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DEPARTMENT OF
BUILDING SCIENCE

DIVISION OF
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Dynamic models for inlet air to roomair or
outlet air temperature

L H Jensen

DYNAMIC MODELS FOR INLET AIR TO ROOMAIR OR OUTLET AIR
TEMPERATURE

L.H. Jensen

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1 Introduction

The main purpose with the work is to study if dynamic models can be obtained from construction data for ventilated rooms. The models can then be used in simulation of control systems and be used to synthesize regulators.

The study is carried out by comparing models based on construction data and models based on experimental data. Three different rooms are used. These are described in section 2.

The model building is carried out in section 3. A first and a second order model are developed and used to determine the gain and the timeconstant.

Models based on experimental data are determined in section 4. The identification method and model are also described.

Finally a comparison is made in section 5 between the models obtained in two different ways.

2 The rooms

2.1 The lecture room (LR)

The lecture room has the following data:

length	11	m
width	7.5	m
height	3.2	m
volume	260	m ³

The number of airchanges is 10.8 per hour.

The room is furnished as normal with 20 tables and 40 chairs. About 80 m² of sound baffles are placed under the ceiling. The inlet air is injected to the lecture room in five cross-sections in the ceiling. The outlet air is evacuated in four cross-sections in between.

The temperature sensors are placed in the inlet and outlet airstreams. The outlet air temperature was chosen as output in this case. This temperature is normally measured to control the room air temperature. It is often rather difficult to find a good room air temperature sensor location in large rooms.

The room and the temperature sensor locations are shown in figure 2.1. Further details can be found in Ekström et al (1974).

3 Models based on construction data

A simple first and second order model will be derived for the input output system inlet air to roomair or outlet air temperature. After this is done data from section 2 will be used to determine the model parameters.

3.1 A first order model

A very simple first order model for the input-output system inlet air temperature to roomair or outlet air temperature can easily be obtained if the room air is assumed to be totally mixed and the heated air, the walls (the ceiling, the floor and the furniture is supposed to be included in "walls") and the outdoor air temperature are assumed to be inputs and not affected by the room air temperature. This type of model will not make any difference between the roomair temperature or the outlet air temperature.

The assumption that the temperature of the walls is not affected by the roomair temperature may be a good approximation if the heat capacity of the walls is far larger than the heat capacity of the roomair. Then the wall temperature will only change very slowly and with small amounts.

The heat balance equation becomes:

$$C \dot{x}(t) = -(nC + Ah) x(t) + nCu_1(t) + Ahu_2(t)$$

Here is $x(t)$ = roomair temperature

$u_1(t)$ = inlet air temperature

$u_2(t)$ = other temperature inputs

A = surface between roomair and walls

C = roomair heat capacity

h = heattransfercoefficient for surface A

n = the number of roomair changes per time unit

If the heat balance equation is Laplace transformed one will get the transferfunction between the input $u_1(t)$ and the output $x(t)$ as:

$$G_1(s) = \frac{K_1}{sT + 1}$$

The relations between parameters in the heat balance equation the ones in the transferfunctions are:

$$\begin{aligned} T &= C/(nC + Ah) \\ K_1 &= nT \end{aligned}$$

3.2 A second order models

In the first order model the walls, the floor, the ceiling and so on were assumed to have infinite thermal capacitance and conductance or were assumed to have the same constant temperature. This is a very crude approximation. If the walls, the floor, the ceiling and so on are assumed to have a fixed mass with a fixed thermal capacitance then the first order model can easily be turned into a second order model. The fast mode of the transferfunction is then of interest. The heatbalance equations for the two masses the roomair $x_1(t)$ and the surrounding mass $x_2(t)$ now becomes

$$C_1 \dot{x}_1(t) = -(Ah + nC_1)x_1(t) + Ahx_2(t) + nC_1u_1(t)$$

$$C_2 \dot{x}_2(t) = -Ahx_2(t) + Ahx_1(t)$$

The surrounding mass can only exchange heat with the roomair mass in this model.

The transferfunction of this second order system can always be written as follows

$$G(s) = \frac{K_1}{sT_1 + 1} + \frac{K_2}{sT_2 + 1}$$

Only the fast part will be of interest when synthesizing a regulator or simulating a regulator. The earlier derived first order model can be regarded as a special case of the second order model when C_2 is assumed to be infinite.

3.3 Application of models

The timeconstant and the gain will be computed for the three rooms. In the second order model the thermal capacitance of the surrounding C_2 will be equal C_1 and $100 C_1$. The heattransfer coefficient between the roomair and the surrounding h has been 1, 2 and 3 $W/m^2 \text{ } ^\circ C$. The result of the different calculations are given in table 3.1-3.6 for the three rooms.

Table 3.1 Computed timeconstant T and gain K as a function of C_2/C_1 and h_{12} for the lecture room with $C_1 = 312 \text{ kJ/}^\circ\text{C}$ $A = 406 \text{ m}^2$ $n = 10.8 \text{ /h}$

C_2/C_1	h_{12} $\text{W/m}^2 \text{ }^\circ\text{C}$	T min	K
-			-
∞	1.	3.9	0.70
∞	2.	3.0	0.54
∞	3.	2.4	0.43
100	1.	3.9	0.70
100	2.	3.0	0.53
100	3.	2.4	0.43
1.	1.	3.5	0.55
1.	2.	2.3	0.32
1.	3.	1.7	0.21

Table 3.2 Computed timeconstant T and gain K as a function of C_2/C_1 and h_{12} for the test room number one with $C_1 = 58 \text{ kJ/}^\circ\text{C}$ $A = 87 \text{ m}^2$ $n = 12.4 \text{ /h}$

