10^{-7}$ to $1.45\cdot10^{-11}$ by the adjustjustments of the parameters.

In Fig. 5 the measured variables and the corresponding output of the one-step ahead prediction are shown. Notice that the data shown in Fig. 5 has not been used to estimate the parameters.

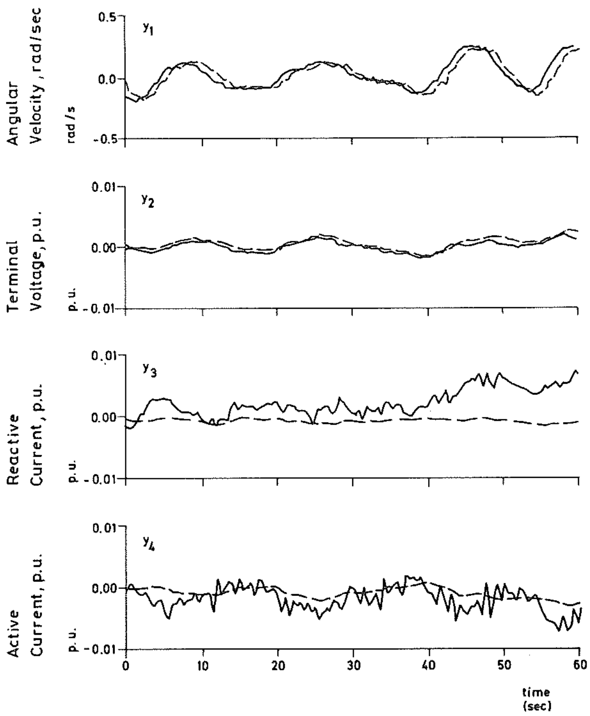


Fig. 5. Comparison of measured variables (solid line) and output from the one-step ahead predictor (broken line). This data has not been used to estimate the parameters.

6. CONCLUSIONS.

The data used for identification has been collected during normal operation. This means in fact that data from a closed loop system is used which requires special attention. Stanton (1965) has, by applying spectral analysis to the same data records, concluded that a first order system describes the dynamics from electric torque to angular velocity. This result does not seem to be consistent with the results obtained in Section 4. The order test indicates that a fifth order model is required to describe the dynamics. The fifth order model obtained from maximum likelihood identification is found to be consistent with results of spectral analysis. An approach to identify systems with feedback has been presented in Section 5. A fifth order state space model has been constructed from physical a priori knowledge. Feedback effects have been included in the structure of the model which has no input but noise. Eight uncertain physical parameters and thirty parameters describing noise have been estimated using a maximum likelihood criterion. The estimated values of the reactances and the damping torque coeffecient are very reasonable. Summarizing, it is found that a model of at least fifth order is required to describe the dynamics of the system. Furthermore a fifth order model has good physical interpretation and can be chosen so that it describes the generator and its environment well.

7. ACKNOWLEDGEMENTS

The authors want to thank professor K.J. Åström, who proposed the problem, for his valuable comments and suggestions, and Dr. K.N. Stanton for providing the data. The authors also want to thank Mrs. G. Christensen, who typed the manuscript, and Mr. B. Lander, who drew the figures.
This work has been partially supported by the Swedish Board of Technical Development under contract 72-202/U137.

8. REFERENCES

Åström, K.J., and Bohlin, T. (1966), "Numerical Identification of Linear Dynamic Systems from Normal Operating Records", In "Theory of Self-Adaptive Systems", P.H. Hammond (ed.) Plenum Press .

Åström, K.J., and Eykhoff, P. (1971), "System Identification - A Survey", Automatica, Vol. 7, p 123.

Eaton, J. (1967), "Identification for Control Purposes", IEEE Winter meeting, New York.

Eklund, K. (1970), "Numerical Modelbuilding", Int. J. Control Vol 11 No. 6, p 973.

Gustavsson, I. (1969), "Parametric Identification of Multiple Input Single Output Linear Dynamical Systems", Report 6907, July 1969, Lund Institute of Technology, Division of Automatic Control.

Jenkins, G.M., and Watts, D.G. (1968), "Spectral Analysis and its Applications", Holden-Day, San Fransisco.

Lindahl, S., and Ljung, L. (1972), "Identification of Power Generator Dynamics from Normal Operating Data", Report 7210, May 1972, Lund Institute of Technology, Division of Automatic Control.

