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Summary

Saito, Quintiere and Williams (1) developed an integral equation derived from quasi–steady thermal flame spread theory to describe spread up a thick solid. Allowance was made in this purely thermal model for a varying rate of pyrolysis and the relationship between flame length and burning rate was linearised. They discussed asymptotic solutions to their equation and derived the requirements for two–dimensional propagation both for a step function pyrolysis rate $m'' =$ constant up to a burnout time and an inverse square root variation.

Here their integral equation is solved analytically by Laplace Transform and is also applied to pyrolysis which decreases exponentially with time. The three conditions for propagation are compared, all being of the type

$$KQ'' = \text{function}\left[\frac{t_{ig}}{t_B}\right]$$

where

K is constant

Q'' is a characteristic heat release rate

t_{ig} is the ignition time

t_B a characteristic of the duration of pyrolysis

Q'', t_{ig} and t_B are obtained under the rate of heating imposed by the flame

In those flames which spread indefinitely a comparison is made with Hasemi, Yoshida and Nohara's (2) asymptotic theory. Some limitations of quasi–steady theory are considered.

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List of symbols

K	=	Flame height constant
$K\rho c$	=	Thermal inertia
m''	=	Rate of production of gaseous fuel per area
q	=	Heat of combustion for unit mass of gaseous fuel
q_w''	=	Heat flux from flame per unit area
Q	=	Energy release rate from ignition source
t	=	Time
T	=	Temperature
T_p	=	Pyrolysis temperature
T_s	=	Surface temperature
T_o	=	Initial temperature
V	=	Velocity of pyrolysis front
W'	=	Total heat release per unit width
χ_f	=	Position of the flame front
χ_{fo}	=	Initial value of χ_f
χ_p	=	Position of the pyrolysis front
χ_{po}	=	Initial value of χ_p
β	=	Decay coefficient for mass loss rate
γ	=	$\mathrm{Ln}(\chi_p/\chi_{po})$ in equation (41)
τ	=	Characteristic ignition time

1. Introduction

Saito, Quintiere and Williams (1) (SQW) have discussed a thermal theory of upward flame spread on thick solids which leads to an integral equation for the velocity of spread.

Certain approximations were required for this equation to be obtained and assumptions, some similar and some different, have been used by Hasemi, Yoshida and Nohara (2) (HYN) in developing another theoretical but related approach.

Saito et.al. discussed the solution of their integral equation at short times and at long times, but here a general analytic solution is obtained and evaluated for certain conditions.

This paper also develops the solutions, considers the approximations and compares the two theories SQW and HYN. The main assumptions (both explicit and implicit) in common are:

1. The material is homogeneous and its thermal properties are constant with temperature.
2. Chemical kinetics are excluded, so very fast (as well as very slow) rates of spread are not fully dealt with and extinction conditions are therefore only discussed approximately.
3. The flame length χ_f depends on a power of W' the rate of convected heat release per unit width of flame front.

The main assumptions which are different are:

1. The model of Hasemi et.al. seeks an asymptotic solution and does not accommodate initial conditions. The SQW theory does include a representation of initial conditions including flame from a pilot or auxiliary burner: the adequacy of this representation is discussed below.

2. The theories differ in their treatment of the relationship between flame length χ_f and total rate of heat release W' per unit width. Both consider

$$\chi_f \sim W'^{2/3}$$

Saito et.al. however choose a linearized form

$$\chi_f \sim W'$$

for mathematical convenience; their data being equally expressible by either form. We shall show below how the 2/3 power law form can be linearized under some useful conditions.

Hasemi et.al. introduce the pyrolysis length χ_p and write

$$\chi_f \sim \chi_p \left[\frac{W'}{\chi_p^{3/2}}\right]^{2/3}$$

and assume, again for mathematical convenience, that $\frac{W'}{\chi_p^{3/2}}$ is a constant. This obviates the need for an integral representation of χ_f as in the SQW theory.

3. Hasemi et.al. also discuss the "steady state" form i.e. when burn out occurs at the same rate as the forward extension i.e. χ_p is constant. They show this gives a lower bound for the dependence of V on χ_p.

4. In the SQW model heat transfer only occurs at constant flux within the region $\chi_f > \chi > \chi_p$. Experimental correlations are used by Hasemi et.al. These include a variable heat flux over the region $\chi_f > \chi > \chi_p$ and heating ahead of the flame.

