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Published in:
IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing

DOI:
10.1109/82.809540

1999

Document Version:
Peer reviewed version (aka post-print)

Link to publication

Citation for published version (APA):
Lau, B. K., Leung, Y. H., Teo, K. L., & Sreeram, V. (1999). Minimax filters for microphone arrays. *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, *46*(12), 1522-1524. https://doi.org/10.1109/82.809540

Total number of authors:
4

Minimax Filters for Microphone Arrays[1]

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ABSTRACT

Conventionally, minimax spatial/spectral filters for microphone arrays are designed by first discretizing the spatial and frequency domains into a finite number of grid points and then performing the optimization over these points. The drawback with this approach is that the response of the spatial/spectral filters in between the grid points can be poor. More recently, an approach that performs the minimax design over the continuum of points in the decision space has appeared in the literature. In this paper, we describe an approach to solving this continuous decision space design problem that is numerically more elegant and efficient. The effectiveness of the new method is illustrated by a numerical example.

1 Introduction

Recently, the design of spatial/spectral filters for broadband receiving antenna arrays operating in the nearfield, such as in microphone array applications, has received much attention in the literature. See, for example, [1] and [2] and the references therein. In this paper, we consider the same filter design problem as that studied in [1] and [2], namely, the design of *minimax* spatial/spectral filters for microphone arrays. This design problem can be stated as follows.

Consider the L-element microphone array shown in Fig. 1 where $\mathbf{r}_l$, $l = 1, \ldots, L$, are the position vectors of the microphones, Δ_l are the corresponding (optional) pre-steering delay elements, and the spatial/spectral filter consists of the L N-tap FIR filters. The pre-steering delay outputs are sampled synchronously at a rate of $1/T$ samples per second. The minimax spatial/spectral filter design problem is defined by

[1] This work was supported by the Australian Research Council under Grant A49601750.

$$\min_{\mathbf{w}\in R^{NL}} \max_{(\mathbf{r},f)\in\Omega} v(\mathbf{r},f)\left|\mathbf{w}^T\mathbf{d}(\mathbf{r},f) - G_d(\mathbf{r},f)\right| \qquad (\mathcal{P}1)$$

where $v(\mathbf{r},f)$ is a positive weighting function, $G_d(\mathbf{r},f)$ is the desired response of the microphone system, Ω is the region in R^4 over which $G_d(\mathbf{r},f)$ is defined, $\mathbf{w}$ is the vector of filter weights,

$$\mathbf{d}(\mathbf{r},f) = \left[A_1(\mathbf{r},f)e^{-j2\pi f\Delta_1}\mathbf{d}_o^T(f) \quad \cdots \quad A_L(\mathbf{r},f)e^{-j2\pi f\Delta_L}\mathbf{d}_o^T(f)\right]^T, \qquad (1)$$

$$\mathbf{d}_o(f) = \left[1 \quad e^{-j2\pi fT} \quad \cdots \quad e^{-j2\pi f(N-1)T}\right]^T, \qquad (2)$$

$A_l(\mathbf{r},f)$ is the transfer function from the spatial point $\mathbf{r}$ to the lth microphone, and the actual response of the overall microphone system is given by $\mathbf{w}^T\mathbf{d}(\mathbf{r},f)$.

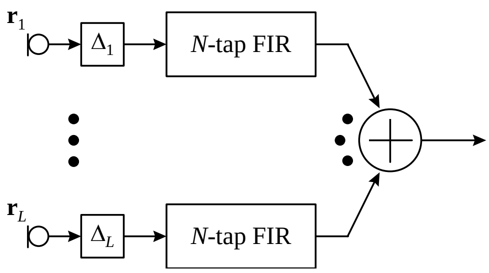

Fig. 1: Microphone array with spatial/spectral filter.

In practice, $A_l(\mathbf{r},f)$ will include the directional characteristics of each microphone. However, for ease of illustration, we shall assume the microphones are identical, omni-directional and all have a flat frequency response. Thus,

$$A_l(\mathbf{r},f) = \frac{1}{\|\mathbf{r}-\mathbf{r}_l\|}\exp\left(-j2\pi f\,\frac{\|\mathbf{r}-\mathbf{r}_l\|}{c}\right),\quad l=1,\ldots,L, \qquad (3)$$

where $c = 340$ m/sec is the speed of sound in air.

