Empirical Essays on Financial Economics

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1 Summary and Conclusions

1.1 Swap Agreements

Swap agreements were first introduced in the 1980s. Since that time the use of swaps has increased at an extraordinary rate, and today swaps are the most used OTC instrument in the market, with an outstanding notional amount that is more than twice the size of world GDP. Even though the papers in this dissertation only concentrate on interest rate swaps, it might be useful to get an insight into the mechanism of other forms of swaps. As the market for swaps in general and credit derivatives in particular is growing very rapidly, I will not try to provide a complete list of all the different types of swaps. Instead I will present the most common types on which most swaps are based. Today there are a number of different types of swaps traded in the market, and most of these can be classified as some variation of interest rate swaps, currency swaps or credit swaps. In an interest rate swap, which is by far the most commonly used form of swap, a fixed interest rate is exchanged for a floating rate on the same notional amount at regular intervals for the duration of the swap. The second most common type of swap agreement (after interest rate swaps) is currency swaps, where interest and principal in one currency are exchanged for the interest and principal in another currency. There are a number of different types of credit swaps; one of the most common is the asset swap in which a risky bond is exchanged for some floating interest rate stream that will continue, even after a default on the risky bond, until the maturity date of the bond. Some other common types of credit swaps are the default swap and the credit default swap. Both of these act as an insurance against losses from a default on some underlying bond. It is not uncommon to combine the swaps described above with some form of call or put provisions.

When trading swaps it is, of course, important to be able to calculate some form of fair price for them. However, since swaps are subject to both interest rate risk and credit risk the calculations are somewhat difficult. Because of this difficulty many market participants claim that they do not concern themselves with the credit risk in a direct way. Instead they only enter into swap agreements with counterparties that have a very low probability of default. In other words they use some form of credit rationing instead of a more efficient form of pricing.

In the thesis we both investigate empirically the way in which swaps are priced in the market today, using a dataset of Swedish swap agreements, and develop the method proposed by Duffie and Huang (1995) into a practically usable form. We also show the implications this has for the swap value.

1.2 Measuring the State of the Economy

We believe that there are many (financial) factors that can convey information on the state of the economy, for example we hypothesize that an undervalued currency affects the economy in an expansionary way. Similar reasoning can be applied to other economic variables. In the second part of this essay we develop a method that attempts to summarize information inferred from financial markets into a practically usable form that can be used as input into the policy making mechanism.

1.3 Summary of the Thesis

In the second chapter entitled "*Swap Agreements under Dual-default Risk: A Simulation Approach*", which is co-authored with Peter Jochumzén, we develop the model for calculating the net expected value of a swap agreement subject to dual-default risk by Duffie and Huang (1996) into a practically usable form. The main explanatory variable for the net expected return of a swap is the default intensity of each party measured by the credit rating of the firm. We derive the net expected return of the swap as a function of the credit rating of the firm receiving the fixed interest rate and credit rating of the firm receiving the floating interest rate. This net expected return will also depend on the stochastic process assumed for the short-term interest rate as well as the initial interest rate, the time to maturity and the time between settlements. We calibrate a counting process, using historical data, for each firm. These processes represent the time to bankruptcy for each firm. We also calibrate a CIR model for the short-term interest rate using historical data. These calibrated processes are used in Monte Carlo simulations to derive the net expected return of the swap. We find that the possibility of default has a large effect on the contract. If both parties are rated AAA, the fixed rate is 6%, the short rate is 6%, the yield curve is flat, the nominal amount is $100 million and the recovery rate is 40%, then the expected returns for both parties are about the same (it is minus $5,000 for the fixed-rate payer). However, if the rating of the party paying the floating rate decreases to B, the expected loss for the fixed-rate payer exceeds $60,000! Clearly, default matters when the counterparty has a low rating.

In the third chapter "*Swap Valuation and Term Structures*" we further investigate the swap pricing model by Duffie and Huang. In order to obtain accurate net expected returns of swap agreements under dual-default risk, it is important to model the process governing the interest rates correctly. In this chapter we present and evaluate different term structure models for the interest rate. The calibration of the term structure model is made with a historical data set of the 1-month Stibor interest rate. We use both a GMM approach and a QML approach to estimate the parameters in the term structure models. For the QML method we use a bootstrapping procedure to obtain variances for the estimated parameters. The predictive ability of the term structure models, based on the GMM estimates, is then compared. We proceed to calculate the net expected value of a 5-year plain vanilla swap using the different term structure models. The conclusion is that the net expected value of the swap is greatly influenced by the choice of term structure model. The differences in the net expected value of the swap between the investigated term structure models are considerable. The Merton model, for example, gives a value of $914 for a swap between a fixed-rate payer with rating AAA and a floating-rate payer with a B rating, whereas the Vasicek model gives a much lower value of $263 for the same swap. There is, however, no clear relationship between the calculated value and the presence or absence of mean-reversion or level-dependent volatility. We also find that the method used when estimating the parameters in the term structure models is of importance.

In the fourth chapter of this thesis "*The Influence of Credit Risk on Swap Agreements*" we analyze the relationship between credit risk and the price of a swap agreement. Our hypothesis is that companies do take the credit risk into consideration when entering swap agreements, either directly through the price or via some form of credit rationing. We have obtained a unique dataset from two Swedish companies containing all relevant information on their swap agreements, as well as full counterparty information. Using this dataset we are able to empirically test our hypothesis. In accordance with our hypothesis it is shown that default risk does matter when companies enter swap agreements. Not only do companies take credit risk into consideration in a direct way by adjusting the price of the swap agreement. They also price the credit risk in a swap agreement in an indirect way; namely through credit lines and by only using counterparties with high creditworthiness for swap agreements with high notional amounts or with a long time to maturity. The companies that provided the used data claim that they do not take credit risk into direct consideration in any systematic way. However, the empirical evidence suggests that the participants in fact do adjust the swap terms depending on the credit rating of the counterparty. This adjustment is probably based on some approximation of the theoretically fair price of the swap.

In the fifth, and final, chapter of the thesis "*Measuring the State of the Swedish Economy: An Equilibrium Conditions Index*" we develop an index for measuring the state of the Swedish economy with financial variables. Instead of using relative deviations from a specified date or level of the exchange rate, interest rate, house prices, and stock prices, as in a standard Financial Conditions Index (FCI), we use deviations from estimated (partial) equilibrium values. This framework makes it possible to differentiate between changes in the variables derived from underlying macroeconomic factors and changes that alter the monetary conditions in an economy. The deviations between actual and equilibrium values for the included variables are weighted together, using a VAR approach, in order to obtain an economic conditions index. The resulting index is able to explain both the larger business cycle movements as well as the devaluations of the krona over the sample period between 1970 and 2002. Interestingly, the index appears to lead the major downturns in the economy in the early 1980s and the early 1990s, making it a useful tool for policy making. We also find that the exchange rate and long-term interest rates currently (2002 Q4) indicate an expanding economy whereas the stock market and house prices affect the economy in the adverse direction. The total effect of the variables in the index is however (slightly) expansionary as of the last quarter of 2002.

References

Duffie D and Huang M, 1996, Swap Rates and Credit Quality, *Journal of Finance*, 51(3), 921- 949.

2 Swap Agreements under Dual-Default Risk: A Simulation Approach[1]

2.1 Introduction

This paper addresses the problems involved in finding the net expected return of swap agreements under dual-default risk. We use a simulation approach where the parameters are calibrated from actual data on interest rates and default risk. Contrary to Huge and Lando (1999), we will not use the model proposed by Duffie and Huang (1996) to obtain swap prices. Instead we will simulate the life of numerous swap agreements to obtain the net expected return. This approach will be superior in the sense that it will not put any restrictions on the relationship between the expected return of the swap and the default characteristics.

We will also answer the following questions:

1. How is the swap's net expected return affected by changes in initial interest rate?
2. How do the parameters in the model for the short term interest rate influence the swap's net expected return?

[1] This paper is co-written with Peter Jochumzén, Department of Economics, Lund University.

3. How will deviations from the long term mean of the interest rate affect the swap's net expected return?

In order to obtain realistic values on the net expected return of swaps it will be necessary to choose the data generating process, DGP, for the short-term interest rate carefully. Chen, Karolyi, Longstaff, and Sanders (1992) compare different models for the short-term interest rate in their paper and come to the conclusion that models with level dependent conditional variances performs well. We will use a Cox, Ingersoll and Ross (1985) model, henceforth CIR, in this paper which we calibrate using historical data. In the second section previous work, within the area of swaps and default risk, is discussed. Section 3 presents the setup of the problem and the definitions used in the paper. The following section defines the stochastic specifications of the short-term interest rate and the default process in a general way. We also compare our results to those found in Chen et al (1992), our aim is however not to replicate their study. We then proceed in section 5 to calibrate the actual model used to obtain the simulated net expected returns of the swaps. The results from the simulations are presented in section 6 with a sensitivity analysis in section 7. The last section contains the conclusions of the simulations.

2.2 Previous Work

In this section we have divided the previous work done on swaps into four categories. In the first category, many of the early papers seek to explain the economic rationale behind swaps. The most know is the paper by Bicksler and Chen (1986) who primarily base their swap theory on the existence of comparative advantages. These comparative advantages arise because of differences in transaction costs among market participants. As the market for swaps expands and becomes more efficient the differences between market participants should diminish. This would lead to a shrinking market for swaps, contrary to the fact that the swap market is still expanding. They also highlight the usefulness of swaps as a liability manager and hedging instrument against interest rate fluctuations. Smith, Smithson and Wakeman (1986) identify four different reasons for the existence of swaps: financial arbitrage, tax and regulatory arbitrage, exposure management and completing markets. The arbitrage reasons can be dismissed as long run explanations. Exposure management is a viable reason if transaction costs associated with swaps are lower than those of other instruments such as options and futures. Finally, the completing market reason argues that swaps provide an instrument that can not be replicated with other instruments. As the maturity of futures and options generally is less than one year and the maturity of swaps, generally, lies between 3 and 5 years, this reason is also plausible.

Turnbull (1987) argues that if the bond market is competitive, not all parties to a swap can benefit. He concludes that externalities must exist since the growth of the swap market suggests that both parts of a swap benefit. Wall and Pringle (1989) give a thorough survey of different motives given for swaps in their paper. They find that no single explanation can account for the existence of swaps.

In the second category, some researchers have developed models for pricing swap agreements under default risk. This is generally done by finding a swap interest rate that makes the expected present value of the fixed and the floating payments equal. Smith, Smithson and Wakeman (1988) address the importance of pricing the default risk in a swap agreement. They put bounds on the swap price where the price of the risk should exceed zero and be below the risk premium of a bond with corresponding default characteristics. The first generation of papers dealt with one-sided default risk. In this group we find

Cooper and Mello (1991) who derive the swap price from market equilibrium conditions and shows how wealth is transferred between shareholders and debt holders. Minton (1997) compares the price of a swap with a chain of overlapping futures. The second generation of swap models deals with dual-default risk. Duffie and Huang (1996) develop a theoretical model that price swaps with dual-default risk. They use a model where the discount rate, relevant to the counterpart with a negative swap value, is used to calculate the present value of the swap. Lang, Litzenberger and Luchuan (1998) take a somewhat different approach and views swaps as a non-redundant security, which creates surplus to be shared by the counterparts to compensate for their risk in the swap. The surplus is created from a reduction in financial cost, exposure of undergraded credits and hedging interest rate risk. They also find that swap spreads contain pro-cyclical elements.

In the third category, there exist a small but growing number of papers dealing with empirical data. Most of the empirical papers use quoted offer and bid spreads on swaps. Sun, Sundaresan and Wang (1993) examine the effect that the swap dealer's credit reputation has on the swap quotations and the bid-offer spread. They find that the dealer with higher credit rating also quotes higher swap offer rates, than the dealer with a lower rating. Brown, Harlow and Smith (1994) seek to explain the volatility in the quoted swap spreads. They find significant differences in the price of short-term swaps compared to long term swaps. They also find that the pricing dynamics changed substantially between 1985 and 1991. Duffie and Singleton (1997) develop a multi-factor model of the term structure of interest rate swap yields. They find that liquidity in the treasury and swap markets as well as the credit ratings of the counterparts is important factors for explaining the variations in swap spreads over the past decade. Cossin and Pirotte (1998) test how some credit risk pricing models fit swap transaction data. They find that the studied models perform poorly when used on transaction data and conclude that further studies with larger data sets are needed. Cossin and Pirotte (1997) examine the effect that the credit rating has on the swap prices. They conclude that the swap spreads differ between rated and non-rated companies. Degrér (1999b) examine rated companies and find significant differences in swap characteristics between companies in different rating groups.

In the fourth category, Hull (1989), Whittaker (1987), Smith, Smithson and Wilford (1990) examine the regulatory framework for swaps in their papers. Hull concentrates on the how contingent claim valuation analysis can facilitate the determination of capital needed for off-balance commitments. Whittaker studies the regulations that affect the swap agreements. Smith, Smithson and Wilford concentrate on the capital requirement that swaps causes. The capital requirements are now formalised in the ISDA Master Agreement and by the Basel Act.

Our paper relates mostly to category three above, since we also deal with default risk. However, contrary to previous work we develop an empirically usable model that lets us calculate the risk neutral price of swap agreements subject to dual-default risk.

2.3 Theoretical Setup of the Simulation Model

In this section we derive an expression for the net expected value of a swap agreement (henceforth *NEV*), subject to dual-default risk. We will start this section by providing an intuitive interpretation and explanation of the factors affecting *NEV*.

The credit risk in a swap agreement comes from the fact that the difference between the fixed and the floating interest rate is exchanged at discrete intervals, typically semi-annually. If, for example, the short term (floating) interest rate has increased to a level higher than the fixed interest rate, then the short term interest rate payer will have a debt to the fixed interest rate payer. If the floating interest rate payer defaults prior to a settlement then the fixed rate payer will loose some proportion of this net debt. It is this credit risk that we want to be able to assign a value to (*NEV*).

In our setup we have five factors that affect *NEV*. The most important factor is the default risk of the counterparts in the swap agreement. If the default risk is zero it is clear that none of the counterparts stands any risk of loosing money, i.e. there is no credit risk. Another important factor is the fractional recovery rate, which governs what proportion of the money, subject to a default, that is repaid in the case of a default. If the recovery rate is 100 percent then it would not matter if a counterpart defaulted. However, typically some portion of the money owed will be repaid. Clearly, the notional amount is also important since a higher notional amount means larger interest rate transfers. A fourth factor is the time to maturity, where a longer time to maturity incurs a higher probability of default (see Table 3). Lastly, the volatility of the short term interest rate is also important since higher volatility means that there is a higher probability of a deviation between the fixed and floating interest rate. In a sense one can compare a swap with an option, which has the same dependence on the volatility.

We will consider a swap agreement with the following characteristics:

- The swap agreement is based on a nominal amount of N
- Party **X** pays fixed interest rate r^{fixed}, determined at the time the contract is written, on this nominal amount.

- Party **Y** pays the floating rate, typically the 3 or 6 month LIBOR rate, $r_t^{floating}$ on the nominal amount where t is measured in years from the start of the contract.
- The agreement lasts for T years.
- We have k settlements at $d_1,\dots,d_k$ where d_i is measured in years from the start of the contract[2].

We begin by defining the size of the interest rate payment. For convenience, we define $d_0 = 0$ even though we have no settlements at $t = 0$[3]. At settlement date d_i, $i = 1,\dots, k$, **X** pays the fixed interest rate r^{fixed} on the nominal amount N and **Y** pays the floating rate $r_t^{floating}$ on the nominal amount. More precisely, the length of the interval between payments is equal to $(d_i - d_{i-1})$ so that party **X** pays $r^{fixed}(d_i - d_{i-1})N$. Party **Y** pays according to the short interest rate *prevailing* d_{i-1} *years after the beginning of the contract,* $r_{i-1}^{floating}$.[4] Thus, **Y** pays $r_{i-1}^{floating}(d_i - d_{i-1})N$. Consequently, there is a net payment at d_i from **X** to **Y** if the fixed rate is above the short rate at d_{i-1}.

We continue by defining indicator functions for the event of a default by X or Y. The indicator I_i^X is equal to 1 if **X** has not defaulted at the settlement date d_i and 0 otherwise. I_i^Y is defined similarly. We assume that the failure dates of the parties are *independent* random variables. This is reasonable as long as the nominal value of the contract is not "too large" in relation to the value of the firm.

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[2] Typically we have settlements semi-annually, which implies that $d_1 = ½$, $d_2 = 1$, …
[3] Except for, in some cases, an initial transfer from one party to the other.
[4] Thus, it is the short-term interest rate at the previous settlement that determines the cash flow at the current settlement. This is by far the most common design of a swap contract.

As mentioned in the beginning of this section only a fraction of the money owed is repaid from the defaulting counterpart in the case of a default. In case of default at time t of say party **X**, we must first, however, decide who is "in the money". If party **Y** is in the money (the fixed rate is higher than the floating rate on the settlement date prior to the default), **Y** pays the full net amount to **X**. If **X** is in the money, only a fraction δ of the net amount is paid to **Y**. After the default, no money is exchanged at the following settlement dates.

Define R_{Xi} as the random return in dollar at settlement date d_i to party **X** from the floating rate payment of **Y**, thus

$$R_{Xi} = r_{i-1}^{floating}\left(d_i - d_{i-1}\right) I_{Xi} I_{Yi} N \tag{2.1}$$

Similarly, R_{Yi} is the random return at settlement date d_i to party **Y** from the fixed rate payment of **X**, thus

$$R_{Yi} = r_{i-1}^{fixed}\left(d_i - d_{i-1}\right) I_{Xi} I_{Yi} N \tag{2.2}$$

Define $t^{default}$ as the time in years to the first default (for party **X** or party **Y**). If there are no failures, we set $t^{default} = T$. The indicator function $I_{default}$ is defined as 1 if there is a failure ($t^{default} < T$) and 0 otherwise ($t^{default} = T$). Define $i^{deafult}$ as the largest i such that $d_i \leq t^{default}$. $i^{deafult}$ will correspond to the number of settlements before the first failure. d_{i^f} is then the time to the last settlement before failure. If $t^{default} < d_1$ (we have a failure before the first settlement), we define $i^{deafult} = 0$ which is consistent with the notation since we have defined $d_0 = 0$. Further, define $r^{default}$ as the short interest rate at the last settlement before failure. This is the short rate that typically will be used at the default date.

Denote by $R_{Xf(Y)}$ the net random amount that **X** receives at the failure date if **Y** fails at $t^{default}$. We set this to 0 if **Y** fails at $t^{default}$ or if $t^{default} = T$. Define $R_{Xf(X)}$,

$R_{Yf(X)}$, and $R_{Yf(Y)}$ in the same manner. Clearly $R_{Xf(X)}$ + $R_{Yf(X)}$ = 0 and $R_{Yf(Y)}$ + $R_{Xf(Y)}$ = 0. For example,

$$R_{Xf(Y)} = \begin{cases} \left(r^{default} - r^{fixed}\right)N \text{ if } r^{default} > r^{fixed} \\ \delta\left(r^{default} - r^{fixed}\right)N \text{ otherwise} \end{cases} \tag{2.3}$$

so that **X** receive the full amount in the first case while there is only fractional recovery for **X** in the second case.

Define R_X as the random net present value of the income streams that **X** receives from **Y** and we define R_Y similarly. We will use the current risk free *t*-year interest rate to discount payments received or paid *t* years from now. This interest rate will be denoted by $r_{0\to t}$ and it is given by the current yield curve. Using this notation,

$$R_X = \sum_{i=1}^{k} \frac{R_{Xi}}{\left(1 + r_{o\to d_i}\right)^{d_i}} + \frac{R_{Xf(X)} + R_{Xf(Y)}}{\left(1 + r^{fixed}\right)^{t^{default}}} \tag{2.4}$$

and

$$R_Y = \sum_{i=1}^{k} \frac{R_{Yi}}{\left(1 + r_{o\to d_i}\right)^{d_i}} + \frac{R_{Yf(X)} + R_{Yf(Y)}}{\left(1 + r^{fixed}\right)^{t^{default}}} \tag{2.5}$$

We would like to determine the *value v of this contract*. We define the value of the contract for party **X** simply as the *expected value of the difference between R_X and R_Y*,

$$v_X = E\left(R_X - R_Y\right) \tag{2.6}$$

The value of the contract for party **Y** is defined as $v_B = -v_A$. It is reasonable to expect party **X** to pay v_X to party **Y** at the beginning of the contract so that the net expected return for both parties from the contract is zero. This is reasonable since with complete markets anything but this amount would give rise to arbitrage possibilities. Notice that it is uncommon for this net payment to actually take place. Instead, the fixed rate is adjusted so that $v_X = v_Y = 0$. With no default probabilities, this fixed interest rate is then typically very close to the long-term interest rate (determined by the length of the contract). When both parties are subject to default, the fixed rate is adjusted appropriately with benefits to the party with the smallest default probability. The difference between the fixed interest rate and the long-term interest rate is then often called “the price of the swap contract” and it is expressed in basis points. Degrer (1999a) and Cossin and Pirotte (1997) examine the relationship between the credit rating of the parties and the price of the contract and they find a significant relationship: the bigger the difference between the credit ratings of the firms, the higher the price of the contract.

The purpose of this paper is then to determine the value of the contract v_X. Let us first look at the factors that affect the value of the contract. First, and most important, the value of the contract will depend on the future short term interest rates since they determine the net settlements at the settlement dates and at the default date (if applicable). The assumptions we make about future short-term interest rates will be crucial for the value of the contract. The stochastic specification of the short-term interest rate is discussed in the next section. Second, the value of the contract will depend on default probabilities for each party[5]. Remember that defaults have two effects. First, an extra settlement is made at the default date where the non-defaulting party may loose money. Secondly, future settlements are cancelled. The expected future short interest rates in relation to the fixed rate will determine who benefit from this. The stochastic specification of the default probabilities is discussed in the next section. Thirdly, the value of the contract will depend on the current yield curve. This is because future settlements must be discounted using the current interest

[5] At first, it may seem obvious that the higher the default probability of party Y, the lower the value of the contract for party X but although this is definitely the typical case it need not be correct. Suppose that party X has zero probability of default and that part X pays a high fixed rate. Suppose that Y pays a low short rate and that we expect the short-term rate to remain low, well below the fixed rate. Then it is clearly to the benefit of part X if Y defaults since it will not have to pay future settlements.

rates[6]. The current yield curve is known when we write the contract so we need not make any assumptions about this. The value of the contract will also depend on the particular characteristics of the contract such as the length of the contract and the regularity of the settlements. For example, the more settlement we have the less important are the default probabilities when it comes to the first effect of default. Finally, the value of the contract will depend on the recovery rate δ. Again, δ only affects the value of the contract through the first effect of default. Note that there is an immediate relationship between the value of the contract and the price of the contract. Everything else given, the value of the contract depends on the fixed interest rate r^{fixed}, $v = v(r^{fixed})$. Therefore, there is a fixed rate $\overline{r}^{fixed}$ such that $v(\overline{r}^{fixed}) = 0$. The price of the contract is then $\overline{r}^{fixed}$ - $r_{0\to T}$.

2.4 Stochastic specification

2.4.1 The Short-Term Interest Rate

Minton (1998) shows that the variance of the short-term interest rate is an important explanatory variable for the value of the contract. This fact makes it very important to choose the data generating process for the short-term interest rate carefully and to calibrate it in an appropriate way. Chan et al shows how different models for the short-term interest rate perform against each other empirically. Using a generalised method of moments, they conclude that models that do allow the conditional volatility to depend on the level of the interest rate outperform the models that do not. The model by Dothan (1978) and the variable-rate Cox, Ingersoll and Ross (1980), CIR, fits the data set used in the best way. We have chosen to use a square root CIR (1985) model since it is easy to work with and since it allows for a dependency between the volatility and the level of the interest rate. It is outside the scope of this paper to consider other

[6] A lengthier explanation of the basic mechanisms in a swap is given in Degrér (1999a)

interest rate models. The CIR model is an affine single-factor general-equilibrium term structure model that has been used extensively in developing models for valuation of interest rate contingent claims. Notably Sundaresan (1989) use a CIR model to derive a model for valuation of non-defaultable swaps. The CIR model may be written using the following notation:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t \tag{2.7}$$

with r_0 given. As equation (2.7) shows, the CIR model implies that the volatility, conditional on changes in r_t, is proportional to the square root of r_t. Important to the CIR model, the short-term interest rate follows a *mean reverting process*. The mean is θ and the deviation from the mean is weighted with κ, which is the adjustment speed parameter, σ is the instantaneous volatility and W is a Wiener process. There are many possible strategies for calibrating the CIR model using real data. Chan et al (1992) use the Generalised Method of Moments by Hansen (1982) to fit a CIR model to US treasury data. With their method the discrete-time specification of the CIR model is used.

$$r_{t+1} - r_t = \alpha + \beta r_t + \varepsilon_{t+1} \tag{2.8}$$

$$E\left[\varepsilon_{t+1}\right] = 0 \quad E\left[\varepsilon_{t+1}^2\right] = \sigma^2 r_t$$

In this model the variance of the interest rate changes, depend directly on the interest rate level. A negative value of β indicates the presence of mean reversion in the interest rate process. Pearson and Sun (1988), as well as Brown and Dybvig (1986), use a maximum likelihood method in order to estimate the parameters in the CIR model. Since we find that the variance of the interest rate heavily influence the value of the swap contract, we try to calibrate the parameters in such a way that the theoretical variance of r_t in the model complies as much as possible with historical values. The parameters are calibrated using an OLS method where ψ in equation (2.8) is minimised with respect to σ and κ. The average interest rate of the UK 1-month Libor rate between 1979 and 1997 is used as an estimate of the parameter θ.[7] The interest rate on the 1st of January 1979 corresponds to r_{d0}. Since we use annual variances the difference d_i-d_{i-1} will equal 262 days, i.e. the number of business days in a year. The general equations used to calibrate the model are shown in equations (2.9)-(2.11).

$$\min_{\sigma,\kappa} \Psi = \Gamma'\Gamma \tag{2.9}$$

where,

$$\Gamma = \begin{bmatrix} (\mathrm{Var}(r_{d_1} \mid r_{d_0}) - \hat{r}_{d_1})^2 \\ (\mathrm{Var}(r_{d_2} \mid r_{d_0}) - \hat{r}_{d_2})^2 \\ . \\ . \\ . \\ (\mathrm{Var}(r_{d_n} \mid r_{d_0}) - \hat{r}_{d_n})^2 \end{bmatrix} \tag{2.10}$$

and,

$$\mathrm{Var}(r_{d_i} \mid r_{d_0}) = r_{d_0} \frac{\sigma^2}{\kappa} \left\{ \mathrm{e}^{-\kappa(d_i - d_0)} - \mathrm{e}^{-2\kappa(d_i - d_0)} \right\} + \theta \frac{\sigma^2}{2\kappa} \left\{ 1 - \mathrm{e}^{-\kappa(d_i - d_0)} \right\}^2 \tag{2.11}$$

This equation follows from the solution of the partial differential equation of the CIR model, see equation (2.7). A complete derivation of equation (2.11) can be found in Cox, Ingersoll and Ross (1985).

2.4.2 Defaults

Defaults are modelled as general counting processes. In particular, we will focus on non-homogeneous Poisson processes where we denote $\lambda_X(t)$ and $\lambda_Y(t)$ as the failure hazards of firm **X** and firm **Y** respectively. If we define T_X as the random failure time of firm **X**, we may define the failure hazard as

$$\lambda_X(t) = \lim_{dt \to 0} \frac{\Pr\left(t \le T_X < t + dt\right)}{dt} \tag{2.12}$$

[7] Another approach would be to estimate all the parameters simultaneously, but since we are mainly interested in the empirical fit of the estimated volatility term structure, our approach is more resonable.