C_2/C_1	h_{12} $\text{W/m}^2 \text{ }^\circ\text{C}$	T min	K
-			-
∞	1.	3.4	0.70
∞	2.	2.6	0.53
∞	3.	2.1	0.43
100.	1.	3.4	0.70
100.	2.	2.6	0.53
100.	3.	2.1	0.43
1.	1.	3.0	0.55
1.	2.	2.0	0.32
1.	3.	1.5	0.21

Table 3.3 Computed timeconstant T and gain K as a function of C_2/C_1 and h_{12} for the test room number one with $C_1 = 58 \text{ kJ/}^\circ\text{C}$ $A = 87 \text{ m}^2$ $n = 10.4 \text{ /h}$

C_2/C_1	h_{12} $\text{W/m}^2 \text{ }^\circ\text{C}$	T min	K
-	-	-	-
∞	1.	3.8	0.66
∞	2.	2.8	0.49
∞	3.	2.3	0.39
100.	1.	3.8	0.66
100.	2.	2.8	0.49
100.	3.	2.5	0.39
1.	1.	3.3	0.49
1.	2.	2.1	0.27
1.	3.	1.6	0.18

Table 3.4 Computed timeconstant T and gain K as a function of C_2/C_1 and h_{12} for the test room number two with $C_1 = 60 \text{ kJ/}^\circ\text{C}$ $A = 80 \text{ m}^2$ $n = 8.2 \text{ /h}$

C_2/C_1	h_{12} $\text{W/m}^2 \text{ }^\circ\text{C}$	T min	K
-	-	-	-
∞	1.	4.6	0.63
∞	2.	3.4	0.46
∞	3.	2.7	0.36
100.	1.	4.6	0.63
100.	2.	3.4	0.46
100.	3.	2.6	0.36
1.	1.	3.9	0.44
1.	2.	2.5	0.24
1.	3.	1.8	0.16

Table 3.5 Computed timeconstant T and gain K as a function of C_2/C_1 and h_{12} for the test room number two with $C_1 = 60 \text{ kJ/}^\circ\text{C}$ $A = 80 \text{ m}^2$ $n = 6.6 \text{ /h}$

C_2/C_1	h_{12} $\text{W/m}^2 \text{ }^\circ\text{C}$	T min	K
-			-
∞	1.	5.3	0.58
∞	2.	3.7	0.41
∞	3.	2.9	0.31
100.	1.	5.3	0.58
100.	2.	3.7	0.40
100.	3.	2.8	0.31
1.	1.	4.3	0.37
1.	2.	2.6	0.19
1.	3.	1.8	0.12

Table 3.6 Computed timeconstant T and gain K as a function of C_2/C_1 and h_{12} for the test room number two with $C_1 = 60 \text{ kJ/}^\circ\text{C}$ $A = 80 \text{ m}^2$ $n = 3.2 \text{ /h}$

C_2/C_1	h_{12} $\text{W/m}^2 \text{ }^\circ\text{C}$	T min	K
-			-
∞	1.	7.5	0.40
∞	2.	4.7	0.25
∞	3.	3.4	0.18
100.	1.	7.5	0.40
100.	2.	4.7	0.25
100.	3.	3.4	0.18
1.	1.	5.2	0.18
1.	2.	2.9	0.09
1.	3.	2.0	0.06

4 Models based on experimental data

In this section models will be obtained by using experimental data.

4.1 Identification experiments

To be able to identify the dynamics of a process, a suitable input signal has to be chosen. This was done by using a PRBS (Pseudo Random Binary Sequence) signal. Details about the PRBS signal can be found in Davis (1970). The signal sequence assumes only two values. The PRBS signal is given by its basic period T and its order n . The sampling interval has been one minute in all experiments. The PRBS signal has usually been controlling the effect to an electrical air heater or the valve position to a water to air crossflow heatexchanger.

Details about the PRBS experiments are given in table 4.1.

Table 4.1 Identification experiments

Room	PRBS		number of samples	number of airchanges per hour
	n	T min		
LR	7	4	400	10.8
TR1	7	4	300	12.4
TR1	5	10	273	10.4
TR2	5	5	130	8.2
TR2	5	5	230	6.6
TR2	5	5	400	3.2

4.2 Model and identification method

Using the experimental data described above, the dynamics from the inlet air temperature (denoted $u(t)$) to the roomair temperature or the outlet air temperature (denoted $y(t)$) was modelled as follows. First the coefficients of a difference equation

$$y(t) + a_1y(t-1) + \dots + a_ny(t-n) = b_1u(t-k-1) + \dots + b_nu(t-k-n) + v(t) \quad (4.1)$$

were determined using a least squares criterion. The model parameters a_i and b_i are thus found by minimizing the lossfunction

$$V = \sum_{t=1}^N v(t)^2 \quad (4.2)$$

Further details about the method are given in Åström (1968).

The identification was performed using the identification program package IDPAC, see Gustavsson, Selander and Wieslander (1973).

4.3 Identification results

Using the technique outlined in section 4.2 models of first and second order have been determined from the experiments given in table 4.1. The delay parameter k has been 0 and 1.

The models turned out to be of first order and without any delay. All the determined first order models are given in table 4.2. The standard deviation of the modelerror is given as λ . The corresponding continuous time timeconstant and static gain

are given in table 4.3.

These first order models describe the process very well which can be seen in the simulations of the models in figure 4.1 - 4.5.

Table 4.2 Identified discrete time model parameters and model error standard deviation λ .