In [1], ($\mathcal{P}$1) was solved by first discretizing the decision space Ω into a number of grid points. The discretized problem was then transformed to an equivalent problem from which an efficient numerical solution method can be derived. The difficulty with the method of [1], as with all multi-grid point methods, is that there are no guidelines on how to choose the grid points. With a poor choice of grid points, the filter response in between the grid points can be unsatisfactory.

In [2], the authors considered the continuum of points in Ω. Like [1], they also transformed the problem in order to derive a numerical method for solving this continuous decision space problem. The numerical method of [2] requires, however, a search in two parameters. Also, no bounds are available on the accuracy of the final solution. In this paper, we combine the ideas of [2] and [3] to overcome these concerns.

2 Problem Reformulation

As with [2], we first write ($\mathcal{P}1$) as a semi-infinite programming problem as follows.

$$\min_{z \in R} z \qquad (\mathcal{P}2)$$

subject to $$E(\mathbf{w}, \mathbf{r}, f) \le z, \ \forall \mathbf{w} \in R^{NL}, \forall (\mathbf{r}, f) \in \Omega$$

where $$E(\mathbf{w}, \mathbf{r}, f) = v(\mathbf{r}, f) \left| \mathbf{w}^T \mathbf{d}(\mathbf{r}, f) - G_d(\mathbf{r}, f) \right|. \tag{4}$$

We next solve ($\mathcal{P}2$) by first applying constraint transcription [4] to the constraint to obtain the following auxiliary function.

$$J_0(\alpha) = \min_{\mathbf{w} \in R^{NL}} \int_\Omega p\big(E(\mathbf{w}, \mathbf{r}, f) - \alpha\big)\, d\mathbf{y} \tag{5}$$

where $$p(t) = \max(0, t). \tag{6}$$

As can be seen, the solution of ($\mathcal{P}2$) is given by the first root of $J_0(\alpha)$, i.e. the smallest α such that $J_0(\alpha) = 0$ [3]. But $p(t)$ is non-smooth. This can lead to numerical difficulties when eq. (5) is solved using standard optimization software. Following [2] and [3], we approximate $p(t)$ with the following smooth function:

$$g_\varepsilon(t) = \begin{cases} 0, & t \le -\varepsilon \\ \dfrac{(t+\varepsilon)^2}{4\varepsilon}, & -\varepsilon < t \le \varepsilon \\ t, & t > \varepsilon \end{cases}. \tag{7}$$

Thus ($\mathcal{P}2$) is solved, approximately, by finding the first root of the following function:

$$J(\varepsilon, \alpha) = \min_{\mathbf{w} \in R^{NL}} \Phi_\varepsilon(\mathbf{w}, \alpha) \tag{8}$$

where $$\Phi_\varepsilon(\mathbf{w}, \alpha) = \int_\Omega g_\varepsilon\big(E(\mathbf{w}, \mathbf{r}, f) - \alpha\big)\, d\mathbf{r}df. \tag{9}$$

Compared to the solution technique of [2], we note that our technique is a *root-catching* technique while the technique in [2] is a constrained minimization technique[2].

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[2] [2] solves the problem $\min_{\mathbf{w}, z} z$ subject to $\Phi_\varepsilon(\mathbf{w}, z) = 0$ using the penalty function method.

3 Computational Procedure

Following [3], we solve ($\mathcal{P}$2) via eqs. (8) and (9) as follows.

Step 1a: Select the parameter ε and set $\delta = \varepsilon^2 / 16B$ (see eq. 11).

Step 1b: Compute the weight vector $\mathbf{w}_o$ as described later.

Step 2: Choose two points $(\mathbf{r}_1, f_1), (\mathbf{r}_2, f_2) \in \Omega$ such that $\alpha_1 < \alpha_2$ where $\alpha_1 = E(\mathbf{w}_o, \mathbf{r}_1, f_1)$ and $\alpha_2 = E(\mathbf{w}_o, \mathbf{r}_2, f_2)$.

Step 3: Calculate $J(\varepsilon, \alpha_1)$ using $\mathbf{w}_o$ as the initial guess.

Step 4: If $J(\varepsilon, \alpha_1) = 0$, decrease α_1 and return to Step 3.

Step 5: Calculate $J(\varepsilon, \alpha_2)$ using the optimum $\mathbf{w}$ found in the previous computation of $J(\varepsilon, \alpha)$ as the initial $\mathbf{w}$.

Step 6a: If $J(\varepsilon, \alpha_2) = 0$, let $\alpha_3 = \alpha_1 + 0.618 \times (\alpha_2 - \alpha_1)$ (method of Golden section). Replace the value of α_2 by the value of α_3. Go to Step 5.