Basically, the intensity $\lambda_X(t)$ may be interpreted as the conditional probability of default on a period after *t* given that the firm did not default before *t*. One way to specify the stochastic processes governing the failures of party **X** and **Y** is to model the intensities and this is what we will do. There are basically two ways of going about this. We may attempt to fully model the relationship between the default intensities and certain explanatory variables (firm specific, industry specific as well as macroeconomic) and use data on actual failures to estimate a functional form. We may also use the credit rating on the firm as the only determinant for the default intensities. The first method is perhaps more desirable but much more difficult to implement. We would need high quality data to figure out the empirical relationship between the explanatory variables and the default hazard. The second method assumes that all information about the default probabilities is incorporated in the rating and this may not be correct. Lack of good data has, however, forced us to use the second method. This implies in practice that we must estimate one failure hazard $\lambda(t)$ for each possible credit rating and that two firms with the same credit rating will have the same failure hazard for all *t*. We will denote the failure hazard for a firm with rating ξ by $\lambda^{\xi}(t)$. Even then, the problem of linking credit ratings to default intensities remains. We decided to use previous investigations that has empirically calculated the proportion of firms with a particular rating that failed after 1, 2, 3, …, 9 and 10 years. These default rates are shown in Table 3. Because we have no information on the proportion of firms that fail after *t* years when *t* is not an integer, it will not be possible to determine the intensities $\lambda^{\xi}(t)$ for a particular rating exactly. All we know is that *n* firms failed in, say, the second year but we have no information on when during that year they failed. Therefore, we may only find the *integrated* values of these default intensities over one year each. Only if we are willing to assume that the failure hazard is constant within the year may are we able to fully estimate $\lambda^{\xi}(t)$ for each *t*.[8]

Define

$$\gamma_t^{\xi} = \int_t^{t+1} \lambda^{\xi}(t)dt \tag{2.13}$$

It is straightforward to show that for integer *t*,

$$S^{\xi}(t) = \Pr(T^{\xi} > t) = \exp\left[\gamma_0^{\xi} + \gamma_1^{\xi} + ... + \gamma_{t-1}^{\xi}\right] \tag{2.14}$$

where $S^{\xi}(t)$ is defined as the survivor function and T^r is the random failure time for a firm with initial rating ξ. Also denote π_t^{ξ} the empirical cumulative proportion of firms that have failed after *t* years given that their original rating was ξ. We would like to set the integrated failure hazards in such a way that the theoretical survival values coincide with the empirical ones for each possible rating,

$$S^{\xi}(t) = 1 - \pi_t^{\xi} \tag{2.15}$$

This implies that we estimate

$$\hat{\gamma}_0^{\xi} = \ln(1-\pi_0^{\xi}) \tag{2.16}$$

$$\hat{\gamma}_t^{\xi} = \ln(1-\pi_t^{\xi}) - \sum_{s=0}^{t-1} \ln(1-\pi_s^{\xi}) \text{ for } t \geq 1 \tag{2.17}$$

To conclude, the value of the contract will depend on the stochastic specification of the short-term interest rate and the failure hazard of each party. The dynamics of the short-term is determined by the parameters κ, θ and σ. The failure intensities are determined by the parameters γ_t^r t = 0,…, T-1 for each rating category *r*. Given the particular contract, a recovery rate and a yield curve, there will be a relationship between these parameters and the value of the contract. Determining the value of an actual contract we attempt to set κ, θ and σ so that

[8] In this case, the failure intensity is equal to the integrated failure intensities.

variance of the theoretical process coincide as close as possible to observed variances of the short-term interest rates and we set γ_t^r so that the expected survival probabilities, year by year, coincide with the empirical observed ones. Calibrating the stochastic processes requires specification of an actual contract and such a contract will be specified in the next section. This section will also consider the calibration problem. Once we have contract and we have calibrated our processes, we consider the actual determination of the value of the contract. This will be done in section 5. In section 6, we will investigate the sensitivity on the value of the contract to the parameters that determine the stochastic process of the interest rate.

2.5 Calibration and Specification of Contract

2.5.1 The Short-Term Interest Rate

We have chosen to focus our interest on swap agreements with a five-year time to maturity. Settlements are made every six months[9]. We assume a completely flat yield curve with an initial interest rate of 6%. We also assume a fixed interest rate of 6% and a recovery rate of 40%[10]. Thus, in addition to the parameters of the stochastic processes, only the credit ratings of the two parties determine the value of the contract.

The time periods used to calibrate the model should reflect the time to maturity, TTM, for the swap agreement, i.e. if the swap has a TTM of 2 years the empirical variances up to 2 years should be used when calibrating the parameters. The reason for this it that only the difference between the fixed and the floating interest rate is exchanged at the settlement dates. As the variance

[9] This is the most common swap construction according to Degrér (1999a)
[10] Follows from estimations made by Altman and Kishore (1996)

increases the uncertainty about future payments increase and therefore the price of the swap will change. The uncertainty about interest rate differentials in periods after the maturity date will not influence the price of the swap. We therefore use the historical five-year volatility to calibrate the parameters in the Cox, Ingersoll, and Ross (1985) square-root model (CIR). Table 1 shows the historical variances used when calibrating the CIR model as well as the predicted variances from the calibrated CIR model. In order to evaluate the estimated parameters we test the predictions made with the estimated model. Specifically we calculate the predicted variance from the CIR model and compare to historical values.

Table 1. Observed and Modeled Variances in the CIR-model

The variances predicted by the estimated CIR model and the actual historical variance of the UK 1-month LIBOR rate between 1979 and 1997. We have chosen to focus on swap agreements with a time to maturity of five years. Therefore we use the historical volatilities of the interest rate for 1 to five years to calibrate the CIR model. We have 262 business days per year and disregard leap days.

TTM	**Observed variance**	**Model Variance**	**Variance by Chan**
1 year	0.0659	0.0960	0.0303
2 years	0.1168	0.1131	0.0303
3 years	0.1603	0.1161	0.0303
4 years	0.1639	0.1166	0.0303
5 years	0.1449	0.1167	0.0303

Our OLS estimation produces values that are somewhat different from those estimated by Chan et al (1992) using daily U.S. treasury yields.[12] Table 2 shows a comparison of the estimates. The largest difference is found in the estimate of

[12] It is of course possible to use other estimation techniques such as GMM or QML. However, preliminary results indicate that it is mainly the standard errors that are affected by changing estimation techniques, not the parameter values. Since it is the parameter values, and not the standard errors, that enters the simulation we have chosen to use OLS.

the adjustment parameter, κ. The difference is probably partly due to the higher volatility in the time period we use as compared to the one used by Chan and the difference in the method of estimation. It is however worth mentioning that the value of κ is larger than zero which implies that we have mean reversion in the interest rate process.

We have tested the estimated model and found that it gives us values whose variance is relatively close to the historical variance compared to the values used by Chan. The CIR model produces close fits for TTM between 2 and 5 years, whereas the variance for 1 year is further from the observed variance.

Table 2. Estimated Parameters in the CIR-model

Estimated parameters in the CIR model using UK 1-month LIBOR January 1979 to December 1997 compared to estimates by Chan et al (1992) using the annualized 1-month U.S. treasury bill yield from June 1964 to December 1989 (304 observations).

Parameter	Estimates	Value by Chan
θ	0.1051	0.0808
κ	0.0033	0.2339
σ	0.08562	0.0854

2.5.2 Defaults

We used the cumulative default probabilities in Table 3 and equations (2.16) and (2.17) to calibrate the default intensities. This resulted in the intensities presented in Table 4. This table show the intensities used in the Poisson process for the different rating categories for TTM between 1 and 10 years. The intensities are calculated using equation (2.16) on the data in Table 3.

Table 3. Cumulative Default Probabilities

Cumulative default probabilities within different time periods measured in percentage. The table is based on actual default data from Moodys of over 4000 rated U.S. and international debt issuers from 1970 through 1990.

	< 1 yr	< 2	< 3	< 4	< 5	< 6	< 7	< 8	< 9	< 10
AAA	0.01	0.02	0.03	0.05	0.07	0.09	0.12	0.16	0.21	0.27
AA	0.02	0.03	0.05	0.08	0.11	0.15	0.20	0.27	0.36	0.46
A	0.02	0.06	0.12	0.20	0.30	0.44	0.61	0.81	1.05	1.34
BAA	0.04	0.08	0.15	0.24	0.37	0.53	0.73	0.99	1.29	1.65
BA	0.05	0.14	0.28	0.47	0.72	1.04	1.42	1.86	2.38	2.96
B	0.06	0.16	0.31	0.52	0.82	1.19	1.66	2.21	2.85	3.57

Source: Fons J and Kimball A, June 1991, *Journal of Fixed Income*.

Table 4. Default Intensities

Default intensities used in the Poisson hazard process. Values are based on the same data set that is used for Table 3. The results are presented in percentages.

	< 1 yr	< 2	< 3	< 4	< 5	< 6	< 7	< 8	< 9	< 10
AAA	0.004	0.004	0.004	0.009	0.009	0.009	0.013	0.017	0.022	0.026
AA	0.009	0.004	0.009	0.013	0.013	0.017	0.022	0.030	0.039	0.043
A	0.009	0.017	0.026	0.035	0.043	0.061	0.074	0.087	0.104	0.126
BAA	0.017	0.017	0.030	0.039	0.056	0.070	0.087	0.113	0.130	0.157
BA	0.022	0.039	0.061	0.083	0.109	0.139	0.165	0.192	0.226	0.253
B	0.026	0.043	0.065	0.091	0.130	0.161	0.205	0.240	0.279	0.314

2.6 Calculating the Net Expected Return of the Swap Agreement

Given the contract and the calibration from section 4, we will find the value of the contract given the credit ratings of the parties. Finding the value is not simple and we will not be able to present a closed form solution. Instead, the solution will be based on simulation techniques.

We will now give a detailed description of the simulation method that we use to find the net expected return of the swap agreement. We have designed a computer program that accepts as input the:

- details of the contract
- fixed interest rate
- parameters of the CIR model
- empirical cumulative survival probabilities for each rating *or* the default intensities for each rating
- current yield curve with the interest rates corresponding to the settlement dates
- rating of the parties
- recovery rate

From this information, the program will come up with a net expected value of the contract and this section describes how it does that.[13] If we are looking for the expected price of the contract, we must instead find the fixed rate such that the value of the contract is zero.

We simulate data for 2N firms (N contracts, two firms for each contract) where we may select N differently. The higher the N, the closer to the true net value of the contract we will get but the longer it will take to get the result. We start by simulating failure times for the companies. This will result in two vectors of default dates with values ranging between 0 and 5 if default has occurred[14] and values above 5 if default has not occurred. One vector represents failure time of the party paying the fixed-interest rate and the other vector that of the floating-rate payer. We then compare the vectors row-wise and drop the higher of the two values. This is done because default of one party terminates the swap and therefore subsequent defaults will not matter. This will result in two categories.

[13] All simulations are carried out in Matlab version 5.5.
[14] Notice that we use 5-year swap agreements in the simulations.

Category 1 where one of the parties default before 5 years (N_1 contracts) and category 2 where there is no default (N_2 contracts). Clearly, as N goes to infinity, N_1/N converges to the true proportion of contracts that will be terminated before five years. The way we have specified the default processes, the probability of default before five years for a firm with rating g is equal to π_t^g from Table 4.

Since we have assumed that the failure times are independent, the theoretical proportion of failures of contracts where party **X** has rating g_X and party **Y** has rating g_Y will be equal to

$$\pi_5^{g_X} + \pi_5^{g_Y} - \pi_5^{g_X} \pi_5^{g_Y} \qquad (2.18)$$

The probability of a failure of a 5-year contract, given the credit ratings of the firms, is presented in Table 5.

Table 5 Swap Contract Default Probabilities

This matrix shows the probability (in percent) that there is a default, by at least one party, within the 5-years swap life. The probabilities are calculated using equation (2.18).

	AAA	**AA**	**A**	**BAA**	**BA**	**B**
AAA	0.135	0.172	0.349	0.414	0.740	0.833
AA	0.172	0.208	0.377	0.439	0.751	0.840
A	0.349	0.377	0.510	0.559	0.804	0.874
BAA	0.414	0.439	0.559	0.603	0.824	0.887
BA	0.740	0.751	0.804	0.824	0.922	0.950
B	0.833	0.840	0.874	0.887	0.950	0.968

For each contract in category 1, we use the calibrated CIR-model to simulate interest rate series for the duration of the contract (one new series for each contract). Using the simulated interest rates, we calculate net transfers (to party

X) at each settlement. We also calculate the net transfer at the failure date. To do this, we calculate the time interval between the last settlement and default. This period is crucial in calculating the value of the swap when we have defaults. The difference between the fixed and the floating rate at the last settlement is used to calculate the net settlement at default. If the fixed rate is higher than the floating, it is the floating-rate payer that stands to lose money if the fixed-rate payer should default. This asymmetry is the most distinguishing element in swaps, and the thing that makes a swap very different from other derivatives. This asymmetry explains why the net value of a contract for firm **X** typically decreases as the credit rating of firm **Y** worsens. We see also, for example, that the more frequent settlements we have, the less the value of the contract will be affected by the default probabilities. For a lengthier discussion of this asymmetry see Duffie and Huang (1996). Finally, we discount all these values back to present time to get the net value of each simulated contract $v_1, \ldots v_{N1}$.

For each contract in category 2 we could have done the same thing: simulate interest rates and discount the net transfers. However, this is not necessary. Because we have assumed a CIR model for the short-term interest rate, we know the distribution at each settlement date. Thus, instead of simulating interest rates for each contract in category 2, we immediately find the expected value (for party **X**) of such a contract which we call $v_{(2)}$. Finally, the value of the contract is estimated with the average:

$$\hat{v}_X = \frac{1}{N}\sum_{i=1}^{N_1} v_i + \frac{N_2}{N} v_{(2)} \tag{2.19}$$

Clearly, plim $\hat{v}_X = v_X$ as $N \to \infty$

Table 6. Expected Swap Values for Different Fixed Interest Rates

The table shows how the net expected return changes when the fixed interest rate changes around the initial floating interest rate, R_0. We use the empirically estimated CIR-parameters from Table 2 and use 50.000 simulations to estimate the values. The values are in basis points of the nominal amount. The amounts are the net amount to be paid to the fixed-rate payer by the floating-rate payer.

Fixed Rate Payer	**AAA**			**A**		
Floating Rate Payer	**AAA**	**A**	**B**	**AAA**	**A**	**B**
R_0 - 0.100 %	-0.902	-0.843	-0.723	-0.853	-0.825	-0.725
R_0 - 0.010 %	-0.091	-0.083	-0.072	-0.084	-0.082	-0.072
R_0 - 0.005 %	-0.045	-0.042	-0.036	-0.042	-0.041	-0.036
R_0 - 0.001 %	-0.009	-0.008	-0.007	-0.008	-0.008	-0.007
R_0	0.000	0.000	0.000	0.000	0.000	-0.000
R_0 + 0.001 %	0.009	0.008	0.007	0.008	0.008	0.007
R_0 + 0.010 %	0.044	0.042	0.036	0.042	0.041	0.036
R_0 + 0.050 %	0.090	0.085	0.072	0.085	0.082	0.072
R_0 + 0.100 %	0.901	0.839	0.723	0.848	0.823	0.720

Table 7. Expected Swap Values for Different Rating Categories

Simulation results using the empirical estimates of the CIR-parameters. The results are based on 500.000 simulations. Values are in basis points, i.e. 100s of percentage of nominal amount. For example a swap between a fixed rate payer with rating AAA (horizontal bar) and a floating rate payer with rating BAA (vertical bar) has a expected net value of 0.00264% of nominal amount. This means that the floating-rate payer shall pay 0.00264% of the notional amount to the fixed-rate payer in order to compensate for the default risk.

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
Floating rate payer	**AAA**	0.005	-0.030	-0.032	-0.134	-0.865	-1.900
	AA	0.045	0.020	0.017	-0.091	-0.822	-1.885
	A	0.054	0.012	0.016	-0.093	-0.818	-1.890
	BAA	0.264	0.220	0.219	0.116	-0.614	-1.698
	BA	1.826	1.791	1.802	1.713	0.968	-0.200
	B	0.005	-0.030	-0.032	-0.134	-0.865	-1.900

The values for $\hat{v}_X$ for different credit rating are presented in Table 6. The corresponding net expected return (in basis points of notional amounts) is presented in Table 7. It is worth to notice that even if the two counterparts have the same rating, the net expected return of the swap is not zero. This is not a spurious result, instead it hinges on the fact that from an initial interest rate of, say 5%, the floating-rate payer will pay between 0% and $+\infty$. The fixed-rate payer will pay 5% at all times. The downside for the fixed-rate payer is smaller than that of the floating-rate payer, all else equal.

2.7 Sensitivity Analysis

In this section we consider some sensitivity analysis on the expected value of the swap as we change the parameters of the CIR process. Table 8 shows the effect of varying the speed of adjustment parameter κ. From this table, we may conclude that a faster adjustment towards the mean is beneficial for the floating-rate payer.

Table 9 shows the effect of varying the long-term average of the short-interest rate θ. Since we are keeping the initial interest rate constant, a θ higher than r_0 means that the short-term interest rate is expected to increase in the short-run period. An increase in the interest rate benefits the fixed-rate payer on the behalf of the floating-rate payer.

Figure 1 illustrates the relationship between the net expected swap return and the initial interest rate when the long-term average in the interest rate is 10%. The relationship is close to linear. This linearity result agrees with the results in Duffie and Huang (1996).

In the following figure we let the long-term mean of the interest rate, θ, be 5% instead of 10%. We then vary the initial interest rate, r_0, between 1% and 10%. The shape of the curve doesn't change notably. We therefore have reason to believe that the change in mean long-term interest rate only causes a shift in the swap curve.

In Figure 3, we let the long-term mean, θ, vary while keeping the initial interest rate constant at 10%. We find a relationship that appears to be linear. The conclusion from this analysis is therefore that the relationship between the net expected return of the swap and the initial interest rate and the long-term mean interest rate is a linear one. It is therefore not necessary to do new simulations every time the r_0 or θ parameters change. Instead it is preferable to use existing values and extrapolate new net expected returns of swaps.

Table 8. Simulation Results with Different κ-values

Simulation results using different values of κ. κ_2 is the estimated kappa, κ_1 is $\kappa_2/2$ and κ_3 is κ_2*2. θ and the initial interest rate are set to 10%. The variance in the CIR is set to the estimated value 0.085615. Each estimate is based on 500,000 simulations, which has been found to be sufficient to produce stable estimates. The estimates are in basis point of percentages, i.e. 1/100 of a percentage point. The asymmetries of the results indicate the difference between paying and receiving a fixed rate. The table shows the expected value of the swap to the fixed-rate payer, i.e. how much the floating-rate payer should pay to the fixed-rate payer. The rating of the fixed-rate payer is shown on the horizontal bar and that of the floating-rate payer on the vertical bar.

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
	AAA						
	κ_1	0.0100	-0.0228	-0.0209	-0.1009	-0.6351	-1.3391
	κ_2	0.0049	-0.0298	-0.0322	-0.1345	-0.8648	-1.8996
	κ_3	0.0004	-0.0362	-0.0399	-0.1597	-0.9478	-2.1696
	AA						
	κ_1	0.0449	0.0187	0.0102	-0.0698	-0.6050	-1.3301
	κ_2	0.0451	0.0199	0.0170	-0.0915	-0.8217	-1.8853
	κ_3	0.0465	0.0128	0.0030	-0.1146	-0.9193	-2.1045
Floating	**A**						
rate	κ_1	0.0371	0.0102	0.0043	-0.0707	-0.6110	-1.3100
payer	κ_2	0.0542	0.0126	0.0160	-0.0932	-0.8181	-1.8904
	κ_3	0.0468	0.0092	0.0101	-0.1127	-0.9257	-2.1336
	BAA						
	κ_1	0.2217	0.1933	0.1997	0.1014	-0.4247	-1.1657
	κ_2	0.2643	0.2202	0.2188	0.1160	-0.6139	-1.6979
	κ_3	0.2637	0.2179	0.2270	0.1104	-0.6870	-1.9301
	BA						
	κ_1	1.6096	1.5877	1.5554	1.4849	0.9687	0.1686
	κ_2	1.8260	1.7906	1.8015	1.7127	0.9684	-0.1998
	κ_3	1.8693	1.8624	1.8314	1.7500	0.9721	-0.3857
	B						
	κ_1	5.6406	5.5586	5.5045	5.4359	4.9282	4.1380
	κ_2	6.1479	6.0975	6.0890	5.9426	5.2567	4.0557
	κ_3	6.3362	6.3006	6.3007	6.1378	5.4011	4.0487

Table 9. Simulation Results with Different θ-values

Simulation results using different values of θ. θ_2 is the estimated theta, θ_1 is $\theta_2/2$ and θ_3 is θ_2*2. κ is set to the estimated value. The initial interest rate is set to 10%. The variance in the CIR is set to the estimated value 0.085615. The table is constructed in the same way as Table 8, with 500.000 simulations per estimate.

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
	AAA						
	θ_1	-0.0164	-0.0896	-0.0889	-0.2520	-1.1463	-3.0033
	θ_2	0.0049	-0.0298	-0.0322	-0.1345	-0.8648	-1.8996
	θ_3	0.0301	0.0137	0.0116	-0.0570	-0.4864	-1.2026
	AA						
	θ_1	0.0011	-0.0638	-0.0651	-0.2398	-1.4172	-2.9966
	θ_2	0.0451	0.0199	0.0170	-0.0915	-0.8217	-1.8853
	θ_3	0.1088	0.0914	0.0821	0.0137	-0.4260	-1.1361
	A						
	θ_1	0.0047	-0.061	-0.0638	-0.2404	-1.4189	-3.0235
Floating	θ_2	0.0542	0.0126	0.01397	-0.0932	-0.8181	-1.8904
rate	θ_3	0.1011	0.0865	0.0834	0.0188	-0.4271	-1.1324
payer	**BAA**						
	θ_1	0.1313	0.0626	0.0664	-0.1255	-1.2614	-2.8747
	θ_2	0.2643	0.2202	0.2188	0.11601	-0.6139	-1.6979
	θ_3	0.4207	0.3822	0.3786	0.3312	-0.0866	-0.8447
	BA						
	θ_1	1.1391	1.0889	1.0663	0.9058	-0.2308	-1.8126
	θ_2	1.826	1.7906	1.8015	1.7127	0.9684	-0.1998
	θ_3	2.6577	2.6144	2.647	2.5554	2.0919	1.2328
	B						
	θ_1	4.2905	4.2366	4.211	4.0557	3.0363	1.5701
	θ_2	6.1479	6.0975	6.089	5.9426	5.2567	4.0557
	θ_3	8.3364	8.2674	8.257	8.2001	7.6465	6.5276

Figure 1. Simulated Net Expected Returns as the Initial Interest Rate Varies, $\theta = 10\%$

The figure illustrates the relationship between the net expected return and the initial interest rate used when simulating the net expected return. We use a swap between a fixed-rate payer with an AA rating and a floating-rate payer with an A rating. The swap has 5 years as time to maturity. The solid line represents the prices when κ=0.0033. The dotted line represents prices when κ=0.0066. We let the initial interest rate, r_0 , in the CIR process vary between 5% and 15%. The long-term mean, θ, is 10%.

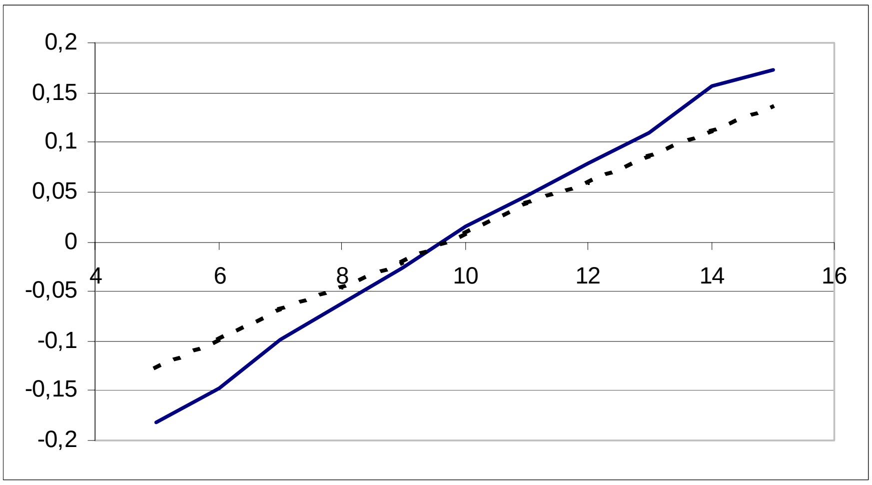


Figure 2. Simulated Net Expected Returns as the Initial Interest Rate Varies, $\theta = 5\%$

The figure illustrates the relationship between the net expected return and the initial interest rate used when simulating the net expected return. The setup is the same as that of Figure 1. We let the long-term mean, θ, in the CIR process vary between 1% and 10%. The initial interest rate, r_0 , is 5%.

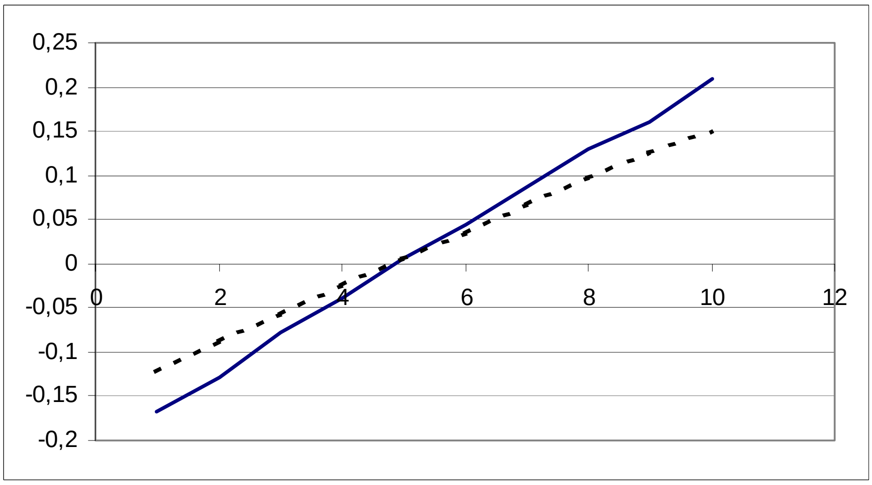
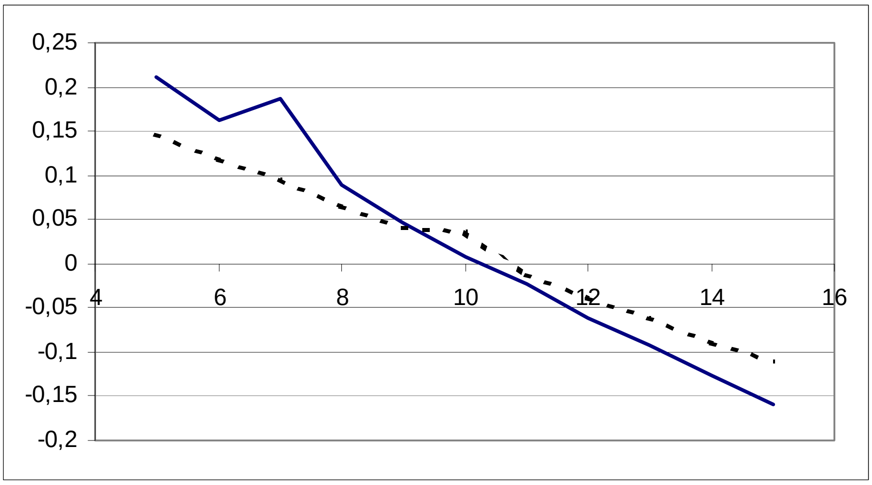


Figure 3. Simulated Net Expected Returns as θ Varies

The figure illustrates the relationship between the net expected return and the initial interest rate used when simulating the net expected return. The setup is the same as in Figure 1. We let the long-term mean, θ, in the CIR process vary between 5% and 15%. The initial interest rate, r_0 , is 10%.