Room	Number of airchanges /h	a_1	b_1	λ
LR	10.8	-0.71	0.074	0.076
TR1	12.4	-0.66	0.106	0.047
TR1	10.4	-0.74	0.095	0.105
TR2	8.2	-0.70	0.079	0.027
TR2	6.6	-0.73	0.069	0.020
TR2	3.2	-0.83	0.038	0.018

Table 4.3 Identified continuous time model parameters

Room	Number of airchanges /h	Timeconstant T min	Static gain K
LR	10.8	2.8	0.25
TR1	12.4	2.4	0.31
TR1	10.4	3.3	0.37
TR2	8.2	2.8	0.26
TR2	6.6	3.2	0.25
TR2	3.2	5.2	0.22

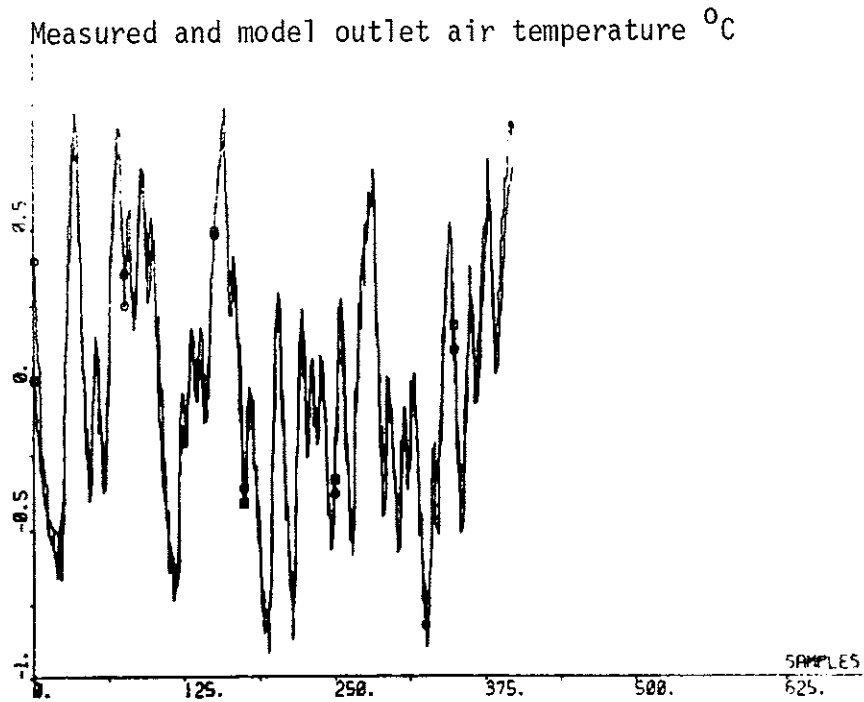


Figure 4.1 Measured and model outlet air temperature for the lecture room. The number of airchanges is 10.8 per hour.

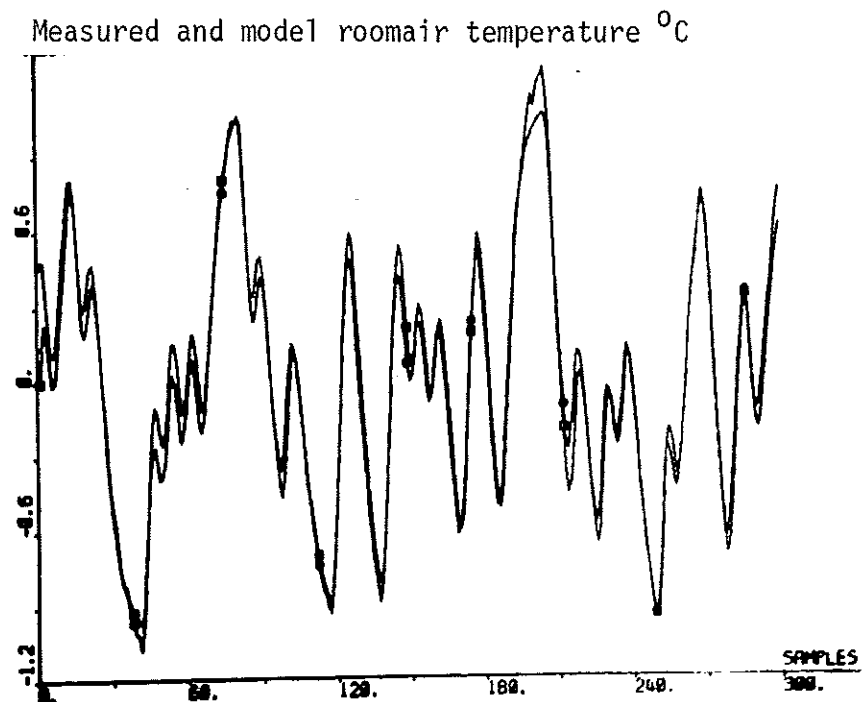


Figure 4.2 Measured and model roomair temperature for the test room number one. The number of airchanges is 12.4 per hour.

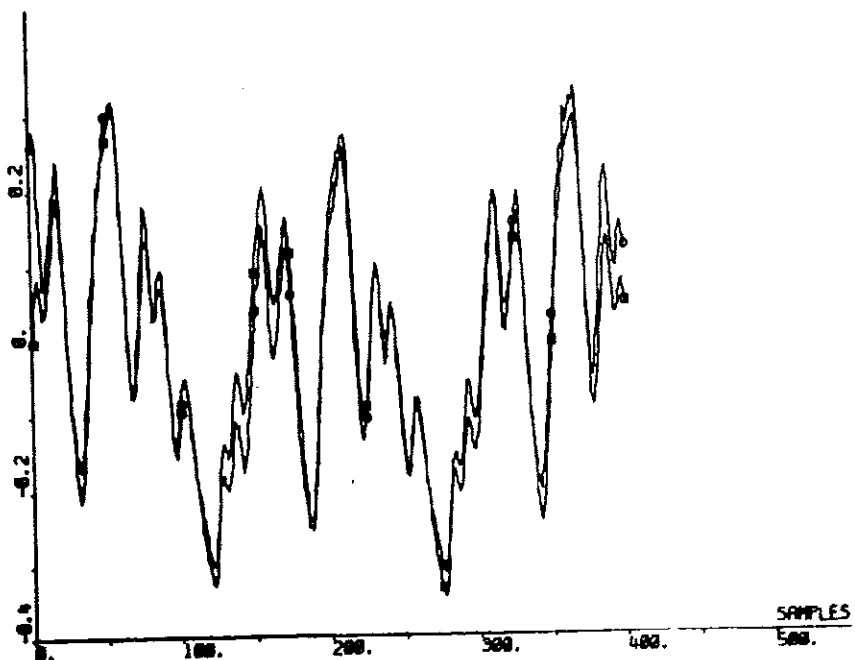
Measured and model roomair temperature $^{\circ}\text{C}$ 

