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Direction-of-Arrival Estimation in the Presence of Correlated Signals with Non-Ideal Uniform Circular Arrays

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Abstract

The Davies transformation is a method to transform the steering vector of a uniform circular array (UCA) to a vector with Vandermonde form. As this form is similar to that of the steering vector of a uniform linear array (ULA), we can apply to UCAs the many tools that have been developed for ULAs. However, the Davies transformation can be highly sensitive to perturbations of the underlying ideal array model. In this paper, we present two different methods for deriving a more robust transformation. The main underlying idea is to trade off robustness against accuracy of the Vandermonde form. In particular, we consider its application to direction-of-arrival estimation in the presence of correlated signals using root Weighted Subspace Fitting (root-WSF). The effectiveness of these methods are illustrated through numerical examples.

1 Introduction

Uniform circular arrays (UCAs), as a result of their two-dimensional array structure, are able to provide all azimuth coverage. Furthermore, when called for, they are able to provide 180° coverage in elevation. In comparison to uniform linear arrays (ULAs), the special features of UCAs come, however, at the cost of a less friendly steering vector: it is non-Vandermonde. It is well known that, as a direct consequence of the Vandermonde property of a ULA's steering vector, many powerful array-processing techniques have been devised. Some examples include root-MUSIC [1], Dolph-Chebyshev pattern synthesis [2] and spatial smoothing/averaging [3], [4]. Significant efforts have been directed to bringing these techniques over to UCAs via preprocessing on the array outputs. The philosophy behind these preprocessing techniques is to transform the steering vectors of UCAs to Vandermonde form.

There are currently two main approaches to the preprocessing step. Bronez et al [5], [6], [7] map the array outputs of a general two-dimensional array (e.g. a UCA) to a virtual uniform linear array, commonly called an interpolated array. However, the large difference in form between the steering vector of a UCA and a ULA means the mapping can be achieved only for a sector of angles to the minimize approximation error. The preprocessing step is then repeated to obtain full azimuth coverage.

A second approach, first devised by Davies [8], makes use of a spatial-DFT based transformation to map the UCA to a virtual array. This virtual array has no physical interpretation except that its steering vector is Vandermonde. Unlike the Bronez approach, the Davies approach uses only a single closed form transformation matrix and the resulting virtual array can provide all azimuth coverage. Different studies have demonstrated the successful applications of this approach in Dolph-Chebyshev pattern synthesis [9], root-MUSIC DOA estimation with spatial smoothing [10], [11] and optimum beamforming [12] in the presence of correlated signals.

In this paper, we apply the Davies transformation to enable DOA estimation with UCAs in the presence of correlated signals using the root Weighted Subspace Fitting (root-WSF) technique, also known as MODE [7], [13]. However, it has been shown that virtual arrays can be highly sensitive to array imperfections such as element position errors, mutual coupling, and gain and phase mismatches [14]. We outline two different techniques to improve the robustness of virtual arrays to array imperfections – a novel semi-infinite optimization technique [14]-[16] and a simple least squares technique. We demonstrate the effectiveness of our approach via numerical examples. In the examples, we also compare the performance of root-WSF against that of root-MUSIC with spatial smoothing. It is well known that the performance of root-WSF approaches that of maximum likelihood (ML) for sufficient number of snapshots. However, root-MUSIC can outperform root-WSF in the presence of array imperfections [17]. Here we investigate the impact of the robustness techniques on both methods.

2 Problem Formulation

2.1 The Davies Transformation

Consider a UCA with N elements, adjacent-element spacing d and radius r. The nth component of the array response (or *steering*) vector $\mathbf{a}(\theta)$, $n=1,\ldots,N$, to a narrowband signal of wavelength λ arriving from angle θ, $\theta\in[-\pi,\pi]$, is given by

$$a_n(\theta)=G_n(\theta)\exp\left[j\frac{2\pi r}{\lambda}\cos\left(\theta-\frac{2\pi(n-1)}{N}\right)\right] \quad (1)$$

where $G_n(\theta)$ is the complex gain pattern of the nth element.