2.8 Summary and Conclusions

In this paper we consider the problem of finding the net expected return of swap agreements when both parties are subject to default risk. We find that the possibility of default has a large effect on the contract. If both parties are rated AAA, the fixed rate is 6%, the short rate is 6%, the yield curve is flat, the nominal amount is $100 million and the recovery rate is 40%, then the expected returns for both parties are about the same (it is minus $5000 for the fixed rate payer, see Table 7). However, if the rating of the party paying the floating rate decreases to B, the expected loss for the fixed-rate payer exceeds $60.000! Clearly, default matters when the counterpart has a low rating.

We also investigate the sensitivity of these results to the processes that governs the short-term interest rate. In this paper we focus on the CIR model and the sensitivity to the parameters of this model. We find that the results are quite sensitive to the parameters that we select although not so sensitive that it will affect the main result that default probabilities matters. This suggests that the interest rate model must be chosen carefully if the results are to be reliable. Future research will investigate the results when other interest rate models are applied.

References

Altman E and Kishore V, 1996, Almost Everything You Wanted to Know about Recoveries on Defaultable Bonds, *Financial Analysts Journal,* Nov/Dec, 57-64.

Bicksler J and Chen A, 1986, An Economic Analysis of Interest Rate Swaps, *Journal of Finance*, 41, 645-655.

Brown S and Dybvig P, 1986, The Empirical Implications of the Cox, Ingersoll, Ross Theory of the Term Structure of Interest Rates, *Journal of Finance*, 41, 617-632.

Brown K, Harlow W and Smith D, 1994, An Empirical Analysis of Interest Rate Swap Spreads, *Journal of Fixed Income*, March, 61-78.

Chan K, Karolyi A, Longstaff F and Sanders A, 1992, An Empirical Comparison of Alternative Models of the Short-Term Interest Rate, *Journal of Finance*, 47, 1209-1227.

Cooper I and Mello A, 1991, The Default Risk of Swaps, *Journal of Finance,* 46, 597-620.

Cox, J., Ingersoll, E., and Ross, S., 1985, A Theory of the Term Structure of Interest Rates, *Econometrica*, 53, 385-407.

Degrér H, 1999a, An Introduction to Swap Agreements, *Licentiate Dissertation*, Departments of Economics, University of Lund, Sweden.

Degrér H, 1999b, The Influence of Credit Risk on Swap Agreements, *Licentiate Dissertation*, Departments of Economics, University of Lund, Sweden.

Cossin D and Pirotte H, 1998, How Well Do Classical Credit Risk Pricing Models Fit Swap Transaction Data?, *European Financial Management Journal*, vol 4, no 1.

Cossin D and Pirotte H, 1997, Swap Credit Risk: An Empirical Investigation on Transaction Data, *Journal of Banking and Finance,* HEC, vol 21, no 10.

Duffie D and Huang M, 1996, Swap Rates and Credit Quality, *Journal of Finance*, 51(3), 921-949.

Duffie D and Singleton K, 1997, Modelling Term Structures of Defaultable Bonds, *Working paper*, The graduate school of business, Stanford University.

Fons J and Kimball A, 1991, Corporate Bond Defaults and Default Rates 1970-1990, *The Journal of Fixed Income*, 36-47.

Hansen L, 1982, Large sample Properties of Generalised Method of Moments Estimation, *Econometrica*, 50, 1029-1054.

Lando D, 1999, Some Elements of Rating-Based Credit Risk Modelling, *Working Paper*, Department of Operations Research, University of Copenhagen, Denmark.

Lang L, Litzenberger R and Liu A, 1998, Determinants of Interest rate Swap Spreads, *Journal of Banking & Finance*, 22, 1507-1532.

Minton B., 1997, An Empirical Examination of Basic Valuation Models for Plain Vanilla U.S. Interest Swaps, *Journal of Financial Economics*, 44, 251-277.

Pearson N and Sun T, 1994, Exploiting the Conditional Density in Estimating the Term Structure: An Application to the Cox, Ingersoll and Ross Model, *Journal of Finance*, 49, 1279-1304.

Smith C, Smithson C and Wakeman L, 1988, The Market for Interest Rate Swaps, *Financial Management*, 17, 34-44.

Sun T, Sundaresan S and Wang C, 1993, Interest Rate Swaps: An Empirical Investigation, *Journal of Financial Economics*, 34, 77-99.

Sundaresan S, 1989, Valuation of Swaps, *Working paper*, Colombia University.

Turnbull S, 1987, Swaps: A Zero Sum Game?, *Financial Management*, Spring, 15-21.

Wall L and Pringle J, 1989, Alternative Explanations of Interest Rate Swaps: A Theoretical and Empirical Analysis, *Financial Management*, Summer, 59-73.

3 Swap Valuation and Term Structures

3.1 Introduction

It is vital to estimate the volatility of the interest rate correctly in order to get accurate estimates of the net expected value of swaps as argued in Degrér & Jochumzén (1999). How the process that generates the term structure is modelled also plays an important role when valuing other interest rate contingent claims. In this paper we study the influence that the choice of term structure model has on the net expected value of the swap. When pricing interest rate contingent claims many practitioners (and academics) often chose an arbitrary term structure model and either calibrate it themselves or use typical values of the parameters. To evaluate the impact of the choice of term structure model on the pricing of contingent claims we have selected some of the most commonly used term structure models. The parameters in these models are calibrated using the Stockholm interbank offer rate (Stibor). We then calculate the net expected values of the swaps, as defined in Degrér and Jochumzén (1999), with the different term structure models. Finally the estimated net expected values are compared. We conclude that even thought the choice of the term structure model affects the net expected value of the swap, it seems more important how the used term structure model is calibrated.

The paper is organized as follows. The second section is a survey of previous related work. In the third section the different term structure models to be estimated and evaluated are presented. In the following section a description of the data set used to evaluate and calibrate the models is given. In the fifth section the calibration of the data generating processes are made. We also compare the estimates to those obtained in Chen et al (1992).[15] Section 6 contains the estimation of the net expected swap returns and an evaluation of the results. Finally, section 7 concludes.

3.2 Short-Term Interest Rate Models

During the last decades there has evolved a number of different models that attempt to generate time-series that in certain ways replicates actual interest rate movements. In one of the most famous papers in the field of term structure models, Chan et al (1992) compares alternative models of the short-term interest rate. They use general method of moments to find the model that best capture the dynamics in the short-term interest rate. They conclude that models which allow the interest rate volatility to depend on the interest rate level perform better than the other models. They do not find any evidence in favour of the use of mean reversion in the models. Dahlquist (1996) also use a GMM approach to compare the performance of different term structure models. They find that mean reversion plays an important part in explaining the behaviour of interest rates. Longstaff and Schwartz (1992) find that term structure models, that let the volatility be a function of the level, fail to model the serial correlation in the interest rates. They also find that GARCH type models fail to capture the relationship between the volatility and the interest rate level. They also introduce a new type of model that allows the interest rate volatility to depend both on the

[15] Our study is however not meant to be a replication of the Chen et al (1992) study.

interest rate level and information shocks. In a more recent study Hördahl (1999) use the Longstaff-Schwartz (1992) model and show that it can be useful for estimating the future distribution of the interest rate. Ait-Sahalia (1996a) estimate the volatility of the interest rate non-parametrically and thus avoids the problems associated with formalizing the dependencies between the volatility and the interest rate. They use the nonparametric model to price interest rate derivatives and find that their model provides different results compared to Cox et al (1985), CIR, and Vasicek (1977) models. Jiang (1998) also use a nonparametric model for the term structure. They find that nonparametric models generate significantly different results that traditional term structure models, i.e. models like those used in this paper. Ait-Sahalia (1996b) compares the implied densities when estimating term structure models parametrically and non-parametrically. They find that the principal source of rejection is the strong non-linearity of the drift component. Stanton (1997) also uses a nonparametric model and finds strong non-linearities in the drift component. Duffee (1996) tries to explain the volatility in another non-traditional way. He divides the volatility into two different components, which behave quite differently. He emphasizes that this is important when estimating the parameters in term structure models.

All in all the previous work emphasises that it is important to model the interest rate in an as flexible way as possible. However, since it is very difficult (possibly impossible) to choose a model that out-performs other models according to all possible criteria, we have chosen to investigate a number of different models to find out the impact that the model selection has on the net expected value of the swaps.

3.3 Models

We deal with term structure models of the generic form shown in equation (2.20) and (2.21). The interest rate depends on a trend component, $\mu(r_t;t)$, and a diffusion component, $\sigma(r_t;t)$. W_s is a standard Brownian motion.

$$\int_{t_0}^{t_N} r_s ds = \int_{t_0}^{t_N} \mu(r_s;s) + \sigma(r_s;s) dW_s \tag{2.20}$$

This equation is often re-written in the partial differential equation (PDE) form:

$$dr_t = \mu\left(r_t;t\right)dt + \sigma\left(r_t;t\right)dW_t \tag{2.21}$$

$$r_0 = r\left(0\right)$$

We restrict the study to deal with models that can be nested in the Chan et al (1992), henceforth CKLS, model.

$$dr_t = (\alpha + \beta r_t)dt + \sigma r_t^{\gamma} dW_t \tag{2.22}$$

In the CKLS model in equation (2.22) the γ parameter is of special interest since it controls the level effect of the interest rate on the volatility. In the CKLS study the γ parameter estimate is 1.449 which (almost) equals the CIR-VR model in Table 1 below. However, Bliss and Smith (1998) re-estimate the γ parameter and include a regime shift parameter. Their estimate of γ then becomes approximately 0.5, which equals the CIR-SR model in Table 1. The β - parameter also has special significance where a negative value indicates the presence of mean reversion in the time series. All models except the CIR-VR, Dothan and Merton models allow for mean-reversion in the interest rate. The continuous time versions of the term structure models under investigation are shown together with their name.

1. Chan et al (1992) (CKLS) $\quad dr_t = (\alpha + \beta r_t)dt + \sigma r_t^{\gamma} dW_t$
2. Brennan-Schwartz (1982) $\quad dr_t = (\alpha + \beta r_t)dt + \sigma r_t \, dW_t$
3. Constant Elasticity of Volatility (CEV) $\quad dr_t = \beta r_t dt + \sigma r_t^{\gamma} dW_t$

4. Cox et al (1985) Square Root (CIR-SR) $dr_t = (\alpha + \beta r_t)dt + \sigma r_t^{1/2} dW_t$

5. Cox et al (1985) Variable Rate (CIR-VR) $dr_t = \sigma r_t^{3/2} dW_t$

6. Dothan (1978) $dr_t = \sigma r_t \, dW_t$

7. Geometric Brownian Motion (GBM) $dr_t = \beta r_t dt + \sigma r_t \, dW_t$

8. Merton (1973) $dr_t = \alpha dt + \sigma dW_t$

9. Vasicek (1977) $dr_t = (\alpha + \beta r_t)dt + \sigma dW_t$

Models 2 to 9 imply the restrictions presented in Table 1 on the CKLS model.

The instantaneous interest rate, r_t, in equation (2.20) is assumed to follow one of the diffusion processes (1) to (9) above. Where W_t is a standard Wiener process and the parameters α, β, γ and σ are taken as constants.

Table 1. Model Restrictions

This table compares an unrestricted model with the restricted models (2-9). Imposed restrictions are shown explicitly. The unrestricted model is the CKLS model in equation (2.22).

Model	α	β	γ	σ
Unrestricted				
Brennan-Schwartz			1	
CEV	0			
CIR-SR			1/2	
CIR-VR	0	0	3/2	
Dothan	0	0	1	
GBM	0		1	
Merton		0	0	
Vasicek			0	

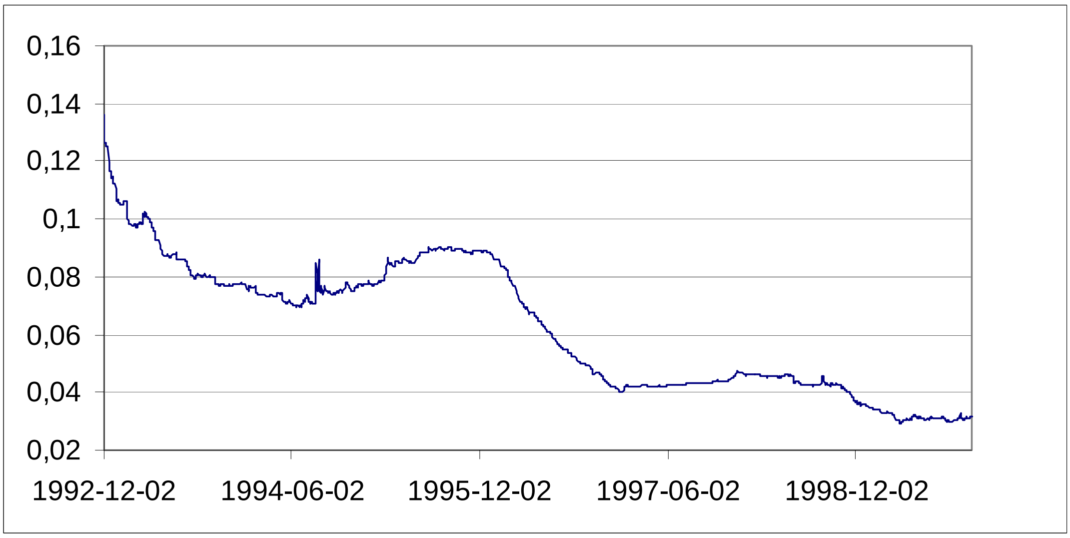
In Table 1 the CKLS, Brennan-Schwartz, CIR-SR and Vasicek models are mean-reverting models, whereas the CEV, GBM and Merton models only incorporates a trend. The CIR-VR and Dothan model consists only of variance components. In all models, except Mertons and Vasiceks, the volatility is allowed to depend on the level of the interest rate.

3.4 Data Set

When estimating the short term interest rate process it is desirable to use an as short term interest rate as possible. However, if we were to choose the shortest (overnight) interest rate then the dataset would contain some unreliable quotes, due to the relatively low liquidity of this market segment. Instead we use the 1-month Stibor middle interest rate (obtained from Datastream). The reason for using the middle rate instead of the rate at some specific time (for example closing rate) is that we want to minimize the possibility of including outliers in the dataset.[16] We use a daily frequency for the interest rate (excluding weekends and holidays).

Figure 1. The Data Set

The figure shows the Stockholm interbank middle offer rate (Stibor) between 92-12-02 and 99-11-03. It consists of 1806 daily observations.



[16] The middle-rate is defined as the average of the highest and the lowest interest rate during the day.

As can be seen in Figure 1, the interest rate exhibits a downward trend over the sample period, mainly due to the falling inflation in Sweden over the period. The interest rate decrease from about 8-12% in 1992-1995 to about 3% in 1999. We have divided the data set into two sub-periods where the first is used for estimation of the term structure model parameters and the second is used for evaluation of the models. By dividing the data in this way, we put the different short term interest rate models to a difficult test. However, we use the same estimation and evaluation periods for all the models which should make the comparison of the models fair.

Table 2. Descriptive Statistics

The table shows descriptive statistics for the 1-month Stibor rate, r_t, for different periods. N indicates the number of observations used.

Period	Time Period	N	Mean	Standard deviation	$\rho_{t,t-1}$
I	921202-960517	903	0.0832	0.0106	0.9964
II	960520-991103	903	0.0419	0.0076	0.9995
I+II	921202-991103	1806	0.0626	0.0226	0.9996

Each sub-period consist of 903 observations. Table 2 shows the mean, standard deviation and auto-correlation of the 1-month Stibor. The average interest rate is 6.26% with a standard deviation of 2.26% for the whole period. For the first sub-period the mean is 8.32% with a standard deviation of 1.06% compared to the second sub-period which has a mean of 4.19% and a standard deviation of 0.76%. It is clear that the mean interest rate as well as the standard deviation has decreased during the whole sample period.

3.5 Estimating the Short Term Interest Rate Process

In this section we describe the models used in the estimation as well as the different methods used to estimate the model parameters. Following Brennan & Schwartz (1982), Dietrich-Campbell and Schwartz (1986), Sanders and Unal (1988), Chan et al (1992) and Dahlquist (1996), we estimate the parameters of the following discrete-time version, equations (2.23)-(2.24) of the stochastic differential equation (2.22).

$$r_t - r_{t-1} = \alpha + \beta r_{t-1} + \varepsilon_t \tag{2.23}$$

$$E\left[\varepsilon_t^2\right] = \sigma^2 r_{t-1}^{2\gamma} \quad E\left[\varepsilon_t\right] = 0 \tag{2.24}$$

The discrete-time model in equation (2.23)-(2.24) is only an approximation of the continuous-time model. It can be shown that the approximation error introduced by using the discrete model is of second order importance if changes in r_t are at a high frequency.[17] The estimation is made with both generalized method of moments and quasi-maximum likelihood (QML).[18] The different approaches are described in some detail below.

3.5.1 Generalized Method of Moment Approach

In this section the GMM approach of Hansen (1982) is used to estimate the parameters in the term structure models. The GMM approach has a number of advantages over other estimation techniques. First, it is not necessary to assume any specific distribution of the disturbances. All GMM results holds asymptotically if the disturbances are stationary and ergodic (see Ogaki (1993)). The assumptions about the disturbances in the models under investigation are very different, for example the Merton model assumes that the disturbances be normally distributed whereas the CIR-SR assumes a non-central χ^2-distibution.

[17] Chan et al (1992) and Dahlquist (1996) uses monthly data in their estimations whereas we use daily data. See also Campbell (1986).
[18] QML is also known as pseudo-maximum likelihood.

This presents no problem for us since, as stated above, the GMM approach does not rely on the distribution of the disturbance term. A second advantage with the GMM approach is that it provides asymptotically efficient estimates, even if the disturbances are conditionally heteroskedastic. The advantages of using GMM is even greater when considering the article by Chan et al (1992) who argue that even if the distribution of the continuous-time model of the term-structure model is known it is not clear that the discrete version of it will follow the same distribution. This is due to the temporal aggregation problem described in Grossman et al (1987), Breeden et al (1989) and Longstaff (1989b, 1990). The final rationale for using the GMM approach is that it has been used extensively in other articles including Gibbons and Ramaswamy (1986), Harvey (1988), Longstaff (1989a), Chan et al (1992) and Dahlquist (1996). We will use the discrete version of equation (2.22) when estimating the parameters. All the other term-structure models can be nested in this model. The model to be estimated follows Chan et al (1992) and Dahlquist (1996).

Define θ to be the parameter vector with elements α, β, γ and σ. We then define the vector of moment conditions, f, as follows:

$$f_t(\theta) = \begin{bmatrix} \varepsilon_{t+\Delta t} \\ \varepsilon_{t+\Delta t} r_t \\ \varepsilon_{t+\Delta t}^2 - \sigma^2 r_t^{2\gamma} \\ \left(\varepsilon_{t+\Delta t}^2 - \sigma^2 r_t^{2\gamma}\right) r_t \end{bmatrix} \tag{2.25}$$

The vector, g, contains the mean of the moment conditions and is defined as:

$$g_T(\theta) = \frac{1}{T} \sum_{t=1}^{T} f_t(\theta) \tag{2.26}$$

In order to obtain the GMM estimates the goal function J is minimized with respect to the parameter vector θ as shown below:

$$\hat{\theta} = \arg\min_\theta J_T(\theta) = g_T^{'}(\theta) W_T(\theta) g_T(\theta) \tag{2.27}$$

The optimal weighting matrix W is defined as (see Hansen, 1982):

$$W_T(\theta) = E\left[f_t(\theta) f_t^{'}(\theta) \right]^{-1} \tag{2.28}$$

The variance/covariance matrix is then given by:

$$asym.VarCov(\theta) = \frac{1}{T}\left(D_{opt}^{'}(\theta) W_{opt}(\theta) D_{opt}(\theta) \right)^{-1} \tag{2.29}$$

In order to evaluate the restrictions imposed by the various models we use the methods developed by Newey and West (1987) where the null-hypothesis is of the form $H_0{:}\rho(\theta)=0$, and where $\rho(\theta)$ is a k-vector of restrictions imposed on the model. The following test statistic is asymptotically χ^2-distributed with k degrees of freedom under the null-hypothesis:

$$R = T\left(J_T\left(\tilde{\theta}\right) - J_T\left(\hat{\theta}\right) \right) \tag{2.30}$$

This test is analogous to the likelihood ratio test. The results from the GMM estimations are presented in Table 3 below.

In the models where the β-parameter has been estimated we find strong evidence of mean reversion in the data, i.e. negative values of β. The *p*-values from the *R*-test suggest that all models except the Vasicek, the Brennan-Schwartz and the CIR-SR model can be rejected at the 95% confidence level. This implies that the other models are miss-specified compared to the CKLS model. The three models that were not rejected are all mean-reverting models whereas the rejected are not. We also find some evidence for mean reversion in the unrestricted CKLS model; estimates of the α and β parameters are both significantly different from zero. These results compare with Dahlquist (1996) who concludes that mean-reversion is an important factor in the specification of the interest rate dynamics. The results do, however, not agree with Chan et al (1992) who argues the mean-reversion is not very important. It is however difficult to comment the differences in the estimated parameter values between this and other studies since the data sets are different.

Table 3. Estimated Parameters in the Term Structure Models (GMM)

The parameters are estimated using the GMM method of Hansen (1982). The data set is the daily middle-rate of the 1-month Stibor between 1992-12-02 and 1996-05-17 (903 observations). The *R*-values are from test proposed by Newey & West (1987). *p*-values for the χ^2-test are presented in parentheses. The CKLS model is estimated with method of moments since it is an exactly identified model. [t-values] are provided for the estimated parameters.

Model	α	β	γ	σ	*R*
CKLS	0.001	-0.016	1.620	0.048	-
	[2.70]	[-2.83]	[0.95]	[0.24]	
Brennan- Schwartz	0.001	-0.015	1.000	0.011	0.122
	[2.85]	[-3.05]	[-]	[6.86]	(0.727)
CEV	0.000	-0.0007	-12.48	0.000	8.255
	[-]	[-3.00]	[-0.62]	[0.02]	(0.016)
CIR-SR	0.0011	-0.014	0.500	0.003	0.37
	2.78	-2.98	[-]	6.75	(0.543)
CIR-VR	0.000	0.000	1.500	0.016	20.745
	[-]	[-]	[-]	[2.41]	(0.000)
Dothan	0.000	0.000	1.000	0.005	20.101
	[-]	[-]	[-]	[2.77]	(0.000)
GBM	0.000	-0.001	1.000	0.005	8.478
	[-]	[-3.59]	[-]	[3.29]	(0.014)
Merton	-0.000078	0.000	0.000	0.00044	9.266
	[-3.33]	[-]	[-]	[2.83]	(0.010)
Vasicek	0.0010	-0.013	0.000	0.0009	0.712
	[2.70]	[-2.91]	[-]	[6.63]	(0.399)

It is worth noticing that the estimate of γ in the CKLS model is close to the values estimated by CKLS in their paper, our point estimate is 1.620 compared to CLKS's estimate of 1.449. The point estimate implies that the volatility is highly dependent on the level of the interest rate. This value is also much higher than the values used in most of the models. In the CKLS model as well as in the Brennan-Schwartz, CEV, CIR-SR, GBM, and Vasicek models we find evidence of mean reversion in the data, i.e. $\beta < 0$. We also observe that the estimated parameters in the CEV model appears to be wrong, since a large and negative value of γ is far from what previously has been found in the literature. This is probably due to problems with the GMM optimization procedure.

3.5.2 Quasi-Maximum Likelihood

Ruiz (1994) shows that, in the case of stochastic volatility models, the relative efficiency of estimators based on the GMM approach is relatively low compared to estimators based on the QML estimators. Andersen and Sørensen (1997) have however challenged some of the results in Ruiz (1994) article by pointing out some cases where the GMM estimator is superior to the QML estimator. However, as Ruiz (1997) argues, the QML estimator is more efficient than the GMM in most cases concerned with the stochastic volatility models.

Because of the similarities between term structure models and stochastic volatility models it appears relevant to re-estimate the parameters using the QML approach. In the unrestricted case the QML approach is identical to the standard ML approach since no misspecification is assumed. The standard errors of the ML estimates are then calculated using a bootstrapping procedure. Bootstrapping is a special Monte Carlo method that reduces the influence of the assumptions made on the distribution of the error term and the parameter values. The maximum likelihood estimates are given in equation (2.31) below. We use the nested CKLS model in the generic maximization formulas below. The restrictions from Table 1 are then imposed in order to obtain ML estimates for the other models.

$$\hat{\theta} = \arg\max_{\theta} L = -\frac{T}{2}\ln\left(2\pi\right) - \frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\varepsilon'\varepsilon \qquad (2.31)$$

The conditions that need to be fulfilled in order for the QML estimates to be asymptotically consistent and have a limiting normal distribution have been discussed in some articles. Bollerslev and Wooldridge (1992) present necessary and sufficient conditions for dynamic models with time-varying covariances. They conclude that if the first two conditional moments are correctly specified the normal log-likelihood function will have the Martingale difference property and therefore generally are consistent and have a limiting normal distribution. These ARCH/GARCH/IGARCH type of models are further studied in papers by Lee and Hansen (1994), Lumsdaine(1996) and González-Rivera and Drost (1999).

It is clearly not a trivial matter to prove consistency and limiting normal distribution for term structure models, which contain both GARCH-type of components as well as trend-components. A different approach, which is also used to some extent in Bollerslev and Wooldridge (1992), is to use a Monte Carlo study to test for consistency. We will follow this approach.

The models under investigation have different distributions, it will therefore be necessary to perform separate Monte Carlo simulations for each model. We have chosen to use parameter values close to those obtained when using the GMM approach with the Stibor data. These estimates are presented in Table 4.

Table 4. Consistency Test for the QML Estimates

This table shows the results from a Monte Carlo simulation of the QML procedure. For each model the first line shows the QML estimates, followed by its standard error. The 95% upper and 95% lower values are the empirical confidence interval boundaries. The Monte Carlo simulations are based on 10.000 simulations with length 1000.

