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INFLUENCE OF BEAM SIZE ON CONCRETE FRACTURE  
ENERGY DETERMINED ACCORDING TO A DRAFT RILEM  
RECOMMENDATION

REPORT TO RILEM TC50-FMC

ARNE HILLERBORG

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**ARNE HILLERBORG**

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## 1. Introduction.

Within the work of RILEM TC 50-FMC (Fracture Mechanics of Concrete) a method has been proposed for the determination of the fracture energy  $G_F$  by means of a stable three-point bend test on a notched beam /1/. The method has been tested by 12 laboratories in different countries /2,3/. When these tests were analysed it appeared that it is uncertain whether the measured quantity can be regarded as a material property or if it is too size dependent. It was therefore decided at a meeting in March 1984 that a new series of tests should be performed with the goal of studying the influence of the beam depth on the measured values.

In the first place it was recommended to test beams with depths of 100, 200, and 300 mm and with the spans proportional to the square root of the depths. The results were reported in July and August and a short summary report has been presented at a committee meeting in September in Evanston.

## 2. Laboratories taking part in the tests.

The following 8 laboratories have reported test results within this test series. The abbreviations within parantheses are used in the table and in the discussions.

Bundesanstalt für Materialprüfung (BAM, Berlin), Winkler, Kleinschrodt.

Ente Nazionale per l'Energia Elettrica (ENEL, Milano), G. Ferrara.

Ecole Polytechnique Fédérale de Lausanne (EPFL, Lausanne), I. Metzener-Gheorghita, F.H. Wittmann.

Italcementi, Bergamo, G.P. Tognon.

Facultad de Ingenieria, Departamento de Construcciones, La Plata, Argentina (La Plata), L.J. Lima, D. Violini, R. Zerbino.

Lund Inst. of Technology, Div. of Building Materials (LTH, Lund), A. Hillerborg, R. Horvath, T. Persson.

Institut für Massivbau und Baustofftechnologie, Universität Karlsruhe (MPA, Karlsruhe), H.K. Hilsdorf, W. Brameshuber.

Universidad Politécnica de Madrid, Departamento de Fisica y Fisica de Materiales (UP, Madrid), M. Elices, H. Corres, J. Planas.

The results reported by the different laboratories are summarised in Table 1.

### 3. Some remarks regarding the tests.

All tests were performed with a ratio of notch depth to beam depth of approximately 0.5. Italcementi used notches which were 5 mm less deep, thus 45, 95, and 145 mm respectively. This small difference is of no importance for the comparisons.

Where more than 3 tests of the same kind have been performed, the standard deviations have been calculated and introduced in the table.

The tests at MPA, Karlsruhe, differ in some respects from the others. These tests had been started before the recommendation had been made by TC50-FMC to perform test in order to study the influence of the beam depth. Some of the beams are extremely large, and they are therefore very useful for the intended purpose. The test procedure used for the large beams is different from the standard procedure in that the beams were tested in an inverted position with the notch in the upper part of the beam and the load acting upwards.

In some of the tests at MPA, Karlsruhe, an unloading followed by a reloading was performed each time that the load-deflection curve became horizontal. This resulted in a cyclic load-deformation curve with a higher deformation and a lower maximum load in each cycle. As no descending branch was recorded, it was easier to perform stable tests with this procedure than with the ordinary one. It proved that the envelope to the cyclic curves practically coincided with the curve from an ordinary test and the  $G_F$ -value calculated from the envelope curve was nearly identical with the value from an ordinary test. This is thus a method which might be used where there are problems with the stability of the tests.

One of the concrete qualities tested at Italcementi, Bergamo, was a very high strength concrete, where the strength was reached by means of high pressure steam curing. This resulted in a cube strength of about 170 MPa, a splitting tensile strength of about 10.5 MPa, a modulus of elasticity of about 45 GPa and a  $G_F$ -value of about 175 N/m. The corresponding characteristic length  $l_{ch}$  is about 70 mm. For ordinary concrete the characteristic length is seldom below 200 mm and often much higher, values around 1000 mm occur /2/. This low value for the very high strength concrete indicates a more brittle behavior and a smaller ratio between e. g. the shear strength of a beam and the tensile strength. On the other hand the difference in this respect between this very high strength concrete and ordinary concretes is not dramatic.