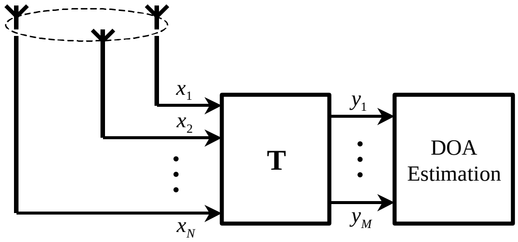

Fig. 1. Transformation for UCAs

Suppose the array elements are all identical and isotropic, i.e., $G_n(\theta)=1$, $n=1,\ldots,N$. Suppose further the antenna element outputs are processed as shown in Fig. 1 where $x_1,\ldots,x_N$ represent the baseband complex output signals of the "real" array and $y_1,\ldots,y_M$, $M<N$, represent the baseband complex output signals of the virtual array. Define the transformation matrix $\mathbf{T}=\mathbf{JF}$ where the matrices $\mathbf{J}\in\mathbb{C}^{M\times M}$ and $\mathbf{F}\in\mathbb{C}^{M\times N}$ are given by

$$\mathbf{J}=\operatorname{diag}\left\{\left[j^{m-1-H}\sqrt{N}J_{m-1-H}\left(\frac{2\pi r}{\lambda}\right)\right]^{-1}\right\} \quad (2)$$

and $$[\mathbf{F}]_{mn}=\tfrac{1}{\sqrt{N}}e^{j2\pi(m-1-H)(n-1)/N}, \quad (3)$$

and where $m=1,\ldots,M$, $n=1,\ldots,N$, $J_k(\cdot)$ denotes a kth order Bessel function of the first kind, and

$$H=(M-1)/2\in\mathbb{Z}. \quad (4)$$

In [10], it is shown that the M-dimensional steering vector of the virtual array, $\tilde{\mathbf{b}}(\theta)$, will take on, approximately, the Vandermonde form

$$\tilde{\mathbf{b}}(\theta)=\mathbf{Ta}(\theta)\cong\left[e^{-jH\theta}\ \cdots\ 1\ \cdots\ e^{+jH\theta}\right]\triangleq\mathbf{b}(\theta). \quad (5)$$

Note, in view of (4), M is odd. Note also that $\mathbf{T}$ is a fixed matrix, i.e., it can be computed off-line.

2.2 Problem Statement

The lack of robustness of the Davies transformation can be traced to the construction of $\mathbf{J}$. As can be seen from (2), for some choices of m, H, and r/λ, the magnitude of one or more of the diagonal elements of $\mathbf{J}$ can approach infinity as the corresponding value of $J_{m-1-H}(2\pi r/\lambda)$ approaches zero. Accordingly, the norm of $\mathbf{T}$ can become very large. But the square of the norm of $\mathbf{T}$ gives a measure of the noise amplification of the transformation matrix. Therefore, for a $\mathbf{T}$ with large norm, small perturbations in $\mathbf{a}(\theta)$ will translate to large perturbations in $\tilde{\mathbf{b}}(\theta)$. Fig. 2 plots $|J_{m-1-H}(2\pi r/\lambda)|$ with zero-crossings in the range of interest for a 15-element UCA and a corresponding 13-element virtual array.

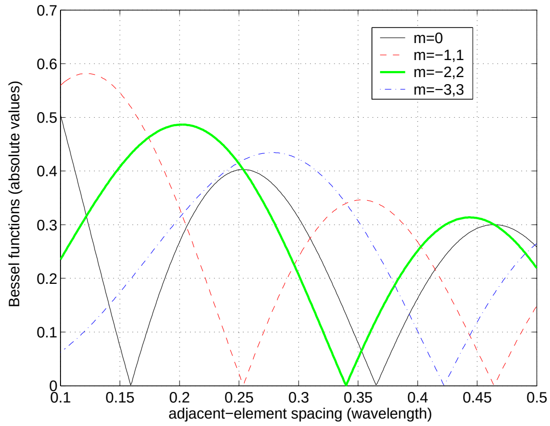

Fig. 2. Bessel functions in $\mathbf{J}$ vs. adjacent-element spacing

Based on the above observation, we formulate the following semi-infinite optimization (SIP) problem to find a more robust transformation matrix. The basic idea is to trade-off the approximation error in the transformation of $\mathbf{a}(\theta)$ for robustness.

