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Andersson, Tommy

2004

Link to publication

Citation for published version (APA):
Andersson, T. (2004). *Nonlinear Pricing under a Balanced-Budget Requirement: The n-Type Case*. (Working Papers. Department of Economics, Lund University; No. 25). Department of Economics, Lund University. http://swopec.hhs.se/lunewp/abs/lunewp2004_025.htm

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PO Box 117
221 00 Lund
+46 46-222 00 00

Nonlinear Pricing under a Balanced-Budget Requirement: The Two-Type Case*

Tommy Andersson†
Lund University

20 September, 2004

Abstract

An economy consisting of two different types of consumers and one publicly owned natural monopoly is under consideration. The preferences of the consumers are assumed to be linear in money and the demand curves are assumed not to cross. We also suppose that the net utility from consumption is so high that we do not have to consider the individual rationality constraints. Given these assumptions, we completely characterize the set of budget-balanced and Pareto efficient nonlinear pricing schedules. This complete characterization has not been presented in the literature before.

JEL Classification: D63, D82, H40.
Key Words: Nonlinear Pricing, Budget-Balance.

*I would like to thank Fredrik Andersson, Anders Borglin, Kristian Jönsson, Bo Larsson, Lars-Gunnar Svensson and Lars Söderström for helpful comments.

†Tommy Andersson, Department of Economics, Lund University, Box 7082, SE-222 07, Lund, Sweden, Phone +46 (0)46 222 49 70, e-mail: tommy.andersson@nek.lu.se

1 Introduction

Selecting an appropriate pricing rule for a natural monopoly is a central problem in public economics. It is often argued that pricing rules for firms operating under increasing returns to scale should be nonlinear. One motivation for this is the well known argument by Willig (1978), which states that nonlinear outlay schedules typically Pareto dominate uniform pricing rules (see also Seade, 1977). A second argument in favor of nonlinear schedules is that nonlinear tariffs can, in stark contrast to the marginal cost pricing rule, be designed to cover costs when there are increasing returns to scale, see e.g. Coase (1946) or Vohra (1990). These two motivations and the observation that nonlinear pricing has an important role in many natural monopoly industries, such as the electricity and water industries, justify theoretical studies of nonlinear outlay schedules. In the literature, theoretical models of nonlinear pricing have received considerable attention, mostly in economies with a continuum of consumers, see e.g. Roberts (1979) or Spence (1977). In this paper, we investigate a finite economy, as in e.g. Brito et al. (1990) and Guesnerie and Seade (1982). We shall also suppose that the natural monopoly operates under a balanced-budget (zero-profit) requirement. This type of finite economy has been considered in the literature by e.g. Laffont (1997) and Sharkey and Sibley (1993). Our main objective is to characterize the set that consists of all Pareto optimal and budget-balanced nonlinear outlay schedules in an economy with two differing consumer types.

A nonlinear outlay schedule or, equivalently, a menu is a list of consumption-outlay pairs. The menu is offered to all consumers in the economy and each consumer chooses a preferred consumption and pays the associated charge. However, if the social planner (or the monopolist) cannot tell consumers apart, he must take into account the possibility of personal arbitrage, i.e. the possibility that a consumer, to whom a specific consumption-outlay pair is directed, picks a consumption-outlay pair that is intended for some other consumer. In order to prevent this possibility, the nonlinear schedule must be restricted by a set of no-envy[1] constraints. Other restrictions such as non-negative profit constraints and individual rationality constraints may restrict the nonlinear outlay schedule further.

The normal procedure to derive Pareto optimal nonlinear outlay schedules is to maximize a social welfare function subject to a set of constraints (e.g. the no-envy constraints). This procedure is adopted by e.g. Maskin and Riley (1984), Sharkey and Sibley (1993), Spence (1977) and Roberts (1979) among others. The approach that we adopt in this

[1]Note that no-envy can be thought of as incentive compatibility or self-selection, see Varian (1989,p.615).

paper is, however, somewhat different from the above procedure. Our starting point is the fairness criterion. An allocation is said to be fair if it is envy-free and Pareto efficient.[2] Varian (1976) argues that since the fairness criterion is an internal measure, which depends specifically on the individual tastes of the agents involved, it is possible to identify fair allocations without the aid of a welfare function and, therefore, the problem of specifying the form of the social welfare function is eliminated. The main disadvantage of the fairness approach is that it is, in contrast to the welfare function approach, of no use in comparing two arbitrary allocations. However, the purpose of this paper is to characterize the set that consists of all fair and budget-balanced nonlinear outlay schedules, and we are, therefore, not primarily interested in how the social planner orders different nonlinear schedules. Hence, the fairness approach fits our purposes better and, as a consequence, we shall adopt it.

In this paper, we investigate fair and budget-balanced nonlinear outlay schedules in an economy with two differing consumer types a publicly owned natural monopoly that is operated by a social planner. We shall assume that the planner knows the characteristics of the consumers but that he is unable tell them apart. The preferences of the consumers are assumed to be linear in money and the demand curves are assumed not to cross. Moreover, we shall suppose that the net utility from consumption is so high that we do not have to consider the individual rationality constraints. This assumption can be justified, for example, in the case when the monopoly supplies water, because it is not unreasonable to believe that most people, at least in the Western World, can afford to consume water, and, moreover, receives a strictly positive net utility from their consumption. This type of environment has been investigated by e.g. Laffont (1997). We contribute to the nonlinear pricing literature in the following senses. Firstly, this paper provides a complete characterization of the set of fair and budget-balanced nonlinear pricing schedules in an economy that is based on the above premises. Our results generalize most of the findings in Sharkey and Sibley (1993) and the second-degree price discrimination results in Laffont (1997). Secondly, instead of adopting the standard procedure to identify fair and budget-balanced schedules (i.e. the welfare function approach), we develop a procedure based on the fairness criterion. Our procedure has, to the best of my knowledge, not been considered in the nonlinear pricing literature before. Thirdly, our assumption that we do not have to consider the individual rationality constraints enable us to provide some

[2]The concept of envy-freeness is attributed to Foley (1967). The existence of fair allocations has been proved in a number of cases under different assumptions about preferences and commodities, see Panzer and Schmeider (1978), Schmeider and Vind (1972), Svensson (1983) and Varian (1974,1976) among others.

results that not has been presented before in the literature. This is, in particular, results that are concerned with distortions in the consumption. Lastly, we reproduce some well known results contained in the literature.

This paper is organized as follows. Section 2 describes the nonlinear pricing environment. In Section 3, we identify and characterize the set of fair and budget-balanced nonlinear price schedules in the two-type economy. Section 4 concludes the paper.