It can also be noted that the very high strength concrete has the highest  $G_F$ -value of all concretes reported in the table. Thus in this case the fracture energy has increased when the strength has increased. It is also possible that for some types of concrete the opposite happens, e g if the aggregate is not strong enough. In such a case the concrete may become very brittle.

At UP, Madrid, the tests with the beams with depths 200 and 300 mm were made with partial weight compensation in order to diminish the relative importance of the correction term in the expression for  $G_F$ . The  $G_F$ -value was then evaluated both with and without weight compensation. The difference between the

two ways of evaluation proved to be about 6 % for the 200 mm beams and 10 % for the 300 mm beams. The values from the proposed standard procedure, without weight compensation, were highest. These values are introduced in the table.

#### 4. Main test results.

The goal of the tests was to study the influence of the beam depth on the measured values of  $G_F$ . Therefore the last column in the table is the most interesting one. These values are illustrated in Fig. 1.

There is a wide scatter in the test results. The values for the 200 mm beams are in some series equal to or even a little smaller than those for the 100 mm beams, whereas in some series the values for the 200 mm beams are up to nearly 60 % higher. The same trend can be found for the 300 mm beams, where the values in some series are nearly 80 % higher than those for the 100 mm beams.

Due to this great scatter it is not possible to draw any definite conclusions regarding the influence of the beam depth on the measured values of  $G_F$ . A statistical analysis shows that as an average the 200 mm beams gave 20 % higher values and the 300 beams 30 % higher values than the 100 mm beams. Then the values for the very large beams tested at MPA, Karlsruhe, have not been taken into account. These tests indicate a much smaller influence of the beam depth.

The reason for the great differences in the results from different test series can not easily be explained. At present it has to be accepted that these great differences have been found and that larger beams as a rule give higher  $G_F$ -values. In the discussion below it is assumed that a typical influence of the beam depth is that the value increases by 20 % when the beam depth is increased by a factor 2.

From the standard deviations and mean values in the table the corresponding coefficients of variation can be calculated. According to the values of Table 1 and the corresponding tables in previous reports /2,3/ the coefficient of variation

is as a mean 10-15 % and in extreme cases about 25 %.

#### 5. Comments on the significance of the results.

The values which have been found for the influence of the beam depth and for the coefficients of variation must be judged with due regard to their importance in the situation where they are to be applied.

As an illustration of this statement let us look at linear elastic fracture mechanics, LEFM. According to LEFM the strength of a structure is proportional to the critical stress intensity factor  $K_c$ .  $K_c$  is in its turn proportional to the square root of the critical strain energy release rate  $G_c$ .

In LEFM the fracture mechanics property of a material can be measured as  $K_c$  or  $G_c$ . An error of 10 % in  $K_c$  will give an error of 10 % in the calculated strength of the structure. An error of 10 % in  $G_c$  will give an error of only 5 % in the calculated strength of the structure, as the strength is proportional to the square root of  $G_c$ .

Thus the calculated strength of a structure is not as sensitive to errors in  $G_c$  as it is to errors in  $K_c$ . This difference can be expressed by means of sensitivity factors  $S$ . The sensitivity factor for a material property is the change in percent in the calculated strength of a structure when that property changes with 1 percent. Thus according to LEFM the sensitivity factors are 1.0 for  $K_c$  and 0.5 for  $G_c$ .

In LEFM the tensile strength  $f_t$  is of no importance, and thus the sensitivity factor for  $f_t$  in this case is 0.