Denote the robust transformation matrix by $\mathbf{U}\in\mathbb{C}^{M\times N}$. We find $\mathbf{U}$ as follows:

$$\min_{\mathbf{U}}\|\mathbf{U}\|_F^2 \quad (\mathcal{P}1)$$

subject to $|\mathbf{Ua}(\theta)-\mathbf{b}(\theta)|\le\mathbf{e}, \quad \forall\theta\in[-\pi,\pi]$

where $\mathbf{e}=[\varepsilon_1\ \varepsilon_2\ \cdots\ \varepsilon_M]^T$, $\varepsilon_m\in\mathbb{R}_+$, $m=1,\ldots,M$, $\|\cdot\|_F$ denotes Frobenius norm, and $|\cdot|$ is the absolute value norm

$$|\mathbf{x}|=\begin{bmatrix}\max\{|\mathrm{Re}\{x_1\}|,|\mathrm{Im}\{x_1\}|\}\\ \vdots\\ \max\{|\mathrm{Re}\{x_M\}|,|\mathrm{Im}\{x_M\}|\}\end{bmatrix},$$

and where $\mathbf{x}\in\mathbb{C}^M$ and x_m is the mth element of $\mathbf{x}$.

Now, since the rows of $\mathbf{U}$ are not related in the above formulation, ($\mathcal{P}1$) can be solved, row-by-row, as follows:

For $m=1,\ldots,M$, $$\min_{\mathbf{u}_m}\|\mathbf{u}_m\|_F^2 \quad (\mathcal{P}2)$$

subject to $\left|\mathrm{Re}\{\mathbf{u}_m^T\mathbf{a}(\theta)-b_m(\theta)\}\right|\le\varepsilon_m$

and $\left|\mathrm{Im}\{\mathbf{u}_m^T\mathbf{a}(\theta)-b_m(\theta)\}\right|\le\varepsilon_m, \quad \forall\theta\in[-\pi,\pi]$,

where $\mathbf{u}_m^T$ is the mth row of $\mathbf{U}$. The advantage of ($\mathcal{P}2$) is that it allows the original problem ($\mathcal{P}1$) to be solved efficiently by the dual parameterization method given in [14]-[16].

Remarks

1. The robustness of $\mathbf{U}$ depends on the choice of ε_m, $m=1,\ldots,M$. The larger we choose ε_m, the greater is the approximation error but correspondingly, the smaller is $\|\mathbf{u}_m\|_F$.
2. If $\varepsilon_m\ge1$, then for that m, ($\mathcal{P}2$) has the trivial solution $\mathbf{u}_m=\mathbf{0}$. This follows since $|b_m(\theta)|=1$.
3. As a rough guide to robustness, the square of the norm of each row of $\mathbf{U}$ should not greatly exceed N/M.

The reasoning is as follows. Suppose array imperfection can be modeled as a (complex) noise output from the antenna elements. Suppose the real and imaginary parts of these "imperfection noise" terms are independent with identical variance σ_x^2, and the imperfection noise terms of all the antenna elements are mutually independent. The total imperfection noise from the array of N elements is then given by $2N\sigma_x^2$. Suppose the transformation matrix has Frobenius norm $\|\mathbf{U}\|_F$. The total imperfection noise at the output of the virtual array is then given by $2\|\mathbf{U}\|_F^2\ \sigma_x^2$. If the transformation is required to not increase noise, then we require $2N\sigma_x^2 \geq 2\|\mathbf{U}\|_F^2\ \sigma_x^2$, or $\|\mathbf{U}\|_F^2 \leq N$. Finally, suppose the noise gain is distributed uniformly over the elements of the virtual array. This yields $\|\mathbf{u}_m\|_F^2 = \|\mathbf{U}\|_F^2 / M \leq N/M$.