Figure 4.3 Measured and model roomair temperature for the test room number two. The number of airchanges is 3.2 per hour.

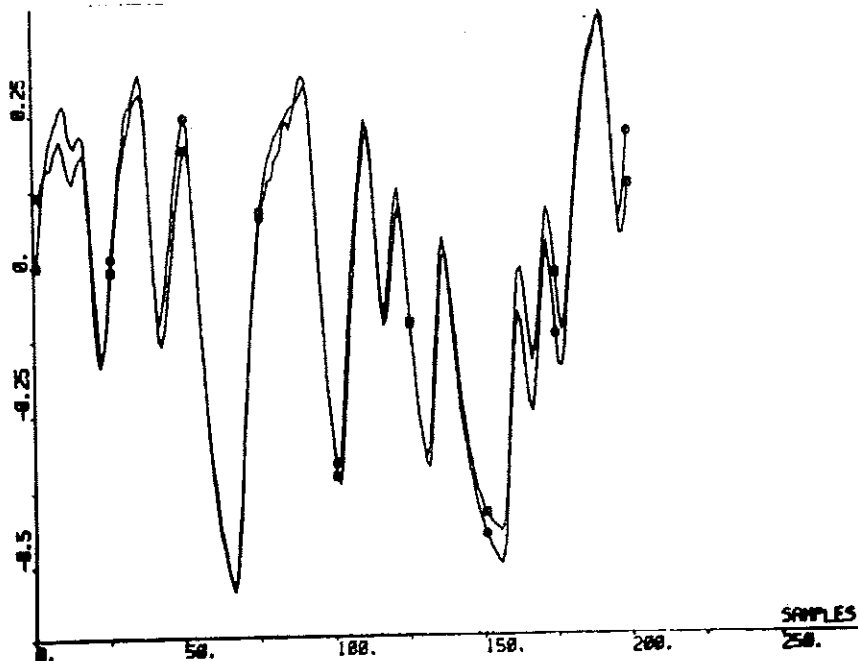
Measured and model roomair temperature $^{\circ}\text{C}$ 

Figure 4.4 Measured and model roomair temperature for the test room number two. The number of airchanges is 6.6 per hour.

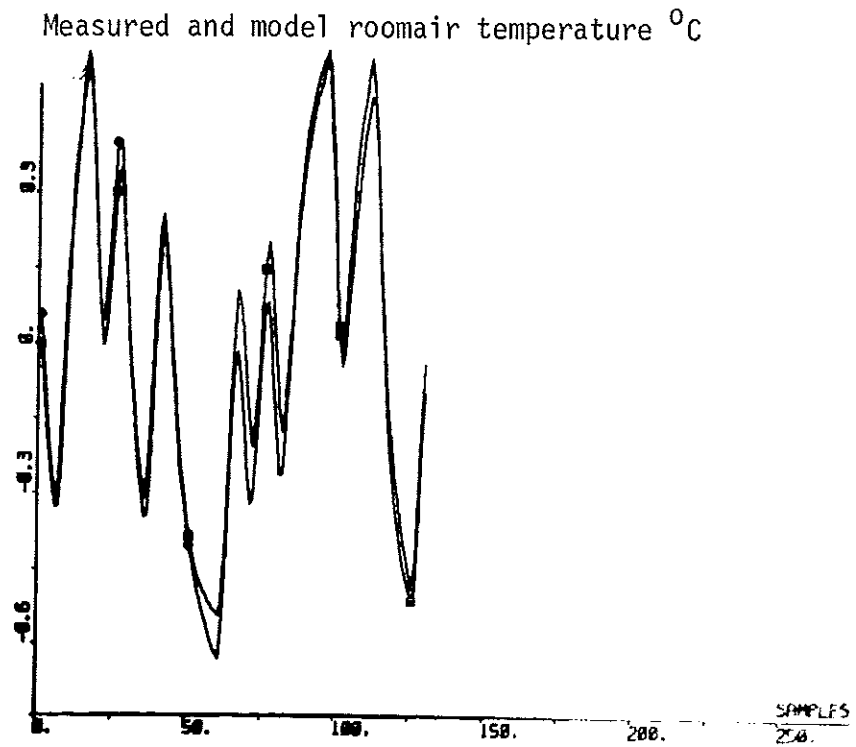


Figure 4.5 Measured and model roomair temperature for the test room number two. The number of airchanges is 8.2 per hour.

5 Comparison and conclusions

A simple method to compare the model parameters obtained in different ways is to plot the ratio between the identified and the construction data model parameters in a diagram. The vertical axis is the static gain ratio and the horizontal axis is the timeconstant ratio. The model parameter based on construction data are given for three values on h the heat transfer coefficient ($h = 1, 2$ and $3 \text{ W/m}^2 \text{ }^\circ\text{C}$) and for two values on C_2 the surrounding heat capacity ($C_2=C_1$ and $C_2=\infty$). The result of this plotting is shown in figure 5.1-5.6.