Step 6b: If $J(\varepsilon, \alpha_2) \geq \delta$, compute $\alpha_3 = \alpha_2 + (\alpha_2 - \alpha_1)\left(\sqrt{\dfrac{J(\varepsilon, \alpha_1)}{J(\varepsilon, \alpha_2)}} - 1\right)^{-1}$. Replace α_2 by α_3 and α_1 by α_2. Go to Step 5.

Step 6c: $J(\varepsilon, \alpha_2) < \delta$. Check whether the solution satisfies $E(\mathbf{w}, \mathbf{r}, f) \leq \alpha_2$. If no, reduce δ and go to Step 6b. Otherwise, stop and the coefficients of the optimum filter are given by the $\mathbf{w}$ found at this stage.

The following two results can be proved in a similar fashion to those in [3].

Result A The computational procedure terminates in a finite number of steps.

Result B Denote the successive α_1's obtained by μ_i, $i = 1, 2, \ldots, (M-1)$ and the last α_2 by μ_M. Then the optimal solution α^* to the original auxiliary function eq. (5) satisfies $\alpha^* \in [\mu_{i0}, \mu_M]$ where the lower bound μ_{i0} is given by

$$J(\varepsilon, \mu_{i0}) \geq \frac{\varepsilon}{4}|\Omega| \quad \text{and} \quad J(\varepsilon, \mu_{i0+1}) < \frac{\varepsilon}{4}|\Omega|, \tag{10}$$

$|\Omega|$ is the Lebesgue measure of Ω, and the upper bound μ_M is given by

$$J(\varepsilon, \mu_M) < \frac{\varepsilon^2}{16B}, \quad B = \frac{4^4}{\varepsilon^3}\prod_{i=1}^{4} B_i, \quad \left|\frac{\partial E(\mathbf{w}, \mathbf{r}, f)}{\partial r_i}\right| \leq B_i \quad \text{and} \quad \left|\frac{\partial E(\mathbf{w}, \mathbf{r}, f)}{\partial f}\right| \leq B_4 \tag{11}$$

Remarks

(i) Result B generalizes a result in [3] where $\Omega \subset R$. Here, $\Omega \subset R^4$. The proof of Result B is given in [5]. The significance of Result B is that it specifies how the accuracy of α^* can be controlled through ε. No similar result was given in [2].

(ii) The problem ($\mathcal{P}$1) does not have any local solutions since it can written as

$$\min_{\mathbf{w} \in R^{NL}} M(\mathbf{w}) \qquad (\mathcal{P}1')$$

and $M(\mathbf{w})$, being a norm (infinite norm), is convex. The global solution may, however, be non-unique. The computational procedure described above will find a **w** whose corresponding cost approximates (depending on ε) the global minimum cost.

4 Design Example

4.1 Problem Description

We consider an example similar to those in [1] and [2]. The array consists of $L = 7$ microphones, spaced 5 cm apart. The microphones are located at $\{(-0.15, 0, 0), (-0.10, 0, 0), \ldots, (+0.15, 0, 0)\}$. Also no pre-steering delays are used and each FIR filter has $N = 33$ taps. The sampling frequency is 8 kHz. The position vector component of the decision space is given by $(r, 1, 0)$, i.e. the set of points 1 m in front of and parallel to the microphone array axis. The desired response is given by $G_d(r, f) = \exp\{-j\omega \cdot [(1/c) + (N-1)T/2]\}$ in the passband region and $G_d(r, f) = 0$ in the stopband region. The passband region is defined by $\{(r, f): -0.4 \le r \le 0.4, 500 \le f \le 3000\}$ and the stopband region is defined by $\{(r, f): -2.5 \le r \le -1.5, 0 \le f \le 4000\} \cup \{(r, f): 1.5 \le r \le 2.5, 0 \le f \le 4000\} \cup \{(r, f): -1.5 \le r \le 1.5, 0 \le f \le 50\} \cup \{(r, f): -1.5 \le r \le 1.5, 3500 \le f \le 4000\}$ where r is in metres and f is in Hz. The passband response was weighted 1.5 relative to the stopband response.

Here, we observe as in [2] that, because of the symmetry of the problem, the filter coefficients of microphone pairs 1 and 7, 2 and 6, and 3 and 5 will be identical.