Alternatively, the idea of trading-off approximation error for a more robust transformation can be captured in the following least squares (LS) formulation:

$$\min_{\mathbf{u}_m}\left\{\alpha\int_{-\pi}^{\pi}\left\|\mathbf{u}_m^H\mathbf{a}(\theta)-b_m(\theta)\right\|_F^2 d\theta+(1-\alpha)\|\mathbf{u}_m\|_F^2\right\} \quad (\mathcal{P}3)$$

where $\alpha \in (0,1)$ is the weighting factor determined by the desired level of trade-off. ($\mathcal{P}3$) can be solved easily by differentiating the objective function w.r.t $\mathbf{u}_m$ and setting the derivative to $\mathbf{0}$.

3 Numerical Examples

3.1 The UCA

We considered a scenario of highly correlated signals (correlation of $0.99\exp(j\pi/4)$), with wavelength λ and a 15-element UCA with $d = 0.46\lambda$. A 13-element virtual array was chosen for root-WSF [7], [13]. A whitening procedure similar to [7] was incorporated into root-WSF since noise in the virtual array is not spatially white [10]. For root-MUSIC, forward-backward spatial smoothing [4] was applied to the 13-element virtual array to restore the rank of the covariance matrix, followed by a whitening procedure [10].

Table 1 summarizes the squared-norm and maximum real and imaginary errors of each row of the Davies matrix for this UCA. As can be seen, the squared-norms of rows 6 and 8 greatly exceed $N/M = 15/13 = 1.1538$. Indeed, it is these very rows that render the Davies matrix non-robust. For the robust transformation matrix, our strategy is to retain as many rows of the Davies matrix as possible except for rows with large squared-norms. Accordingly, for the SIP technique, we replaced rows 6 and 8 with rows found by solving ($\mathcal{P}2$) with ε_m set to 0.55[1]. Note that the resulting ε_m in Table 1 is slightly different from 0.55 due to the finite number of iterations in the optimization procedure. The characteristics of the robust transformation matrix are summarized also in Table 1. Note the increase in approximation error in rows 6 and 8. Note also that the resulting transformation vectors for rows 6 and 8 are related by a phase rotation. This also holds true when the optimization was carried out for any other similar pairs of rows in Table 1. In other words, we only need to compute ($\mathcal{P}2$) once for each such pairs of rows.

	Davies Matrix		**Robust Matrix**	
Row #	**Squared-norm**	**Max Error**	**Squared-norm**	**Max Error**
1, 13	0.5896	0.1677		
2, 12	0.5375	0.0634		
3, 11	2.2956	0.0460		
4, 10	2.7382	0.0159		
5, 9	0.7161	0.0023		
6, 8	174.437	0.0102	18.7656	0.5534
7	0.7433	0.0003		

Table 1. Characteristics of the Davies and robust transformation matrices for $N = 15$, $M = 13$, $r = 1.106\lambda$

3.2 Case Study 1

First we look at the case of 2 signals arriving from directions 0º and 30º. The signal-to-noise-ratios (SNRs) of both signals are varied between 0 and 15 dB. We used 2 subarrays of 12 elements each for spatial smoothing. The performance is measured in terms of the average root mean squared error (RMSE) of the DOAs over all signals present. Fig. 3 summarizes the average RMSE of the DOA estimates for the ideal, non-ideal and robust UCAs. The non-ideal and robust UCAs were subject to perturbations drawn from standard normal distributions with zero means. The standard deviation for either real or imaginary part of gain perturbations is 0.05 (relative to 1); for either *x*- or *y*-axis element position perturbation, 0.01λ; and for either real or imaginary part of mutual coupling, 0.01 (relative to 1). We assumed that mutual coupling is significant only for adjacent UCA elements. Here we used 200 snapshots and 1000 Monte-Carlo trials, where each Monte-Carlo trial corresponds to a realization of array perturbations. From Fig. 3, it is clear that in this case root-WSF with robust UCA has significantly better performance than that with non-ideal UCA. At low SNR, root-WSF with robust UCA appears to outperform root-WSF with ideal UCA. This could be due to the sensitivity of root-WSF to amplification of "noise" (for rows in Davies matrix with large norm) resulting from the number of finite snapshots at low SNR.