Model	α	β	γ	σ
CKLS				
Coefficient	0.001	-0.016	1.620	0.048
Std.error	0.0003	0.004	1.250	0.180
Upper 95%	0.0016	-0.0012	2.220	0.382
Lower 95%	0.0005	-0.0023	0.058	0.000
Brenna-Schwartz				
Coefficient	0.001	-0.015	1	0.011
Std.error	0.0003	0.005	-	0.000
Upper 95%	0.0015	-0.005	-	0.012
Lower 95%	0.0006	-0.029	-	0.011
CEV				
Coefficient	0	-0.001	-12.480	0.001
Std.error	-	0.0003	20.130	2.251
Upper 95%	-	-0.0006	14.725	5.190
Lower 95%	-	-0.0017	-34.289	-3.752
CIR-SR				
Coefficient	0.0001	-0.014	0.5	0.003
Std.error	0.00003	0.005	-	0.001
Upper 95%	0.00015	-0.007	-	0.005
Lower 95%	0.00006	-0.028	-	0.001
CIR-VR				
Coefficient	0	0	1.5	0.016
Std.error	-	-	-	0.007
Upper 95%	-	-	-	0.032
Lower 95%	-	-	-	0.002
Dothan				
Coefficient	0	0	1	0.005
Std.error	-	-	-	0.002
Upper 95%	-	-	-	0.008
Lower 95%	-	-	-	0.003
GBM				
Coefficient	0	-0.0010	1	0.005
Std.error	-	0.0002	-	0.002
Upper 95%	-	-0.0006	-	0.009
Lower 95%	-	-0.0015	-	0.001
Merton				
Coefficient	-0.0010	0	0	0.0010
Std.error	0.0003	-	-	0.0004
Upper 95%	-0.0005	-	-	0.0021
Lower 95%	-0.0019	-	-	0.0001
Vasicek				
Coefficient	0.0010	-0.013	0	0.0010
Std.error	0.0003	0.004	-	0.0002
Upper 95%	0.0018	-0.001	-	0.0019
Lower 95%	0.0003	-0.021	-	0.0002

Since all models can be nested in the CKLS model we will only present the log-likelihood function used for that model:

$$\log L_t = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log\left(\sigma^2 r_{t-1}^{2\gamma}\right) - \frac{1}{2\sigma^2 r_{t-1}^{2\gamma}}\left(r_t - \alpha - (1+\beta) r_{t-1}\right)^2 \qquad (2.32)$$

The log-likelihood function can easily be verified if the CKLS model is re-written as:

$$r_t = \alpha + (1+\beta) r_{t-1} + \varepsilon_t \qquad (2.33)$$

$\varepsilon_t = \omega_t \eta_t$ where η_t is $\mathrm{N}(0,1)$ and

$\omega_t = \sigma r_t^{2\gamma}$

The log-likelihood function is then maximized in order to obtain estimates of the parameters. The estimation is done on a data set that is simulated using the corresponding model. For example when the GBM model is tested, the GBM model is first used to simulate a data set and then used again in the log-likelihood function to estimate the parameters. This procedure is performed 10.000 times on data sets of length 1000. The results are presented in Table 4 above. As shown in appendix A, the resolution (i.e. the number of steps used when approximating the continuous model) of the process does not appear to matter when using a Monte Carlo approach. We have therefore chosen a low resolution in order to save computing time. Table 4 reveals no obvious inconsistencies in the QML method when applied to the used models. The true parameter values lies well inside the 95% confidence interval bounds of the estimated parameters. We therefore conclude that the QML provides us with apparently consistent estimates for the parameter values used in the test. Since the QML appears to provide consistent estimates we now proceed with the bootstrapping, introduced by Efron (1979), in order to obtain variances of the estimates.[19] As shown in Flanchaire (1999), bootstrapping can yield large asymptotical improvements over other numerical methods of calculating test

[19] Another approach would be to use the algebraic method presented in Gourieroux et al (1984), White (1982) and Wooldridge (1991).

statistics. As can be seen in the papers by Lii and Maddala (1996) and Hinkley (1997) there are a number of other variations of the bootstrap. The different variants deal mainly with heteroscedasticity and autocorrelation in the empirical distribution. We use block bootstrapping method with a block length of 4 in order to incorporate the autoregressive components in the models. We have tried different block lengths but not found any improvements when using larger the block sizes. The method used is explained in some detail in appendix B. The *t*-values from the bootstrapping procedure are presented together with the parameter estimates from the QML in Table 5. We find that the QML estimates of α and β are very similar to those obtained from the GMM approach in Table 4. The estimates of γ and σ are, however, somewhat different from the GMM estimates found in the previous section.

We also find that the bootstrapped variances of the parameter estimates are much lower than those obtained with the GMM approach. We find that mean-reversion is important and that all α and β parameters are significantly different from zero (except for the Vasicek model). This is the same result that we obtained with the GMM method.

When comparing the estimates from the QML approach to the GMM approach we find that we still have evidence of mean reversion in the CKLS model (as well as in the other models). However, the γ-estimate from the QML approach (0.398) is now much closer to 0.5 as found in Bliss and Smith (1998) than to the value of 1.449 as found in the CKLS study.

Table 5. Estimated Parameters in the Term Structure Models (QML)

The parameters are estimated using the QML. The data set is the daily middle-rate of the 1-month Stibor between 1992-12-02 and 1996-05-17 (903 observations). The [*t*-values] are obtained from the bootstrapping procedure.

Model	α	β	γ	σ
CKLS	0.001	-0.014	0.398	0.002
	[5.43]	[-5.83]	[6.70]	[5.05]
Brennan- Schwartz	0.001	-0.012	1	0.011
	[4.37]	[-4.64]	[-]	[52.94]
CEV	0	-0.001	0.479	0.003
	[-]	[-3.87]	[8.18]	[5.42]
CIR-SR	0.001	-0.014	0.5	0.003
	[5.30]	[-5.68]	[-]	[59.22]
CIR-VR	0	0	1.5	0.038
	[-]	[-]	[-]	[Inf]
Dothan	0	0	1	0.011
	[-]	[-]	[-]	[58.13]
GBM	0	-0.001	1	0.011
	[-]	[-2.82]	[-]	[62.06]
Merton	-0.0001	0	0	0.001
	[-3.52]	[-]	[-]	[5.97]
Vasicek	0.001	-0.016	0	0.001
	[0.80]	[-0.80]	[-]	[0.81]

3.5.3 Evaluation

In the following sections we evaluate the GMM and the QML estimates in order to choose which estimates to use in the remainder of the paper.

3.5.3.1 GMM Estimates

There are many possible ways to evaluate the different term structure models. Chan at al (1992) use the method developed by Newey and West (1987) to test the restrictions imposed on the unrestricted model when estimating the other models. In this test the null hypothesis takes the form, $H_0: \alpha(\theta)=0$, where $\alpha(\theta)$ is a vector of order k (number of restrictions). The test statistics is:

$$R = T\left(J_T\left(\tilde{\theta}\right) - J_T\left(\hat{\theta}\right)\right) \tag{2.34}$$

R is χ^2-distributed with k degrees of freedom. This test is analogous to the likelihood ratio test. $J_T\left(\tilde{\theta}\right)$ is the objective function for the restricted efficient GMM-estimator and $J_T\left(\hat{\theta}\right)$ is the objective function for the unrestricted efficient GMM-estimator. Both estimates use the same weighting matrix W from the unrestricted model. A high value of R means that the restricted model is miss-specified compared to the unrestricted. However, it says nothing about the predictive ability of the different term structure models. In order to be able to make some comparison of the models we use the mean residual squared sum (*MRSS)* as defined in equation (2.35) below.

$$MRSS = \frac{1}{N}\sum_{j=1}^{N}\sum_{i=1}^{903}(r_i - \hat{r}_{i,j})^2 \tag{2.35}$$

Where $\hat{r}_{i,j}$ is the predicted value of the interest rate at time I for simulation j and r_i is the value of the 1-month Stibor at time i. Further,

$$\rho_{t,t+1} = \frac{1}{N}\sum_{j=1}^{N}\frac{\operatorname{cov}(d\hat{r}_{i,j}, d\hat{r}_{i+1,j})}{\operatorname{var}(d\hat{r}_{i,j})}\forall i = 2,3,...,902 \tag{2.36}$$

where,

$$d\hat{r}_{i,j} = \hat{r}_{i,j} - \hat{r}_{i-1,j} \forall i = 2,3,...,903, \; j = 1,2,...,N$$

This measure is equivalent with the *RSS* in least square estimation. The *MRSS* measure is however not normally distributed in most of the tested models and it is therefore not possible to statistically test which model that gives the best fit. Nonetheless it provides us with a rough idea of the predictive efficiency of the models. The predictive power of the different models is especially important when the models are being used to price long-term derivatives such as swaps since it is the future interest rate that determines the value subject to default risk. This is explained in detail in Degrér (1999). In Table 6 we also present the mean interest rates, first order autocorrelation of the first differences and the standard deviation of the simulated series. These measures will also help in the comparison of the different models.

We find that there are large differences in the predictive power of the used term structure models. A higher value of *MRSS* means that the model performs less well. If we compare the models based solely on the *MRSS* we find that CEV, GBM, Merton, CIR-VR and Dothan perform considerably better than the rest of the models. The models that perform poorly are all mean-reverting models. The reason that these models perform poor is that they miss-specify the long-term mean of the interest rate process. This is illustrated in appendix C. It is clear that the predictive power of the term structure models is crucially dependent on the ability to correctly predict the future interest rate level.

Table 6. Goodness-of-Fit Statistics in the GMM Estimated Term Structure Models

Summary statistics for the goodness-of-fit for the term structure models using parameters estimated with the GMM approach. Parameters are estimated using the 1-m Stibor rates between 1992-12-02 and 1996-05-17 (903 observations). The models are evaluated using 1-m Stibor rates between 1996-05-20 and 1999-11-03 (903 observations). We have simulated 100.000 series of length 903 for each model. The *MRSS* is the mean residual sum of squares between the actual interest rate and the simulated interest rates. The autocorrelation, $\rho_{t,t-1}$, of the simulated interest rates is calculated using first differences. The numbers inside parenthesis shows the *t*-values.

Model	*MRSS*	*Mean(r)*	*Std(r)*	$\rho_{t,t-1}$
Brennan-Schwartz	1.8603	0.0790	0.0129	0.0073
	(1.83)	(5.99)	(4.69)	(1.33)
CEV	0.0389	0.0432	0.0112	0.9990
	Inf	Inf	Inf	Inf
CKLS	1.4395	0.0744	0.0113	0.0085
	(1.71)	(6.77)	(4.35)	(1.32)
CIR-SR	1.7264	0.0771	0.0130	0.0064
	(1.67)	(5.87)	(4.92)	(1.91)
CIR-VR	0.5876	0.0656	0.0030	-0.0011
	(49.25)	(279.00)	(47.47)	(-0.74)
Dothan	0.6034	0.0656	0.0038	-0.0012
	(46.30)	(261.45)	(48.14)	(-0.81)
GBM	0.0743	0.0430	0.0115	0.0017
	(8.04)	(28.79)	(11.33)	(0.96)
Merton	0.3910	0.0304	0.0206	-0.0010
	(44.64)	(123.9)	(147.5)	(-0.95)
Vasicek	1.7453	0.0765	0.0136	0.0052
	(1.65)	(5.55)	(5.16)	(1.99)
True r-process	-	0.0419	0.0076	0.1355

3.5.3.2 QML Estimates

Most of the estimates from the QML method are similar to those from the GMM methods. This is especially true for the α- and the β- parameters. For the Brennan-Schwartz model, all the parameters from the QML method are equal to the estimates from the GMM method, except for a noise term. These similarities are to be expected since the consistency test did not reveal any inconsistent estimates. The differences in the *t*-values from the GMM and the QML are much larger than the differences in the estimated parameter values as can be seen in Table 5. Both estimation methods provide *t*-values higher than 2.5 for some of the models, namely the Brennan-Schwartz, CIR-SR, CIR-VR, Dothan and GBM models.

Since the estimates from both the GMM and the QML methods are similar we have chosen to only use the GMM estimates in the following analysis.

3.6 Net Expected Value of Swap Agreements

In the following we will use the GMM estimates from above when simulating the net present values of the swap agreements. We start with hazard rate processes that replicate the default rate processes of the counterparts in the swap. The result from these processes provides us with default dates for combinations of counterparts with different credit ratings. We then proceed to simulate interest rate series and calculate the net expected loss/gain for all combinations of counterparts. A full description of the setup is given in Degrer & Jochumzen (1999).

We use an interest rate swap where a fixed interest rate is exchanged for a floating interest rate. The net of the interest rates is exchanged twice a year. We use swaps with a notional amount of $100 million in the simulations. These simulations are repeated 500.000 times for each combination of counterparts with different and/or same credit ratings. It is necessary to have a high number of simulations in order to correctly estimate the net expected values of the swaps.[20] This is especially true when the rating of the counterparts are high since that implies that they rarely default, e.g. a swap between two AAA counterpart

[20] All simulations and estimations are done in Matlab.

has a cumulative default probability of roughly 0.13%. We calculate the net expected values for each term structure model. The results are presented in Table 7a-h below.

Table 7. Net Expected Value from the Different Term Structure Models

The tables show the net expected value of a $100 million 5-year plain-vanilla swap with semi-annual settlements. We use the values estimated with the GMM approach from Table 1. A negative value indicates a negative net expected value to the fixed rate payer. The rating of the fixed rate payer is shown on the horizontal bar and floating rate payers rating is on the vertical bar. The calculations are based on 500.000 simulations of each value. The CEV model has been excluded since the GMM method gave unrealistic estimates. The last table shows the results obtained in Degrér and Jochumzen (1999).

Table 7a. The CKLS Model

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
Floating rate payer	**AAA**	-1.91	-7.12	-7.00	-20.62	-114.54	-242.75
	AA	-1.89	-7.13	-6.99	-20.22	-113.45	-242.57
	A	-1.89	-7.18	-6.91	-20.99	-114.45	-241.81
	BAA	3.82	-0.56	0.13	-15.13	-106.83	-233.53
	BA	56.83	52.94	51.95	39.84	-47.91	-174.05
	B	261.92	264.54	264.62	251.04	173.24	52.82

Table 7b. The Brennan-Schwartz Model

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
Floating rate payer	**AAA**	-1.92	-7.16	-7.05	-20.73	-114.98	-243.14
	AA	-1.91	-7.17	-7.03	-20.31	-113.85	-242.95
	A	-1.96	-7.23	-6.96	-21.11	-114.89	-242.19
	BAA	3.81	-0.59	0.10	-15.24	-107.25	-233.89
	BA	55.43	51.41	53.34	38.65	-47.73	-178.57
	B	267.45	264.87	264.48	249.40	171.82	50.14

Table 7c. The CIR-SR Model

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
Floating rate payer	**AAA**	-1.92	-7.16	-7.05	-20.71	-114.72	-242.08
	AA	-1.90	-7.17	-7.03	-20.26	-113.58	-241.89
	A	-1.90	-7.23	-6.95	-21.09	-114.65	-241.11
	BAA	3.82	-0.58	0.11	-15.20	-106.99	-232.83
	BA	55.50	51.47	53.43	38.74	-47.39	-177.44
	B	267.63	265.06	264.67	249.59	172.25	56.19

Table 7d. The CIR-VR Model

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
Floating rate payer	**AAA**	0.34	-0.67	-0.56	-2.86	-20.17	-40-26
	AA	1.47	0.35	0.39	-1.96	-18.97	-38.80
	A	1.35	0.30	0.37	-2.20	-19.55	-39.16
	BAA	9.67	9.28	9.90	6.68	-10.10	-30.21
	BA	79.04	79.16	78.53	76.31	60.35	34.96
	B	304.26	309.83	310.10	305.41	290.45	260.14

Table 7e. The Dothan Model

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
Floating rate payer	**AAA**	0.33	-0.91	-0.81	-3.77	-25.50	-50.78
	AA	1.73	0.34	0.40	-2.59	-23.95	-49.06
	A	1.58	0.28	0.36	-2.88	-24.69	-49.46
	BAA	10.57	9.95	10.59	6.66	-14.43	-39.85
	BA	83.21	82.27	85.12	79.79	60.27	27.74
	B	320.12	320.17	319.71	313.05	294.00	259.88

Table 7f. The GBM Model

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
Floating rate payer	**AAA**	3.23	3.15	2.97	2.54	2.32	2.02
	AA	7.08	6.76	6.47	7.19	6.24	5.43
	A	17.31	17.97	17.76	17.82	16.09	13.87
	BAA	16.34	16.17	15.81	16.90	15.04	12.93
	BA	50.20	50.68	51.78	50.59	48.26	42.22
	B	280.56	280.25	282.82	280.78	267.50	258.92

Table 7g. The Merton Model

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
Floating rate payer	**AAA**	11.74	11.32	10.71	11.74	10.35	9.08
	AA	28.39	29.36	28.99	29.02	26.33	22.60
	A	25.75	25.64	24.96	26.83	23.73	20.12
	BAA	76.87	77.17	78.68	76.98	73.23	63.21
	BA	411.19	410.33	413.33	411.07	388.08	340.91
	B	914.05	912.62	907.39	901.77	871.44	786.34

Table 7h. The Vasicek Model

		Fixed rate payer					
		AAA	**AA**	**A**	**BAA**	**BA**	**B**
Floating rate payer	**AAA**	-1.91	-7.14	-7.02	-20.61	-114.04	-240.12
	AA	-1.89	-7.14	-7.00	-20.16	-112.89	-239.91
	A	-1.89	-7.20	-6.92	-21.00	-113.99	-239.13
	BAA	3.85	-0.52	0.17	-15.08	-106.31	-230.83
	BA	57.07	53.19	52.19	40.13	-47.21	-171.11
	B	262.61	265.22	265.33	251.78	174.39	56.08

Table 7a-h above contains all the estimated net expected values, henceforth NEV, from the different term structure models except the CEV model. The CEV model has been excluded from the evaluation since the GMM estimation gave unrealistic values, i.e. zero variance. The estimations of the NEV are based on the estimated parameters from the GMM approach in chapter 3.5.1. Compared to the values presented in Degrér and Jochumzen (1999) it is evident that the values in Table 7 are much lower. This indicates a lower risk for the counterparts in the swap agreement. The difference occurs because the model in Degrér and Jochumzen (1999) is calibrated using a different approach, which induces a larger variance in the term structure model.[21] The results from Degrér and Jochumzen (1999) are also presented in Table 7 above. One other factor that influences the NEV is the existence or non-existence of a trend in the interest rate. All the models except the CIR-VR and the model by Dothan allow for a trend. If we investigate the values in Table 7 we see that the highest values of the NEV is found when using the model by Merton. The NEV for two B companies is nearly three times higher than those obtained when using the model that incurs the second highest values, i.e. the CIR-VR model. The NEV results can be categorized into three different groups. The first group contains the model by Merton, which provides the highest values. The GBM, Dothan and CIR-VR model enter into the second group, which provides NEV that are in between group one and three. The third group contains the Vasicek, Brennan-Schwartz, CKLS and CIR-SR model. The difference between the NEV from the Brennan-Schwartz and the Merton model are very large. The Merton model gives us a NEV of $914 for a swap with a fixed rate payer with rating AAA and a floating rate payer with rating B. The corresponding value for the Brennan-Schwartz model is only $262.

[21] Degrér & Jochumzen (1999) calibrates a CIR-SR model in such a way that the sum of the squared differences between the modulated variance at 1, 2, 3, 4 and 5 years and the corresponding empirical variance are minimised.

3.7 Conclusions

This paper illuminates the importance of choosing the correct model for the term structure when calculating the net expected value of swap agreements (NEV). The differences in the NEV between the investigated term structure models are very considerable. The Merton model, for example, gives a NEV of $914 for a swap between a fixed rate payer with rating AAA and a floating rate payer with a B rating whereas the Vasicek model only gives a NEV of $263 for the same swap. There is however no clear relationship between the calculated NEV and the presence or absence of mean-reversion or level dependent volatility.

We have also shown that the method used when estimating the parameters in the term structure model is of importance. The differences between the GMM approach and the QML approach are most apparent in the estimation of the γ, which influences the relationship between the variance and the level of the interest rate, and the parameter for the variance, σ. The consistency check for the QMLE also indicates that the γ and σ parameters seems to be the most difficult to estimate.

For further research we would recommend a test for a structural break in the dataset that takes account of a possible regime shift in the Swedish monetary policy between 1994 and 1996.

References

Ait-Sahalia Y., 1996a, Non-Parametric Pricing of Interest Rate Derivative Securities, *Econometrica*, 64(3), 527-560

Ait-Sahalia Y., 1996b, Testing Continuous-Time Models of the Spot Interest Rate, *The Review of Financial Studies*, 9(2), 385-426

Andersen T.G. and Sørensen B., 1997, GMM and QML Asymptotic Standard Deviations in Stochastic Volatility Models: Comments on Ruiz (1994), *Journal of Econometrics*, 76, 397-403.

Bera A. and Jarque, 1982, Model Specification Tests: A Simultaneous Approach, *Journal of Econometrics*, 20, 59-82.

Bliss, R. and Smith, D., 1998, The Elasticity of Interest Rate Volatility – Chan, Karolyi, Longstaff and Sanders Revisited, *Journal of Risk*, 1(1), 21-46.

Bollerslev T. and Wooldridge J., 1992, Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances, *Econometric review*, 11(2), 143-172.

Breeden D.T., Gibbons M.R. and Litzenberger R.H., 1989, Empirical Tests of the Consumption-Oriented CAPM, *Journal of Finance*, 44, 231-262.

Brennan M. and Schwartz E., 1982, An Equilibrium Model of Bond Pricing and a Test of Market Efficiency, *Journal of Financial and Quantitative Analysis*, 17, 301-329.

Brown S.J. and Dybvig P.H., 1986, The Empirical Implications of the Cox, Ingersoll, Ross Theory of the Term Structure of Interest Rates, *Journal of Finance*,41, 617-632.

Campbell J., 1986, A Defence of Traditional Hypothesis about the Term Structure of Interest Rates, *Journal of Finance*, 41, 183-193.

Chan K., Karolyi G., Longstaff F. and Sanders A., 1992, An Empirical Comparison of Alternative Models of the Short-Term Interest Rate, *Journal of Finance*, 47, 1209-1227.

Cox J., Ingersoll E. and Ross S., 1980, An Analysis of Variable Rate Loan Contracts, *Journal of Finance,* 35, 389-403.

Cox J., Ingersoll E. and Ross S., 1985, A Theory of the Term Structure of Interest Rates, *Econometrica,* 53, 385-407.

Dahlquist M., 1996, On Alternative Interest Rate Processes, *Journal of Banking & Finance*, 20, 1093-1119.

Degrér H., 1999, An Introduction to Swap Agreements, *mimeo*, Department of Economics, University of Lund.

Degrér H. and Jochumzen P., 1999, The Net Expected Value of Swap Agreements: A Simulation Approach, *Licentiate Dissertation*, Department of Economics, University of Lund

Dietrich-Campbell B. and Schwartz E., 1986, Valuing Debt Options: Empirical Evidence, *Journal of Financial Economics*, 16, 321-343.

Dothan L., 1978, On the term Structure of Interest Rates, *Journal of Financial Economics*, 6, 59-69.

Duffee G., 1996, Idosyncratic Variation of Treasury Bill Yields, *The Journal of Finance*, 51(2), 527-551.

Efron B., 1979, Bootstrap Methods: Another Look at the Jackknife, *The Annals of Statistics*, 7, 1-26.

Fitzenberger B., 1997, The Moving Blocks Bootstrap and Robust Inference for Linear Least Squares and Quantile Regressions, *Journal of Econometrics*, 82, 235-287.

Flanchaire E., 1999, A Better Way to Bootstrap Pairs, *Economic Letters*, 64, 257-262.

Gibbons M.R. and Ramaswamy K., 1986, The Term Structure of Interest Rates: Empirical Evidence, *Working Paper*, Stanford University.

González-Rivera G. and Drost F., 1999, Efficiency Comparisons of Maximum-Likelihood-Based Estimators in GARCH Models, *Journal of Econometrics*, 93, 93-111.

Gourieroux C., Monfort A. and Trognon A., 1984, Pseudo Maximum Likelihood Methods: Theory, *Econometrica*, 52(3), 681-700.

Grossman S.J., Melino A. and Shiller R.J., 1987, Estimating the Continuous-Time Consumption-Based Asset Pricing Model, *Journal of Business and Economic Statistics*, 5, 315-327.

Hall P., Horowitz J. and Jing B., 1995, On Blocking Rules for the Bootstrap with Dependent Data, *Biometrika*, 82, 561-574.

Hansen L., 1982, Large Sample Properties of Generalized Method of Moments Estimators, *Econometrica*, 50(4), 1029-1054.

Harvey C., 1988, The Real Term Structure and Consumption Growth, *Journal of Financial Economics*, 22, 305-333.

Hinkley D., 1997, Discussion of Paper by H. Li & G.S. Maddala, *Journal of Econometrics*, 80, 319-323.

Hull J. and White A., 1990, Pricing Interest-Rate-Derivative Securities, *Review of Financial Studies*, 3, 573-592.

Hördahl P., 1999, Estimating the Implied Distribution of the Future Short Term Interest Rate Using the Longstaff-Schwartz Model, *Working Paper*, Riksbanken, Sweden.

Jiang G., 1998, Nonparametric Modelling of U.S. Interest Rate Term Structure Dynamics and Implications on the Price of Derivative Securities, *Journal of Financial and Quantitative Analysis*, 33(4), 465-497.

Kunsch H., 1989, The Jackknife and the Bootstrap for General Stationary Observations, *Annals of Statistics*, 17, 1217-1241.

Lee S. and Hansen B., 1994, Asymptotic Theory for the GARCH(1,1) Quasi-Maximum Likelihood Estimator, *Econometric Theory*, 10, 29-52.

Li H. and Maddala G.,1996, Bootstrapping Time Series Models, *Econometric Reviews*, 15(2), 115-195 including commentary.

Longstaff F.A., 1989a, A Non-Linear Equilibrium Model of the Term Structure of Interest Rates, *Journal of Financial economics*, 23, 195-224.

Longstaff F.A., 1989b, Temporal Aggregation and the Continuous-Time Capital Asset Price Model, *Journal of Finance*, 44, 871-887.

Longstaff F.A., 1990, Time Varying Term Premia and the Traditional Hypothesis about the Term Structure, *Journal of Finance*, 45, 1307-1314.

Longstaff F.A and Schwartz E.S., 1992, Interest Rate Volatility and the Term Structure: A Two Factor General Equilibrium Model, *Journal of Finance*, 47, 1259-1282.

Lumsdaine R., 1996, Consistency and Asymptotic Normality of the Quasi-Maximum Likelihood Estimator in IGARCH(1,1) and Covariance Stationary GARCH(1,1) Models, *Econometrica*, 64(3), 575-596.

Merton R., 1973, Theory of Rational Option Pricing, *Bell Journal of Economics and Management Science, 4, 141-183.*

Newey W. and West K., 1987, Generalized Method of Moment Specification Testing, *Journal of Econometrics*, 29, 229-256.

Ogaki M., 1993, Generalized Method of Moments: Econometric Applications, in: Maddala G., Rao C. and Vinod H., eds., Handbook of statistics, Vol. 11 455-488.

Ruiz A., 1994, Quasi-Maximum Likelihood Estimation of Stochastic Volatility Models, *Journal of Econometrics*, 63, 289-306.

Ruiz A., 1997, QML and GMM estimators of Stochastic Volatility Models: Response to Andersen and Sørensen, *Journal of Econometrics*, 76, 405.

Sanders A. and Unal H., 1988, On the Intertemporal Stability of the Short Term Rate of Interest, *Journal of Financial and Quantitative Analysis*, 23, 417-423.

Stanton R., 1997, A Non-Parametric Model of Term Structure Dynamics and the Market Price of Interest Rate Risk, *The Journal of Finance*, 52(5), 1973-2002.

Vasicek O., 1977, An Equilibrium Characterization of the Term Structure, *Journal of Financial Economics*, 5, 177-188.

White H., 1982, Maximum Likelihood Estimation of Miss-Specified models, *Econometrica*, 50(1), 1-25.

Wooldridge J., 1991, Specification testing and Quasi-Maximum-Likelihood Estimation, *Journal of Econometrics*, 48, 29-55.

Appendix A: The Effect of the Step Length in the Discrete Versions of Term Structure Models.