The timeconstant ratio for the first order model is 1.0 if the heat transfer coefficient is chosen suitable. This value is close to $2 \text{ W/m}^2 \text{ }^\circ\text{C}$. The ratio changes with the number of airchanges per hour (see figure 5.4-5.6). It should however be pointed out that the temperature difference between the air and the surroundings has not been the same in the different experiments (see figure 4.3-4.5). The heat transfer coefficient depends also on the temperature difference.

One exception is the experiment with test room two with 10.4 airchanges per hour (see figure 5.2). The identified model parameters indicate that the heat transfer coefficient h should be about $1.5 \text{ W/m}^2 \text{ }^\circ\text{C}$ and not $2-3 \text{ W/m}^2 \text{ }^\circ\text{C}$ as in the experiment with 12.4 airchanges per hour. The identified timeconstant is however sensitive to trends in data. Only the mean value has been removed at the identification in this case.

The timeconstant ratio for the second order model is usually one for a heat transfer coefficient between $1 \text{ W/m}^2 \text{ }^\circ\text{C}$ to $2 \text{ W/m}^2 \text{ }^\circ\text{C}$.

A suitable value on the heat transfer coefficient to compute the timeconstant would be to choose $2 \text{ W/m}^2 \text{ }^\circ\text{C}$, if the first order model is used.

The gain ratio is far from one. The second order model is closer to the right value. A simple formula which computes an approximate gain, has been found and is given below

$$K_a = \frac{nC}{(Ah + 2nC)}$$

The gain can never exceed 0.5. The gain and the timeconstant has been computed and is compared in table 5.1.

Another conclusion is that a room cannot be regarded as a pure mixing chamber with gain 1 and a timeconstant equal to the time of one airchange. This is also shown by the figures in table 5.1.

Table 5.1 Approximate gain K_a and timeconstant T_a for a first order model with $h = 2 \text{ W/m}^2 \text{ } ^\circ\text{C}$, identified gain K_i and timeconstant T_i for a first order model and time per airchange T_{ac} for different rooms.

Room	Number of airchanges per hour	K_a	K_i	T_a min	T_i min	T_{ac} min
LR	10.8	0.35	0.25	3.	2.8	5.5
TR1	10.4	0.33	0.37	2.8	3.3	5.8
TR1	12.4	0.35	0.31	2.6	2.4	4.8
TR2	3.2	0.20	0.22	4.7	5.2	18.7
TR2	6.6	0.29	0.25	3.7	3.2	9.1
TR2	8.2	0.32	0.26	3.4	2.8	7.3

A summing up gives:

- The process air inlet temperature to room air temperature or air outlet temperature has been identified as first order system or can be regarded as a mixing chamber with heat exchange with the surrounding surfaces.
- The time constant can be computed approximately with the formula $T = C / (nC + Ah)$ where $h = 2 \text{ W/m}^2 \text{ } ^\circ\text{C}$.
- The gain can be computed approximately with the formula $K = nC / (Ah + 2nC)$ where $h = 2 \text{ W/m}^2 \text{ } ^\circ\text{C}$.

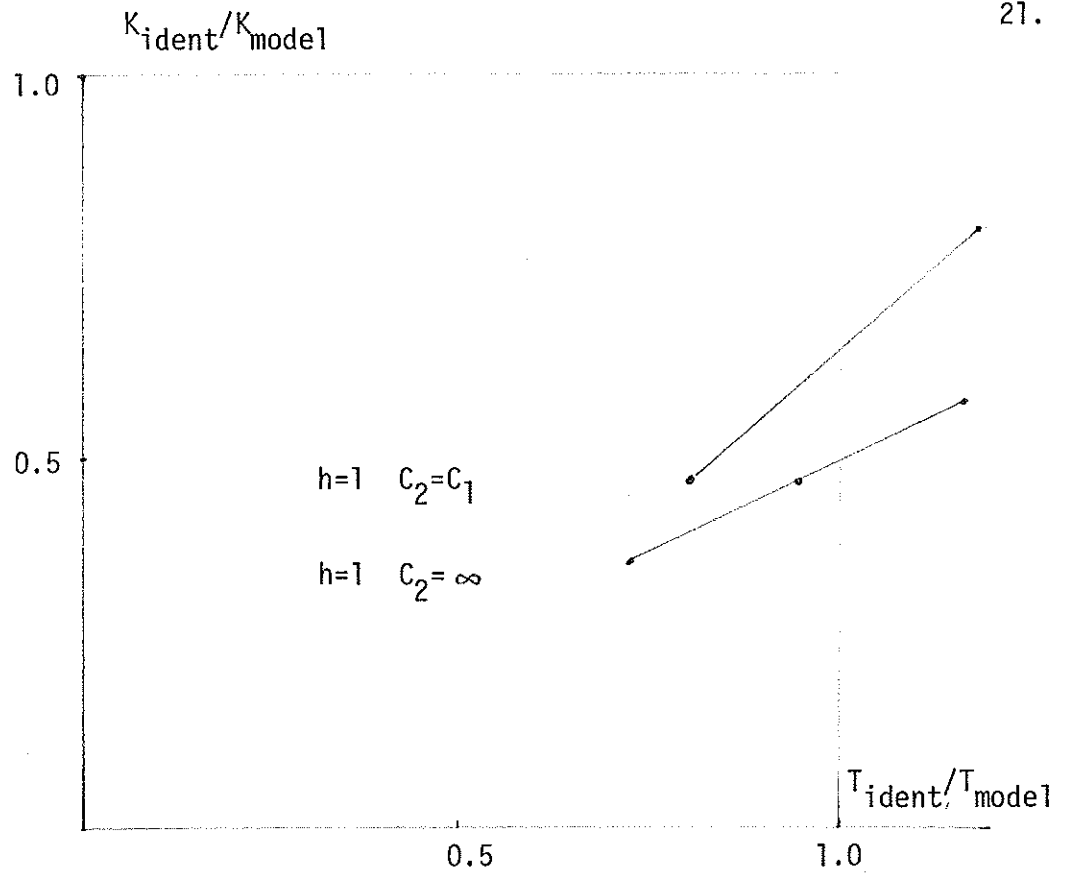


Figure 5.1 The ratios between identified and model parameters for the lecture room ($n=10.8/h$)