According to non-linear fracture mechanics the strength  $f$  of a structure is proportional to  $f_t$  and a function of  $d/l_{ch}$ , where  $d$  is a typical dimension of the structure and

$$l_{ch} = EG_F / f_t^2$$

Figs 2-4 show examples of theoretical relations between  $f/f_t$  and  $d/l_{ch}$ . In all these diagrams logarithmic scales are used on the axes. It must be noticed that the scale on the vertical axes is extended four times in comparison to the scale on the horizontal axis.

The influence on the structural strength  $f$  of a small change in  $f_t$ ,  $G$ ,  $E$  or  $d$  may be found from the slope of the curve in the following way.

For a small change we can represent the curve by its tangent, the slope of which is denoted by  $-B$ , and thus write

$$\ln(f/f_t) = A - B \ln(d/l_{ch}) = A - B \ln(df_t^2/EG_F)$$

or

$$\ln f = A - B \ln d - (2B-1) \ln f_t + B \ln E + B \ln G_F$$

A differentiation of this expression yields

$$\frac{df}{f} = -B \frac{dd}{d} + (1-2B) \frac{df_t}{f_t} + B \frac{dE}{E} + B \frac{dG_F}{G_F}$$

If the sensitivity of  $f$  with regard to  $G_F$  is denoted  $S(G_F)$ , etc, we find that

$$S(G_F) = B$$

$$S(f_t) = 1 - 2B$$

$$S(f_t) = 1 - 2S(G_F)$$

which can also be written

$$S(f_t) + 2S(G_F) = 1$$

One extreme case is represented by LEFM, where  $S(f_t) = 0$  and  $S(G_F) = 0.5$ . In this case  $G_F = G_C$  and the values coincide with the values discussed above for LEFM.

The other extreme case is represented by the ordinary theory of strength of material without cracks, where  $S(f_t) = 1$  and  $S(G_F) = 0$ .

In practical applications  $S(G_F)$  is always smaller than 0.5. From Fig 2 it can be found that  $S(G_F)$  is never greater than 0.33 for a notched beam (of a reasonable size) and 0.17 for an unnotched beam. Fig 3 shows that  $S(G_F)$  is not greater than 0.35 for shear fracture in the studied type of beam. Fig 4 shows that  $S(G_F)$  is not greater than 0.34 for crushing failure and 0.13 for bending failure of an unreinforced pipe of the type shown.

From the above it can be concluded that the sensitivity factor for  $G_F$  is never higher than about  $1/3$ , whereas the sensitivity factor for  $f_t$  varies between  $1/3$  and 1.

An influence of 20 % on the measured  $G_F$ -value due to a change in the depth of the specimen by a factor 2 thus even in the worst case will influence the calculated strength of a structure by no more than 7 %. This value is of the same order as the change in measured compressive strength when the specimen size is changed with a factor 2. Thus the uncertainty in the measured values which are caused by the influence of the size of the specimens have about the same significance for the calculated strength of a structure in both cases.

In the same way a coefficient of variation of 10-15 percent in the measured value of  $G_F$  gives a coefficient of variation of 3-5 % or less in the calculated strength of a structure. Thus the influence of the scatter in the measured values is not greater than the influence of the normal scatter in the values of compressive strength.

As a matter of fact in most cases the uncertainty in the determination of the tensile strength  $f_t$  has a greater influence on the calculated strength of a structure than the uncertainty in the determination of  $G_F$ .

## 6. Conclusions.

The size of the specimen has an influence on the measured value of  $G_F$ . As an average the value seems to increase by about 20 % when the depth increases by a factor 2 and by about 30 % when the depth increases by a factor 3. These values however show a large scatter between different test series.

The scatter in measured values in the same test series is as an average 10-15 %, and in extreme cases 25 %.

The sensitivity factor for  $G_F$ , i.e. the change in percent in the calculated strength of a structure when  $G_F$  is changed by 1 percent, is in practice never greater than  $1/3$ . When this fact is taken into account, the uncertainties in the determination of  $G_F$  and in the determination of the compressive strength are of about the same significance for the calculated strength of a structure. Thus from this point of view the proposed  $G_F$ -test seems to be as acceptable as the ordinary cube or cylinder tests for compressive strength.