A CIR-SR process is used to test how the step length in the discrete version of the model affects some of the moments of the distribution. The model is specified in equation (2.37). The step length *Δt* is gradually decreased. In a continuous time model: lim $\Delta t \to 0$. Since it is not possible to obtain data on continuous processes it is necessary to use discrete approximations of the processes. When approximations are used it is sometimes uncertain which properties that are inherited from the continuous processes. We have chosen to use only the four first moments to illustrate the relevancy / irrelevancy of the step length. The result is shown for the data set in first differences as well as in level. For each step length data sets of 100.000 days are simulated using the parameters from Table 3.

$$r_{t+\Delta t} = r_t + \alpha + \beta r_t + \sigma r_t^{1/2} \varepsilon_{t+\Delta t} \; r_{t+\Delta t} = r_t + \alpha + \beta r_t + \sigma r_t^{1/2} \varepsilon_{t+\Delta t} \qquad (2.37)$$

Δt=1 daily data

Δt=0.5 (2 observations per day)

Δt=0.1 (10 observation per day)

Δt=0.01 (100 observation per day)

Table A1. The Series in Level

	Δt = 1	**Δt = 0.5**	**Δt = 0.1**	**Δt = 0.01**
Mean	7.83%	7.83%	7.83%	7.83%
Std. Dev.	0.54%	0.49%	0.53%	0.54%

Table A2. The Series in 1st Difference

	Δt = 1	Δt = 0.5	Δt = 0.1	Δt = 0.01
Mean	0.000	0.000	0.000	0.000
Std. Dev.	0.000	0.000	0.000	0.000
Kurtosis	3.021	3.006	3.013	3.013
Skewness	-0.020	-0.007	-0.003	-0.012
Bera-Jarque	6.71	0.82	0.16	2.41

The Bera and Jarque (1980) test can not reject normality when using a 99% confidence interval. It is worth noting that a rejection of the normality hypothesis does not confirm normality since the test is only of symmetry and mesokurtosis.

Appendix B: The Bootstrapping Procedure

In the bootstrapping procedure we follow the method of Kunsch (1989) and Fitzenberger (1997). They introduced the moving blocks bootstrap, which is an extension of the standard bootstrap method introduced by Efron (1979). This method is robust to heteroskedasticity and autocorrelation of unknown forms. The method follows the steps below:

Calculate the residuals using the estimated parameters from the QML methods (see Table 4).

Draw overlapping blocks, with replacement, from the series with residuals. We continue to draw blocks until the drawn series has approximately the same length as the original series, i.e. 903 observations. If the length of the drawn series is not 903 the results are adjusted to compensate for the difference in length. The following scheme is followed:

1. Adjust the original series with the drawn residual series.
2. Re-estimate the parameters from the adjusted series using QML.
3. Repeat step 2 and 3 a large number of times, we repeat the steps for 10.000 times.
4. Calculate the variance of the estimates. This variance is the bootstrap variance.

There are many different ways to choose the length of the blocks in the bootstrap, we follow Hall et al (1995) and chose $T^{1/5}$ as the length. We have also tried other block lengths but found that the results are relatively invariant to block lengths close to $T^{1/5}$.

In our case with 903 observations this means that the block length should be 4, i.e. $903^{1/5}$.

Appendix C: Confidence Bounds

Figure C1. In-Sample Confidence Bounds on Term Structure Models

The figure shows the daily actual 1-month Stibor rate, for the period used for the estimation of the parameters, and 99% and 95% confidence intervals for the simulated interest rates using the CKLS model with the parameters from the GMM estimation. The confidence intervals are calculated using the actual distribution of the interest rate under the term structure model, i.e. from 1.000 Monte Carlo simulations.

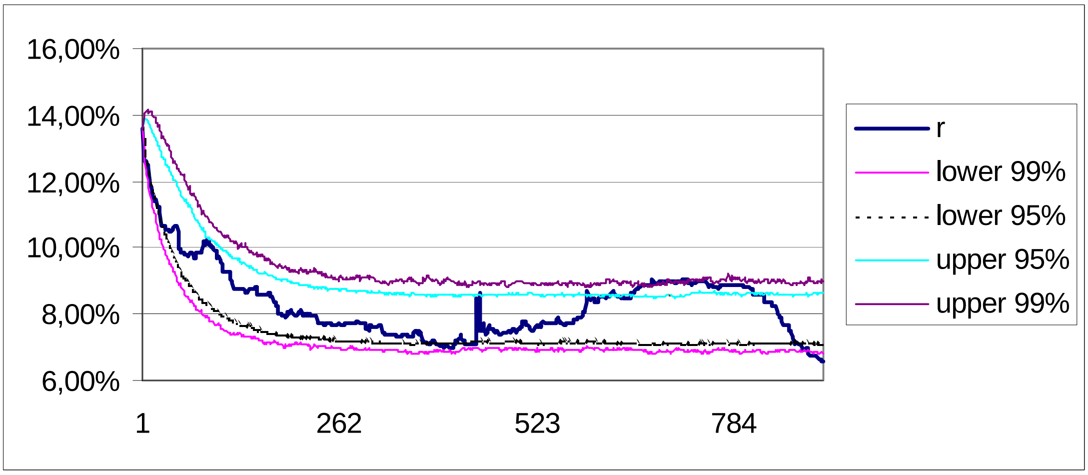
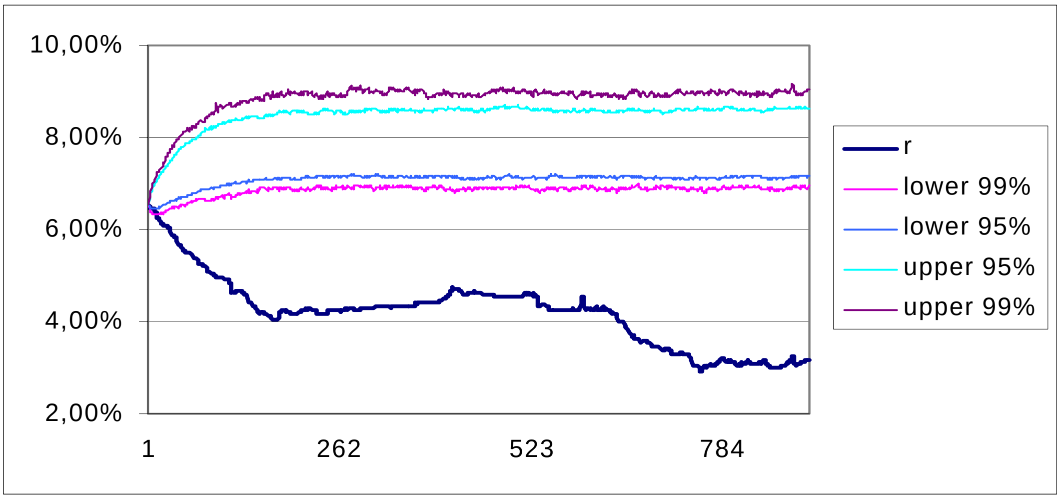


Figure C2. Out-of-Sample Confidence Bounds on Term Structure Models

The figure shows the daily actual 1-month Stibor rate, for the period after the estimation period, and 99% and 95% confidence intervals for the simulated interest rates using the CKLS model with the parameters from the GMM estimation. The confidence intervals are calculated using the actual distribution of the interest rate under the term structure model, i.e. from 1.000 Monte Carlo simulations.



4 The Influence of Credit Risk on Interest Rate Swap Agreements

4.1 Introduction

In an interest rate swap agreement two parties agree to exchange interest rate payments. In the standard (plain-vanilla) swap this means that a payment based on a floating interest rate is paid from one party to the other, who in return pays an interest, based on a fixed rate. Both payments are based on the same principal, which is generally not exchanged. The exchanges of payments are done at regular (for example semi-annual) intervals for the duration of the swap. At first glance this type of agreement would appear to be superfluous since each of the two counterparties could just as well borrow money with an interest that corresponds to their preferred cash flow scheme. One of the most predominant motivations for the existence of swap agreements is comparative advantages, where one party is said to have a comparative advantage in the floating market and the other in the fixed market. By exploiting these comparative advantages a profit can be made.

No matter what the motive for the existence of swap agreement is, it is clear that the market for swap agreements is very large. As of 2002, the total outstanding notional amount of swap agreements (henceforth swaps) exceeded US$ 68 trillion, more than double the world gross domestic product. In terms of outstanding notional amount swaps are the most important OTC derivative[22].

Swap agreements are subject to default risk, much in the same way as bonds are. The main difference is that the principal can not be lost. Another difference is that only the *difference* between the fixed and floating interest rate payments can be lost, should there be a default. The way in which default risk affects the price of swap agreements is discussed in papers by Duffie and Huang (1996), Cossin and Pirotte (1997) and to some extent in Duffie and Singleton (1997). Default risk and the pricing of swaps under dual-default risk is the fastest growing research area in the field of swaps in today. One reason for this growth is that an efficient pricing of swaps would make the market more efficient and make it possible for many new firms to enter swaps. It is also vital that all swap-dealers take the default risk into account when entering swaps, especially if the counterpart has a low rating, otherwise the market will not be efficient and firms with low rating will find it hard to find counterparts for their swaps.

In a more empirically oriented branch of the research on swaps and default risk, Cossin and Pirotte (1998) examine the performance of classical models for pricing swaps using a transaction data sample (of less than 300 swaps). Sun et al (1993) examine the effect of swap dealers credit rating on swap spreads. Minton (1997) uses quoted swap rates to test some of the available models for pricing swaps. Brown, Harlow and, Smith (1994) also use quoted rates to analyse the volatility of the interest rate swap spreads. Most of the empirical papers on the subject use quoted offer and bid swap rates. These rates offer no company specific information such as the rating of the counterparts. In this paper we, however, work with a unique data set containing over 1500 swap contracts from

[22] Data from Bank for International Settlement (BIS) 2002 Press Release 8 November 2002. Figures are corrected to eliminate double-counting.

two separate Swedish financial institutions that includes full rating history and swap terms.[23] In the paper we examine if there is a relationship between the swap rate (i.e. the price of the swap) and the credit worthiness of the parties involved in the agreement, similarly to Cossine and Pirotte (1998). We expect that a company engaging in a swap agreement would demand compensation for taking on credit risk.

From our tests we conclude, as expected, that a higher compensation is required as the rating of the counterpart decrease. We also find that there is a form of indirect pricing made by the investigated companies, where the notional amount and the time to maturity is significantly lower for the swap counterparts with lower credit ratings.

The paper is organised as follows. In section 4.2 the theory behind credit risk and swap rates are discussed. Section 4.3 describes the data set. The analysis of the data is done in section 4.4. Section 4.5 contains the summary.

4.2 Theory of Pricing Default Risk

The price of a swap is mainly determined by the credit risk of the counterparty and by the liquidity of the swap itself.[24] Take for example two companies where one (called A) has a zero probability of default and the other (called B) has a positive default probability. If these companies enter a plain vanilla swap agreement, using unadjusted (for example the market rates) interest rates, then the initial value of the swap will be positive for B and hence negative for A. The reason is that when the value of the swap fluctuates between settlement dates, A

[23] This set is superior to the set used by Cossin and Pirotte (1996) which is made up of less than 300 observations and with less detailed information. Our paper is the only other paper than Cossin and Pirotte (1996), that the author is aware of, that uses actual transaction data.
[24] Lang, Litzenberg and Liu (1998) summarize some of the different explanations found in the literature to explain the swap price.

will stand a strictly positive probability of losing money, in case B should default. It must therefore be that the lower the default probability of the counterpart (relative ones own), the higher the value of the swap will be to you. If the counterpart defaults when the value of the swap is negative for him you will only receive a fraction of the total amount owed, called the fractional recovery rate. This fraction, in combination with the default probability, constitutes the components of the credit risk. Briefly stated, higher default probability and lower fractional recovery rate of the counterpart decreases the value of the swap to you. One theoretical prediction of this paper will therefore be that a counterpart with a high rating will be able to negotiate better conditions than a counterpart with a low rating. This can be manifested either trough a better swap rate or some other kind of favourable condition such as higher notional amount or longer time to maturity.

As hinted in the introduction there is many different ways to model the default risk when pricing swaps. In the model presented by Duffie and Huang (1996) asymmetric dual-default risk is considered. In their model of counterparts with different probability of default, the promised cash flow of a swap agreement are discounted by a switching discount rate that, at any state and time, is equal to the discount rate of the counterpart for whom the swap is currently out of the money (i.e. has a negative value). They make the simplification that the default-risk and fractional recovery rate is an exogenously given processes to simplify the calculations. Without this simplification it would be very cumbersome to calculate a price for swap agreements. First, the value of the swap for the counterparts would have to be calculated. Second, the swap would influence the value of the firms, possibly in a non-linear way, and so on. However, this line is none the less followed by Merton (1974) who views the credit risk as equivalent to a short position on a put option on the other company. The result is a closed form expression of the swap value. The major drawback of this model is the need to examine the debt structure of the counterparts of the swap agreement, making it somewhat unpractical to use. Merton's model also disregards the effect that dual-default risk has on the swap value. Longstaff - Schwartz (1994) continues Merton's work and treat default-risk in the same way, but add a stochastic interest rate, making it more realistic. In a more empirically oriented paper Cossin and Pirotte (1998) examine how the relative difference between actual rates for traded swaps and quoted rates are affected by the rating of the counterparts of the swap. In this paper we extend their study using a larger, more detailed data set to analyse both how rating affects the swap rate and how time to maturity and notional amount are affected by the credit rating of the counterparts in the swap. Similarly to Cossin and Pirotte (1998), we conclude that the default risk affects the swap. Contrary to Cossine and Pirotte, we use actual swap rates instead of quoted rates in our calculations. Following Duffie and Huang's (1996) conclusion, that a lower rating of a counterpart will stipulate a higher swap price for the lower rated counterpart, our hypothesis in this paper is that when the rating of the counterpart is low, then the time to maturity and the notional amount will also tend to be low. This is a form of indirect pricing of the swap that is made in order to reduce the risk that arises from a higher default probability. In Table 1 below the cumulative default probability for companies rated by Moody's is shown.

It is clear that the probability of default increases as time to maturity increases and as the credit rating decreases. Companies that do not take this into consideration stands a chance to loose money either by not getting compensation for credit risk taken, or by discarding potential swap counterparts with "too" low credit rating. By excluding these low rated companies they might not get the most advantageous swap agreement.

Table 1. Cumulative Default Probabilities

Default probability within different time periods measured in percent. The probabilities show the risk that a company has defaulted within the time period. Based on international data from Moodys during 1970 to 1990.

	< 1 yr	**< 2**	**< 3**	**< 4**	**< 5**	**< 6**	**< 7**	**< 8**	**< 9**	**< 10**
AAA	0.01	0.02	0.03	0.05	0.07	0.09	0.12	0.16	0.21	0.27
AA+	0.02	0.03	0.05	0.08	0.11	0.15	0.20	0.27	0.36	0.46
AA	0.02	0.06	0.12	0.20	0.30	0.44	0.61	0.81	1.05	1.34
AA-	0.04	0.08	0.15	0.24	0.37	0.53	0.73	0.99	1.29	1.65
A+	0.05	0.14	0.28	0.47	0.72	1.04	1.42	1.86	2.38	2.96
A	0.06	0.16	0.31	0.52	0.82	1.19	1.66	2.21	2.85	3.57
A-	0.09	0.28	0.59	1.01	1.56	2.23	3.02	3.91	4.90	5.98

Source: Fons J and Kimball A, 1991 who extend the work done by Lucas and Lonski, 1992.

Two different tests to check the responsiveness of the credit risk on the swaps are made in the paper. The first test aims to investigate whether the companies take credit risk into account in a direct way when pricing the swap agreements. We postulate the following hypotheses:

H_0: *Swap rate is not affected by the credit rating*

H_1: *Swap rate is affected by the credit rating*

In the second test we first regress the credit rating on the notional amount in order to see if the companies take the credit risk into account in an indirect way. The following hypotheses are tested:

H_0: *Notional amount do not depend on the credit rating*

H_1: *Notional amount do depend on the credit rating*

We also test the hypothesis that the time to maturity is lower for swap agreements entered with counterparts of lower creditworthiness.

H_0: *Time to maturity do not depend on the credit rating*

H_1: *Time to maturity do depend on the credit rating*

4.3 The Data Set

The data set contains 1476 swap contracts with full counterpart information. As far as we know this is the largest swap transaction data set ever analyzed. The size of the data set makes it possible to do more reliable and precise evaluation of the influence of the credit rating on swap agreements than what has been possible before. The data was collected from two major Swedish financial institutions. Since the information is sensitive, both of the contributors have requested to be anonymous.

The information we have collected on the contracts are:

i. Trade date, the date that the contract was negotiated
ii. Effective date, the date that the swap agreement is commenced
iii. Termination date, the final day of the swap agreement
iv. Notional amount of the swap
v. Fixed and floating interest rates
vi. Rating of the parties, including a full history of the rating[25]
vii. Whether default has occurred or not (which never happened for any of the examined companies)

Aside from the above information we also use data on Swedish government bond interest rates.[26]

The 1476 contracts in our data set make up a total value of over US$ 100 billion, with a median swap notional value of US$ 45 million and an average value of US$ 71 million. Figure 1 and Table 2 show the distribution of the notional amount of the swaps. More than 50% of the swap agreements have notional

[25] Retrieved from Standard and Poor's Credit Analysis Reference Disc, version 1 1997
[26] Provided by Datastream.

amounts between US$ 20 million and US$ 80 million. The notional amount of the individual swaps range from US$ 1 million up to over US$ 200 million.

Figure 1. Histogram of the Notional Amounts in the Sample

The histogram shows how the notional amounts of the swap agreements are dispersed. The notional amount of the swaps is shown on the horizontal bar in millions of US$. Ten observations with large notional amounts have been omitted from the histogram in order to make it more legible. The omitted values are US$ millions: 319, 385, 412, 454, 513, 735, 886, 906 and 1517.

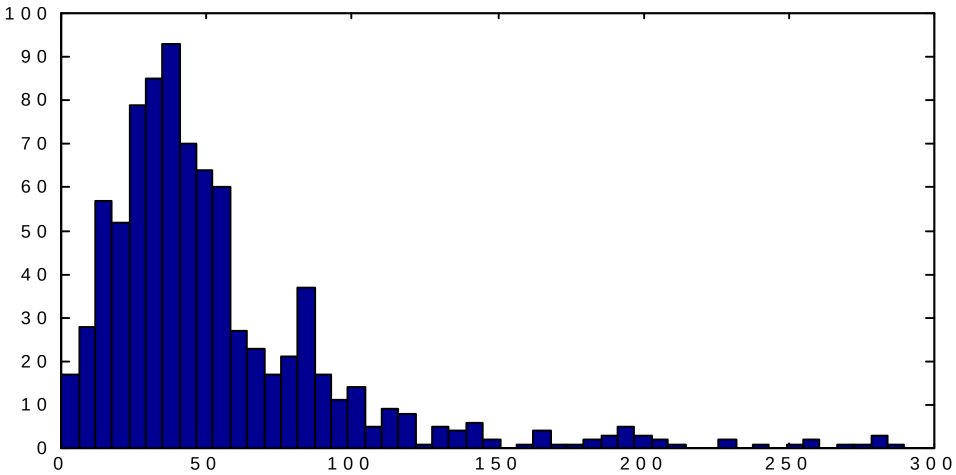


Table 2. Statistics on the Notional Amounts in the Sample

Dispersion of the swap agreements between different notional amount categories. Amounts are in US$ million.

Amount	**< 10**	**11 – 20**	**21 - 30**	**31 - 40**	**41 - 50**	**51 – 60**	**61 - 70**	**71 - 80**	**81 - 90**	**91 – 100**	**> 101**
Share	3%	5%	8%	12%	13%	10%	8%	8%	4%	3%	26%

It is clear from Table 2 that the notional amounts of the swaps vary over a wide range. The notional amounts are however typically in the range between USD 30 and 80 millions.

The ratings of the two companies, which provided the data, have fluctuated between AAA and AA+. A summary of the ratings of their counterparts is shown in Table 3. One third of the counterparts have ratings below AA+, and no swaps have been entered into with counterparts with ratings below BBB[27]. It is also notable that over 60 percent of the swaps are with companies with in the two highest rating categories. The rating of the contributing companies has fluctuated between AAA and AA+ over the investigated period. None of the contributing companies' counterparts has had a higher rating than the contributing companies at any time during the sample period. If the pricing of the swaps had been made in an efficient way we would expect that the ratings of the counterparts would not be so biased toward high rating categories. Analogously to the bond market, one can observe efficiency when many diverse parties participate in the market and the prices are sensitive to the credit worthiness of the counterparts. This leads to the conclusion that the pricing of swaps made by the examined companies is not efficient.

Table 3. Statistics on the Ratings on the Companies in the Sample

Dispersion of ratings in sample, the rating Lower contains the categories from BB+ through C. The ratings of the investigated companies fluctuate between AAA and AA+ under the sample period.

Rating of company	Share
AAA	36.0%
AA+	31.2%
AA	8.7%
AA-	11.5%
A+	5.4%
A	1.3%
A-	4.3%
BBB+	0.0%
BBB	0.4%
BBB-	0.0%
Lower	0.0%

[27] Bonds rated below BBB are regarded as having significant speculative characteristics.

In Table 4 we present the distribution of the time to maturities of the swap agreements. Almost all of the swaps have a time to maturity that is above 1 year and lower than 10 years.

Table 4. Statistics on the Time to Maturity on the Swaps in the Sample

Dispersion of time to maturity of the swap agreements. Time to maturity is relative to the time when the swaps were entered into.

Time to Maturity	Share of sample
< 1 yr	1%
1 - 2 yrs	7%
2 - 3 yrs	5%
3 - 4 yrs	7%
4 - 5 yrs	14%
5 - 6 yrs	8%
6 - 7 yrs	3%
7 - 8 yrs	12%
8 - 9 yrs	3%
9 - 10 yrs	19%
10 - 15 yrs	20%
> 15 yrs	1%

Trading dates range from the beginning of 1985 until 1997, with times to maturity between 100 days up to over 15 years. The mean time to maturity is 7 years. In Table 5 the swaps are divided into different categories corresponding to their time to maturity at the trading date. Less than a third of the swaps have times to maturity of less than 5 years and about 21% of the swaps have times to maturity over 10 years. The most common swap agreement in the data set is one where the floating rate is the 6-month Libor and both the fixed and the floating interest are payable semi-annually.

4.4 Analysis

In order to find out if default risk matters, we test to see if there is any (linear) relation between the default risk of the counterpart and the different swap characteristics. If there is not any significant relationship it must be because one of three reasons:

1. Default risk does not matter
2. The parties in the swap agreement have not considered or do not know how to handle the risk of default.
3. Default risk affects the swaps in a way that can not be observed with the tests used in this paper, e.g. compensation for risk taken may be given by adjusting the interest on another loan or swap.[28] The relationship could also be non-linear.

If default risk does matter, then models of credit risk should have an important part to play in the pricing of swap. It is also the case that as the time to maturity for swaps increase so does the risk for default, which makes it even more important to price the credit risk in an efficient way.[29] It is obvious, when studying Table 1, that the default risk increase as time to maturity increase. It is also evident that the lower the rating of the counterpart, the higher the risk of default. If a company where to take this into consideration when entering a swap agreement it would mean that when the credit rating of the counterpart decreases, the notional amount should decrease and/or the swap rate of the swap agreements should increase and/or the time to maturity should decrease .

This should be especially apparent when the rating of the counterpart changes from AA+ to AA where the probability of default increases dramatically in

[28] When collecting the data used we found some swaps with fixed rates of zero percent and other with floating rates of zero percent between the same counterparts, which has been omitted from the analysis.
[29] Lucas and Lonski (1992) and Altman (1998) discuss this so-called ageing effect.

comparison to other one-step rating transitions. If the notional amount, time to maturity and/or swap rate is not adjusted in accordance to the discussion above, then it must be the case that the examined companies do not use the information conveyed through the credit rating or that they use the information incorrectly.

We use two different methods to test if credit risk matters when pricing swaps. In the first test, we use ordinary least square (OLS) to test the hypothesis that credit rating affects the swap rate, i.e. the spread between the market interest rate and the swap interest rate.[30] This tells us if the investigated companies use the information that the credit rating conveys about default risk in a direct way, when pricing the swaps. In the second test we use OLS to regress the notional amount on the credit rating. We also regress the time to maturity on the credit rating. These tests makes it possible for us to see if the credit rating affects the swaps in an indirect way, either via credit rationing or via adjustments of the time to maturity.

4.4.1 The Influence of Credit Risk on the Swap Rate

For the first test in the analysis, we use a dummy which equal 0 if the counterpart belong to the low credit risk group and 1 if it belong to the high credit risk group. The low credit risk group includes the companies with a rating of AA+ or AAA. In the high credit risk group, companies without rating and companies with a rating of AA or lower can be found. This division is based on the fact that the risk for default increases dramatically (in comparison to the other transitions) between the AA+ and the AA rating according to Table 1 above. Another approach would be to use separate dummies for all rating

[30] Hull and White (1995) explore the implications of one-sided default risk on options and other derivative securities. Their paper presents a model for valuing derivative securities under default risk. They come to the conclusion that the variables concerned with default risk are independent of the variables underlying the value of the derivative in a default free world. This conclusion makes it possible for us to use the credit rating as a proxy for the default risk, without concerning ourselves with the value process of the firm.

categories. However, as shown above there are very few contracts with counterparts with ratings lower than A, which makes it difficult to obtain accurate estimates of the parameters.

We define the swap rate, s, as the difference between the fixed interest rate for the swap, r^{swap} and the corresponding market interest rate, r^{market}:

$$s = \begin{cases} r^{swap} - r^{market} & \text{if } X \text{ recieves a fixed rate} \\ r^{market} - r^{swap} & \text{if } X \text{ pays a fixed rate} \end{cases} \quad (3.1)$$

In equation (3.1) the contributing companies are called *X*. The definition above of the swap rate assures us that we always measure the swap rate in relation to *X*, i.e. as the positive or negative compensation that *X* receives. We would expect that the swap rate increases as the credit risk of *X*'s counterparts increases.

The market interest rate has been calculated as the effective interest rate received on a government bond with time to maturity corresponding to that of the swap. When no government bond of corresponding maturity exists we have used a bootstrapping procedure to estimate a compatible interest rate.[33]

Table 5. Descriptive Statistics of the Sample Swaps

Summary statistics over the sample of swap agreements. Notional amounts are in millions of USD. Time to maturity is in years and swap rates are in basis points.

	Notional Amount	Swap rate	Time to Maturity
Mean	71.1	13.2	7.52
Standard deviation	76	5.4	3.26
Minimum	0.2	-10.4	0.50
Maximum	1517	22.8	18.9
Observations	1476	1476	1476
Sum	104 944	-	-

[33] For a description of the bootstrapping procedure see for example Anderson et al (1996).

Table 5 shows the summary statistics for the sample used for the tests. The mean swap rate in the used sample is 13.2 basis points (i.e. 1/100 of a percent) with a standard deviation of 5.4 basis points and has a range between -10.4 and 22.8 basis points.

The proposed hypothesis is:

H_0: Swap rate is not affected by the credit rating, i.e. $\beta=0$

H_1: Swap rate is affected by the credit rating, i.e. $\beta \neq 0$

A positive value of β, in equation (3.2), implies that the swap rate increase in order to compensate for higher credit risk as the credit rating of the counterpart decrease.[34] In order to test the hypothesis above (if the credit rating affects the swap rate, *s*) we estimate the following equation:

$$s_i = \alpha + \beta D_i + \varepsilon_i \qquad (3.2)$$

$$\varepsilon_i \sim N\left(0, \sigma^2\right)$$

where the dummy, D_i, assumes the value 1 if the counterpart belongs to the high credit risk group and 0 otherwise. We also include a constant, α.

Using the method of ordinary least square we obtain the following estimated relationship between the swap rate and the rating category of the counterpart of the swap:

$$\hat{s}_i = 11.1 + 6.4 D_i \qquad (3.3)$$

According to the regression above, the swap rate increases by 6.4 basis points as the rating of the counterpart moves from the low credit risk group (AAA to AA+) to the high credit risk group (below AA+).

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[34] When interpreting the results in this chapter it is important to keep in mind a *high* credit rating implies a *low* credit risk and vice versa.