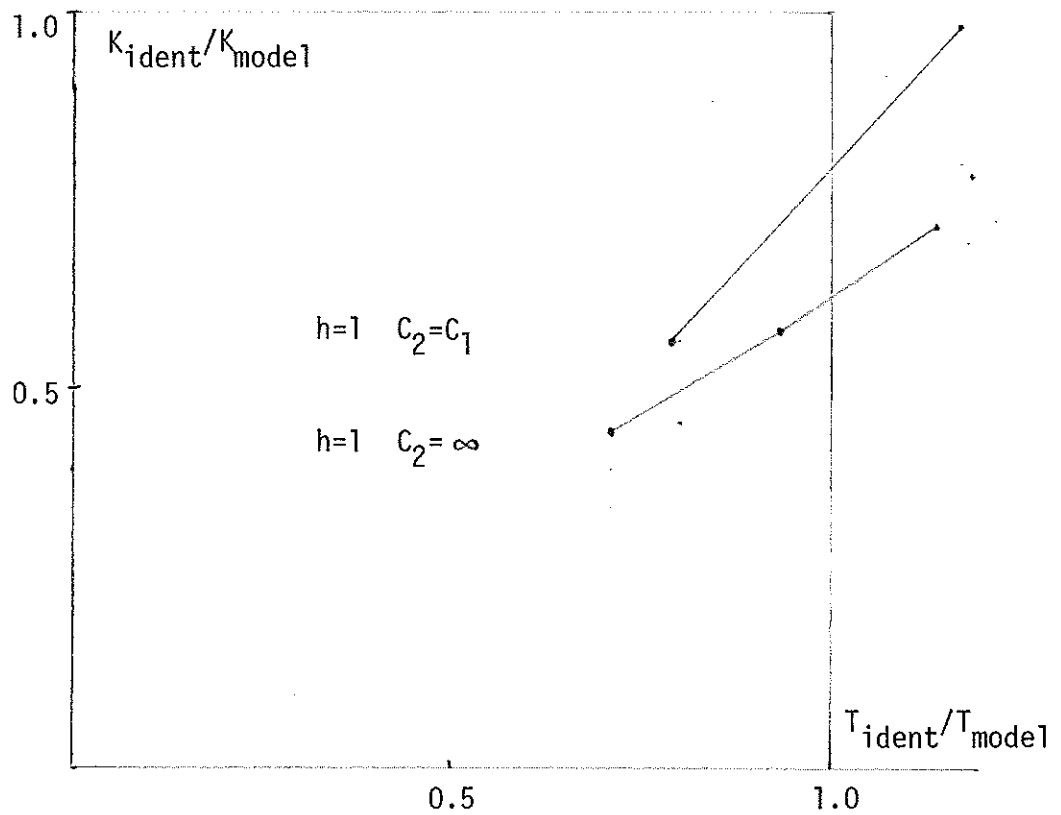


Figure 5.2 The ratios between identified and model parameters for the test room number one ($n=12.4/h$)