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Table 1.

Laboratory	w/C ratio	Cement cont. kg/m	D <sub>max</sub> mm	Age Days	Cu-ring	Number	b mm	d mm	l mm	G <sub>F</sub> N/m	Relative value
BAM Berlin	0.63		16	28	A	3	100	100	800	91	1.08
						3	100	200	1131	98	
						3	100	300	1386	118	
	0.45		16	28	A	4	100	100	800	114±10	0.98
						4	100	200	1131	112±12	
						4	100	300	1386	114±11	
	0.42		16	28	A	4	100	100	800	113±12	1.27
						4	100	200	1131	144±13	
						4	100	300	1386	135±10	
ENEL Milano	0.47	490	10	28	A	4	100	100	800	65±3	0.89
						4	100	150	980	58±2	
						4	100	200	1130	66±6	
						4	100	250	1260	70±3	
	0.47	400	20	28	A	4	100	100	800	90±15	0.96
						4	100	150	980	86±12	
						4	100	200	1130	89±7	
						4	100	250	1260	99±17	
EPFL Lausanne	0.58	300	30	34	A	5	100	100	800	100±8	1.20
						5	100	200	1148	121±15	
						5	100	300	1415	163±16	
	0.40	400	30	31	A	4	100	100	800	130±33	1.28
						5	100	200	1148	167±33	
						5	100	300	1415	234±30	
Italcementi Bergamo	0.48		20	30	B	16	100	100	800	77±10	1.36
						9	100	200	1150	105±11	
						2	100	300	1350	94	
	0.35		20	30	B	12	100	100	800	102±12	1.57
						11	100	200	1150	160±15	
						2	100	300	1350	150	
	0.33		20	30	C	14	100	100	800	174±27	1.25
						9	100	200	1150	218±29	
						2	100	300	1350	221	

Table 1, cont.

La Plata	0.48	423	9,5	30	A	7 7	100 100	100 200	800 1148	113±11 151±19	1.34
	0.58	350	9.5	30	A	7 7	100 100	100 200	800 1148	101±18 149±18	1.48
	0.68	300	9.5	30	A	6 7	100 100	100 200	800 1148	95±14 111±12	1.17
LTH Lund	0.40	613	8	28	D	6	100	100	800	113± 8	1.27 1.46
						3	100	200	1131	143	
						1	100	300	1386	165	
	0.80	284	8	28	D	6	100	100	800	122±10	0.93 1.01
2						100	200	1131	113		
3						100	300	1386	123		
0.40	612	12	28	D	6	100	100	800	91± 9	1.53 1.62	
					2	100	200	1131	139		
					2	100	300	1386	147		
0.80	286	12	28	D	6	100	100	800	64±11	1.16 1.16	
					3	100	200	1131	74		
					3	100	300	1386	74		
MPA Karlsruhe	0.54	315	32	28	E	16	100	100	500	142±30	1.00 1.20 0.96
						6	200	400	2000	142±12	
						5	400	800	4000	170±12	
	0.54	535	2	28	E	4	400	800	2000	137±15	
8						100	100	500	53± 5	0.92 0.83	
7						200	400	2000	49± 5		
4	400	800	4000	44± 3							
UP Madrid	0.58	300	20	28	A	13	100	100	800	123±22	1.07 1.14
						7	100	200	1131	132±29	
						8	100	300	1386	140±36	

Curing: A) Moist cured one day, then in lime-saturated water, B) >90 percent RH, C) High pressure autoclaving, which gave a very high strength concrete, cube strength about 170 MPa, D) In wet burlap surrounded by a plastic foil, E) Wetted and sealed in plastic sheeting.