Table 6. Coefficients in the regression of Swap Rate on Rating

Estimation using OLS, the unit is in basis points. The dummy is zero for rating categories AAA and AA+, and one otherwise. The presented *p*-value for the intercept corresponds to a two-sided confidence band.

	Coefficients	**Standard deviation**	***t*-value**	***p*-value**
Intercept	11.1	4.1	2.71	0.01
Dummy	6.4	2.3	2.83	0.01

As can be seen in Table 6 both the constant and the rating-dummy are significantly different from zero at all conventional levels. It is therefore possible to reject our hypothesis that the swap rate is not affected by the rating of the counterpart. We therefore conclude that the rating actually is taken into consideration when the swap agreements are negotiated. Further, we also find that the adjustment of the swap rate is done in a way that is consistent with our hypothesis that compensation is required in order to compensate for higher credit risk.

4.4.2 The Influence of Credit Risk on Notional Amount and Time to Maturity

For the second test in the analysis we also use a dummy which equals 0 if the counterpart belongs to the low credit risk group and 1 if it belongs to the high credit risk group.

We want to test the following hypothesis:

H_0: Notional amount do not depend on the credit rating, i.e. $\beta=0$

H_1: Notional amount do depend on the credit rating, i.e. $\beta \neq 0$

To test this hypothesis we regress the notional amount, *NA*, (in USD millions) of the swap agreements on the rating-dummy, *D*:

$$NA_i = \alpha + \beta D_i + \varepsilon_i \tag{3.4}$$

$$\varepsilon \sim N\left(0, \sigma^2\right)$$

where α and β are parameters to be estimated.

Ordinary least squares give the following result:

$$N\hat{A} = 75.5 - 13.6 D_i \tag{3.5}$$

Table 7 Coefficients in the Regression of Notional Amount on Rating

Estimation using OLS, units in millions of US$. The dummy is zero for rating categories AAA and AA+, and one otherwise. The presented *p*-value for the intercept corresponds to a two-sided confidence band.

	Coefficients	Standard deviation	*t*-value	*p*-value
Intercept	75.5	7.12	10.60	0.00
Dummy	-13.6	6.77	2.01	0.04

As seen in Table 7, all of the coefficients are significantly different from zero using a 5 percent confidence interval. When the rating of the counterpart is in the lower rating category the notional amount is on average USD 13.6 millions lower than for a swap with a counterpart of higher credit worthiness. This leads to the conclusion that companies take credit rating into consideration when entering into swap agreements in both a direct way, as shown in the previous section, and in an indirect way as shown above.

To further strengthen our findings we also test the following hypothesis:

H_0: Time to maturity do not depend on the credit rating, i.e. $\beta = 0$

H_1: Time to maturity depend on the credit rating, i.e. $\beta \neq 0$

To test this hypothesis we regress the time to maturity, *TTM*, (in years) of the swap agreements on the rating-dummy, *D*:

$$TTM_i = \alpha + \beta D_i + \varepsilon_i \tag{3.6}$$

$$\varepsilon \sim N\left(0, \sigma^2\right)$$

where α and β are parameters to be estimated.

Ordinary least squares give the following results:

$$\hat{TTM}_i = 8.3 - 2.4 D_i$$

Table 8 Coefficients in the Regression of Time to Maturity on Rating

Estimation using OLS, the unit is in millions of US$. The dummy is zero for rating categories AAA and AA+, and one otherwise. The presented *p*-value for the intercept corresponds to a two-sided confidence band.

	Coefficients	Standard deviation	*t*-value	*p*-value
Intercept	8.3	1.18	7.04	<0.01
Dummy	-2.4	1.17	2.05	0.04

At the 5 % significance level we reject the null hypothesis that the rating does not affect the length of swap contract. Further, the estimates in Table 8 implies that the time to maturity decreases from 8.3 years for swaps with counterparts of high credit worthiness to 5.9 years for swaps with counterparts of low credit worthiness. This also indicates that some form of credit rationing is present in the dataset.

4.5 Summary

When trading swap agreements, which are subject to default risk, it is important to be able to price the default risk in a correct way. Today very few of the participants in the swap market use any theoretical model that take default risk into account when pricing swaps; instead they use some kind of credit rationing. The main reasons that default risk is not considered when pricing swaps is that the models that do exist have not been empirically tested and that some of the valuation models use the value of the counterpart (which is cumbersome to calculate and re-calculate each time that a swap agreement is entered). However, even though market participants claim that they do not use the default risk directly to price the swap agreements, we have found evidence that suggests that the rating of the counterparts is taken into account when swap agreements are made. This leads us to believe that the participants in fact do adjust the swap terms depending on the rating of the counterpart. This adjustment is probably based on some approximation of the theoretically fair price of the swap.

References

Altman E, 1998, The Importance and Subtlety of Credit Rating Migration, *Journal of Banking & Finance*, 22, 1231-1247.

Anderson, N., Breedon, F., Deacon, M., Derry, A., and Murphy, G., 1996, Estimating and Interpreting the Yield Curve, *John Wiley & Sons Ltd.*, England.

Brown K, Harlow W and Smith D, 1994, An Empirical Analysis of Interest Rate Swap Spreads, *Journal of Fixed Income*, March, 61-78.

Cossin C and Pirotte H, 1998, How Well Do Classical Credit Risk Pricing Models Fit Swap Transaction Data?, *European Financial Management Journal*, vol 4, no 1.

Degrér, H. and Jochumzén, P., 1999, Swap Agreements under Dual-Default Risk: A Simulation Approach, *Licentiate Dissertation*, University of Lund.

Duffie D and Huang M, 1996, Swap Rates and Credit Quality, *Journal of Finance*, 51(3), 921- 949.

Duffie D and Singleton K, 1997, Modelling Term Structures of Defaultable Bonds, Working paper, The Graduate School of Business, Stanford University.

Fons J and Kimball A, 1991, Corporate Bond Defaults and Default Rates 1970-1990, *The Journal of Fixed Income*, vol 1, no 1, 36-47.

Lang L, Litzenberger R and Liu A, 1998, Determinants of Interest Rate Swap Spreads, *Journal of Banking & Finance*, 22, 1507-1532.

Longstaff F and Schwartz E, 1994, A Simple Approach to Valuing Risky Fixed and Floating Rate Debt and Determining Swap Spreads, *Journal of Finance,* 50(3), 789-820.

Lucas D and Lonski J, 1992, Changes in Corporate Credit Quality 1970-1990, *The Journal of Fixed Income*, Vol 1, No 4, 7-14.

Merton R, 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance*, 29, 449-470.

Minton B., 1997, An Empirical Examination of Basic Valuation Models for Plain Vanilla U.S. Interest Swaps, *Journal of Financial Economics*, 44, 251-277.

Sun T, Sundaresan S and Wang C, 1993, Interest Rate Swaps: An Empirical Investigation, *Journal of Financial Economics*, 34, 77-99.

5 Measuring the State of the Swedish Economy: An Equilibrium Conditions Index

5.1 Introduction

Market participants in general and policy makers in particular have shown an interest in indicators that summarise the state of the economy in a timely manner. Indicators such as the output gap are only available on a quarterly basis, whereas most financial variables are updated much more frequently. In order to extract information from financial variables, different forms of indices have been developed. Economic conditions have often been summarized in a Monetary Conditions Index (MCI), which consists of two variables: the short interest rate and the exchange rate.[35,36] MCI indices have been described in many studies, see for example Freedman (1995) and Fung and Yuan (1999) on Canadian data, Batini and Turnbull (2000, 2002) on UK data, and Hanson and Lindberg (1994) on Swedish data. Many central banks have used these indicators as an aid in their policy making, for example the central banks in Sweden, Finland, Norway and Iceland. The Bank of Canada and the Reserve

[35] Sometimes a bond yield is included in MCIs.
[36] Ericsson and Kerbeshain (1997) contain an excellent survey of the work done on MCIs.

Bank of New Zealand have even used an MCI index as an operational target for their monetary policy. As described by Smets (1997), an MCI has the potential drawback that it may fail to take (potentially) important factors into account, such as the value of the stock market and housing prices. In an attempt to incorporate asset prices in the policy index, Smets (1997) develops a Financial Conditions Index (FCI), which includes house prices and stock prices.[37] The reason for including asset prices in the FCI is that it is believed that the wealth effect of changed asset prices changes the level of consumption and investment in the economy and thereby affects economic conditions. As pointed out in Freedman (1995), however, it is important to differentiate between shocks to the index that affect the desired index level, such as supply shocks, and shocks that do not, such as purely financial shocks (for example the LTCM crisis in 1997 when one of the largest hedge funds went bankrupt). In other words, a change in the conditions index does not necessarily mean that conditions in the economy have changed, since the change could also be due to changes in some fundamental determinants of the economy. One way to take this distinction into account is to use deviations from (partial or general) equilibrium values of the included variables instead of their values relative to some base period. For example, with an inflation targeting regime, nominal exchange rate depreciation may call for different policy responses depending on what has caused the movement in the exchange rate. First, if the equilibrium real exchange rate has not changed, then the depreciation in the nominal exchange rate can be expected to lead to inflation since it is a higher (relative) inflation rate that has caused the depreciation.[38] In that case monetary policy might need to be tightened if an inflation targeting regime is followed and the resulting inflation is higher than the inflation target. If, on the other hand, the equilibrium real exchange rate has

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[37] See also Mayes (2001), Goodhart and Hofmann (2000), and Dudley and Hatzius (2000) for more applications of financial conditions indices.

[38] We assume that inflation is persistent in which case a higher inflation in period *t* leads to higher inflation in period *t+1*.

depreciated there is not necessarily any reason to change monetary policy since a tightening will/might not cause a real appreciation of the currency.[39] This highlights the importance of differentiating between fluctuations derived from changes in the equilibrium value of financial variables and changes derived from "non-fundamental" factors that can/should be offset by monetary policy.

In the paper an equilibrium conditions index is estimated for Sweden over the period between 1970 and 2002. It is found that the index appears to do well in explaining periods with unusually contractionary and expansionary conditions in the economy. As an extra bonus the estimated index also appears to be able to predict larger movements in the output gap for Sweden.

The paper is organized the following way. In Section 2 we discuss the theoretical motivation for the economic conditions index used. We also describe the individual partial equilibrium models used to calculate the fundamental values for the included variables. In Section 3 we construct an equilibrium conditions index based on the deviations between actual and equilibrium values of the financial variables with weights obtained from an impulse response methodology. We also evaluate the index. Section 4 summarizes and concludes.

5.2 Theory and Estimation

The index developed in this paper focuses on economic conditions. We measure economic conditions as the output gap, which is the difference between actual and potential output (estimated as a filtered trend). An output higher than the economy's potential is believed to cause inflation and lead to overheating. On

[39] This is in line with Obstfeld and Rogoff (1995) who claims that, for example, changes in real exchange rates that are caused by changes in relative productivity should not cause an altered monetary policy.

the other hand, output lower than the economy's potential gives rise to unemployment and may even cause deflation. In other words, it is desirable to keep actual and potential output on a par with each other. In order to gauge how different financial factors affect economic conditions we develop an economic conditions index that aims to capture these factors. Economic conditions are believed to depend on a number of factors. However, we have decided to use only a few potentially influential factors in the analysis. The chosen factors are the real interest rate and the real exchange rate as in an MCI. We have chosen to divide the interest rate into two maturity categories, long- and short-term, and to include asset prices as in an FCI. Case et al (2001) argue that there are a number of reasons why economic conditions might be affected differently by changes in different asset classes. For example, increases in asset values of a certain kind might be viewed as temporary or uncertain. It has also been argued in Shefrin and Thaler (1988) that people divide asset wealth into different "mental accounts" which are viewed quite differently and thereby have different marginal effects on consumption. According to Shefrin and Thaler the marginal effect on consumption of changes in house prices is higher than that from stock prices. Following the arguments above we have divided the asset prices into house prices and stock prices.

It is of vital importance to measure the difference between current values of the factors in the index and some form of equilibrium value of the factors. If we were to focus only on, for example, the stock price level in Sweden compared to some base period we would conclude that economic conditions have become continuously more expansionary during the period studied (disregarding the downturn since March 2000). However, this does not necessarily mean that economic conditions actually have become more expansionary; it could also be due to the trending nature of stock prices or changes in underlying fundamental factors. The same reasoning could to a large extent be used for house prices and the exchange rate (see Appendix A). We use the hypothesis that only deviations in the factors away from their fundamental value, and not changes in the fundamental value itself, affect economic conditions. The main challenge with this approach is of course to develop methods to determine the fundamental or equilibrium values of the factors. One approach would be to develop a single macroeconomic model that incorporates all the factors above. However, no general equilibrium model that takes all the mentioned financial variables into account is (yet) to be found in the literature. We have therefore chosen a different path where the fundamental value of the interest rate, exchange rate, house prices and stock prices are calculated separately in different partial equilibrium models. The main drawback with this approach is that it does not assure that the different partial models are consistent with each other. However, the included partial models are all, separately, consistent with economic theory.

As mentioned above, the conditions index is meant to measure if the economy is in a contractionary or expansionary state. We calibrate the index in such a way that a neutral value of the index (chosen to equal 100) should conform to a real GDP growth in line with its (unobserved) potential growth rate. Ideally the index should also be forward-looking and thereby be able to predict future movements in the output gap. We have however chosen to optimize the index in such a way that it follows the output gap as closely as possible. Below we describe the models used to calculate fundamentally justified values of the exchange rate, interest rates, and asset prices.

5.2.1 Exchange rate

Recent literature on purchasing power parity (PPP) indicates that it is not a very good method in itself for calculating equilibrium exchange rates.[40] It appears to be necessary to model the deviations from PPP in order to obtain well behaved models for the real exchange rate. One popular way to model the exchange rate is the behavioural equilibrium exchange rate (BEER) approach; see for example Clark and MacDonald (1999). In the BEER-type models the deviations from PPP are assumed to depend on real economic factors such as relative growth, the terms of trade, the trade balance and the current accounts. In choosing a model for the real Swedish trade-weighted exchange rate we use a general-to-specific approach, within a Johansen cointegration framework, using all of the theoretically justifiable variables (see Juselius (2002) for a theoretical justification of the methodology). We exclude variables that exhibit signs that contradict economic theory.

When modelling the exchange rate we start with the Clark and MacDonald (1999) model which, in turn, is based on the Frenkel and Mussa (1986) stock-flow consistent model of the equilibrium exchange rate, $\overline{q_t}$ (measured as real SEK/TCW):

$$\overline{q_t} = f\left(nfa_t, tot_t, tnt_t\right) \qquad (3.7)$$

where nfa_t is net foreign assets, tot_t the terms of trade and tnt_t is a measure of the Balassa (1964) and Samuelson (1964) relative productivity effect (as measured by the price ratio of tradable and non-tradeable goods). Instead of tnt_t it is possible to use some other type of relative productivity measure, for example the relative (real) GDP growth, y_t. Clark and MacDonald (1999) also include a real uncovered interest rate parity condition using the real interest rate differential when estimating the equilibrium exchange rate in their estimation

[40] See for example MacDonald and Marsh (1999) and MacDonald (2000).

procedure. Other studies, for example Lindblad and Sellin (2003), include the structural budget deficits, g_t, and demographic factors, dep_t, in the explanatory vector.

We use all of the variables above in a Johansen (1995) cointegration framework and apply a general-to-specific approach in order to obtain a parsimonious model of the exchange rate (all variables are described in appendix A).

Generic model:

$$\overline{q_t} = f\left(nfa_t, tot_t, tnt_t, y_t, g_t, dep_t\right) \tag{3.8}$$

A general-to-specific approach leaves us with a model where the exchange rate depends on (is cointegrated with) the terms of trade and (log) relative real GDP.

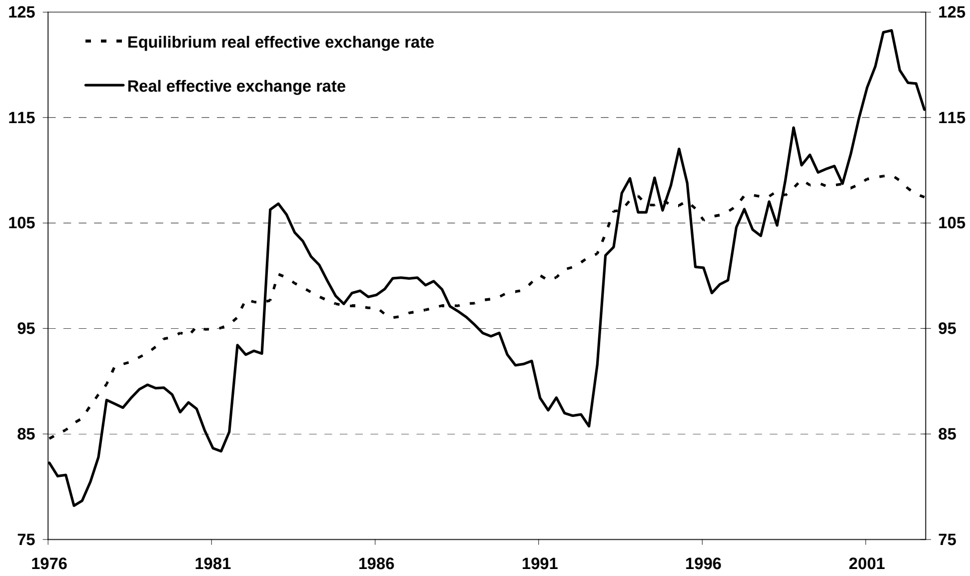
The equilibrium correction representation of the estimated VAR(1) exchange rate model (the Johansen cointegration test indicates one cointegrating vector and the Akaike information criterion indicates one lag):

$$\Delta q_t = \alpha_1\left(q_{t-1} - \beta_0 - \beta_1 tot_{t-1} - \beta_2 y_t\right) + \gamma_{1,0} + \gamma_{1,1}\Delta q_{t-1} + \gamma_{1,2}\Delta tot_{t-1} + \gamma_{1,3}\Delta y_{t-1} + \varepsilon_1$$
$$\Delta tot_t = \alpha_2\left(q_{t-1} - \beta_0 - \beta_1 tot_{t-1} - \beta_2 y_t\right) + \gamma_{2,0} + \gamma_{2,1}\Delta q_{t-1} + \gamma_{2,2}\Delta tot_{t-1} + \gamma_{2,3}\Delta y_{t-1} + \varepsilon_2$$
$$\Delta y_t = \alpha_3\left(q_{t-1} - \beta_0 - \beta_1 tot_{t-1} - \beta_2 y_t\right) + \gamma_{3,0} + \gamma_{3,1}\Delta q_{t-1} + \gamma_{3,2}\Delta tot_{t-1} + \gamma_{3,3}\Delta y_{t-1} + \varepsilon_3$$

The chosen model, which includes the relative growth between Sweden and a trade-weighted average of our largest trading partners and the Swedish terms of trade, is estimated within a cointegration framework. Figure 1 below shows the real equilibrium exchange rate as derived from the model.[41]

[41] The real exchange rate, q_t, is obtained as the nominal exchange rate, e_t, adjusted for differences in the price level between the domestic, p_t, and foreign market, p_t^*, i.e. $q_t = e_t + p_t - p_t^*$ where variables are expressed in logarithms.

Figure 1. Real Effective (Trade-Weighted) Exchange Rate for Sweden 1976:1 – 2002:4



The estimated parameters are shown in Table 1 below.

Table 1: Estimated Parameters

The presented standard errors are the White's heteroskedasticity-consistent standard errors.

Variable	Coefficient	std err	t-value
α_0	-0.052	0.012	4.34
α_1	0.872	0.784	1.11
α_2	0.030	0.019	1.58
β_0	4.879	-	-
β_1	1.023	0.81	1.26
β_1	0.015	0.006	2.50
$\gamma_{1,0}$	0.002	0.003	0.77
$\gamma_{1,1}$	0.149	0.107	1.39
$\gamma_{1,2}$	-0.109	0.219	0.50
$\gamma_{1,3}$	-0.001	0.002	0.69
$\gamma_{2,0}$	-0.002	0.001	2.16
$\gamma_{2,1}$	-0.016	0.040	0.41
$\gamma_{2,2}$	-0.533	0.081	6.54
$\gamma_{2,3}$	0.0004	0.001	0.61
$\gamma_{3,0}$	0.140	0.150	0.96
$\gamma_{3,1}$	12.941	5.675	2.28
$\gamma_{3,2}$	-1.424	11.642	0.12
$\gamma_{3,3}$	0.345	0.098	3.53

Note. R2=0.65

The chosen fundamental model is able to explain the movement in the actual exchange rate over the period fairly well. In particular it shows the depreciation pressure building up before the floating of the Swedish krona in the fourth quarter of 1992. According to this model the krona appears to have been undervalued in 2002, which is consistent with the views of the Swedish central bank as well as market participants. In line with economic theory an undervalued currency should help domestic exporters and thereby lead to an expanding economy, ceteris paribus.

5.2.2 Stock Prices

Many different approaches have been suggested in order to calculate an equilibrium or fair value of stock prices. Lettau and Ludvigson (2001) use a common trends approach to calculate a value of the stock market that is consistent with growth in consumption, labour income and asset holdings. Although their approach appears to be viable it has nonetheless met with some criticism, notably from Rudd and Whelan (2002) and Brennan and Xia (2002), who claim that Lettau and Ludwigson use an incorrect measure of income. When a correct measure is used, the cointegrating relationship vanishes. Campbell and Shiller (1988, 1998) and Fama and French (1988) use different methods that utilise present value calculations based on Gordon (1962). Lee and Swaminathan (1999), among others, use forecasted earnings in the Gordon model and thereby claim to obtain more accurate forecasts of stock prices than if historical earnings are used. We use the version of Gordon's model that is presented in *IMF Economic Outlook* (May 2000). In this version of Gordon's model the fundamental value of the stock market is assumed to depend on the ratio between (historical) earnings and stock prices, the growth rate in the economy, an equity premium and the real interest rate. It would be preferable to use forecasts of earnings; however no data exists on forecasted earnings for Sweden before the beginning of the 1990s.

We assume that investors are rational and that the current equity price is determined by the present value of future expected dividends:

$$P_t = \sum_{i=0}^{\infty} \frac{E_t\left[D_{t+i}\right]}{\left(1+\left(r_{t+i}+e_{t+i}\right)\right)^i} \tag{3.9}$$

where P_t is the current real equity price, $E_t[D_{t+i}]$ is the expected dividend at time $t+i$, $(r_{t+i}+e_{t+i})$ the equity discount factor at time $t+i$, r the risk-free real interest rate, and e is the equity premium. Furthermore, we assume i) that dividends grow at the same rate as potential GDP, g_t, in the long run, ii) that the equity premium is constant and that the share of total earnings, c, used for dividends is constant, iii) a flat yield curve, which implies that $E(r_{t+i})=r_t$. This gives the following equation:

$$P_t = \frac{c \cdot EA_t\left(1+g_t\right)}{\left(1+r_t+e-g_t\right)} \tag{3.10}$$

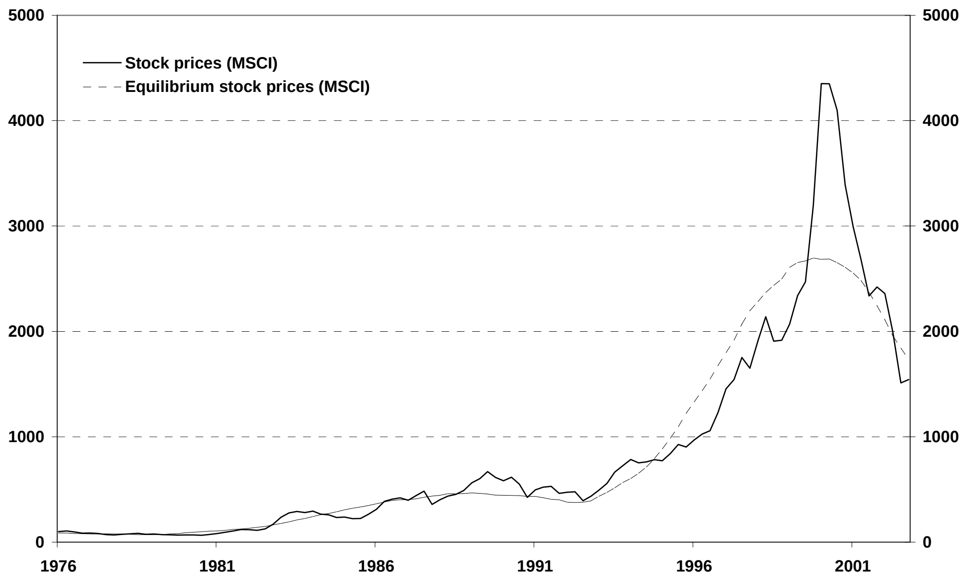
where EA is total earnings.

In order to obtain the fundamental value of the stock market we need to estimate (3.10). For this purpose we use data between 1976Q1 and 2002Q4 for Sweden. The stock prices are measured using the Morgan Stanley Capital Index (MSCI) for Sweden deflated with the Swedish CPI. We follow the approach taken in *IMF Economic Outlook* and proxy the real risk-free interest rate with a long-term nominal interest rate (5-year government bond) deflated by CPI inflation. The equity premium is assumed to be constant at 0.8 % (which is the result from the IMF study for Sweden).[42] The P/E ratio (i.e. the ratio of stock prices to earnings) has been smoothed using a Hodrick-Prescott filter (with smoothing

[42] By using IMF's estimated equity premium, we implicitly assume that the stock market has been correctly valued (on average) over the period between 1980 and 1999.

parameter equal to1600) in order to remove some of the roughness in the series that is partly due to technical factors such as accounting practices, since most dividends are paid out in February. The P/E ratio captures the relationship between stock prices, P_t, and dividends expressed as a fraction, c, of total earnings, EA_t. Finally, the potential growth rate is calculated using a Hodrick and Prescott (1997) filtering method (with the smoothing parameter set to 1600) for real Swedish GDP.

Figure 2. Real and Estimated Equilibrium Stock Prices for Sweden 1976 -- 2002



Note. R2=0.97

As can be seen in Figure 2 above, the model appears to be able to detect the equity bubble in the stock market in 2000. It also appears as if the stock market was close to its fair value, according to the estimated model, at the end of 2002.

5.2.3 House Prices

In many OECD countries real house prices have shown a tendency to move in cycles. Hort (1997) finds an autoregressive pattern in the prices with a positive autocorrelation in the short term and a negative autocorrelation over longer periods. Because of these cycles in the housing market, the question is whether house prices have been driven only by fundamental factors or also by speculative factors. Following Hort (1997) we develop an error correction model for the fundamental value of the housing stock which seems to be able to explain a significant part of the variations in house prices. In the underlying theoretical model the supply side of the housing market is explained by the cost of new constructions and the demand side is explained by the disposable income and the user cost.[43] The user cost depends on the interest rate after tax deductions, property taxes, wealth taxes and depreciations. In the cointegrating relationship we include the log of real house prices, the log of real construction costs, the log of real disposable income and real user costs.[44] The estimation period used is between 1976:1 and 2002:4. We also include a trend in the relationship which, according to Hort, can be thought of as a proxy for some omitted fundamental variables. All variables are integrated of order one according to an augmented Dickey-Fuller test. Further, a Johansen cointegration test reveals one cointegrating vector, which makes it possible to estimate the error-correction relation with a single equation method (i.e. Engle and Granger's 2-step method). With this method we first estimate the following long-term relationship:

$$hp_t = \alpha_0 + \alpha_1 \cdot rcc_t + \alpha_2 \cdot rdi_t + \alpha_3 \cdot ruc_t + \alpha_4 \cdot t + \varepsilon_t \qquad (3.11)$$

$$\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right),$$

where hp_t is the log of real house prices, rcc_t the log of real construction costs, rdi_t the log of real disposable income, ruc_t the log of real user costs, t is a time trend, and ε_t is a residual. Furthermore, α_0 to α_4 are parameters to be estimated and the residual is assumed to be normally distributed with variance equal to σ_ε^2. In the second step we use the estimated residuals, $\hat{\varepsilon}_t$, from equation (3.11) to estimate the dynamics (or short-term relationship) of the model:

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43 See appendix for a description of the variables.