As for root-MUSIC which is inherently more robust towards array imperfections [17], the performance of the robust UCA is roughly the same as the non-ideal UCA for $\varepsilon_m = 0.55$. A very marginal improvement may be obtained by optimizing ε_m for the best robust UCA performance. We also note that the approximation involved in the transformation introduces a small bias in the DOA estimates of both methods [11] which has been taken into account in the RMSE measure.

[1] A simple search procedure is used to find the value of ε_m that gives the optimum robust performance w.r.t root-WSF.

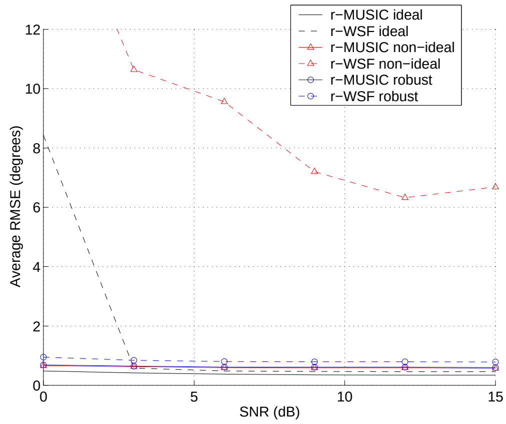

Fig. 3. RMSE DOA performance of UCAs: 2-signal case

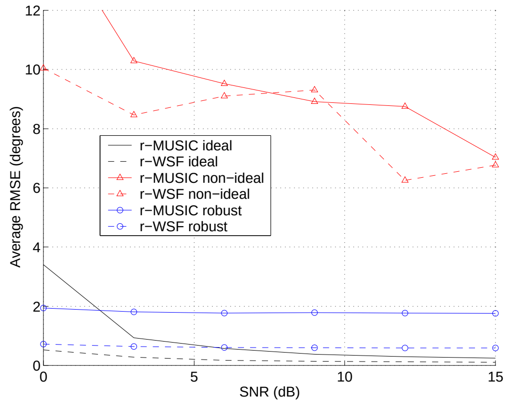

Fig. 4. RMSE DOA performance of UCAs: 5 signal case

3.3 Case Study 2

Next, we had 5 signals arriving from the directions -150°, –90º, 0º, 37º, 85º with SNRs varied between 0 and 15 dB. For this case, we used 5 subarrays of 9 elements each for the spatial smoothing. For this case, setting $\varepsilon_m = 0.55$ again gives the best performance for root-WSF. In this case, we observe significant improvement in the robust UCAs for both root-WSF and root-MUSIC. The presence of more highly correlated signals presents a bigger challenge for root-MUSIC as more subarrays of a smaller subarray size are used. As a result, root-MUSIC is less robust than the 2-signal case. In fact, if the correlation between signals is real or close to real, then the performance of root-MUSIC (with spatial smoothing) deteriorates further as the forward-backward smoothing effectively reduces to forward only smoothing under this condition [18].

3.4 Discussions

With the LS technique to find the optimum $\mathbf{u}_m$ and α, we found in our simulations that the procedure, though computationally simple and also effective, does not perform as well as the SIP technique[2]. This could be due to the constrained formulation of ($\mathcal{P}2$) which gives a more uniform approximation error for the virtual array.

It is observed from the case studies that the relative performance of root-WSF and root-MUSIC with spatial smoothing depends on several parameters which include the number of highly correlated signals present (which also influences the required size of subarray), the nature of signal correlation (real or complex) [18] and the sensitivity to array imperfections [17]. While root-WSF has higher computational complexity than root-MUSIC, root-WSF demonstrates a more consistent RMSE performance than root-MUSIC for the robust UCAs where ε_m is optimized separately for each method.