2 The Model and Basic Definitions

The economy consists of two differing consumer types and one publicly owned (natural) monopoly. The set of consumers $\{1, 2\}$ is denoted N. Consumer $i \in N$ has preferences over *consumption-outlay bundles* $x_i = (q_i, t_i) \in \mathbb{R}^2_+$, where q_i denote type i's consumption of a perfectly divisible good and t_i is a monetary transfer from consumer $i \in N$ to the publicly owned firm. We assume that consumer i's preferences can be represented by a quasi-linear *utility function*: $u_i = \phi_i(q_i) - t_i$, where $\phi_i(q_i)$ is supposed to be continuous and (at least) twice differentiable with: $\phi_i(0) = 0$, $\phi_i'(q_i) > 0$ and $\phi_i''(q_i) < 0$. We shall also make the sorting assumption that marginal willingness-to-pay for a given quantity is increasing in type, i.e.:

$$\phi_2'(z) > \phi_1'(z) \text{ for all } z \in \mathbb{R}_{++}. \tag{1}$$

Assumption (1) together with the assumption that $\phi_i(0) = 0$ implies that total willingness-to-pay is increasing in type, i.e.: $\phi_2(z) > \phi_1(z)$ for all $z \in \mathbb{R}_{++}$. The cost of producing output $q = \sum_{i \in N} q_i$ is given by $C(q) = \beta q + F$, where β is the constant *marginal cost* of production and F is a *fixed cost*. A *social planner* operates the publicly owned monopoly, which is assumed to be restricted by a *balanced-budget* requirement, i.e.: $\sum_{i \in N} t_i = C(q)$.

Consumption is said to be *first-best* for agent $i \in N$ if: $\phi_i'(q_i^*) = \beta$. A *menu* or, equivalently, a *nonlinear outlay schedule* $x \in \mathbb{R}^4_+$ is a list of two bundles, i.e. $x = \{x_i\}_{i=1}^2 = \{(q_i, t_i)\}_{i=1}^2$. Agent $i \in N$ is said to *envy* agent $j \neq i$ if he prefers agent j's bundle $x_j \in x$ to his own bundle $x_i \in x$. x is said to be *envy-free* if no agent envies any other agent. x is said to be *Pareto efficient* if there is no other $\widetilde{x}$ such that $u_i(\widetilde{x}_i) \geq u_i(x_i)$ for all $i \in N$ and $u_i(\widetilde{x}_i) > u_i(x_i)$ for some $i \in N$. x is said to be *fair* if it is envy-free and Pareto efficient. We shall, finally, assume, as in e.g. Laffont (1997), that:

$$\phi_1(q_1) - \frac{C(q)}{n} \geq 0, \tag{2}$$

for all fair and budget-balanced nonlinear price schedules. This will be the case if the valuations of the good under consideration are "sufficiently high". As we later demonstrate, the implication of this assumption is that we do not have to consider the *individual rationality constraints*, i.e. every consumer $i \in N$ obtains a non-negative net utility by consuming the good.

The main objective of this paper is to characterize the set of fair and budget-balanced nonlinear pricing schedules. We denote this set X^*. In order to characterize X^*, the following procedure is adopted. Firstly, we identify the set that consists of all envy-free and budget-balanced menus. This set is denoted X. We then exclude all menus in X that are not Pareto efficient, i.e.:

$$X^* = \{x \in X \mid x \text{ is Pareto efficient}\}. \tag{3}$$

Note that when we impose the Pareto efficiency criterion, we only consider the set of nonlinear schedules that is constrained by the envy-freeness criterion and the balanced-budget requirement (i.e. the menus in X). Therefore, our measure of Pareto efficiency is, strictly speaking, not Pareto efficiency in the ordinary (first-best) sense but rather Pareto efficiency in the constrained (or second-best) sense, see e.g. Mas-Colell et al. (1995,pp.445).

3 The Characterization Results

In this section we characterize the set of fair and budget-balanced nonlinear schedules.

3.1 Envy-free Nonlinear Pricing

In the two-type case, envy-freeness requires that the following two constraints are satisfied:

$$\phi_1(q_1) - t_1 \geq \phi_1(q_2) - t_2, \tag{4}$$

$$\phi_2(q_2) - t_2 \geq \phi_2(q_1) - t_1. \tag{5}$$

We shall refer to condition (4) as the *upward no-envy constraint* and to condition (5) as the *downward no-envy constraint*. The upward (downward) no-envy constraint guarantees that type 1 (type 2) does not pick the bundle that is directed to type 2 (type 1), i.e. the type that is located just above (below) him.

In Lemma 1, we gather a few results that are well known in the nonlinear pricing literature (see e.g. Cooper, 1984, or Laffont and Tirole, 1993, pp.299).

Lemma 1 *Suppose that the menu of bundles* $\{(q_i, t_i)\}_{i=1}^2$ *is envy-free. Then:*

(i) $q_2 \geq q_1$.

(ii) *Conditions* (4) *and* (5) *are simultaneously binding if* $q_2 = q_1$.

(iii) *Conditions* (4) *and* (5) *cannot be simultaneously binding if* $q_2 > q_1$.

We next describe the intuition behind Lemma 1. Type 2 values the consumption of the good higher than type 1, by assumption, and, therefore, he is willing to pay more than type 1 for any given quantity. As a consequence, type 2 will always pick bundle $x_1 = (q_1, t_1)$ if $q_1 > q_2$. Hence, $q_2 \geq q_1$ in every envy-free menu. We also know that every envy-free menu must satisfy conditions (4) and (5). But these conditions give us the following relation between t_2 and t_1:

$$\phi_1(q_2) - \phi_1(q_1) \leq t_2 - t_1 \leq \phi_2(q_2) - \phi_2(q_1). \tag{6}$$

Note first that if condition (4) (condition (5)) is binding, the left (right) inequality in condition (6) holds with equality. In the case when $q_2 = q_1$, condition (6) reduces to $0 \leq t_2 - t_1 \leq 0$, implying that $t_2 = t_1$. So if $q_2 = q_1$ then $t_2 = t_1$ and, therefore, conditions (4) and (5) are simultaneously binding. In the case when $q_2 > q_1$ it is immediate from assumption (1) and condition (6) that $t_2 > t_1$ and, hence, at least one inequality in condition (6) must be strict. Using this fact we can rewrite condition (6) to:

$$t_2 - t_1 = (1 - k)\left(\phi_1(q_2) - \phi_1(q_1)\right) + k\left(\phi_2(q_2) - \phi_2(q_1)\right) \subset \mathbb{R}_+, \tag{7}$$

where $k \in [0, 1]$. In the bounding cases when $k = 0$ and $k = 1$ conditions (4) and (5) are binding, respectively. If $k \in]0, 1[$, conditions (4) and (5) hold with strict inequality. Note next that even if $q_1 = q_2$, condition (7) is valid since $t_1 = t_2$ for any $k \in [0, 1]$ in this case. We conclude that the set that consists of all envy-free menus is characterized by condition (7) for $k \in [0, 1]$ and $q_2 \geq q_1$.