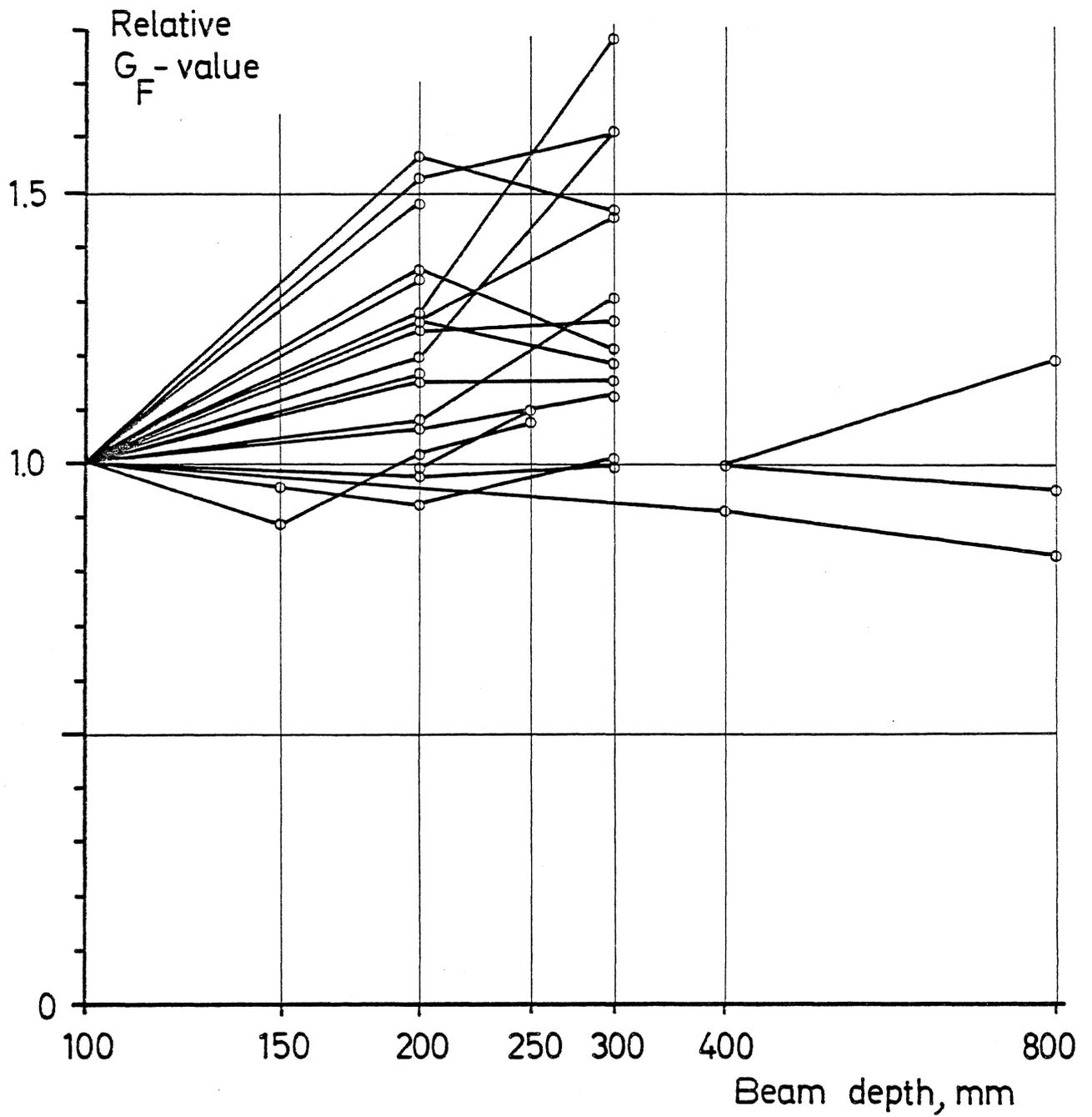


Fig 1 Variations in relative values of  $G_F$  with beam depths.