2. The model of Saito, Quintiere and Williams

Saito, Quintiere and Williams develope an integral equation for V, the propagation velocity of the pyrolysis front, when the relationship between heat release and flame length is made linear. They discuss their solution both for the initial condition and for long times. Criteria for propagation are derived. Hasemi et.al. only discuss asymptotic solutions, one for a flame in which χ_p remains constant – i.e. the rear burns on as far as the front advances and the other for extending χ_p with no burn out. Before comparing these two theories we shall give a formal solution to the integral equation derived by Saito et.al. Starting from

$$V(t) = \frac{\chi_f - \chi_p}{\tau} \tag{1i}$$

they obtain[1]

$$V(\epsilon) = \int_0^{\epsilon} V(\epsilon')F(\epsilon - \epsilon')d\epsilon' + G(\epsilon) \tag{1ii}$$

where $\epsilon = t/\tau$

and $$\tau = \left[\frac{4}{\pi}\, \frac{q_w''^2}{K\rho c(T_p - T_s)^2}\right]^{-1} \tag{2}$$

q_w'' = the uniform flux from the flame to the material over the distance $\chi_f - \chi_p$. This flux is taken as zero beyond the flame, a restriction not imposed in Hasemi's model.

K, ρ, c are the thermal conductivity, density and specific heat, all assumed constant and uniform.

T_p is the pyrolysis temperature ($= T_{ig}$ in Hasemi's model), T_s is the initial surface temperature beyond the pyrolysis zone.

$$V(\epsilon) = V(t)/V(o)$$

where $$V(o) = \frac{\chi_{fo} - \chi_{po}}{\tau} \tag{3}$$

χ_{fo} is the initial value of χ_f, the flame length, and χ_{po} is the initial value of χ_p, the position of the pyrolysis front. Note $KQ_o = \chi_{po}$.

$$F(t) = Kqm''(t) - 1$$

where K is given by

$$\chi_f = K\left[(Q + q\int_0^{\chi_p} m''(t)d\chi)\right] \tag{4i}$$

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[1]Further details are given in connection with equations (51) and (52).

q is heat release for unit mass of gaseous fuel – assumed constant. m''(t) is the rate of production of gaseous fuel. Various forms for m''(t) are considered by Saito et.al.

$$\text{i.e.}\quad \chi_f = K(Q + qm''(t)\ \chi_{po} + \int_o m''(t - t_p)V(t_p)dt_p) \tag{4ii}$$

$$\text{and}\quad G = 1 + \frac{Kq\left[m''(\epsilon) - m''(o)\right]\chi_{po} - K\left[Q(o) - Q(\epsilon)\right]}{\chi_{fo} - \chi_{po}} \tag{5}$$

Note that this formulation derived form equation (1i) does not allow the flame to die out when V becomes zero. We shall however continue with these equa– tions.

Saito, Quintiere and Williams point out that for small ϵ

$$V(t) \doteq V(o)[1+\left[G'(o) + F(o)\right]\epsilon]$$

whilst if propagation persists

V(t) becomes proportional to $V(o)e^{\alpha t}$

where $\int_0^\infty F(\epsilon)e^{-\alpha\epsilon}\, d\epsilon = 1$ determines α. It is thus possible to discuss the roles of a [= Kqm''(o)] and parameters in the form of m''(t).

However, there is a formal solution of equation (1ii) viz if we take the Laplace Transform

$$\overline{w} = \int_0^\infty we^{-p\epsilon}d\epsilon$$

then

$$\overline{V} = \frac{\overline{G}}{1-\overline{F}} \tag{6}$$

The inversion theorem (for ϵ as the time variable) is

$$W = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \overline{W}\, e^{\lambda\epsilon}\, d\lambda$$

where γ exceeds the real part of all singularities i.e. the roots of

$$\int_0^\infty F(\epsilon)e^{-p\epsilon}d\epsilon = 1$$

which is akin to the result given by Saito, Quintiere and Williams for an asymptotic form.