3.2 Envy-free and Budget-Balanced Nonlinear Pricing

Our next aim is to characterize the set of envy-free and budget-balanced menus. The balanced-budget requirement is given by:

$$t_1 + t_2 = C(q). \tag{8}$$

Using conditions (7) and (8) to solve for t_1 and t_2 yields:

$$t_1 = \frac{1}{2}\left(C(q) - (1-k)\left(\phi_1(q_2) - \phi_1(q_1)\right) - k\left(\phi_2(q_2) - \phi_2(q_1)\right)\right), \tag{9}$$

$$t_2 = \frac{1}{2}\left(C(q) + (1-k)\left(\phi_1(q_2) - \phi_1(q_1)\right) + k\left(\phi_2(q_2) - \phi_2(q_1)\right)\right), \tag{10}$$

where $k \in [0,1]$. From the above two equations, two important conclusions can be drawn. Firstly, it is clear that the outlays for consumer 1 and 2 can be expressed as functions of q_1 and q_2. Hence, $t_i = t_i(q_1, q_2)$ for all $i \in N$. Secondly, if the nonlinear schedule is envy-free and budget-balanced, net utility will always be largest for type 2. To see this, note that if the outlays are given by conditions (9) and (10), then:

$$u_2 - u_1 = (1-k)(\phi_2(q_2) - \phi_1(q_2)) + k(\phi_2(q_1) - \phi_1(q_1)). \tag{11}$$

But from assumption (1) it follows that $u_2 - u_1 > 0$ for all $k \in [0,1]$ and $q_2 \geq q_1$. We conclude that no menu, where $q_2 \geq q_1$ and the outlays are given by conditions (9) and (10), violates the envy-freeness criterion or the balanced-budget rule.

3.3 Fair and Budget-Balanced Nonlinear Pricing

We next study menus that, on top of the envy-freeness criterion and the balanced-budget rule, satisfy the Pareto efficiency criterion. We demonstrate that this additional requirement leads to the facts that (i) more restrictions must be imposed on q_1 and q_2 and that (ii) all fair and budget-balanced menus are individual rational when assumptions (1) and (2) are satisfied. We start by demonstrating the latter result, noticing that assumption (1), equation (9) and the restriction $q_2 \geq q_1$ imply that: $t_1 \leq \frac{C(q)}{2}$. But $\phi_1(q_1) - \frac{C(q)}{2} \geq 0$ for all fair and budget-balanced menus, by assumption (2), implying that $u_1 \geq 0$. Then since $u_2 > u_1$, it follows directly that $u_2 > 0$. Hence, all fair and budget-balanced menus are individual rational when assumptions (1) and (2) are satisfied.

In the rest of this section, we shall characterize how q_1 and q_2 must be selected for every $k \in [0,1]$ in order to guarantee a fair and budget-balanced outcome. This will result in four lemmas and one proposition. In Example 2, below, these results are illustrated with the aid of a simple numerical example. The first observation in our characterization process is that if $q_1 = q_2$, then it is possible to make a Pareto improvement and at the same time respect the envy-freeness criterion and the balanced-budget rule, i.e. a pooling contract can never be optimal. To see this, suppose that $q_1 = q_2$ and recall that $t_1 = t_2$ in this case, i.e. both consumers are offered the same bundle. Moreover, by the balanced-budget rule, it follows that $t_1 = t_2 = \beta q_1 + \frac{F}{2}$. But this schedule is Pareto dominated by the schedule

where $q_i = q_i^*$ and $t_i^* = \beta q_i^* + \frac{F}{2}$ for all $i \in N$ since $q_i^* = \arg\max_{q_i}\{\phi_i(q_i) - \beta q_i - \frac{F}{2}\}$ and $q_1 \neq q_i^*$ for $i = 1$ and/or $i = 2$ by condition (1). This conclusion is formally stated in Lemma 2 and can, for example, be found in Laffont and Tirole (1993,p.299).

Lemma 2 *Let the outlays be given by equations* (9) *and* (10) *for* $k \in [0,1]$ *and note that* $t_i = t_i(q_1, q_2)$ *for all* $i \in N$. *Then* $x \notin X^*$ *if* $q_1 \geq q_2$.

We next observe that every nonlinear price schedule where total utility (i.e. $u_1 + u_2$) is maximized is Pareto efficient. This conclusion follows trivially by definition of Pareto efficiency, since there exists no other nonlinear schedule where one type is strictly better off and the other type (at least) is not worse-off. We next note that total utility is maximized, by definition, when consumption is first-best for both types. But if consumption is first-best for both types and if the outlays are given by equations (9) and (10), it follows that total utility is maximized for all $k \in [0,1]$. The latter conclusion follows directly from the solution to the maximization problem:

$$\max_{q_1,q_2}\{\phi_1(q_1) - t_1 + \phi_2(q_2) - t_2\}, \tag{12}$$

since $t_1 + t_2 = C(q) = \beta(q_1 + q_2) + F$ for all $k \in [0,1]$ by the balanced-budget rule. Hence, $q_1 = q_1^*$ and $q_2 = q_2^*$ solve the above maximization problem for all $k \in [0,1]$.

Lemma 3 *Let the outlays be given by equations* (9) *and* (10) *for* $k \in [0,1]$ *and note that* $t_i = t_i(q_1, q_2)$ *for all* $i \in N$. *Then* $x \in X^*$ *if* $q_1 = q_1^*$ *and* $q_2 = q_2^*$.

Our next observation is that the envy-free and budget-balanced nonlinear price schedule that maximizes utility for type $i \in N$ is Pareto efficient. The reason for this is that when utility is maximized for type i, he will be worse-off by any other nonlinear price schedule and, as a consequence, no Pareto improvements are possible. A natural question is then: what are the characteristics of the type i utility maximizing schedule? Two conclusions can be drawn immediately. Firstly, the envelope theorem reveals that net utility for type 1 (type 2) is increasing (decreasing) in k. Therefore, it must be true that $k = 1$ ($k = 0$) in the type 1 (type 2) utility maximizing schedule. Secondly, if $k = 1$ ($k = 0$), so the downward (upward) incentive constraint is binding, type 2 (type 1) consumes the first-best quantity. The reason for this is that the upward (downward) incentive constraint is made inactive, by Part (iii) of Lemma 1, since $q_2 > q_1$, by Lemma 2, and, as a consequence, there is no reason to distort "on the top" ("on the bottom"). Hence, $q_2 = q_2^*$ ($q_1 = q_1^*$). We will, however, demonstrate that $q_i \neq q_i^*$ in the type i utility maximizing programme. This conclusion follows directly from our next definition.