Stanton, K.N. (1964), "An Incremental Wattmeter for Use in Interconnected Power Systems", Elect. Mech. Engng. Trans. Instn. Engrs., Aust., May 1964, p 20.

Stanton, K.N. (1965), "Estimation of Turboalternator Transfer Functions Using Normal Operating Data", Proc. I.E.E., Vol 112, p 1713.

APPENDIX

The time derivative of the difference between a fixed point on the rotor and the standard time phasor $(\theta_1-\omega_0 t)$ is given by

$$f_1 = \frac{d\theta_1}{dt} - (\omega_1-\omega_0) = 0$$

where ω_1 is the angular velocity at the rotor. Conservation of the angular momentum of the rotor implies

$$f_2 = \frac{d\omega_1}{dt} - [P_{m1}/\omega_1 - M_{e1} - D(\omega_1-\omega_2)]/J_1 = 0$$

where P_{m1} is the mechanical power from the turbine, M_{e1} is electric torque, D is the damping torque coefficient, and J_1 is the combined turbine-rotor inertia. The time derivative of the field flux (ψ_f) is obtained from the induction law

$$f_3 = \frac{d\psi_f}{dt} - (v_f - i_f r_f) = 0$$

where v_f is the field voltage, i_f is the field current, and r_f is the field resistance. The angle of the terminal voltage of the equivalent generator (θ_2) is assumed to coincide with a fixed point on the rotor. Denoting the angular velocity of the rotor by ω_2 the following differential equations describe the equivalent generator

$$f_4 = \frac{d\theta_2}{dt} - (\omega_2-\omega_0) = 0$$

$$f_5 = \frac{d\omega_2}{dt} - [P_{m2}/\omega_2 - M_{e2} - D(\omega_2-\omega_1)]/J_2 = 0$$

where P_{m2} is the mechanical power from turbine 2, M_{e2} is the electric torque, and J_2 is the combined turbine-rotor inertia. The angular velocity (ω_1) and the terminal voltage (V) of generator 1 are measured:

$$g_1 = y_1 - \omega_1 = 0$$

$$g_2 = y_2 - \sqrt{v_d^2 + v_q^2}$$

where v_d and v_q denote d-axis and q-axis component of the terminal voltage. The active and the reactive part of the armature current $(I_r=y_3$ and $I_q=y_4)$ are given by

$$g_3 = I_q - (\cos \delta i_d - \sin \delta i_q) = 0$$

$$g_4 = I_r - (\sin \delta i_d + \cos \delta i_q) = 0$$

where δ is the load-angle, i_d and i_q are the d-axis and q-axis components of the armature current. The electric torque (M_{e1}) is given by

$$h_1 = M_{e1} - (\psi_d i_q - \psi_q i_d) = 0$$

where ψ_d and ψ_q are the d-axis and q-axis components of the armature flux linkage. The total power demand is assumed to be a stochastic process with mean P_d which is given by

$$h_2 = P_d - \left[\omega_2 M_{e2} - \omega_1 M_{e1} - r_a(i_d^2+i_q^2)\right] = 0$$

where r_a is the armature resistance of generator 1. The flux linkages are assumed to be linear functions of the currents

$$h_3 = \psi_f - (X_{ff} i_f - X_{af} i_d)/\omega_0 = 0$$

$$h_4 = \psi_d - (X_{af} i_f - X_d i_d)/\omega_0 = 0$$

$$h_5 = \psi_q + X_q i_q/\omega_0 = 0$$

where X_{ff}, X_{af}, X_d and X_q denote reactances of generator 1. The load-angle is given by

$$h_6 = \mathrm{tg}\ \delta - v_d/v_q = 0$$

The induced voltages are given by

$$h_7 = v_q + r_a i_q - \omega_1 \psi_d = 0$$

$$h_8 = v_d + r_a i_d + \omega_1 \psi_q = 0$$

Finally we have expressions for the terminal voltages (v_2) of generator 2:

$$h_9 = v_q - v_2 \cos(\theta_1-\theta_2) + X_\ell i_d = 0$$

$$h_{10} = v_q - v_2 \cos(\theta_1-\theta_2) + X_\ell i_d = 0$$

where X_ℓ denotes the reactance of the lossless transmission line.