If we consider ignition being effected by radiation so that

$$\dot{Q}(o) = \dot{Q}(t) = 0$$

then

$$\frac{\chi_{fo}-\chi_{po}}{\chi_{po}} = Kqm''(o)-1 = a - 1 \qquad (7)$$

$$\text{and } \overline{G} = \frac{a\left[\frac{\overline{m}''}{m''(o)} - \frac{1}{p}\right]}{a - 1} + \frac{1}{p} = \frac{a\frac{\overline{m}''}{m(o)} - \frac{1}{p}}{a - 1} \qquad (8)$$

When $\epsilon \longrightarrow 0$, $\overline{F}$ is $0(1/p)$, $p \longrightarrow \infty$

and $\overline{V} = \overline{G} + \overline{GF}$

$$\text{i.e.} \quad V = G(\epsilon) + \int_0^\epsilon G(\epsilon)F(\epsilon'-\epsilon')d\epsilon' = 1 + (G'(o) + F(o))\epsilon \qquad (9)$$

the result discussed by Saito et.al.

Saito et.al. discussed two forms of $m''(t)$ – viz

$$m''(\epsilon) = m''(o) \quad 0 < \epsilon < \epsilon_0 = 0 = \epsilon > \epsilon_0 \qquad (10)$$

referred to as Curve 1

and $m''(\epsilon) = \dfrac{E}{\sqrt{\epsilon}}$ referred to as Curve 2. (11)

However here we discuss primarily a form used by Magnusson and Karlsson (3) viz

$$m''(\epsilon) = m''(o)e^{-\beta t} = m''(o)\ e^{-\gamma\epsilon} \tag{12}$$

where $\gamma = \beta\tau$ and to which we refer as Curve 3.

For Curve (1)

$$\overline{F} = \frac{a}{p}\left[1-e^{-p\epsilon_o}\right] - \frac{1}{p} \tag{13}$$

and
$$\overline{G} = \frac{\frac{a}{p}\left[1-e^{-p\epsilon_o}\right]-\frac{1}{p}}{a-1} \tag{14}$$

$$= \frac{1}{p} - \frac{a}{p(a-1)}\ e^{-p\epsilon_o} \tag{15}$$

For $\epsilon_o \longrightarrow \infty \quad \overline{V} = \frac{1}{p-(a-1)}$

$$\text{i.e. } V(t) = V(o)e^{(a-1)t/\tau} \tag{16}$$

This has close similarities with Hasemi's model which we discuss below. The requirement for continued burning is clearly $a>1$. For finite ϵ_o inversion is algebraically cumbersome.

The condition for a real root

$$1 - \overline{F} = 0$$

is that a $p_o > 0$ exists where

$$p_o + 1 - a + ae^{-p_o\epsilon_o} = 0 \tag{17}$$

and the limiting condition is obtained from this and

$$1 = a\epsilon_o e^{-p_o\epsilon_o} \tag{18}$$

results given by Saito et.al.

For $\frac{m''(\epsilon)}{m''(o)} = \frac{E}{\sqrt{\epsilon}}$

$$1 - \overline{F} = 1 + 1/p - \frac{E\sqrt{\pi}}{\sqrt{p}} \qquad (19)$$

$$\overline{G} = \frac{\frac{aE\sqrt{\pi}}{\sqrt{p}} - 1}{a - 1} \qquad (20)$$

i.e. $\overline{V} = \frac{aE\sqrt{\pi}\sqrt{p} - 1}{(a-1)[p+1-aB\sqrt{\pi}\sqrt{p}]}$ (21)

The denominator can be factorised into

$$\left[\sqrt{p} - e + \sqrt{e^2-1}\right] \text{and} \left[\sqrt{p} - e - \sqrt{e^2-1}\right]$$

where $e = aE\sqrt{\pi}/2$ with inversion into analytic functions involving the error function.

Propagation leads to an asymptotic solution

$$V(\epsilon) \sim e^{\left[e+\sqrt{e^2-1}\right]t/\tau} \qquad (23)$$

For Curve 3

$$\frac{m''(\epsilon)}{m''(o)} = e^{-\gamma\epsilon}$$

and

$$\overline{V} = \frac{1}{(a-1)} \frac{(a-1)p-\gamma}{p^2+(\gamma+1-a)p+\gamma} \qquad (24)$$

The denominator has real roots only if

$$a > (1+\sqrt{\gamma})^2 \qquad (25)$$

and the asymptotic solution is then

$$\exp\left[\frac{a-\gamma-1}{2} + \frac{1}{2}\sqrt{\left[a-(\gamma+1)^2\right]-4\gamma}\right]\epsilon \tag{26}$$