4.2 Selection of $\mathbf{w}_o$

We describe here a method for choosing the initial **w** required in Steps 1b and 3 of the computational procedure. The motivation is to find a $\mathbf{w}_o$ that is as close as possible to the optimum solution to reduce the number of iterations that are required. The basic idea is to derive a "conventional" spatial/spectral filter that is focussed at the central speaker position, i.e. $\mathbf{r} = (0, 1, 0)$, with frequency response $G_d(0, f)$, $0 < f < 4000$, and spatial response $G_d(r, 1750)$, $-2.5 < r < 2.5$[3].

(1) Use the Remez exchange algorithm [6] to design the FIR filter of the centre microphone (microphone 4). The desired magnitude response of this filter is given by $|G_d(0, f)|$ (with passband weighted 1.5 times relative to the stopband), and the desired phase response is given by $-\omega \cdot (N-1)T/2$, i.e. the phase of $G_d(0, f)$ less the propagation delay from the central speaker position to the centre microphone. Denote the impulse response of the filter by $\{h_i,\ i = 0, \ldots, 32\}$.

(2) Derive the other FIR filters by interpolating, time-advancing, and re-sampling $\{h_i\}$. We illustrate this via microphones 3 and 5.

(i) Interpolate $\{h_i\}$ to obtain the continuous-time function

$$\bar{h}(t) = \sum_{i=0}^{32} h_i \frac{\sin \pi (t - iT)/T}{\pi (t - iT)/T}. \tag{14}$$

(ii) Time-advance $\bar{h}(t)$ to get $\bar{g}(t) = \bar{h}(t + \tau_{3,5})$ where $\tau_{3,5} = \left(\sqrt{1^2 + 0.05^2} - 1\right) \Big/ c$ is the difference in propagation delays from the central speaker position to microphone 4 and from the same position to microphones 3 and 5. (iii) Sample $\bar{g}(t)$ at $t = iT$, $i = 0, \ldots, 32$, to obtain the coefficients of the FIR filters for microphones 3 and 5.

(3) Adjust the gains of the FIR filters so that at 1750 Hz, the spatial response approximates $G_d(r, 1750)$, $-2.5 < r < 2.5$, in a weighted least squares sense.

4.3 Results

The design obtained from the computer program, for $\varepsilon = 10^{-8}$ and $\delta = 2.98 \times 10^{-7}$, is shown in Fig. 2. We shall refer to this design as Design 1. The corresponding bounds for α are given by $\mu_{i0} = 0.105$ and $\mu_M = 0.126$. In our program, we evaluate eq. (9) with the double Simpson's rule (as recommended in [2]), and solve eq. (8) with the sequential quadratic programming (SQP) method.

[3] In view of Remark (ii), Section 3, one may choose other $\mathbf{w}_o$ s such as $[\mathbf{w}_o]_i = 1/NL$, $i = 1, \ldots, NL$, to find other equally optimum solutions.

In Design 2, we repeat the design procedure but with all the elements of $\mathbf{w}_o$ set to $1/NL$. This yielded a response similar to Design 1. The design time was, however, about 1.2 times longer.

For comparison, we implemented the multi-grid method of [1] and repeated the design (Design 3). The filter response obtained is shown in Fig. 3. The grid spacing used was as recommended in [1], with a minor modification to accommodate the stopband region from 0 Hz to 50 Hz. As can be seen, the performance is quite poor. In Design 4, we increased the 345 grid points of Design 3 to 2429 points[4]. The response of the filter is now much better and closely approximates the response shown in Fig. 2. However, the design time was more than 4 times longer than that of Design 1.

Finally, we implemented the design method of [2] (Design 5). Using the same integration program as in Design 1, the filter response obtained again closely approximates that of Design 1. The design time was, however, more than 1.3 times longer.

5 Conclusion

In this paper, we have successfully applied a new optimisation technique to solve a minimax spatial/spectral filter design problem. Unlike the conventional techniques which discretize the decision space, our technique performs the design over all points in the space. Moreover, compared to a recent continuous decision space design technique, our technique is numerically more elegant. Bounds on the performance of our new technique and a method to initialize the design procedure are also presented.

References

[1] S. Nordebo, I. Claesson and S. Nordholm, “Weighted Chebyshev Approximation for the Design of Broadband Beamformers Using Quadratic Programming,” *IEEE Signal Processing Letters*, vol. 1, no. 7, pp.103-105, July 1994.