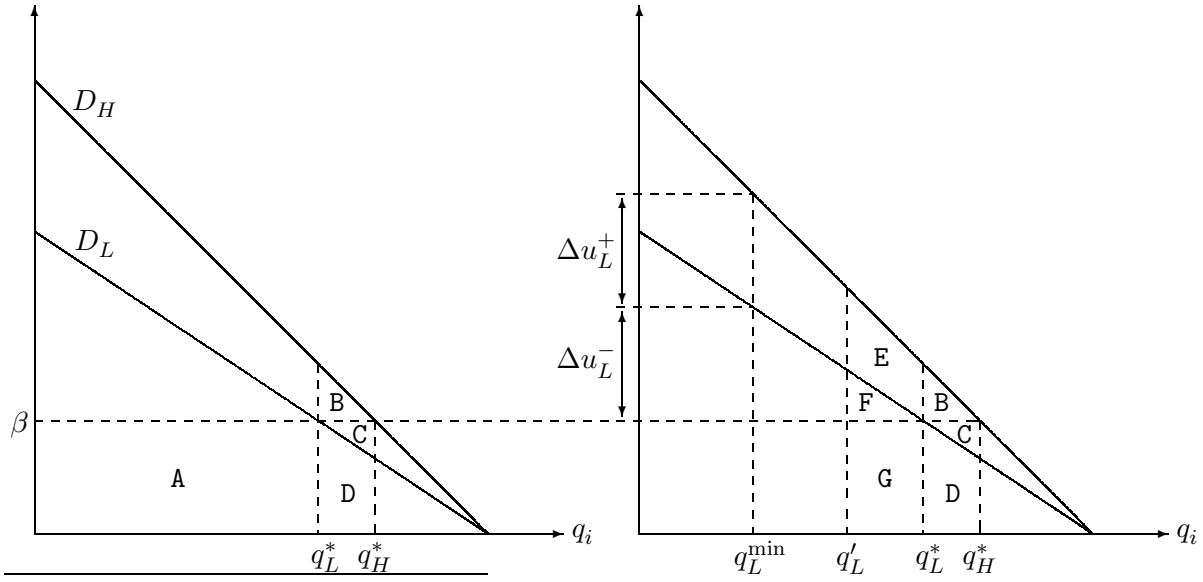
Definition 1 *Let the outlays be given by equations* (9) *and* (10) *for* $k \in [0,1]$ *and note that* $t_i = t_i(q_1, q_2)$ *for all* $i \in N$. *Define* $q_1^{\min}$ *as the solution to* $\max_{q_1} \{\phi_1(q_1) - t_1\}$ *for* $k = 1$ *and* $q_2^{\max}$ *as the solution to* $\max_{q_2} \{\phi_2(q_2) - t_2\}$ *for* $k = 0$, *i.e.:*

$$\phi_1'(q_1) - \beta - (\phi_2'(q_1) - \phi_1'(q_1)) = 0 \text{ if } q_1 = q_1^{\min}, \tag{13}$$

$$\phi_2'(q_2) - \beta + (\phi_2'(q_2) - \phi_1'(q_2)) = 0 \text{ if } q_2 = q_2^{\max}. \tag{14}$$

Note first that it is clear from the definition of first-best and the sorting condition (1) that: $q_1^{\min} < q_1^*$ and $q_2^{\max} > q_2^*$. We next illustrate Definition 1 for the type 1 utility maximizing schedule. Suppose first that both individuals are offered consumption of the first-best quantity. This situation is illustrated in the left panel of Figure 1, where the demand curves of the two consumers are given by D_1 and D_2. Note first that since the budget is balanced and individual 2 is indifferent between the bundle that is designed for him and the bundle that is intended for type 1, by the above arguments (i.e. since $k = 1$), the outlays must be given by: $t_1 = \mathtt{A} - \frac{\mathtt{B}}{2}$ and $t_2 = \mathtt{A} + \mathtt{C} + \mathtt{D} + \frac{\mathtt{B}}{2}$.[3] However, this schedule does not maximize type 1 utility. To see this, consider the right panel of Figure 1 where q_1^* is reduced to the new level q_1'. This reduction will affect the net utility for type 1 in the following manner. Firstly, utility decreases by area $\mathtt{F} + \mathtt{G}$ due to the reduction in

Figure 1. The Type 1 Utility Maximizing Policy



[3]In this example, the fixed cost is set to zero. If the fixed cost is positive, half of the fixed cost should be added to both outlays.

consumption. Secondly, it is possible to reduce the outlay for individual 1 by area $\frac{1}{2}(\mathtt{E}+\mathtt{F})+\mathtt{G}$ without risking that individual 2 picks the bundle that is directed to individual 1. Hence, type 1 is better off if $\frac{1}{2}(\mathtt{E}+\mathtt{F})+\mathtt{G} > \mathtt{F}+\mathtt{G}$ or, equivalently, if $\mathtt{E} > \mathtt{F}$. From the figure it is clear that $\mathtt{E} > \mathtt{F}$. Hence, individual 1 is best off when he consumes a quantity which is strictly lower than the first-best. Continuing in this way, the planner will reduce consumption for type 1 until the marginal gain from a reduced consumption, Δu_1^+, is equal to the marginal loss from a reduced consumption, Δu_1^-. In the right panel of Figure 1, we see that $\Delta u_1^+ = \Delta u_1^-$ when $q_1 = q_1^{\min}$.

Throughout this paper, we shall assume, as e.g. Sharkey and Sibley (1993), that there exist unique solutions to equations (13) and (14) and that the necessary first-order conditions for a maximum are also sufficient conditions. In the following example, we specify a few assumptions on $\phi_i(q_i)$ that guarantee the existence of unique maximum solutions to equations (13) and (14). Note also that this is just one example. The class of utility functions where the necessary first-order conditions for a unique maximum also are sufficient conditions is, of course, much larger.