Note that in SQW notation

$$V(\epsilon) = \frac{V(t)}{V(o)} = \frac{V(t)\cdot\tau}{\chi_{fo}-\chi_{po}} = \frac{\tau V(t)}{\chi_{po}(a-1)} \tag{27}$$

In what follows we shall refrain from using the variable $\epsilon = t/\tau$ and use the Laplace Transform defined in term of denoting the operator by "s" instead of as "p" above.

i.e. $\quad W_s = \int_0^\infty \bar{e}^{st}\ dt$

From equation (1i) and the same quasi–steady assumption that $V = d\chi_p/dt$ we have

$$\bar{V} = s\bar{\chi}_p - \chi_{po} = \frac{\bar{\chi}_f - \bar{\chi}_p}{\tau} \tag{28}$$

and, from equation (4ii)

$$\bar{\chi}_f = K\bar{Q} + Kq\chi_{po}\ \overline{m}''\ Kq\overline{m}''\bar{V} \tag{29}$$

from which

$$\bar{\chi}_f = \frac{K\bar{Q} + Kq\overline{m}\chi_{po} - \chi_{po}}{1/s - Kq\overline{m}'' + \tau} \tag{30}$$

Further development depends on the forms of m" and Q.

In terms of the original variables with the more general form

$$m''(t) = m''_a\ e^{-\beta t} + m''_c\ . \tag{31}$$

and with Q(t) assumed to be a constant Q_o, we have

$$\bar{V} = (As + B)/(s^2 + Cs + D)$$

i.e.

$$V(t) = \frac{1}{Y_1 - Y_2}\left[(B+AY_1)e^{Y_1 t} - (B+AY_2)e^{Y_2 t}\right] \tag{32}$$

where, since $KQ_o = \chi_{po}$

$$A = \frac{K}{\tau}[\chi_{po}q(m_a'' + m_c'')] = \frac{\chi_{fo} - \chi_{po}}{\tau} = V(o) \tag{33i}$$

$$B = \frac{\beta}{\tau}(K\chi_{po}qm_c'') \tag{33ii}$$

$$C = \frac{1}{\tau}[1 - Kq(m_a'' + m_c'') + \tau\beta] \tag{33iii}$$

$$D = \frac{\beta}{\tau}(1 - Kqm_c'') \tag{33iv}$$

$$Y_1 = \frac{1}{2}\left[-C + \sqrt{C^2 - 4D}\right] \tag{34}$$

$$Y_2 = \frac{1}{2}\left[-C - \sqrt{C^2 - 4D}\right] \tag{35}$$

If $C^2 > 4D > 0$ and C is –ive both Y_1 and Y_2 are +ive, if $C^2 > 4D > 0$, and C is +ive, both are negative and spread must eventually stop.

Initially Y_1t and $Y_2t << 1$

and $V(t) \doteqdot A + \left[B + A(Y_1 + Y_2)\right]t \doteqdot A + (B - AC)t...$

Because A is +ive spread can start, but only accelerates if

$$B > AC \tag{36}$$

The time of peak V(t) is obtained as

$$\tau_{vmax} = \frac{1}{\chi_1 - \chi_2} \text{ Ln } \frac{(B+AY_2)Y_2}{(B+AY_1)Y_1} \tag{37}$$

which is real and positive if B > AC.

When $4D > C^2$ the exponentials $e^{Y_1 t}$ and $e^{Y_2 t}$ become trigonometric functions. Propagation will only initially accelerate if $B > AC$ but eventually it ceases. Unlike the case when Y_1 and Y_2 are negative and the propagation dies out asymptotically it ceases at a definite time when $C^2 < 4D$. This time can be shown after some algebraic manipulations of equation (32) to be

$$t_o = \left[\sqrt{4D-C^2}\right]^{-1} \ \mathrm{Tan}^{-1} \ \frac{A\sqrt{4D - C^2}}{AC - 2B} \tag{38}$$

3. <u>Linearisation</u>

The SQW approximation of a linear relation between χ_f and heat release can be regarded as an approximation to a non–linear relationship.