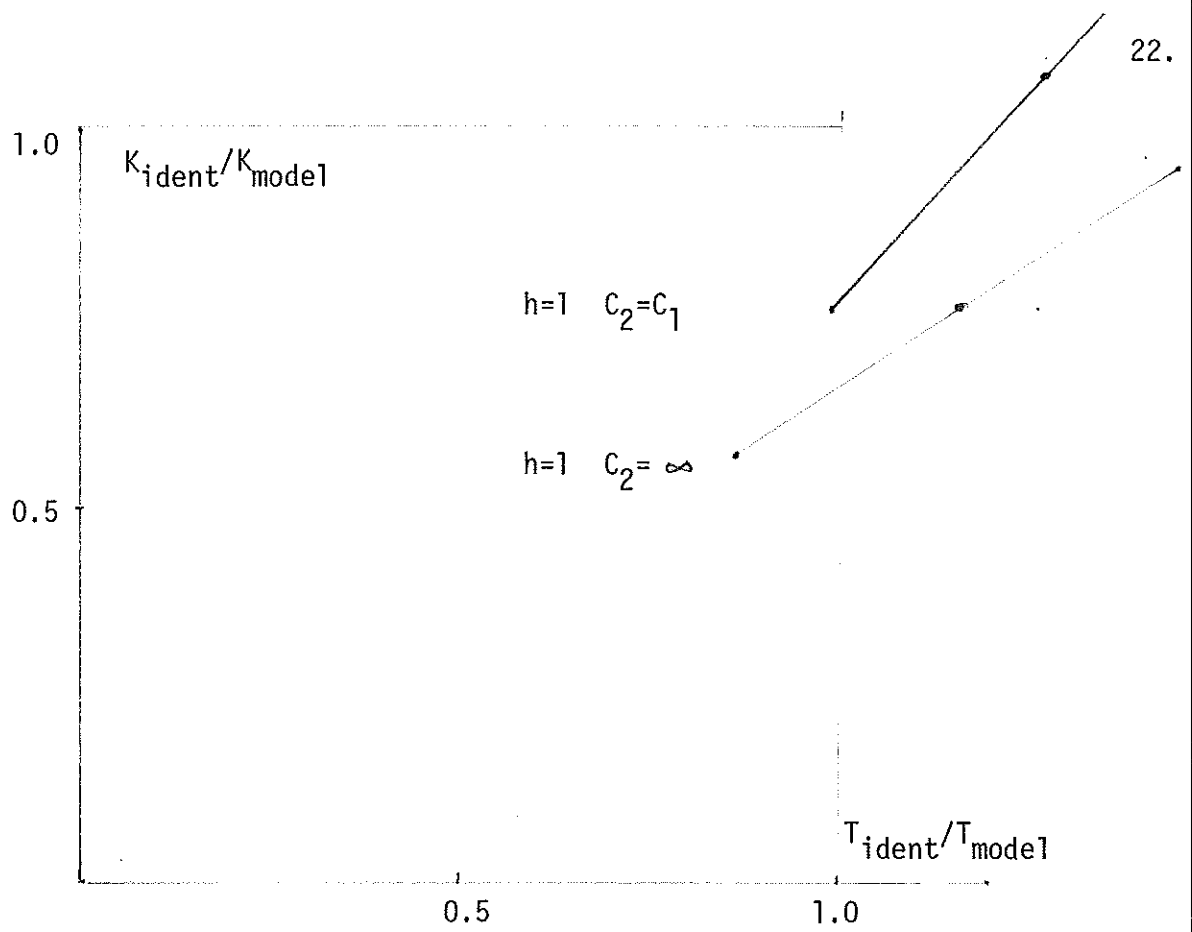


Figure 5.3 The ratios between identified and model parameters for the test room number one ($n=10.4 /h$)

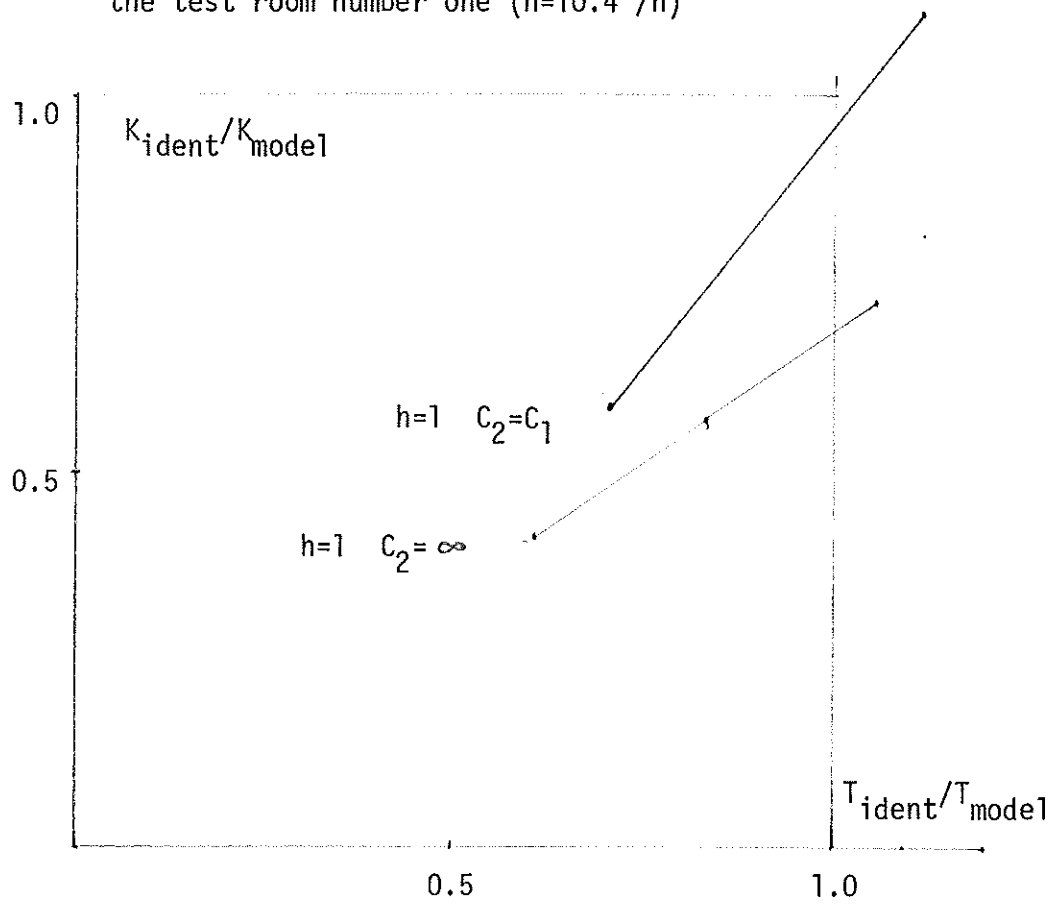


Figure 5.4 The ratios between identified and model parameters for the test room number two ($n=8.2 /h$)