Example 1. Assume that $\phi_i(q_i) = \theta_i\gamma(q_i)$ is a continuous function with $\gamma'(q_i) > 0$, $\gamma''(q_i) < 0$ for all $q_i \in \mathbb{R}_{++}$. Suppose also that $\lim \gamma'(q_i) \to \infty$ as $q_i \to 0$, $\lim \gamma'(q_i) \to 0$ as $q_i \to \infty$ and that θ_i is a constant parameter in $\mathbb{R}_{++}$, where $\theta_2 > \theta_1$ by condition (1). Let $\varphi(q_1) = \theta_1\gamma'(q_1) - \beta - (\theta_2 - \theta_1)\gamma'(q_1)$, so the first order condition (13) is given by $\varphi(q_1) = 0$. Note next that if $\theta_1 \leq \frac{\theta_2}{2}$, then $\varphi(q_1) < 0$ for all q_1. In this case, there is no solution to equation (13). If, however, $\theta_1 > \frac{\theta_2}{2}$ it is immediate from the above properties of $\gamma(q_1)$, together with the definition of first-best consumption, that $\varphi(q_1) > 0$ as $q_1 \to 0$ and $\varphi(q_1) < 0$ when $q_1 = q_1^*$. Then since $\gamma(q_1)$ is a continuous function, it follows from the intermediate-value theorem that there exists a $q_1^{\min} \in]0, q_1^*[$ such that $\varphi(q_1^{\min}) = 0$. This solution is a unique maximum since $\varphi'(q_1) < 0$ for all $q_1 \in \mathbb{R}_{++}$. Hence, there is a unique maximum to equation (13) if and only if $\theta_2 > \theta_1 > \frac{\theta_2}{2}$. This condition is also sufficient for the existence of a unique maximum to equation (14). ■

Lemma 4 *Let the outlays be given by equations* (9) *and* (10) *for* $k \in [0,1]$ *and note that* $t_i = t_i(q_1, q_2)$ *for all* $i \in N$. *Then* $x \in X^*$ *if* x *is the type* $i \in N$ *utility maximizing nonlinear outlay schedule.*

The results in Lemma 4 can e.g. be found in Laffont (1997). From Lemmas 3 and 4 we know that if $k = 1$, $q_2 = q_2^*$ and $q_1 \in \{q_1^{\min}, q_1^*\}$, then the menu is fair and budget-balanced. As our next lemma establishes, this conclusion is not only true for $q_1 \in \{q_1^{\min}, q_1^*\}$, it is in fact true for all $q_1 \in [q_1^{\min}, q_1^*]$.

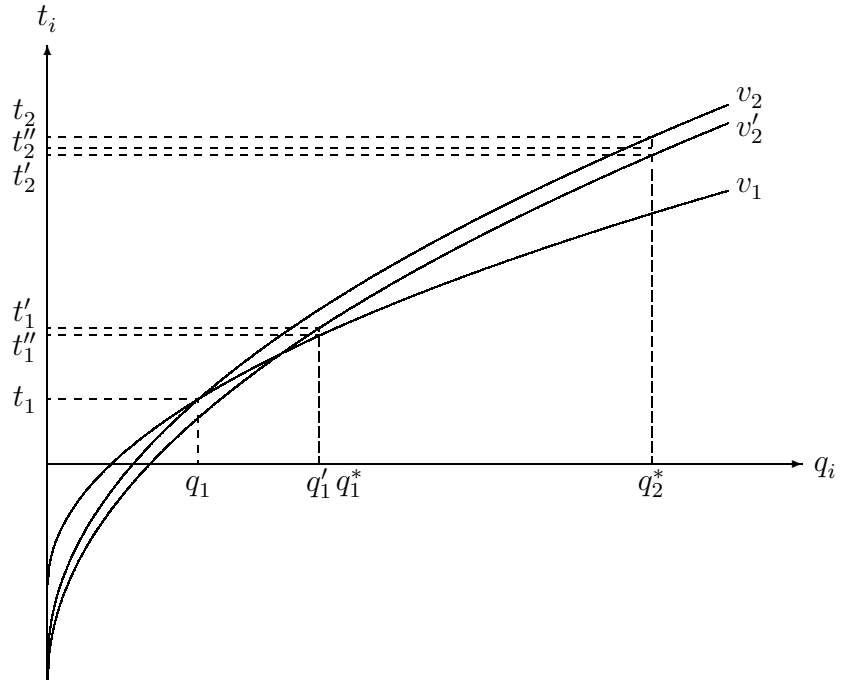
Lemma 5 *Let the outlays be given by equations* (9) *and* (10) *for* $k \in [0,1]$ *and note that* $t_i = t_i(q_1, q_2)$ *for all* $i \in N$.

(i) *In the case when* $k = 1$, *then* $x \in X^*$ *if and only if:* $q_1 \in [q_1^{\min}, q_1^*]$ *and* $q_2 = q_2^*$.

(ii) *In the case when* $k = 0$, *then* $x \in X^*$ *if and only if:* $q_1 = q_1^*$ *and* $q_2 \in [q_2^*, q_2^{\max}]$.

Part (i) of Lemma 5 states that if the downward no-envy constraint is binding, the nonlinear outlay schedule is fair and budget-balanced if and only if $q_1 \in [q_1^{\min}, q_1^*]$ and $q_2 = q_2^*$. The intuition behind this result is as follows. Since the upward no-envy constraint is made inactive, by Part (iii) of Lemma 1, there is no reason to distort "on the top". Hence, $q_2 = q_2^*$. But since q_2 is first-best it is not possible to make a Pareto improvement, and at the same time respect the envy-freeness criterion and the balanced-budget rule by adjusting q_2. The same argument goes for q_1 in the case when $q_1 = q_1^*$. To see that the nonlinear schedule is fair and budget-balanced when $q_1 \in [q_1^{\min}, q_1^*[$ consider Figure 2, where the indifference curves, v_i, of the two consumers are represented in the consumption-outlay space. Net utility is increasing in the south-east direction and the indifference curves are parallel-shifted since utility is linear in money. The downward incentive constraint is binding and, therefore, type 2 is indifferent between bundle (q_1, t_1) and bundle (q_2^*, t_2), i.e. both (q_1, t_1) and (q_2^*, t_2) lie on the same indifference curve, v_2.

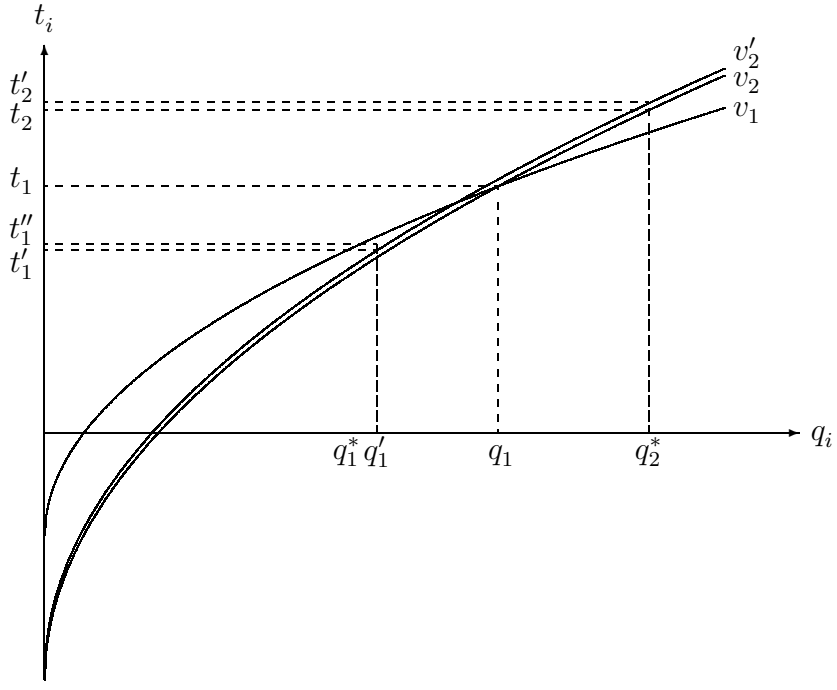
Figure 2. A Fair and Budget-Balanced Policy