If $$\chi_f = K_n\left[Q(t) + q\chi_{po}m''(t) + q\int_o^t m''(t-t_p)V(t_p)dt_p\right]^n \tag{39}$$

and since we need to emphasize behaviour near to the initial condition then

$$\chi_f = \frac{n\ K_n}{[Q(t)+q\chi_{po}m''(t)]^{1/3}} \left[\frac{Q(t)+q\chi_p m''(t)}{n} + q\int_o^t m''(t-t_p)V(t_p)dt_p)\right] \tag{40}$$

and we need to define a mean value for

$$\frac{n\ K_n}{[Q(t)+q\chi_{po}m''(t)]^{1/3}}$$

and introduce an extra fictitious heat source equal to

$$(\frac{1}{n} - 1)\ \left[Q(t) + q\chi_{po}m''(t)\right]$$

If flames are regarded as a region of uniform temperature then in the lower parts gas will accelerate under buoyancy and for a line plume the inflow velocity of entrainment air will, at a height z, be proportional to $z^{1/2}$, so that below a height z_a the entrainment air is proportional to $z_a^{3/2}$ per unit width of flame. If the pyrolysis rate is uniform the mass flow up to the height of the pyrolysis zone is proportional to z_p. Then a crude identification of flame height z_f as the height to a particular degree of dilution or air/fuel ratio leads to

$$z_f^{3/2} \;\alpha\; z_p$$

i.e. $$z_f \;\alpha\; z_p^{2/3}$$

This is the conventional relationship. However the pyrolysis rate increases with height eg. as z^{α} $(\alpha > 0)$ then

$$z_f^{3/2} \;\alpha\; \int_0^{z_p} z^{\alpha}\, dz \;\alpha\; z_p^{1+\alpha}$$

and $$z_f \;\alpha\; z_p^{2/3(1+\alpha)}$$

and the dependance of z_f on z_p moves from a 2/3 power dependence towards one nearer unity (if $\alpha < 3/2$).

4. The model of Hasemi et.al.

In the HYN model there is no difference between initial and asymptotic conditions and an exponential solution is introduced into the equations to solve them. Thus writing the conventional heat condition equation

$$T - T_o = \frac{1}{\sqrt{\pi}} \int_0^{\infty} q_w''(\chi/\chi_f) \,\frac{d\mu}{\sqrt{t-\mu}} \tag{41}$$

where T becomes T_{ig} $(=T_p)$ when $\chi = \chi_p$, they introduce an experimental function for $q''(\chi/\chi_f)$ and the relationship

$$d\chi_p/dt = \alpha\chi_p \tag{42}$$

which leads to

$$\frac{d\chi_p}{dt} = V_p = \frac{\chi_p}{\pi K\rho c(T_{ig}-T_o)^2} \cdot \left[\int_o^\infty \left[q_w''(\chi/\chi_f) \cdot \frac{d\lambda}{\sqrt{\lambda}}\right]\right]^2 \qquad (43)$$

If now we introduce $q_w'' =$ constant for $\chi_p < \chi < \chi_f$ and $= 0$ for $\chi > \chi_f$ we can compare the two models. Bearing in mind they use

$$\lambda = \mathrm{Ln}\chi_p/\chi_{po} \qquad (44)$$

we obtain

$$\frac{d\chi_p}{dt} = \frac{\chi_p}{\tau} \,\mathrm{Ln}\, \frac{\chi_f}{\chi_p} \qquad (45)$$

where τ has the same definition as given by Saito et. al.

Equation (45) has been derived on the presumption that $\frac{d\chi_p}{dt}$ is exponential whilst the pyrolysis front moves from one value of χ_p to where the flame front then was i.e. χ_f.

$$\mathrm{Ln}\frac{\chi_f}{\chi_p} \doteqdot \frac{\chi_f - \chi_p}{\chi_p} + \text{higher powers of } \frac{\chi_p}{\chi}$$

The first term leads to

$$\frac{d\chi_p}{dt} = \frac{\chi_f - \chi_p}{\tau} \qquad (46)$$

which is the equation used by Saito et.al.

The origin of this equation is a steady relate theory in which the temperature rise is zero at $\chi = \chi_f$ and $T_p - T_o$ at $\chi = \chi_p$. In between it rises accor– ding to the conventional square root law (in time) associated with the heating of α semi infinite solved by a constant flux.

Hence $\frac{T_\chi - T_o}{T_P - T_o} = \sqrt{\frac{\chi_f - \chi}{\chi_f - \chi_p}}$ (47)

No constraints can be applied to the surface temperature at $t = 0$ ecxept that $T_\chi = T_p$ for $\chi \leq \chi_p$ but clearly in discussing the initial behaviour of the flame this problem has to be considered. Clearly if $T_\chi = T_o$ $\chi > \chi_p$ one could expect some step by step process since the pyrolysis zone could not extend until all the zone $\chi_p < \chi < \chi_f$ were heated at once: a process which, in a different context has been discussed by Emmons (4).