It is worthwhile to note that while the robustness procedure is able to improve the performance of non-ideal UCAs with critical array parameters, the best robust UCA performance is still poorer than that of UCAs with non-critical parameters. Here, we give an example of the performance of a UCA with non-critical parameters, viz. $N = 15$, $M = 13$, $d/\lambda = 0.4$. There are 5 signals and 5 subarrays of 9-element each as described in case study 2. In Fig. 2, we see that $d/\lambda = 0.4$ is far from any zero-crossings. Indeed, Table 2 further confirms that the norm values are acceptable. In Fig. 5, we show the RMSE performance for the ideal, non-ideal and robust UCAs. The robust UCA operates on rows 4 and 10 and uses $\varepsilon_m = 0.1$ for the best performance. The degradation in performance in non-ideal UCA is smaller than case studies 1 and 2, and the robust procedure, as expected, gives only very marginal improvements, if at all. Further we note that in this case the performance of root-WSF is better than that of root-MUSIC in the ideal, non-ideal and robust UCAs.

	Davies Matrix		**Robust Matrix**	
Row #	**Squared-norm**	**Max Error**	**Squared-norm**	**Max Error**
1, 13	1.0587	0.0888		
2, 12	0.5020	0.0203		
3, 11	0.5380	0.0062		
4, 10	6.4713	0.0058	5.2178	0.101
5, 9	1.0560	0.0006		
6, 8	0.9297	0.0001		
7	2.5200	0.0001		

Table 2. Characteristics of the Davies transformation matrices for $N = 15$, $M = 13$ and $r = 0.962\lambda$

The above observation also indicates that while robustness may be introduced in the vicinity of critical parameters, it is prudent to avoid them altogether if possible. Nevertheless, for large arrays with more frequent zero crossings than that of Fig. 2, and where these parameters are not easily avoided, our proposed method can be effective.

[2] A simple search procedure is used to find the value of α that gives the optimum robust performance w.r.t root-WSF.

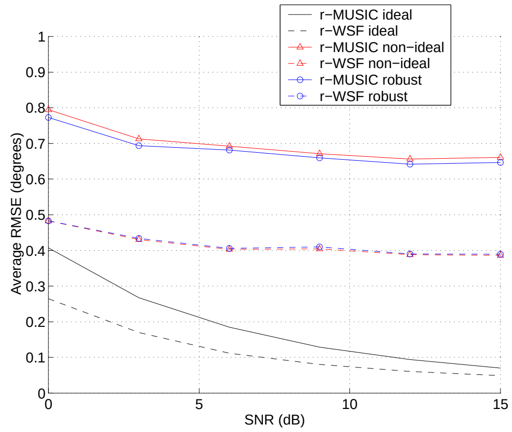

Fig. 5. RMSE DOA performance of UCAs: 5 signal case for $r = 0.962\lambda$

5 Conclusions

In this paper, we addressed the important problem of direction finding with imperfect UCAs in correlated signal environments. We showed that a crucial step in the solution is to find a robust transformation matrix to transform the steering vector of the UCA to one with Vandermonde form. The robust matrix is found by posing and solving a quadratic semi-infinite optimization problem which trades-off the Vandermonde approximation error with a matrix of lower norm. We showed that, by an appropriate formulation, we can decompose the problem into a set of much simpler optimization problems which can then be solved efficiently using the dual parameterization method of [14]-[16]. Each sub-problem yields a row of the robust transformation matrix. Alternatively, we also proposed a least squares formulation that operates on similar principles. The robustness of the new transformation matrix is demonstrated by numerical examples where we investigated the performance of the robust UCA with respect to root-WSF and root-MUSIC with spatial smoothing. We note that the robustness procedure appears to be particularly effective for root-WSF. Finally, we remark that it is trivial to extend the presented work to elevation angles for hemispherical coverage [11] where a different **J** is used for each elevation angle of interest.

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