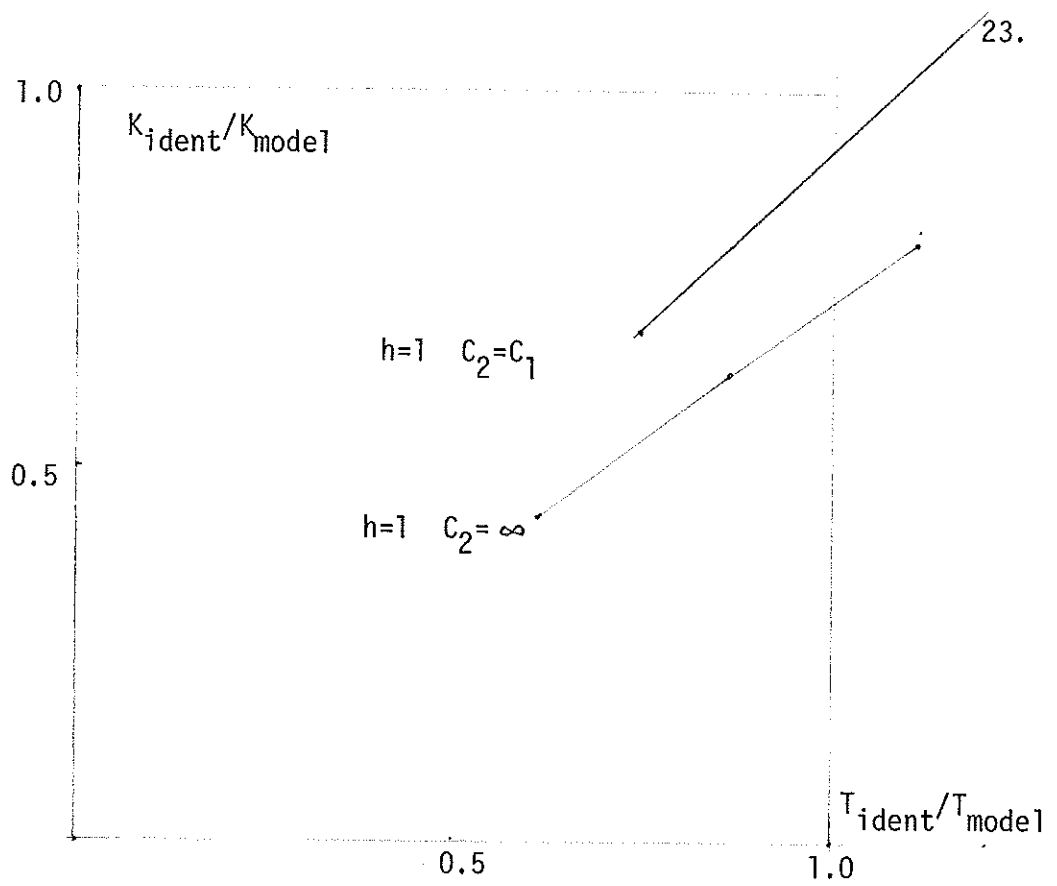


Figure 5.5 The ratios between identified and model parameters for the test room number two ($n=6.6/h$)

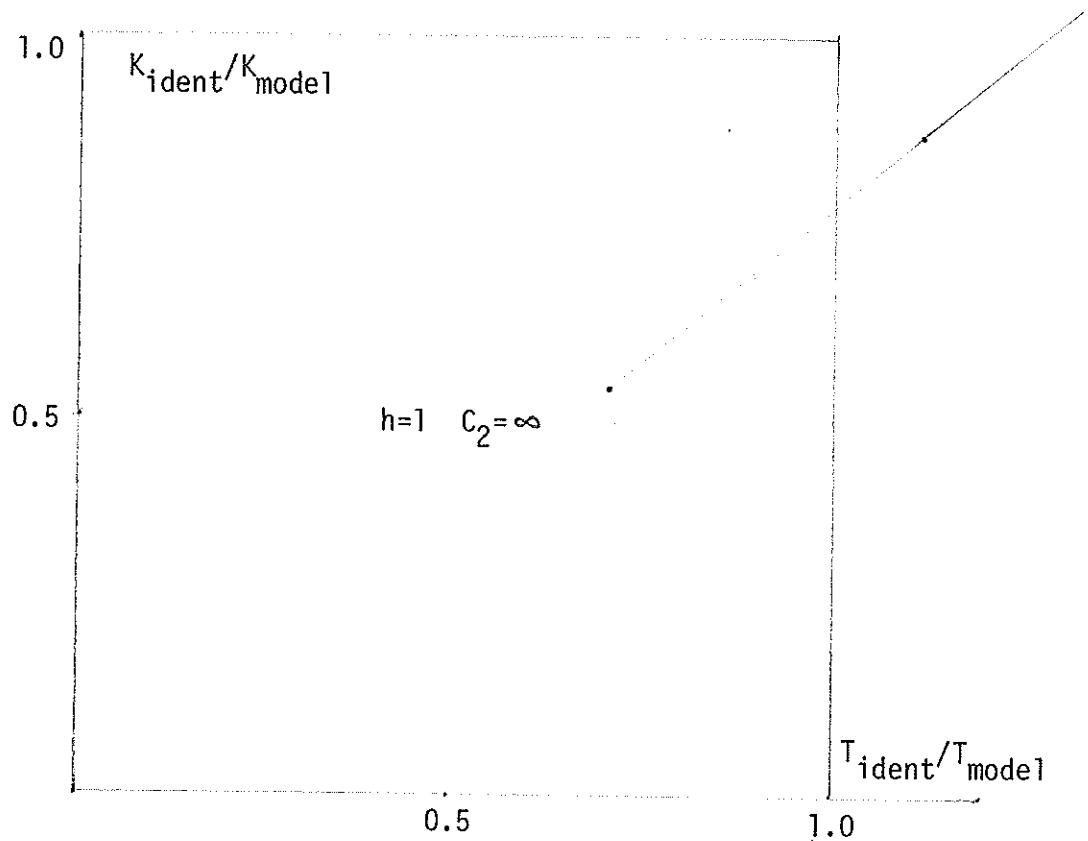


Figure 5.6 The ratios between identified and model parameters for the test room number two ($n=3.2/h$).
 $h_1=1$ $C_2=C_1$ gives $K_{ident}/K_{model}=1.20$ and $T_{ident}/T_{model}=1.00$

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