5. The relationship between χ_f and χ_p

Saito et. al use the linear forms

$$\chi_f = K\left[Q(t) + qm''(t)\chi_{po} + q\int_0^t m''(t-t')V_p(t')dt'\right] \tag{48}$$

$$\chi_p = \chi_{po} + \int_0^t V_p dt \tag{49}$$

Hence

$$\chi_f - \chi_p = KQ + (a - 1)\chi_p - a\int_0^f \frac{m''(t-t')}{m''_o} V(t')dt' \tag{50}$$

The HYN model omits the first and last terms, the pilot and the effect of the lessening value of m"(x), so permitting the approximation χ_f/χ_p = constant. As illustrated above there is an implied requirement that $(a- 1) << 1$ so as to make the two models similar $\frac{\chi_f-\chi_p}{\chi_p} \doteqdot \mathrm{Ln}\,\frac{\chi_f}{\chi_p}$.

Both models can be extended to include a transient variation in T_s, if it is a uniform one resulting from a uniform imposed radiation flux. This cannot describe a real initial condition where there is little likelyhood of a discontinuity between T_{ig} or T_p and T_s just above the zone of pyrolysis. In this context it is im—

portant to note that for $T_s \rightarrow T_{ig}$, $\tau \rightarrow 0$ and the importance of λ is reduced – i.e. the role of charring is lessened (see equations (29) → (35)).

In this sense ($\tau \rightarrow 0$) the exponential asymptotic begins earlier in real time and one is then only concerned with the asymptotic solution.

6. Conditions inhibiting upward spread

For a general discussion of the failure of the flame to spread we omit m''_c and consider $Q(t) = Q_o$ so that $B = 0$. The condition for upward spread ($B > AC$) is now that C is –ive; were C +ive there would be no acceleration and propagation would slow down and eventually stop with the reservations made regarding the formulation of the theory – especially as it effects the initial stages and the extinction (with $\chi_f = \chi_p$ indefinitly).

We write integrating equation (32)

$$\chi_p = \chi_{po} + \frac{1}{(Y_1-Y_2)}\left[\frac{B+AY_1}{Y_1}\,(e^{Y_1 t}-1) - \frac{B+AY_2}{Y_2}\,.\,(e^{Y_2 t}-1)\right]$$

which with $B = 0$

$$= \chi_{po} + \frac{A}{Y_1-Y_2}\left[e^{Y_1 t} - e^{Y_2 t}\right] \qquad (51)$$

For the condition C +ive and $C^2 > 4D$, Y_1 and Y_2 are negative and the pyrolysis does not grow. From equation (1i)

$$\chi_f = \chi_p + \tau\frac{d\chi_p}{dt}$$

$$= \chi_{po} + A/(Y_1 - Y_2)\left[(1+Y_1\tau)\; e^{Y_1 t} - (1+Y_2\tau)e^{Y_2 t}\right] \qquad (52)$$

We note that at $t = 0$

$$\left[\frac{d\chi_f}{dt}\right]_o = V(o) + \tau\left[\frac{dV}{dt}\right]_o = V(o)\;\;[1-C\tau]$$

Since $B = 0$, the pyrolysis zone does not spread for $C\tau > 1$ and the flame only grows, later to subside from its initial χ_{fo} ($> \chi_{po}$) to the lower distance χ_{po} if $0 < C < 1/\tau$.

If however Y_1 and Y_2 are complex we have $V = 0$ at t_o give by equation (38), which for $B = 0$ gives

$$t_o\sqrt{4D-C^2} = \mathrm{Tan}^{-1}\left[\frac{\sqrt{4D-C^2}}{C}\right] \tag{53}$$

The important condition $C^2 = 4D$ that seperates an extinction mode from a propagating mode (when $C < 0$) or extinction at t_o or asymptotically ($C>0$) is obtained readily

$C\tau$ can be written as $1 - a + \beta\tau$ and $D\tau^2$ as $\beta\tau$ (with $B = 0$)

so extinction results if

$$4\beta\tau > (1 - a + \beta\tau)^2$$

i.e. $\beta\tau$ lies between $(1 - \sqrt{a})^2$ and $(1 + \sqrt{a})^2$. For $\beta\tau = (1 - \sqrt{a})^2$, (see also equation 25)

$$C\tau = 2\sqrt{a}(1-\sqrt{a}) < 0 \text{ whereas for } \beta\tau = (1+\sqrt{a})^2,\ C\tau = 2\sqrt{a}(1+\sqrt{a}) > 0$$

The various zones are shown in Fig (1).