44 The real variables are calculated as the nominal variables deflated by the CPI.

$$\Delta hp_t = \beta_0 + \beta_1 \Delta rcc_{t-1} + \beta_1 \Delta rdi_{t-1} + \beta_1 \Delta ruc_{t-1} + \gamma \hat{\varepsilon}_{t-1} + \eta_t \qquad (3.12)$$

$$\eta \sim N\left(0, \sigma_\eta^2\right),$$

where Δ denotes that first differences are used. The parameters β_0 to β_3 and γ are to be estimated and the residual, η_t, is assumed to be normally distributed with variance equal to σ_η^2. In the short-term equation , the gamma parameter, γ, is of special interest since it determines the speed of adjustment of any deviations from equilibrium. In our case, however, we are not interested in making any forecasts of house prices and therefore only focus on the long-term equation, (3.11).

Table 2. Estimated Long-Term Relationship

Dependent variable is the log of Swedish real house prices; sample period is 1976:1 to 2002:4

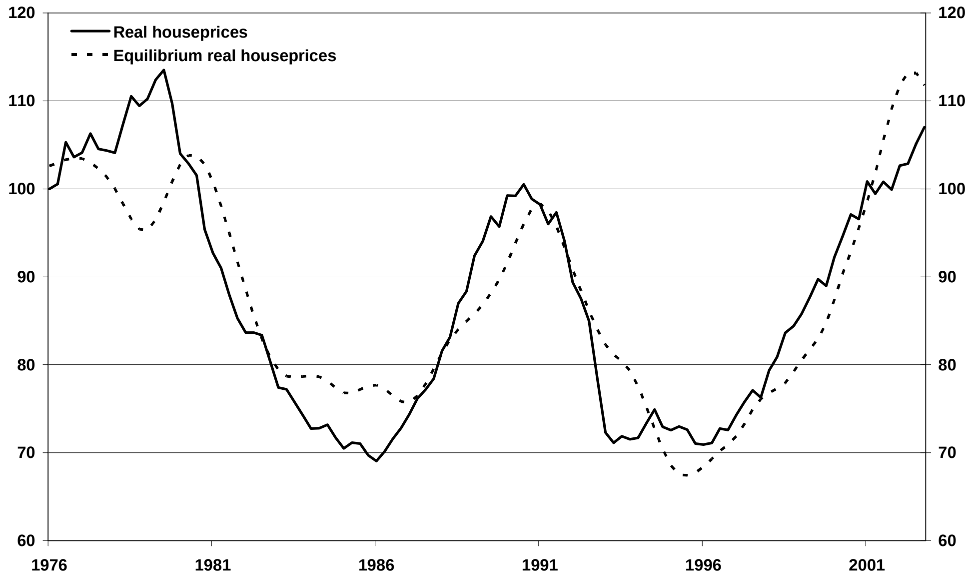
Variable	**Coefficient**	**Standard Error**	**t-value**
Constant	-5.69	0.87	-6.54
Log Real Construction Costs (*rcc*)	0.55	0.08	6.59
Log Real Disposable Income (*rdi*)	1.69	0.23	7.50
Log Real User Costs (*ruc*)	-0.02	0.004	-5.49
Trend, *(T)*	-0.01	0.001	-7.15

Note. R2 = 0.86

All included variables have the expected sign and are significant at all conventional levels. An augmented Dickey-Fuller test on the residual rejects a unit root; we therefore conclude that the series are cointegrated.

From Figure 3 it appears as if the equilibrium house price index is able to track the actual house price index fairly well over the estimated period (although the equilibrium price appears to be lagging the actual price somewhat).

Figure 3. Real House Prices for Sweden 1976--2002



Interestingly, the model also seems to do a good job in explaining the surge in house prices from the end of the 1980s to 1991 when both actual and equilibrium house prices fell sharply to levels comparable to those seen in the middle of the 1980s. The model indicates that the housing market is slightly undervalued at present. The working hypothesis in this paper is that this undervaluation affects the economy in a contractionary way. The reason is that there is a positive wealth effect from house prices to consumption, which in turn affects economic conditions.

5.2.4 Interest Rates

In almost all economic conditions indexes the short-term interest rate is included. The reason for including this variable is that central banks use this variable in their conduct of monetary policy. Thus, in order for the indices to be of use to monetary policy makers, it is important to be able to measure the increase or decrease of the monetary policy rate that is needed to force the economy back to a neutral stance. However, the interest rate on longer-term bonds often need not follow the shorter rate. In order to capture the effect of deviations in longer interest rates from their equilibrium values, we divide the interest rate into two segments according to time to maturity. The long rate (as well as the short rate) is assumed to depend on a number of fundamental factors. The modelling of the two interest rates is explained in more detail below.

5.2.4.1 The Long-Term Interest Rate

Both the long- and short-term interest rates are assumed to depend on fundamental macroeconomic variables. However, only the long-term interest rate is estimated as a function of fundamental variables since no viable model was found for the short-term interest rate. For the long-term interest rate we follow Masson (1998), who extends a theoretical equilibrium function for the interest rate from a simple model, found in Knight and Masson (1988), to separate the net savings in the economy into private and public savings. In the model, savings and investment are assumed to depend on a real interest rate, the domestic real structural budget deficit in relation to the structural budget deficit of the rest of the world, and the demographic situation in the domestic economy compared to that of the rest of the world.[45] These factors govern savings and investment in the economy in their model. In addition to the factors above, Masson (1998) also includes the (relative) output gap, as in the Laubach and Williams (2001) natural interest rate model, in order to derive an equilibrium interest rate. We apply a general-to-specific methodology in a cointegration framework in order to obtain a parsimonious model for the long interest rate, i. The resulting error correction model uses the Swedish GDP growth rate, y, the Swedish structural budget deficit in relation to GDP, g, and the world structural budget deficit, g^*, (proxied by the EMU countries) to explain the equilibrium real interest rate. The resulting model (estimated over the period between 1976:1

[45] See appendix for a more detailed description.

and 2002:4) has one cointegrating vector according to Johansen's (1995) rank test. Estimated error correction equation:

$$\Delta i_t = \alpha\left(i_t - \beta_0 - \beta_1 y_t - \beta_2 g_t - \beta_3 g_t^*\right) + \gamma_0 \Delta y_t + \gamma_1 \Delta g_t + \gamma_2 \Delta g_t^* + \varepsilon_t \qquad (3.13)$$

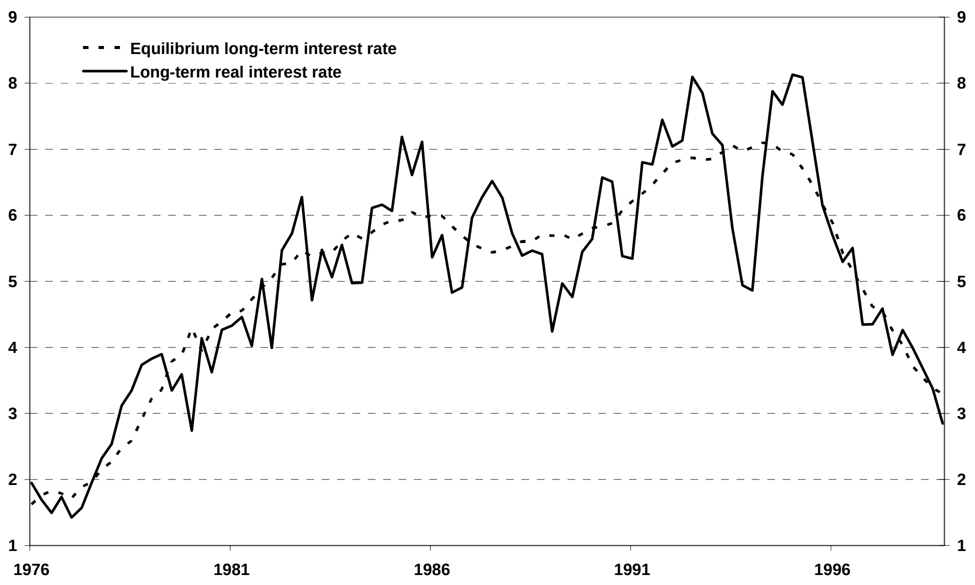
where an asterisk indicates foreign (EMU) variables.

Table 3. Estimated Parameters for the Error Correction Equation of the Swedish Long Interest Rate.
The presented standard errors are the White's heteroskedasticity-consistent standard errors.

Variable	Coefficient	Std.dev	T-value
α	-0.46	0.08	5.5
β_0	-33.20	2.73	12.2
β_1	6.69	052	12.8
β_2	-0.11	0.03	4.24
β_3	-1.14	0.08	14.8
γ_0	-0.68	3.91	0.2
γ_1	-0.16	010	-1.6
γ_2	-0.41	0.72	-0.6

In Figure 4 below the estimated equilibrium interest rate is shown together with the actual real long interest rate.

Figure 4. Real Long-Term Interest Rate for Sweden 1976 – 2002

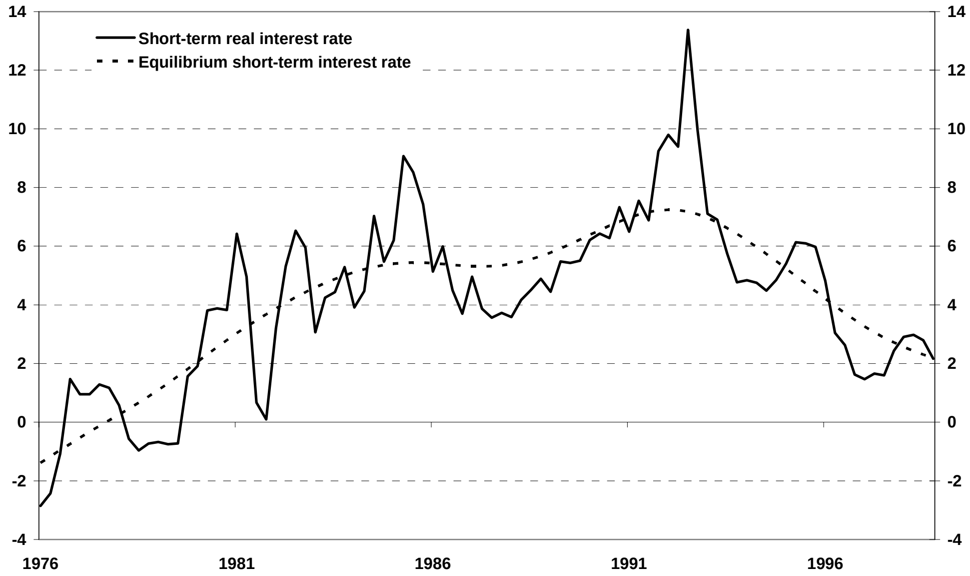


It is clear from Figure 4 that the long interest rate in the fourth quarter of 2002 was below the estimated equilibrium interest rate. An interest rate below the equilibrium value is assumed to affect the economy in an expansionary manner.

5.2.4.2 The Short-Term Interest Rate

Theoretically we could chose to model the short interest rate in the same way as the long interest rate. However, we believe that the mechanism underlying the short interest rate is different from that of the long interest rate. The main reason for this is that the short end of the yield curve is largely controlled by the central bank. The central bank's instrumental rate responds to expected as well as actual policy shocks following some kind of policy rule, see for example Svensson (1997) and Söderström et al (2002). The problem with using policy rules to estimate an equilibrium rate is that we then have to use the dependent variable in the conditions index, the output gap, in the model. Instead we use the same approach as in Woodford (2002) who estimates the equilibrium real interest rate using a filter method. We have chosen to use a simple Hodrick and Prescott (1997) filter with a smoothing parameter equal to 1600 on quarterly data for the period between 1976:1 to 2002:4 . We augment the 3-month money market interest rate with consensus forecasts of the short interest rate in order to circumvent end-point problems, and deflate the series with the Swedish CPI in order to obtain the equilibrium (or neutral) short-term real interest rate.

Figure 5. Real Short-Term Interest Rate for Sweden 1976 – 2002



According to Figure 5 above, the short-term interest rate is presently at its equilibrium value. Interest rates above equilibrium values should affect the economy in a contractionary way and vice versa, which follows standard macroeconomic theory and policy.

5.3 The Equilibrium Conditions Index

It is of course of great importance to be able to weight the different financial variables in a coherent way according to their relative importance for the economy. There are many different ways to obtain weights for a conditions index. For example, Walton and Masson (1999) and Jen and Yilmaz (2002) use an output gap that is assumed to be endogenously determined and estimate a VAR model of the output gap, interest rate, the exchange rate, stock prices and house prices where the accumulated impulse response effects are used to estimate the relative weights of the index. Another approach is applied by the OECD which use weights derived from a larger macroeconomic model, i.e. the OECD Interlink model. The IMF, on the other hand, calibrates their model in such a way that the chosen weights in their MCI index are in line with estimated weights found in the literature. In this paper we follow the approach taken by Walton and Masson, which also has the desirable property that all variables are treated as endogenous.

In order to obtain weights for the equilibrium conditions index we estimate a VAR (vector autoregression) model for the deviations between actual values and equilibrium values of the components of the equilibrium conditions index.

$$\mathbf{Y_t} = \mathbf{A_0} + \mathbf{A_1}\mathbf{Y_{t-1}} + \mathbf{A_1}\mathbf{Y_{t-2}} + \mathbf{A_1}\mathbf{Y_{t-3}} + \mathbf{A_1}\mathbf{Y_{t-4}} + \varepsilon_\mathbf{t} \quad (3.14)$$

$$\mathbf{Y_t} = \begin{bmatrix} \hat{y}_t \\ \hat{e}_t \\ \hat{s}_t \\ \hat{lr}_t \\ \hat{sr}_t \\ \hat{hp} \end{bmatrix} \quad , \quad \boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \\ \varepsilon_{6,t} \end{bmatrix}$$

where $\mathbf{A_0},\mathbf{A_1},\ldots,\mathbf{A_4}$ are matrices of coefficients with conforming dimension. The variables in the VAR(4) system above are defined as the relative deviations between actual and equilibrium values for the real exchange rate, e_t, real stock prices, s_t, the real long-term interest rate, lr_t, the real short-term interest rate, sr_t and real house prices, hp_t. It is essential that some form of standardization occur lest highly volatile series, such as stock prices, should obtain a disproportionately large weight in the index.[46] The lag length in the VAR specification is chosen using the standard Akaike information criterion. The chosen VAR model is also tested for possible misspecification. Specifically, we test for skewness and serial correlation in the residuals.[47]

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[46] We have standardized the variables by dividing them with their respective standard deviations.
[47] Skewness and serial correlation are the most important sources of bias in a VAR model; see for example Juselius (2002).

In order to obtain weights for the equilibrium conditions index we first convert the VAR system to its VMA (Vector Moving Average) form and estimate the impulse response functions using the Cholesky decomposition.[48] We then track the accumulated effect on the output gap of a (one standard deviation) shock in the included variables. The relative weights, W_i, are then calculated as the relative accumulated effect, $imp_{i,k}$, over i periods that a standardized shock in each of the, k variables has on the output gap.[49]

$$W_i = \sum_{i=1}^{40} imp_{i,k} \Big/ \sum_{k=1}^{5} \sum_{i=1}^{40} imp_{i,k} \qquad (3.15)$$

Both the nominal and effective weights of the components in the equilibrium conditions index are presented in Table 9 below.[50]

According to the estimated (effective) weights in the index, it is mainly the exchange rate and house prices that influence the economic conditions index. These two variables together with the short-term interest rate account for over 80 percent of the variation in the index. Interestingly, it is also found that the long-term interest rate accounts for less than one percent of the variation in the index. There are several possible explanations that can account for the surprisingly low weight that is put on the long-term interest rate in the estimated index. First, since the long- and short-term interest rates are correlated, it is possible that variations in the long-term interest rate are taken into account indirectly via the short-term interest rate.

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[48] The obtained weights are found to be robust against the ordering of the variables in the VAR system.
[49] We accumulate the impulse responses over 40 quarters when steady state appears to be regained.
[50] The effective weight of a variable is the weight that the variable has on the dependent variable when the relative volatility is taken into consideration. The effective weight of a highly volatile variable, such as stock prices, is thereby much higher than its nominal weight, whereas interest rate variables will have a lower effective weight.

Table 4. Estimated Weights for the Equilibrium Conditions Index

The weights in the second column are based on the standardized variables (mean = 0 and normalized to have σ = 1) and reflect the effective weights whereas the weights in the third column are based on the non-standardized variables and thus reflect the nominal weights.

Variable	Effective weights	Nominal weights
Real exchange rate	33.5 %	21.4 %
Real stock prices	12.4 %	2.2 %
Real long-term interest rate	0.2 %	1.2 %
Real short-term interest rate	21.4 %	55.8 %
Real house prices	32.5 %	19.4 %

Secondly, following Taylor (2003) there is a relationship between the exchange rate and the long term interest rate via a monetary policy rule. More specifically, a deviation between the equilibrium and actual exchange rate indicate an expected change in the exchange rate that will cause inflationary or deflationary pressure on the economy trough the impact of the exchange rate on both output and the price of imported goods. With a forward-looking monetary rule future inflation/deflation will be met by a higher/lower interest rate that will affect the long term interest rate today, according to the expectations hypothesis and rational expectations. In other words this means that changes in the exchange rate and the long term interest rate might be correlated.

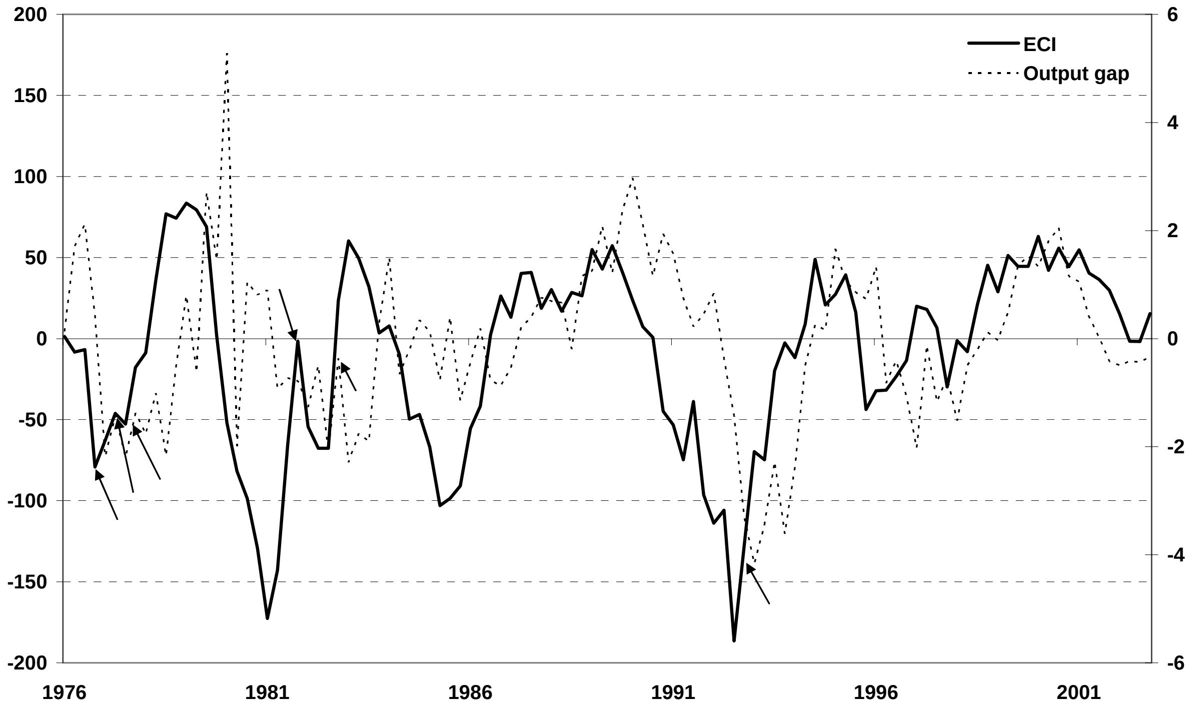
Looking at the nominal weights we can confirm the 3:1 relation between short interest rates and exchange rates that are broadly used in the MCI literature (see Batini and Turnbull, 2002).

To obtain the equilibrium conditions index in Figure 6 below, we apply the nominal weights from Table 1 on the (non-standardized) deviations from equilibrium values for the financial variables in the following way:

$$ECI_t = 0.214\hat{e}_t + 0.022\hat{s}_t + 0.012\hat{i}_t^{long} + 0.558\hat{i}_t^{short} + 0.194\hat{h}_t \,, \qquad (3.16)$$

where a hat denotes relative deviations between the actual and equilibrium real effective exchange rate, e_t ,real stock prices, s_t , the real long-term interest rate, i_t^{long} , the real short-term interest rate, i_t^{short} , and real house prices, h_t .

Figure 6. The Equilibrium Conditions Index and the Swedish Output Gap



Note. The arrows indicate the devaluations of the Swedish krona, and the depreciation in 1992 after the regime shift between a fixed and a floating currency. The output gap is calculated using the real Swedish GDP and the potential Swedish GDP obtained with a Hodrick and Prescott (1997) filter.

In order to be able to rely on the validity of the equilibrium conditions index, it is important to be able to understand the evolution of the index over the sample period. Below we try to explain the reasons for the movement in the equilibrium conditions index and connect the movements with other economic events in Sweden.

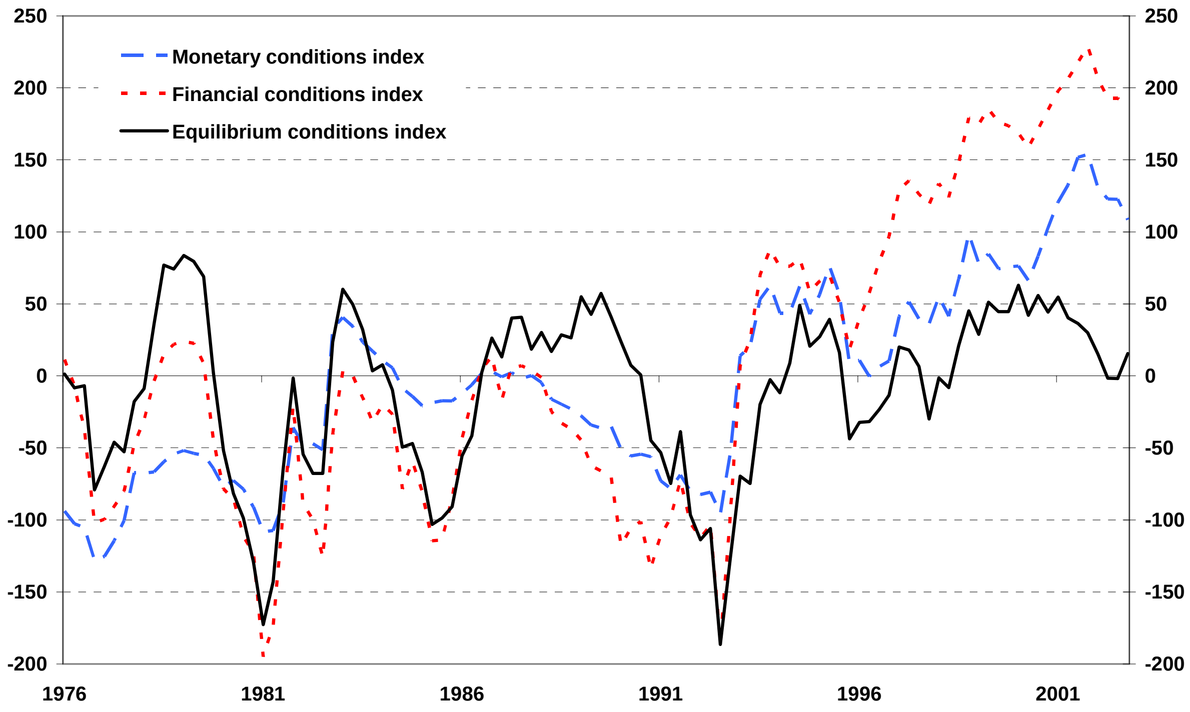
During the late 1970s, the equilibrium conditions index rose after three devaluations of the Swedish krona in November 1976, April 1977 and August 1977. The increases in real house prices also contributed to more expansionary conditions in the economy over this period. From 1980-1981, the equilibrium conditions index fell sharply due to an appreciating real exchange rate and falling real house prices. This drop was partly neutralized by monetary stimulus via lower real short-term interest rates together with two new devaluations in 1981 and 1982. Around 1983, the real Swedish exchange rate appreciated again and the real short-term interest rate rose, thus causing contractionary pressure on the economy. From 1986, real house prices began to climb upwards at the same time as real long-term interest rates fell, which helped to create expansionary conditions in the economy up until the late 1980s when the real Swedish exchange rate began to appreciate again. The effect of the appreciating real Swedish exchange rate became painfully apparent in November 1992 when the krona was floated and depreciated by over 20 percent. During the second half of the 1990s, the high valuation of the stock market contributed to keeping the economy in an expansionary condition. Low real short-term interest rates in the wake of lower inflation expectations, and the weaker krona, also put an expansionary pressure on the economy.

It is worth noticing that the index appears to predict the two major economic downturns in Sweden in the early 1980s and early 1990s. This is particularly important since it opens up the possibility of using the index as an indicator for large movements in the economy.

Under the last quarter of 2002 (the last observation in the sample period), the real exchange rate was undervalued by 7 percent, real house prices were 4 percent under their equilibrium values, real stock prices were about 10 percent under equilibrium values, and both short- and long-term real interest rates were under their equilibrium levels according to the models used. As can be seen in Figure 6, this indicates close to neutral conditions for the Swedish economy.

Figure 7. Comparison of the Equilibrium Conditions Index, an MCI and an FCI.

For the MCI and FCI: 1987 = 0.



In Figure 7 the equilibrium conditions index is compared to an MCI and an FCI over the period 1976 to 2002.[51] During the period from 1976 to the middle of the 1990s, all indices co-vary to a high degree. During the latter part of the 1990s, the different series' start to diverge. The reason for this divergence comes especially from the trending nature of the stock and house markets, as well as from the Swedish real exchange rate. Both the MCI and the FCI indicate (extremely) expansionary conditions in the economy in 2001 and 2002 when the ECI declined towards neutral values. When comparing the recent development of the different indices with the output gap in Figure 6, it is clear that neither the MCI nor FCI are very accurate measures of economic conditions in Sweden. It

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[51] We use the same type of MCI found in Freedman (1994) where only the short interest rate and the real exchange rate are included. In Freedman's MCI the exchange rate is weighted by a factor of 1/3. This means that a 1 percent change in the short interest rate has the same effect as a 3 percent change in the exchange rate on monetary conditions. The FCI follows Jen and Yilmaz (2002) with the following weights: short-term interest rate (0.47), real exchange rate (0.12), real house prices (0.37), and real stock prices (0.04). As in Freedman, 1987 is used as a base/neutral year, i.e. for the MCI and FCI, 1987 = 0.

is only the ECI that is able to track (and to some extent predict) economic conditions.

5.4 Concluding Remarks and Summary

In this paper we develop an economic conditions index (ECI) for the Swedish economy. The index is based on the deviations between the actual value and the estimated equilibrium value of the real exchange rate, long- and short-term real interest rates, and real stock and house prices. The financial variables are weighted in an index with weights obtained from the impulse response function of the output gap from a VAR model. The relative weights obtained for the exchange rate and the short-term interest rate conform to typical values from the literature on monetary conditions indices (MCIs).