Let now $q_1 \in [q_1^{\min}, q_1^*[$ and note that, in this case, Pareto improvements that respect the envy-freeness criterion and the balanced-budget rule may only be possible for increases in q_1. Suppose, therefore, that q_1 is increased to the "new" level $q_1' \leq q_1^*$. Type 2 is indifferent between the "new" bundles (q_2^*, t_2') and (q_1', t_1'), since the downward no-envy constraint is binding (i.e. $k = 1$). Total net utility is increasing since we are moving toward the first-best frontier, and net utility for type 1 is, by Definition 1, negatively affected by the change in q_1. Hence, type 2 must, by the balanced-budget rule, compensate type 1 so that type 1 is (at least) indifferent to the change in q_1. In Figure 2, the compensating bundles are denoted (q_2^*, t_2'') and (q_1', t_1''). But these bundles can not be envy-free because condition (5) was binding before type 1 was compensated, $t_1' > t_1''$ and $t_2'' > t_2'$, which implies that type 2 strictly prefers bundle (q_1', t_1'') over (q_2^*, t_2''). This is also obvious from the figure.

If on the other hand $q_1 \notin [q_1^{\min}, q_1^*]$, the menu can not be fair and budget-balanced. To see this, note first that if the downward no-envy constraint is binding, type 2 is best off when $q_1 = q_2^*$ and type 1 is best off when $q_1 = q_1^{\min}$.[4] Hence, by applying Lemma 2, we need only demonstrate that the menu can not be fair and budget-balanced if $q_1 \in]q_1^*, q_2^*[$.

Figure 3. A Pareto Improving Policy



Suppose, therefore, that $q_1 \in]q_1^*, q_2^*[$ and reduce q_1 to the new level $q_1' \geq q_1^*$. Condition (5) is unaffected by the reduction in q_1 and is still binding, so type 2 is indifferent between bundle

[4]The first conclusion follows directly from the first-order condition: $\left.\frac{\partial u_2}{\partial q_1}\right|_{k=1} = \left.\frac{-\partial t_2(q_1,q_2)}{\partial q_1}\right|_{k=1} = \frac{1}{2}(\beta - \phi_2'(q_1)) = 0$. The second conclusion follows from Definition 1.

(q_2^*, t_2') and bundle (q_1', t_1'). Total utility increases but type 2's utility decreases, so type 2 must receive a compensation from type 1 in order to remain on the original indifference curve, v_2. The compensating bundles are given by (q_2^*, t_2) and (q_1', t_1''). In contrast to the above case, this "new" schedule is Pareto improving, envy-free and budget-balanced since $t_1'' > t_1'$, $t_2 < t_2'$ and condition (5) was binding before type 2 was compensated. The intuition behind Part (ii) of Lemma 5 is the same.

The results from Lemma 1-5 are gathered in Proposition 1, where we completely characterize the set of fair and budget-balanced nonlinear outlay schedules in the two-type case in an economy that is based on the premises in Section 2.

Proposition 1 *Let the outlays be given by equations* (9) *and* (10) *for* $k \in [0,1]$ *and note that* $t_i = t_i(q_1, q_2)$ *for all* $i \in N$.

(i) *In the case when* $k = 1$, *then* $x \in X^*$ *if and only if:* $q_1 \in [q_1^{\min}, q_1^*]$ *and* $q_2 = q_2^*$.

(ii) *In the case when* $k \in]0,1[$, *then* $x \in X^*$ *if and only if:* $q_1 = q_1^*$ *and* $q_2 = q_2^*$.

(iii) *In the case when* $k = 0$, *then* $x \in X^*$ *if and only if:* $q_1 = q_1^*$ *and* $q_2 \in [q_2^*, q_2^{\max}]$.

We shall end this section with a simple numerical example that illustrate the results of this paper.

Example 2. Assume that the marginal and the fixed costs are given by $\beta = 1$ and $F = 7$, respectively, and that:

$$\phi_i(q_i) = \theta_i \sqrt{q_i}, \tag{15}$$

where $\theta_1 = 5$ and $\theta_2 = 7$. Straightforward calculations yields $q_1^{\min} = 2.25$, $q_1^* = 6.25$, $q_2^* = 12.25$ and $q_2^{\max} = 20.25$. We first calculate u_1 and u_2 for every possible combination of $q_i \in \{1, 2, ..., 25\}$ and $k \in \{0, 0.25, 0.5, 0.75, 1\}$, subject to the restrictions that (i) $q_2 \geq q_1$ and (ii) that the outlays are given by equations (9) and (10). The resulting utility pairs are marked with a small dot $(\cdot)$ in Figure 4. The utility pairs in the figure are related to the results in Lemma 2-5 in the following manner:

- The utility pairs that correspond to the menus where $q_1 = q_2 \in [1, 25]$ are displayed along the curve `ABCD`. It is clear from the figure that every utility pair on the curve `ABCD` is Pareto dominated by utility pair `E`, where $q_i = q_i^*$ and $t_i^* = \beta q_i^* + \frac{F}{2}$ for all $i \in N$. Hence, if $q_1 = q_2$ the nonlinear schedule can not be fair and budget-balanced. This conclusion is formally stated in Lemma 2.