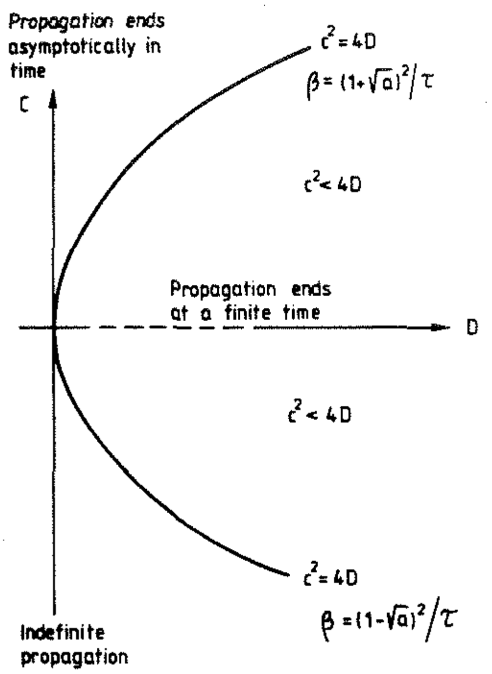

Fig. 1 Zones of propagation & non–propagation.

From a practical point of view the conditions at the start of upward spread are of more practical importance than the asymptotic behaviour of faster and faster spread because the time of spread is more determined by the slow initial spread. It is therefore essential to develop the theory with a view to examining the early behaviour in particular.

If we cease distinguishing between the two kinds of non–propagation and consider only the condition for indefinite propagation we can compare the conditions de–rived by Saito et.al. for two forms of $m''(t)$ and the third form used by Magnusson & Karlsson. We shall compare them on the basis of equal total heat output (or M mass loss) and introduce a time $t_{1/2}$ at which half this total has been delivered. We take for simplicity $m''_c = 0$. For the SQW Curve (1) $m'' = m''_o$ constant for $0<t<t_B$.

Hence $$M = m''_o \; t_B \qquad (54i)$$
$$t_{1/2} = t_B/2 \qquad (54ii)$$

The corresponding mass flux $m^{*}{''}$ defined

by $$m^{*}{''} t_{1/2} = 1/2 \int_0^{t_B} m'' \, dt \qquad (54iii)$$

is $m^{*''} = m''_o$

For the SQW Curve (2)

$$m'' = m''_o \sqrt{\frac{t_c}{t}} \tag{55i}$$

and $$M = 2m''_o \sqrt{t_c t_B} \tag{55ii}$$

$$\text{i.e. } t_c = t_B/4 \tag{55iii}$$

Note this differs from Saito et.al.'s own choice of t_c. The characteristic $m^{*''}$ is $2m''_o$ and $t_{1/2} = t_{B/4}$.

For the exponential form we assume there is no cut off in m'' so that

$$M = \frac{m''_o}{\beta} \tag{56i}$$

$$\text{Hence } \beta = 1/t_B \tag{56ii}$$

and $$\frac{m''_o}{\beta}(1 - e^{-\beta t_{1/2}}) = \frac{m''_o}{2}$$

$$\text{i.e. } t_{1/2} = \frac{0.69}{\beta} = 0.69 t_B = \frac{t_B}{1.45} \tag{56iii}$$

For curve (1) Saito et.al. give the minimum value of $a = Kqm''_o$ for propagation as

$$\frac{a}{X} = 1 + 1/X + L_n\,(a/X) \tag{57}$$

where $X = t_{ig}/t_B$ ($t_{ig} = \tau$).

In terms of $a' = Kqm''_o$ and $X' = t_{ig}/t_{1/2}$ we have $a' = a$ and $X' = 2X$.