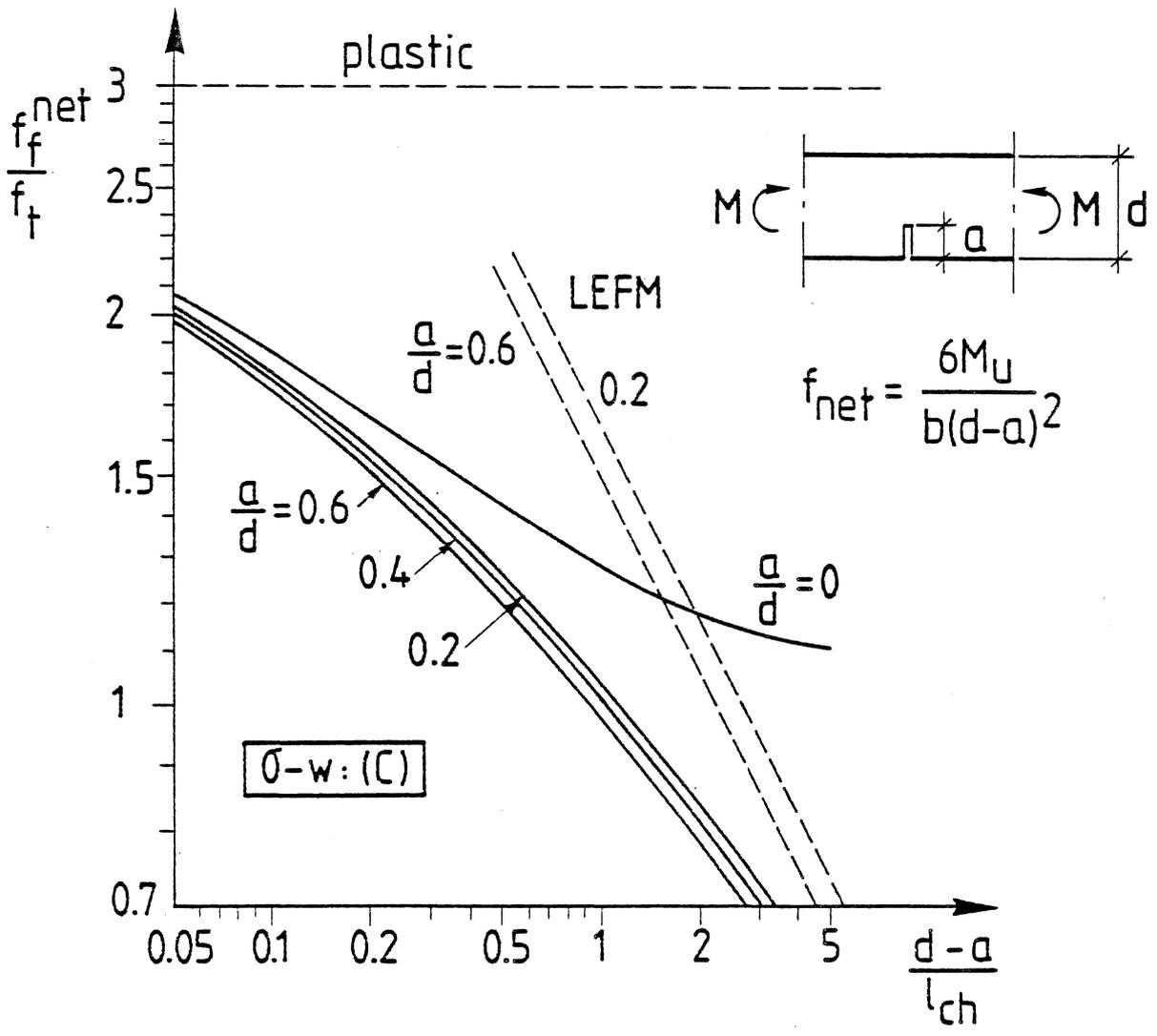


Fig 2 Theoretical bending strength variations of notched and unnotched concrete beams /4/.