The resulting index is able to explain both the larger business cycle movements as well as the devaluations of the krona over the sample period between 1970 and 2002. Interestingly, the index appears to predict the major downturns in the economy in the early 1980s and the early 1990s, (potentially) making it a useful tool for policy making.

When compared with an MCI and a financial conditions index (FCI), the ECI is clearly better at tracking economic conditions in Sweden (as measured by the output gap) over the latest downturn in the economy (2001 – 2002). This is mainly because the MCI and the FCI do not include the trend found in the exchange rate, the stock market or the housing market in any way, which leads to trends in the indices. This ability of the ECI to compensate for fundamentally motivated trends in the financial variables is one of the major advantages of the ECI over MCIs and FCIs.

Eika et al (1996) conclude that there is a strong possibility that the weights in MCIs are time-varying, which would make the resulting index inappropriate as a measure of economic conditions. Furthermore, they point out that the variables included in the MCIs might not be cointegrated and that important economic factors might be excluded. Some of this criticism has been circumvented in our approach where we include more variables than in a standard MCI and test for cointegration. However, future research should investigate the possibility of time-varying weights in the index.

References

Balassa, B., 1964, The Purchasing Power Parity Doctrine: A Reappraisal, *Journal of Political Economy*, 72, 584-596.

Batini, N. and Turnbull, K., 2000, Monetary Condition Indices for the UK: A Survey, *Bank of England discussion paper from the external MPC unit*, no.1.

Batini, N. and Turnbull, K., 2002, A Dynamic Monetary Conditions Index for the UK, *Journal of Policy Modelling*, 24:3, 257-281.

Campbell, J.and Shiller, R., 1988, Stock Prices, Earnings, and Expected Dividends, *Journal of Finance*, Vol. 43, no. 3, 661-676.

Campbell, J.and Shiller, R. ,1998, Valuation Ratios and the Long term Stock Market Outlook, *Journal of Portfolio Management*, Vol. 24, no. 2, 11-26.

Case, K., Quigley, J., and Shiller, R., 2001, Comparing Wealth Effects: The Stock Market versus the Housing Market, *Working paper 8606*, NBER.

Clark, P. and MacDonald, R., 1999, Exchange Rates and Economic Fundamentals: A Methodological Comparison of BEERs and FEERs, in: R. MacDonald and J. Stein (eds.), *Equilibrium exchange rates*, Amsterdam: Kluwer.

Dudley, W. and Hatzius, J., 2000, The Goldman Sachs Financial Conditions Index: The Right Tool for a New Monetary Policy Regime, *Goldman economics papers*, Goldman Sachs, no. 44.

Eika, K., Ericsson, N., and Nymoen, R., 1996, Hazards in Implementing a Monetary Conditions Index, *Oxford Bullentin of Economics of Statistics*, 54(4), 765-790.

Ericsson, N. and Kerbeshain N., 1997, Monetary Condition Indexes: A Bibliography with Synopses of Articles, *mimeo*, Federal Reserve Board, Washington D.C.

Fama, E. and French, K., 1988, Dividend Yields and Expected Stock Returns, *Journal of Financial Economics*, October, 3-25.

Frenkel, J. and Mussa, M., 1986, Asset Markets, Exchange Rates, and the Balance of Payments, in: E. Grossman and K. Rogoff (eds.) in: *Handbook of International Economics*, Vol. 2, Amsterdam.

Freedman, C., 1995, The Role of Monetary Conditions and the Monetary Conditions Index in the Conduct of Monetary Policy, *Bank of Canada Review*, Autumn.

Fung, B. and Yuan, M., 1999, Measuring the Stance of Monetary Policy, *Conference Proceedings*, Bank of Canada.

Goodhart, C. and Hofmann, B., 2000, Financial Variables and the Conduct of Monetary Policy, *Working Paper no. 112*, Sveriges Riksbank.

Gordon, M., 1962, The Investment, Financing, and Valuation of the Corporation, Homewood, Ill.

Hansson, B. and Lindberg, H., 1994, Monetary Conditions Index – A Monetary Policy Indicator, *Quartely Review*, Sveriges Riksbank, no. 3, 12-17.

Hodrick, R. and Prescott, E., 1997, Post-War US Business-Cycles: An Empirical Investigation, *Journal of Money, Credit and Banking*, 29, 1-16.

Hort, K., 1997, The Determinants of Urban House Price Fluctuations in Sweden 1968-1994, *Journal of Housing Economics*, 7, 93-120.

Jen, S. and Yilmaz, F., 2002, An Asset and Monetary Conditions Index & USD, *mimeo*, Global Equity Research, Morgan Stanley.

Juselius, K, 2002, The Cointegrated VAR model: Econometric Methodology and Macroeconomic Application, unpublished manuscript.

Knight, M. and Masson, P., 1988, Fiscal Policies, Net savings, and Real Exchange Rates. In: *International Aspects of Fiscal Policies*, eds. J. Frenkel, University of Chicago Press.

Laubach, T. and Williams, J., 2001, Measuring the Natural rate of Interest, Finance and Economics Discussion Series 2001-56, Board of Governors of the Federal Reserve System.

Lee, C., Myers, J. and Swaminathan, 1999, What is the Intrinsic Value of the Dow?, *Journal of Finance*, Vol. 54, no. 5, 1693-1741.

Lettau, M. and Ludvigson, S., 2001, Consumption, Aggregate Wealth, and Expected Stock Returns, *Journal of Finance*, vol. LVI, no.3, 815-849.

Lettau, M. and Ludvigson, S., 2001, Understanding Trend and Cycle in Asset Values: Bulls, Bears, and the Wealth Effect on Consumption, *CEPR Discussion paper 3104*.

Lindblad H. and Sellin, P., 2003, Equilibrium Rate of Unemployment and Real Exchange Rate: An Unobserved Components System Approach, Working Paper no 152, Sveriges Riksbank.

MacDonald, R., 2000, Concepts to Calculate Equilibrium Exchange Rates: An Overview, *Discussion paper 3/00*, Economics Research Group of the Deutsche Bundesbank.

MacDonald, R. and Marsh, I., 1999, Exchange Rate Modelling, *Advanced Studies in Theoretical and Applied Econometrics Series*, Kluwer Academic Publishers.

Masson, P., 1998, A Globally Consistent Conceptual Framework, Chapter IV in *Exchange Rate Assessment: Extensions of the Macroeconomic Balance Approach* edited by P. Isard and H. Faruqee, IMF Occasional paper No 167.

Mayes, D., 2001, Financial Conditions Indexes, *Bank of Finland Discussion Papers*, no. 17/2001.

Obstfeld, M. and Rogoff, K., 1995, The Mirage of Fixed Exchange Rates, *Journal of Economic Perspective*, 9, 73-96.

Samuelson, P., 1964, Theoretical Notes on Trade Problems, *Review of Economics and Statistics*, 46, 147-154.

Shefrin, H. and Thaler, R., 1988, The Behavioural Life-Cycle Hypothesis, *Economic Inquiry*, 26, 609-643.

Smets, F., 1997, Financial Asset Prices and Monetary Policy: Theory and Evidence, *CEPR Discussion Paper*, No. 1751.

Svensson, L, 1997, Optimal Inflation Targets, “Conservative” Central Banks, and Linear Inflation Contracts, American Economic Review, Vol. 87, no. 1, 98-114.

Taylor, J., 2003, The Role of Exchange Rate in Monetary Policy Rules, *Unfinished Manuscript*, Stanford University, USA.

Rudd, J. and Whelan, K., 2002, A Note on the Cointegration of Consumption, Income, and Wealth, *Working Paper*, Federal Reserve Board, USA.

Söderström, U., Söderlind, P., and Vredin, A., 2002, Can a Calibrated New-Keynesian Model of Monetary Policy Fit the Facts?, Working Paper no 140, Sveriges Riksbank.

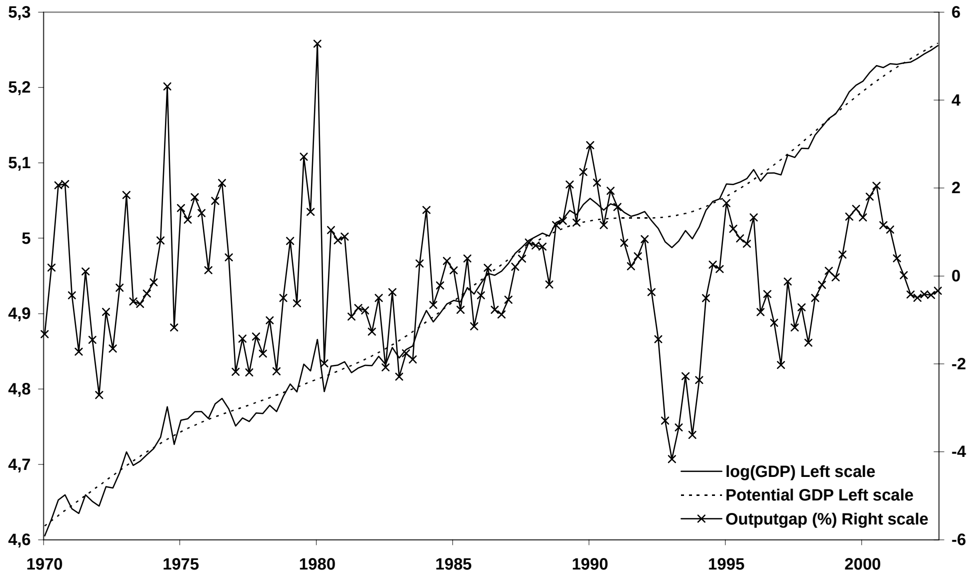
Walton, D. and Massone, M., 1999, Sterling and the Conduct of UK Monetary Policy, *The UK Economics Analyst*, July/August, Goldman Sach.

Woodford, M., 2002, A Neo-Wicksellian Framework for the Analysis of Monetary Policy, *mimeo*, Department of Economics, Princeton University

Appendix A: Data

Figure A1. Real and Potential Swedish Output and the Swedish Output Gap

Potential output is calculated using a HP (1600) filter on real Swedish GDP.

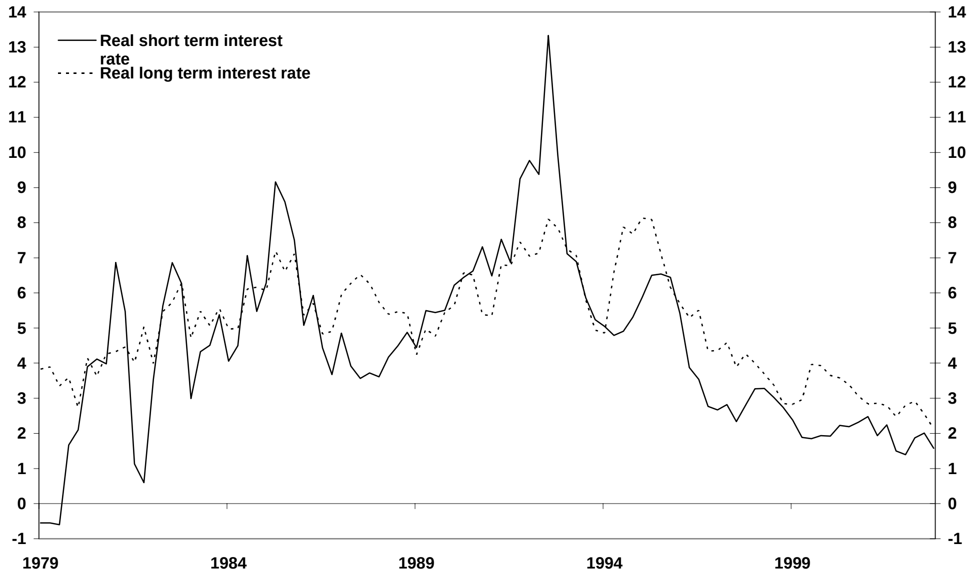


Sources: Statistics Sweden and Sveriges Riksbank

The series are seasonally adjusted.

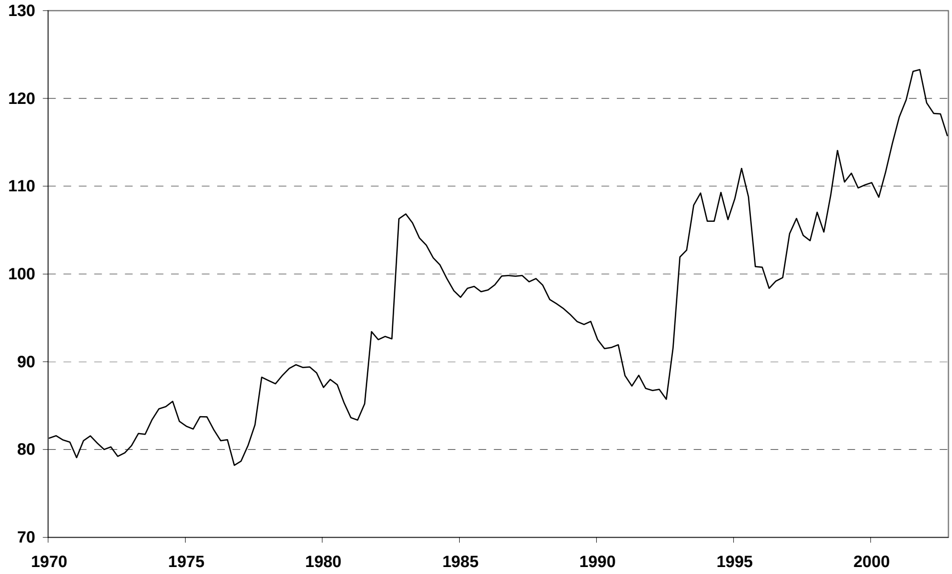
Figure A2. Real Short- (3-month) and Long-Term (10-year) Swedish Interest Rates

Nominal series deflated by the Swedish CPI.



Source: Sveriges Riksbank

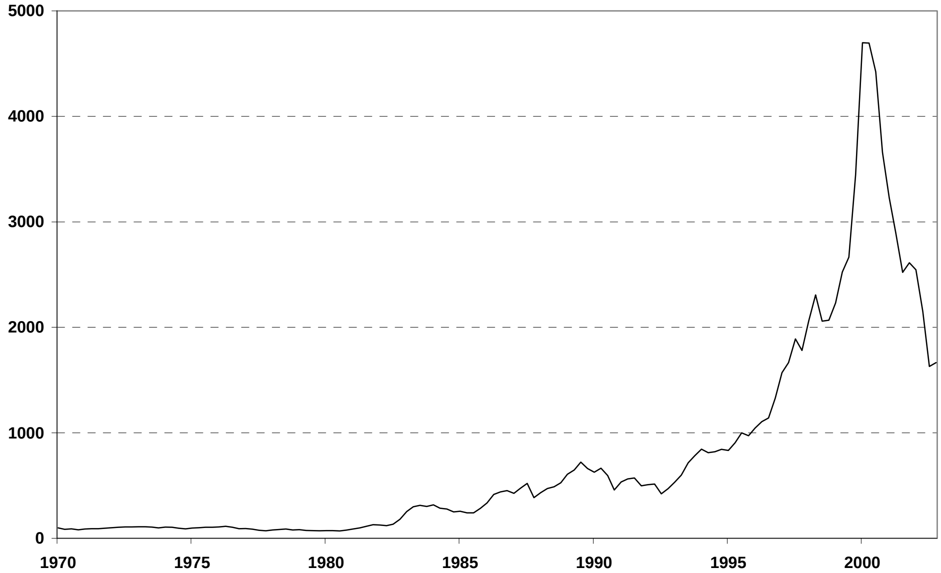
Figure A3. Real Swedish Trade- Weighted Exchange Rate (TCW)



Source: Sveriges Riksbank

Figure A4. Real Stock Prices (MSCI)

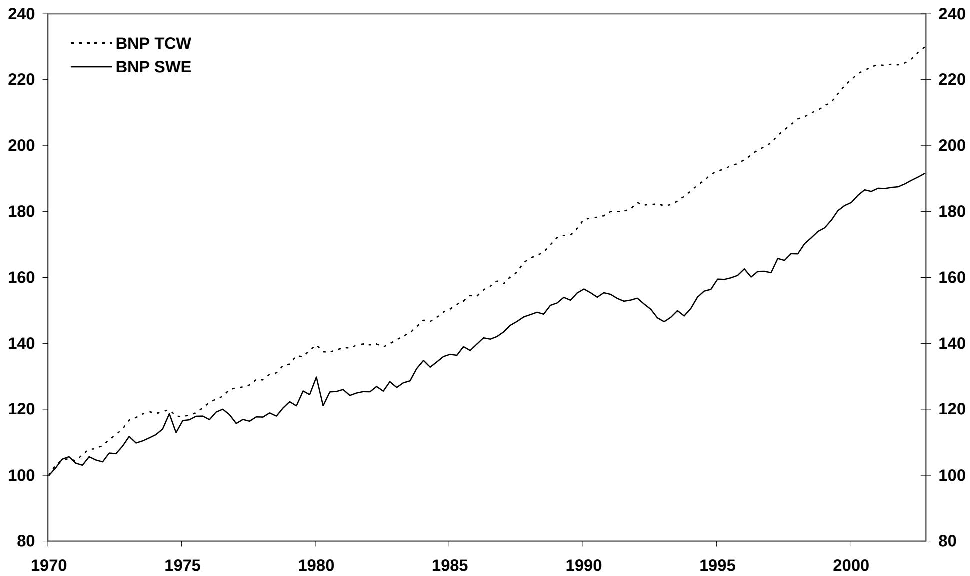
Nominal MSCI Sweden deflated by Swedish CPI.



Sources: Datastream and Statistics Sweden

Figure A5. Real Swedish GDP and Real GDP for the TCW area (TCW for Sweden)

Nominal series deflated by Swedish CPI and the CPI for the TCW area respectively.

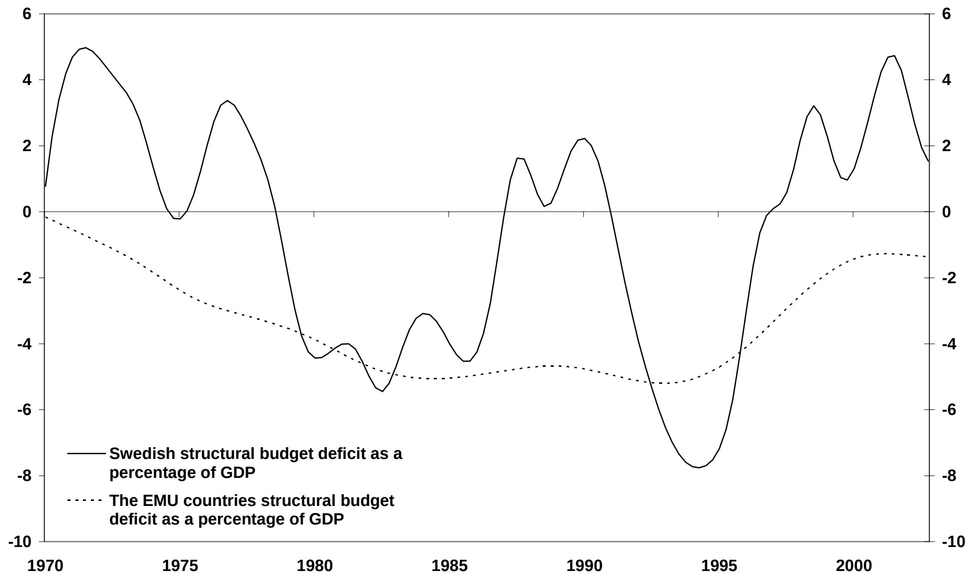


Source: Statistics Sweden

The series are seasonally adjusted.

Figure A6. Swedish and EMU Structural Budget Deficit in Relation to GDP

Calculated as the Hodrick and Prescott (1600) filtered budget deficit series.

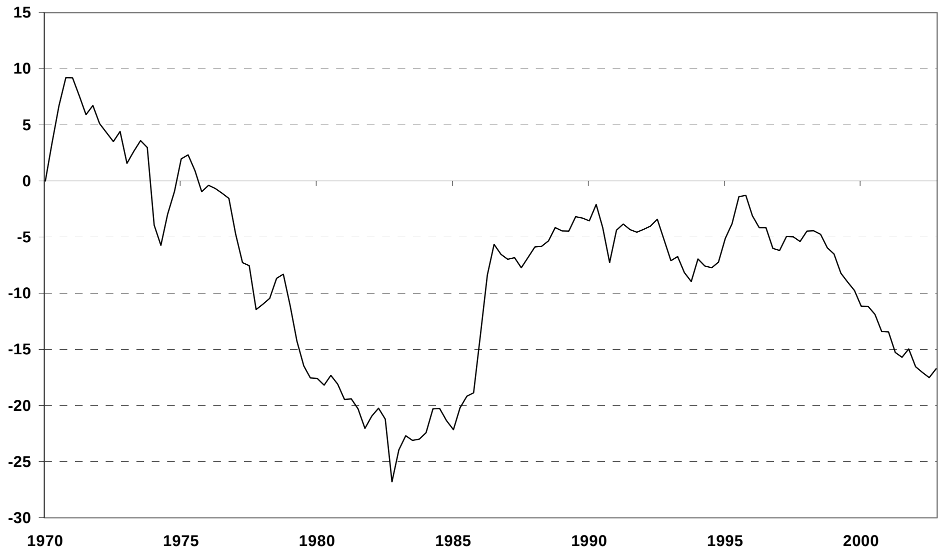


Sources: Statistics Sweden, OECD and Sveriges Riksbank

The series are seasonally adjusted.

Figure A7. Swedish Terms of Trade

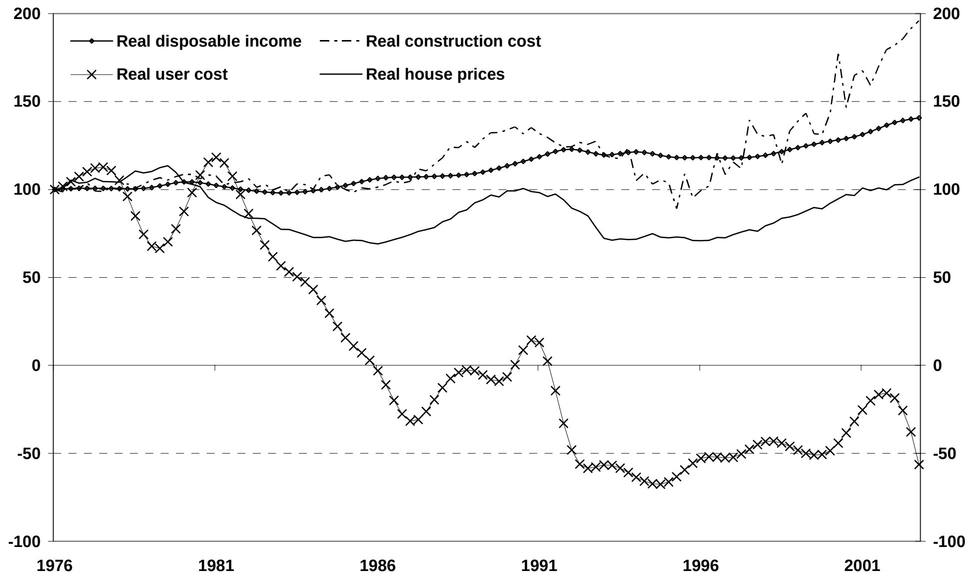
Calculated as the Swedish export price index divided by the Swedish import price index.



Source: Statistics Sweden

The series are seasonally adjusted.

Figure A8. Real House Prices, Real Disposable Income, Real User Costs and Real Construction Costs



Sources: Statistics Sweden, Hort (1997) and Sveriges Riksbank

Note. Real house prices are measured as constant quality house prices deflated by the consumer price index. Real user cost is calculated as $[(1-t)i-\pi^e+t_h]$, where t denotes the marginal tax rate determining the tax deduction, i the annual average interest rate offered on 4-5 year mortgages, π^e is the expected inflation rate calculated as the average inflation rate for the present and previous year, and t_h is the effective property tax. Real disposable income is interpolated annual nominal disposable income deflated with the consumer price index. Real construction costs is a construction cost index taken from Statistics Sweden deflated with the consumer price index. Real disposable income is seasonally adjusted.

Appendix B: Estimation of the Equilibrium Conditions Index

Table B1: χ^2-test for Skewness of Estimated VAR

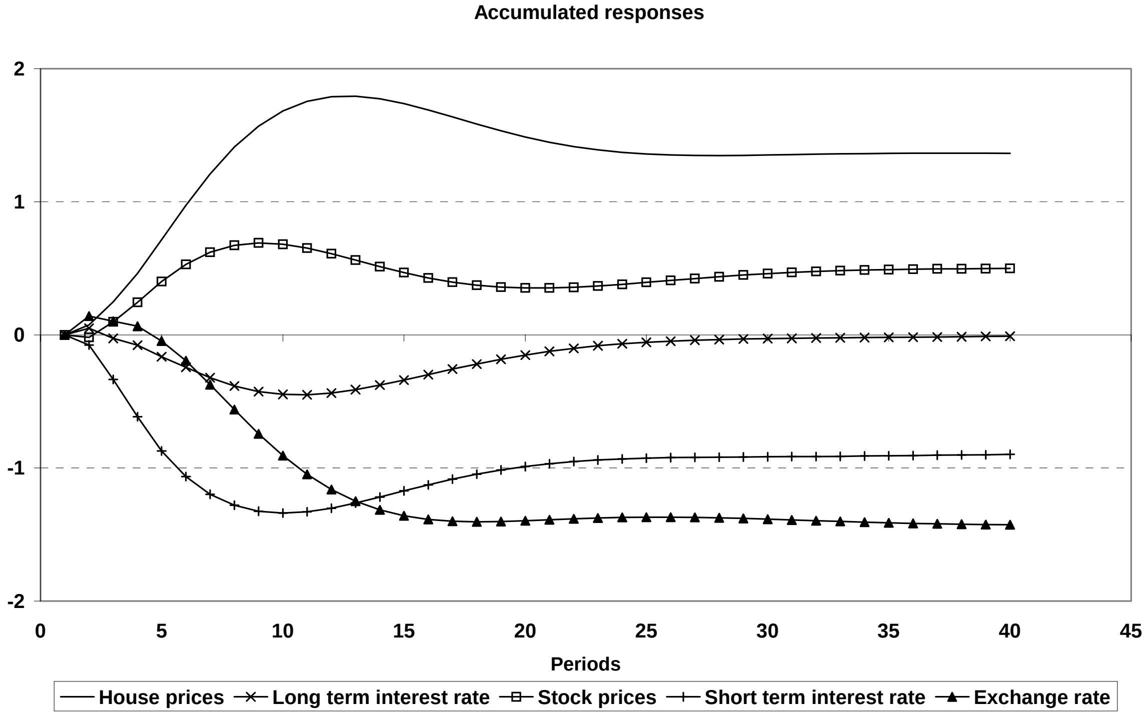
Variable	Skewness	Chi-sq	df	Prob.
Output gap	0.086	0127	1	0.722
House prices	0.085	0.125	1	0.724
Long-term interest rate	-0.206	0727	1	0.394
Stock prices	0.061	0.064	1	0.801
Short-term interest rate	0.190	0.618	1	0.432
Exchange rate	-0.379	2.471	1	0.116
Joint		4.132	6	0.659

Table B2: LM-test for Autocorrelation in Estimated VAR

H_0: no autocorrelation at lag s

Lags	LM-Stat	Prob
1	41.21	0.253
2	30.84	0.712
3	39.45	0.319
4	46.75	0.108
5	36.70	0.436
6	33.86	0.571
7	45.13	0.142
8	45.91	0.125
9	38.65	0.351
10	37.41	0.404

Figure B3: Impulse Responses of the Output Gap from a One Standard Deviation Shock (Cholesky) to the Variables in the Equilibrium Conditions Index



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