For curve (2) Saito et.al. give

$$a \; > \; \frac{4}{\sqrt{\pi}} \; \sqrt{X} \tag{58}$$

where $a' = 2a$ and $X' = 4X$ i.e. $a' = \dfrac{8}{\sqrt{\pi}} \sqrt{X'}$

For curve (3) $\beta = (1 - \sqrt{a})^2/\tau$ and $\sqrt{\beta\tau} = \sqrt{\alpha} - 1$

$$\text{i.e. } a \; > \; (\sqrt{\beta\tau} + 1)^2 = (1 + \sqrt{X})^2 \tag{59}$$

$$a' = a \cdot 0.72$$
$$X' = X \cdot 1.45$$

The three relationships of a'(X') appear in Fig (2).

The differences are small except near the origin because there the inverse square root gives an instantaneously infinite m'' at t= 0.

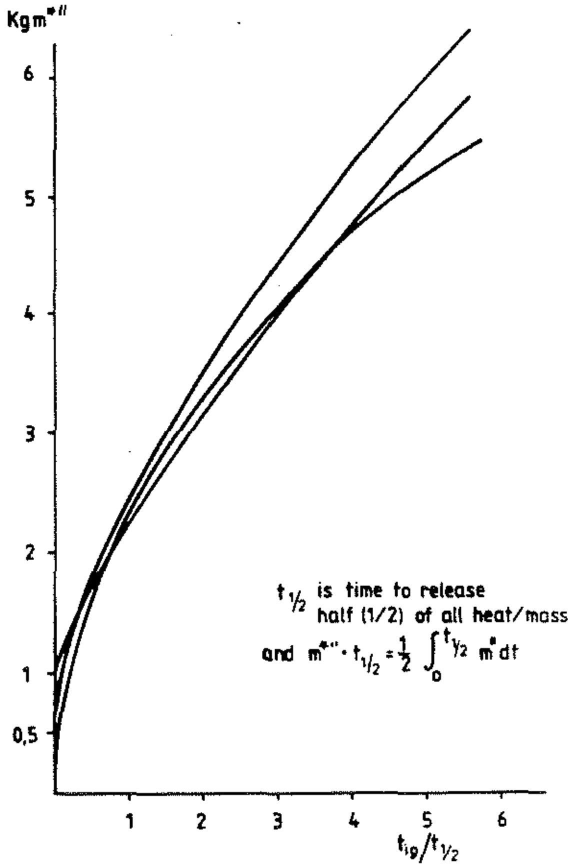

Fig. 2

All the quasi steady models give a single functional result

$$Kqm^{*}" = \text{funct}(t_{ig}/t_{1/2}) \tag{60}$$

K is a constant.

All the quantities are measurable qm" being Q" the rate of heat release. All quantities m", t_B, t_{ig} are to be obtained under the same heat flux as from the flame. One could choose a different characteristic time but it has to be related to t_B by the variables included above of which one is implicit viz the $\int Q"\,dt$ the total heat release "H" which defines $Q^{*} = H/2t_{1/2}$.

One could therefore use, as a means of correlating data

$$\frac{K.H.}{t_{ig}} = \text{function}(t_{ig}/t_{1/2}) \tag{61}$$

The choice of definition of $t_{1/2}$ has been somewhat arbitrary, but clearly the calculated results indicate that a relationship of the above form can be obtained which (provided the initial values of m" are not infinite) will be expected to be very similar for a range of m"(t) characteristics.

7. Limitations to quasi steady theory.

Were the initial conditon on the surface $\chi_{po} < \chi < \chi_{fo}$ to be a zero rise above ambient instead of that corresponding to the quasi–steady model we could not expect any propagation for a constant m" unless $t_{ig} < t_B$. The shaded area on Fig (2) shows the extent of the "error", for this case. This is a problem yet to be examined.

8. Conclusion

Fairly obviously, because of approximations in and omissions from the mathematical description given here, one cannot expect too close an agreement between theory and physical reality. However it does seem that the theory developed by Saito, Quintiere and William can be profitably extended. It would seem that the condition $\alpha > 0$ discussed by Saito et.al. and elaborated there can be adapted to provide a criterion of the form $KQ_0'' <$ function $(t_{ig}/t_{1/2})$ for inhibition to indefinite propagation. Q_0'' and t_{ig} are obtainable from small scale heat release data as can t_{ig}. Differences between the SQW and the HYN models have been discussed and whilst the SQW model appears to deal with initial conditions an examination shows that implicit in them is the unstated constraint $\frac{\chi_f - \chi_p}{\chi_p} << 1$. Further theoretical studies allowing for initial temperatures between T_s and T_p outside the initially pyrolysing zone, need to be made.

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