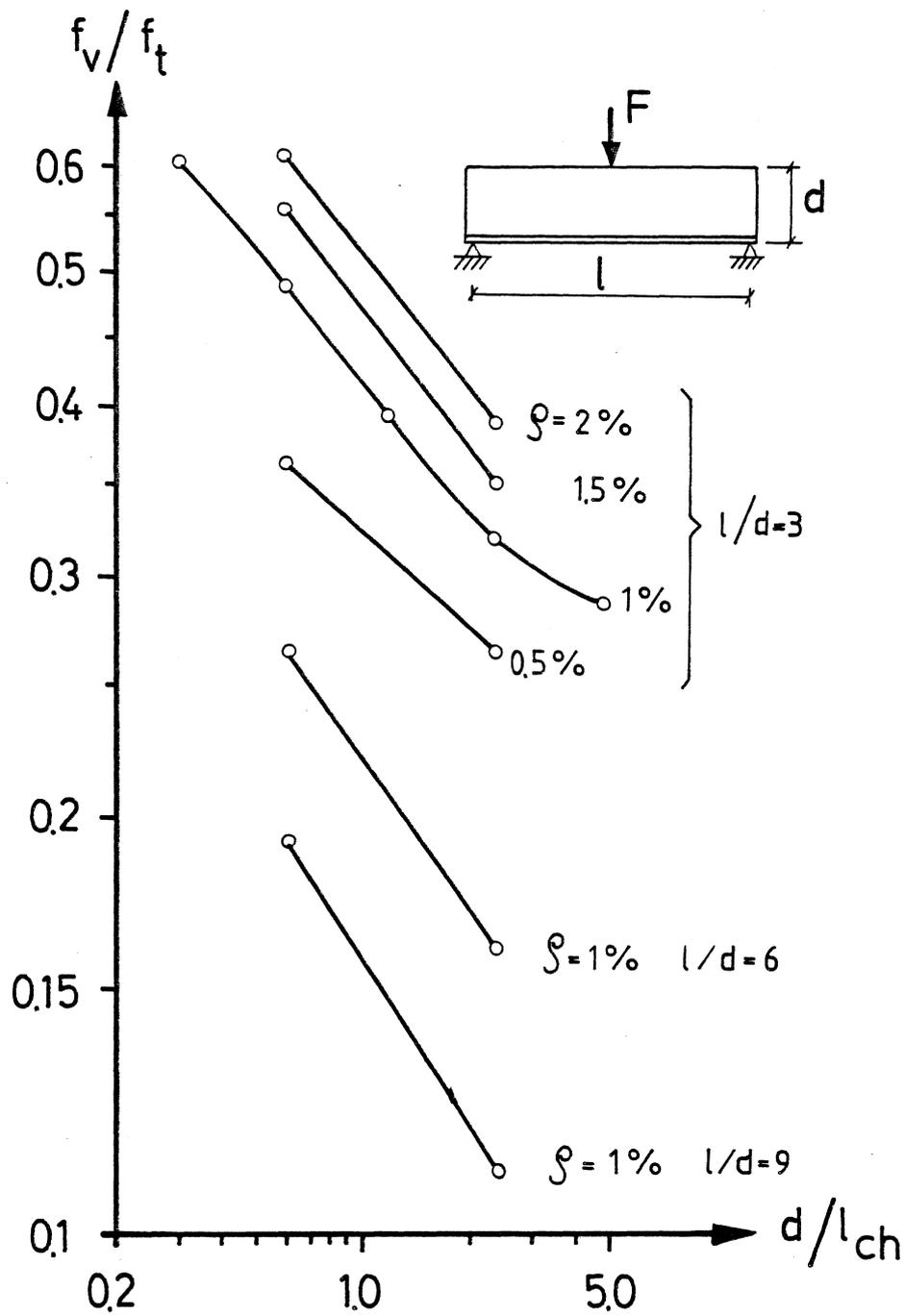


Fig 3 Theoretical shear strength variations of reinforced beams /4/.