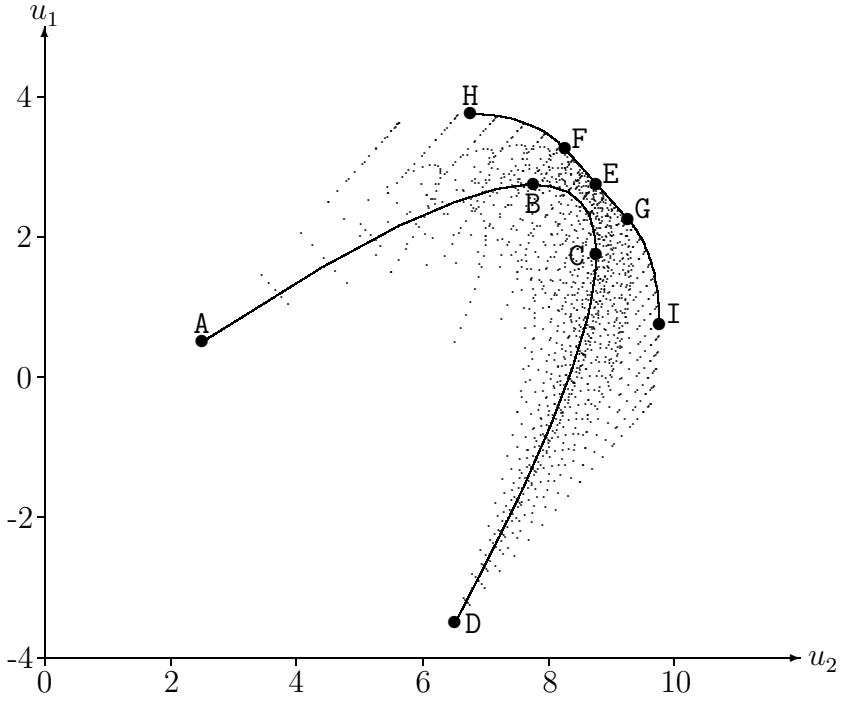
- The utility pairs that correspond to the menus where q_1 and q_2 are first-best are displayed along the line FEG. Lemma 3 reports that these utility pairs are fair and budget-balanced. This is also obvious from the figure.

- The utility pairs that correspond to the type 1 and type 2 utility maximizing schedules are marked with H and I, respectively. From Lemma 4, we know that these schedules are fair and budget-balanced. This is also apparent from the figure.

- The utility pairs that correspond to the menus where $q_1 \in [q_1^{\min}, q_1^*]$ and $q_2 = q_2^*$ are located along the curve HF. Part (i) of Lemma 5 reports that these utility pairs are fair and budget-balanced. This is also clear from the figure. Similarly, the utility pairs that correspond to Part (ii) of Lemma 5 are located along the curve GI.

The above conclusions are gathered in Proposition 1, which states that a nonlinear outlay schedule is fair and budget-balanced if and only if the corresponding utility pair is located along the curve HFEGI. ■

Figure 4. The Utility Pairs from Example 2



4 Conclusions

In this paper, we have characterized the set of fair and budget-balanced nonlinear price schedules in an economy with two differing consumer types. We have not been concerned with welfare functions. Instead, the set of fair and budget-balanced nonlinear pricing schedules has been characterized from the viewpoint of the fairness criterion. To my knowledge, a nonlinear pricing environment has never been subject to an investigation from this perspective before.

Some of the results that we have presented are well known in the literature. These include the characterization of the type 1 and type 2 utility maximizing schedules, see e.g. Laffont (1997). We have also demonstrated that the marginal price facing the largest consumer (i.e. type 2) need not be equal to marginal cost. This result is not "standard" in the literature, but has e.g. been demonstrated by Sharkey and Sibley (1993). Our compete characterization has, however, not been presented in the literature before. Apart from our characterization results, the main contribution to the literature is the procedure that we have developed in order to analyze fair and budget-balanced nonlinear outlay schedules.

Throughout this paper we have assumed that there exist unique solutions to the first order conditions and that the necessary conditions for a maximum are also sufficient conditions. A neglected but important topic for future research is to explore these assumptions in a finite economy. For the continuous case with one-dimensional types, see e.g. Guesnerie and Laffont (1984). Moreover, the characterization results in this paper depend on the properties of the utility functions. Future research should include characterization results that do not depend on specific assumptions of the utility functions. A good starting point is Brito et al. (1990). It would also be desirable to generalize these results to an finite economy, consisting of $n \geq 2$ differing consumer types.

Appendix

In this appendix, we prove Lemma 5 and Proposition 1.

Proof Lemma 5. In the proof, we consider a change in q_i with a "small" $\varepsilon \in \mathbb{R}$, so $dq_i = \varepsilon$, $du_i = \left(\phi_i'(q_i) - \frac{\partial t_i}{\partial q_i}\right)\varepsilon$ and $du_j = -\frac{\partial t_j}{\partial q_i}\varepsilon$ for $j \neq i$ and $i, j \in N$.[5]

Part (i) "if". Suppose that the outlays are given by equations (9)-(10) for $k = 1$. We need demonstrate that if $q_1 \in [q_1^{\min}, q_1^*]$ and $q_2 = q_2^*$, then $x \in X^*$. Note first that since

[5]This procedure is also adopted in e.g. Laffont and Tirole (1993,pp.299) and Salanié (1999,p.23). Note also that the lemma is of type "A if and only if B". To prove a lemma of this type, we first prove that $B \Rightarrow A$ ("if") and then prove that not $B \Rightarrow$ not A ("only if").

$k = 1$, condition (5) is binding. But then since $q_2 > q_1$, the upward no-envy constraint (4) is satisfied and non-binding for "sufficiently small" changes in q_i, by Part (iii) of Lemma 1. Hence, type 1 will always pick the bundle that is intended for him when ε is "sufficiently small". Note next that, by the standard argument, it is not possible to make a Pareto improvement and at the same time respect the envy-freeness criterion and the balanced-budget rule by adjusting q_i when $q_i = q_i^*$. As a consequence, we need only demonstrate that it is not possible to make a Pareto improvement that respects the envy-freeness criterion and the balanced-budget rule by adjusting q_1 when $q_1 \in [q_1^{\min}, q_1^*[$.

If the outlays are given by equations (9) and (10) for $k = 1$ then: $\frac{\partial u_1}{\partial q_1} \geq 0$ if and only if $q_1 \leq q_1^{\min}$ and $\frac{\partial u_2}{\partial q_1} \geq 0$ if and only if $q_1 \leq q_2^*$. Thus, if we increase $q_1 \in [q_1^{\min}, q_1^*[$ by a "small" $\varepsilon \in \mathbb{R}_{++}$, it follows that: $du_1 < 0$, $du_2 > 0$ and $\sum_{i=1}^{2} du_i = (\phi_1'(q_1) - \beta)\,\varepsilon > 0$. Since condition (5) holds with equality, type 2 is indifferent between bundle (q_2, t_2) and $(q_1 + \varepsilon, t_1)$. But $du_1 < 0$ and, therefore, type 1 must be compensated in order to be (at least) indifferent to the change in q_1. Type 1 is indifferent to the change in q_1 if the menu is given by: $\{(q_1 + \varepsilon, t_1 + du_1), (q_2, t_2 - du_1)\}$. But this menu can not be envy-free since $du_1 < 0$, i.e. type 2 prefers bundle $(q_1 + \varepsilon, t_1 + du_1)$ over bundle $(q_2, t_2 - du_1)$. Hence, it is not possible to make a Pareto improvement that respects the envy-freeness criterion and the balanced-budget rule by adjusting q_1 when $q_1 \in [q_1^{\min}, q_1^*[$.