[2] S. E. Nordholm, V. Rehbock, K. L. Teo and S. Nordebo, “Chebyshev Optimization for the Design of Broadband Beamformers in the Near Field,” *IEEE Trans. Circuits Syst. - II*, vol. 45, no. 1, pp. 141-144, January 1998.

[3] D. C. Jiang, K. L. Teo and W. Y. Yan, “A New Computational Method for the Functional Inequality Constrained Minimax Optimization Problem,” *Computers Math.*

[4] We note here a potential problem with the method of [1]. Large design problems will require many grid points (constraints [1]). This can cause the program to not run because of insufficient memory.

Applic., vol. 33, no. 6, pp. 53-63, 1997.

[4] K. L. Teo, C. J. Goh and K. H. Wong, *A Unified Computational Approach to Optimal Control Problems*, Burnt Mill, Essex, UK: Longman Scientific & Technical, 1991.

[5] B. K. Lau, Y. H. Leung, K. L. Teo and V. Sreeram, "Optimum Filters for Microphone Arrays," SPL-TR-012, ATRI, Curtin University of Technology, July 1999.

[6] A. Antoniou, *Digital Filters: Analysis, Design and Applications*, 2nd ed., McGraw-Hill, 1993.

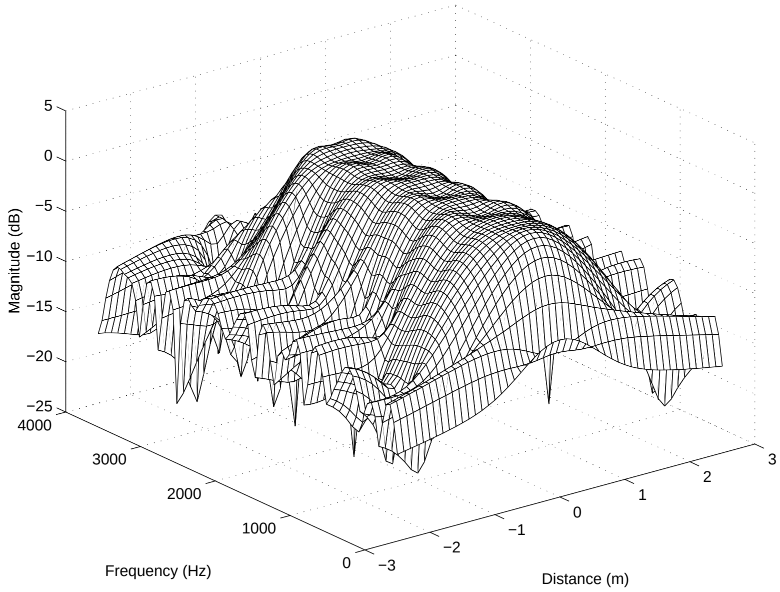

Fig. 2: Magnitude response of minimax 7-microphone 33-tap filter designed using the root-catching technique.

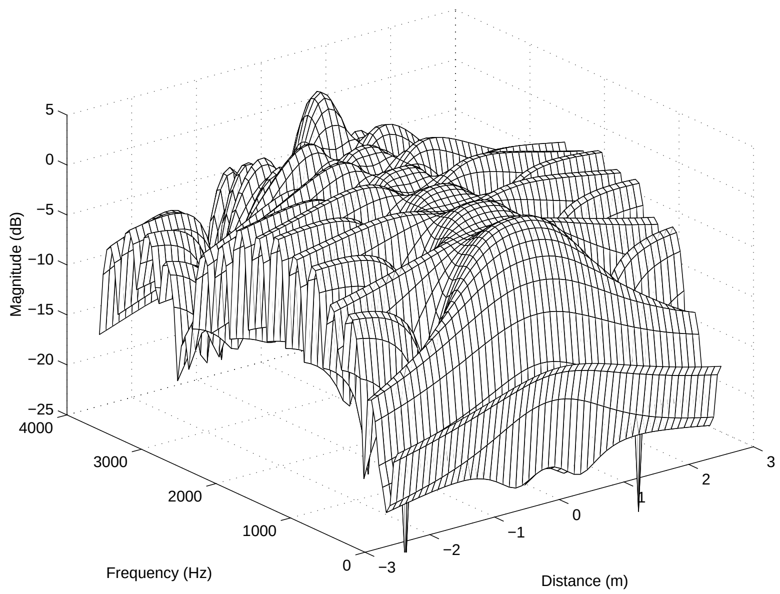

Fig. 3: Magnitude response of minimax 7-microphone 33-tap filter designed using the multi-grid point method.