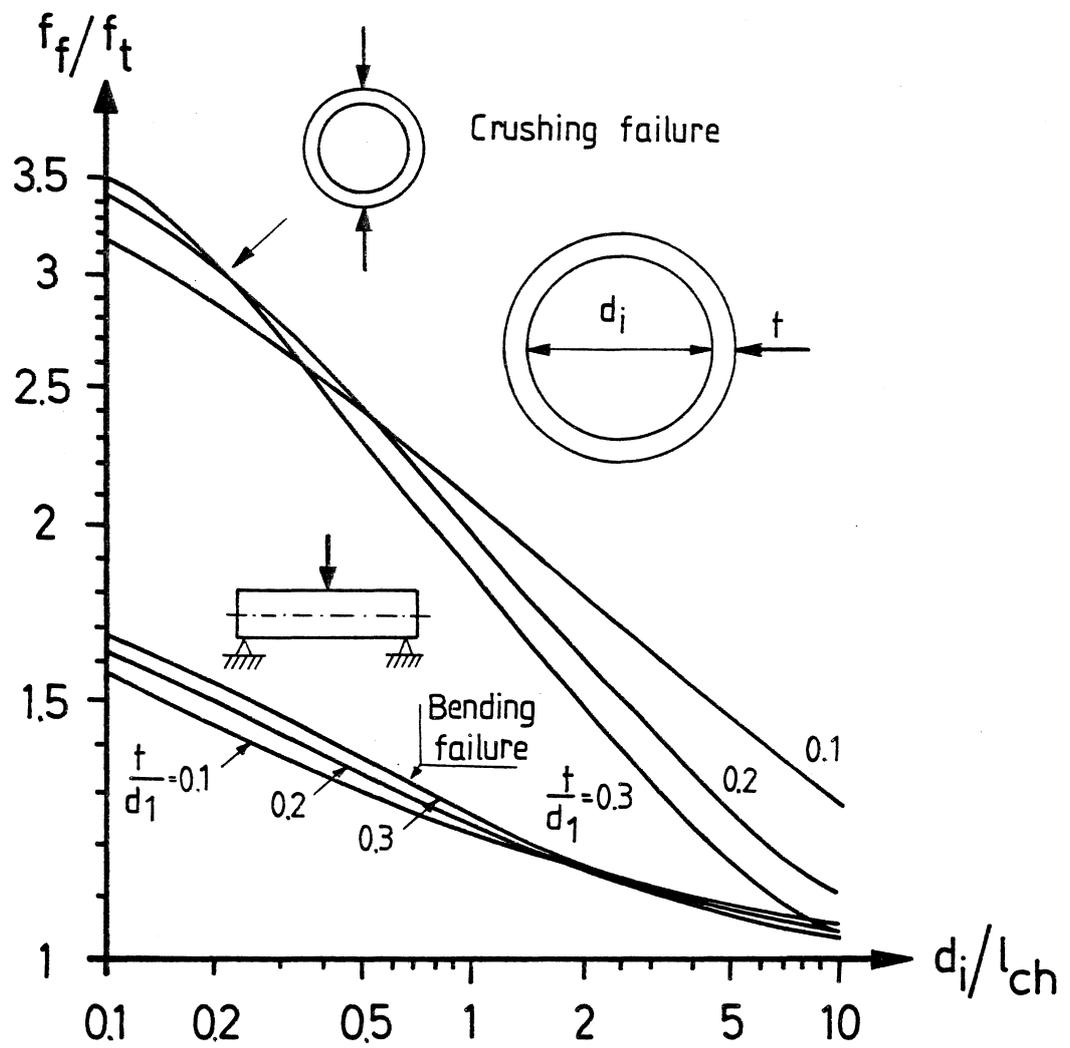


Fig 4 Theoretical strength variation of unreinforced pipes /4/.