Part (i) "only if". Suppose that the outlays are given by equations (9)-(10) for $k = 1$. We need demonstrate that if $q_1 \notin [q_i^{\min}, q_1^*]$ or $q_2 \neq q_2^*$ then $x \notin X^*$. Note first that if $k = 1$ and if the outlays are given by equations (9)-(10) then: $\frac{\partial u_1}{\partial q_2} = \frac{\partial u_2}{\partial q_2} = \frac{1}{2}(\phi_2'(q_2) - \beta)$, i.e. both types are best off when $q_2 = q_2^*$, so if $q_2 \neq q_2^*$ then $x \notin X^*$. From the properties of the partial derivatives, it is also clear that if $q_1 < q_1^{\min}$ or $q_1 > q_2^*$ then $x \notin X^*$. Thus, if we can demonstrate that $q_1 \in]q_1^*, q_2^*]$ imply that $x \notin X^*$, the conclusion follows. Suppose that $q_1 \in]q_1^*, q_2^*]$ and adjust q_1 with a "small" $\varepsilon \in -\mathbb{R}_{++}$. It follows that: $du_1 > 0$, $du_2 < 0$ and $\sum_{i=1}^{2} du_i = (\phi_1'(q_1) - \beta)\,\varepsilon > 0$. Since condition (5) holds with equality, type 2 is indifferent between bundle (q_2, t_2) and bundle $(q_1 + \varepsilon, t_1)$. But $du_2 < 0$ and, therefore, type 2 must be compensated in order to be (at least) indifferent to the change in q_1. Type 2 is indifferent to the change in q_1 if the menu is given by: $\{(q_1 + \varepsilon, t_1 - du_2), (q_2, t_2 + du_2)\}$. We next note that this menu is envy-free, budget-balanced and Pareto improving since $du_2 < 0$ and condition (4) is inactive. Hence, if $q_1 \in]q_1^*, q_2^*]$ then $x \notin X^*$.

Part (ii) "if". Suppose that the outlays are given by equations (9) and (10) for $k = 0$. We need demonstrate that if $q_1 = q_1^*$ and $q_2 \in [q_2^*, q_2^{\max}]$ then $x \in X^*$. In this case, condition (4) is binding since $k = 0$ and it follows from Part (iii) of Lemma 1 that the downward no-envy constraint (5) is satisfied but non-binding since $q_1 < q_2$. Hence, type 2 will always pick the bundle that is intended for him when ε is "sufficiently small". By the

same arguments as in Part (i) of this proof, we need only establish that it is not possible to make a Pareto improvement and at the same time respect the envy-freeness criterion and the balanced-budget rule by adjusting q_2 when $q_2 \in]q_2^*, q_2^{\max}]$.

If the outlays are given by equations (9) and (10) for $k = 0$ then: $\frac{\partial u_1}{\partial q_2} \geq 0$ and $\frac{\partial u_2}{\partial q_2} \geq 0$ if and only if $q_2 \leq q_1^*$ and $q_2 \leq q_2^{\max}$, respectively. Hence, if we adjust $q_2 \in]q_2^*, q_2^{\max}]$ with a "small" $\varepsilon \in -\mathbb{R}_{++}$ it follows that: $du_1 > 0$, $du_2 < 0$ and $\sum_{i=1}^{2} du_i = (\phi_2'(q_2) - \beta)\,\varepsilon > 0$. Type 1 is indifferent between bundle (q_1, t_1) and bundle $(q_2 + \varepsilon, t_2)$ due to the fact that $k = 0$. But since $du_2 < 0$, type 2 must be compensated. If the menu is given by: $\{(q_1, t_1 - du_2), (q_2 + \varepsilon, t_2 + du_2)\}$, type 2 is indifferent to the change in q_2. However, $du_2 < 0$ and thus type 1 chooses bundle $(q_2 + \varepsilon, t_2 + du_2)$ over bundle $(q_1, t_1 + du_2)$ so this menu can not be envy-free. Hence, it is not possible to make a Pareto improvement and at the same time respect the envy-freeness criterion and the balanced-budget rule by adjusting q_2 when $q_2 \in]q_2^*, q_2^{\max}]$.

Part (ii) "only if". Suppose that the outlays are given by equations (9)-(10) for $k = 0$. We need demonstrate that if $q_1 \neq q_1^*$ or $q_2 \notin [q_2^*, q_2^{\max}]$ then $x \notin X^*$. We first observe that if the outlays are given by equations (9)-(10) for $k = 0$, then: $\frac{\partial u_1}{\partial q_1} = \frac{\partial u_2}{\partial q_1} = \frac{1}{2}(\phi_1'(q_1) - \beta)$. Hence, if $q_1 \neq q_1^*$ then $x \notin X^*$. Using the same arguments as in Part (i) of this proof, we need only demonstrate that if $q_2 \in [q_1^*, q_2^*[$ then $x \notin X^*$. For this purpose, assume that the outlays are given by equations (9) and (10) for $k = 0$ and that $q_2 \in [q_1^*, q_2^*[$. Next, increase q_2 by a "small" $\varepsilon \in \mathbb{R}_{++}$. It is clear that: $du_1 < 0$, $du_2 > 0$ and $\sum_{i=1}^{2} du_i = (\phi_2'(q_2) - \beta)\,\varepsilon > 0$. Because condition (4) is binding, type 1 is indifferent between bundle (q_1, t_1) and bundle $(q_2 + \varepsilon, t_2)$. But since $du_1 < 0$, type 1 must be compensated. If the menu is given by: $\{(q_1, t_1 + du_1), (q_2 + \varepsilon, t_2 - du_1)\}$, type 1 is indifferent to the increase in q_2. We next note that this menu is envy-free, budget-balanced and Pareto improving since $du_1 < 0$ and condition (5) is inactive. Hence, if $q_2 \in [q_1^*, q_2^*[$ then $x \notin X^*$. ■

Proof Proposition 1. The proof of Part (i)-(iii) of the proposition follows from Lemma 5. To see that Part (ii) follows from Lemma 5, note that if $k \in]0, 1[$, conditions (4) and (5) are non-binding and, therefore, none of the "small" changes considered in Lemma 5 will violate the envy-freeness criterion. Hence, it is always possible to make a Pareto improvement when $k \in]0, 1[$ and at the same time respect the envy-freeness condition and the balanced-budget rule, unless, of course, q_1 and q_2 are first-best. ■

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