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IDENTIFICATION AND
ADJOINT PROBLEMS OF
PROCESS COMPUTER CONTROL

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Lund Institute of Technology
Division of Automatic Control

INSTITUTT FOR ATOMENERGI
OECD Halden Reactor Project

IDENTIFICATION AND ADJOINT PROBLEMS OF
PROCESS COMPUTER CONTROL

by

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1. INTRODUCTION

Computer-based control and supervision strategies for complex technical processes, like those in nuclear plants, make to an increasing degree use of mathematical models of the respective process. Depending on the specific application, the approach to modelling the process may be quite different. Therefore, when modelling a nuclear reactor plant, the purpose of the model must be stated clearly. It is very difficult to use a detailed differential equation model, based on only physical assumptions for controlling purposes. Rather a much less detailed model might give results which are accurate enough to design a suitable controller.

In the case of the Halden reactor there are different needs of models. They should be used partly for load follow control laws, for adaptive control or flux control, partly for steady state stochastic control. In the former case a linear model with time variable coefficients is relevant, in the latter case a linear stochastic model with constant coefficients could be suitable.

Models can be established with the help of physical principles, by evaluating measured input and output signals or by a combination of the two ways. The latter method is of particular importance in the case of processes with varying characteristics, e.g. during large transients of individual variables. Here, a basic structure of the process description would be set up from physical considerations or from some preceding analysis of measured data, while the current changes of the model parameters had to be tracked by on-line identification routines. In view of the increasing importance of suitable techniques for process identification from measured data - both for flexible on-line modelling and in order to save extensive physics calculations - an attempt has been made in this paper to present an outline of possible applications of on-line and off-line identification methods and some associated problems related to the acquisition of process data. The presented techniques refer to applications in connection with the research programme on computer control at the Halden Project.

In the first part of the paper an on-line identification algorithm is presented. The model is derived recursively from input - output measurement data during normal operation. A recursive algorithm based on generalized least square method was used. This method is able to track parameters of time varying processes. Multivariable processes were investigated on simulations and on real plant data.

In the second part a Maximum Likelihood technique is applied to fit input - output experimental data to a linear stochastic model with constant parameters.

The third part of the paper considers design of digital filters used for prefiltering of data. These data were then used to prepare the data for the reported on-line identification calculations.

A digital spectrum density analyser based on digital filters, and a new method, which allows to determine the ratio between the prompt and delayed signal response of self-powered vanadium neutron detectors is described.

The problem of exciting the process with different perturbation signals during operation was considered. Digitally designed low-pass-, band-pass-, and time-derivative filters were used for filtering the process input and output signals. When identifying the parameters of a continuous model, the unmeasurable time derivatives of the signals are obtained by a set of time derivative filters.

In simulations, process parameters were estimated in steady state and during transients. Using arbitrary initial values for the parameter vector, it converges to the correct value. To test the identification algorithm with real process data, perturbation experiments on the HBWR plant were

performed, to obtain data at different operating conditions. The process parameters resulting from these data by the on-line estimator are comparable with those generated by off-line calculations.

2. IDENTIFICATION OF THE HBWR-DYNAMICS

2.1 On-Line Identification

2.1.1 Choice of the Process Description

The process is assumed to be given in its functional form as a mathematical model. The parameters of this model have to be estimated. An arrangement can always be found for the form of the model to conform with the general relationship,

$$\begin{pmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} \underline{x}_1^T \cdot \underline{a}_1 \\ \vdots \\ \underline{x}_i^T \cdot \underline{a}_i \\ \vdots \\ \underline{x}_m^T \cdot \underline{a}_m \end{pmatrix} \quad /1/$$

where*

$$\underline{x}_i^T = [x_{i1} \ x_{i2} \ \dots \ x_{in}] ; \underline{a}_i^T = [a_{i1} \ a_{i2} \ \dots \ a_{in}]$$

The a_{ij} are the unknown parameters and the x_{ij} are a set of functions that can be derived from the normal operating data available from the process. y_i are observations contaminated by noise. The measured process input - output data are passed through digital filters, before entering the On-Line Estimator.

Two among the better known dynamical models were chosen:

Difference equation model

Differential equation model

Both models are "structural models". The estimated process parameters will therefore be affected by noise-induced bias. This can be corrected by different means. If the statistical properties of the noise are available, some suitable estimation methods, e.g. the Instrumented Variable method could be chosen (5). If these properties are not known, then some prefiltering can help to reduce the noise/signal ratio of the measurements.

- Difference Equation Form

The general mathematical description for a multivariable process in the discrete form can be written in the vector-matrix form containing a set of first order difference equations:

$$\begin{pmatrix} v_1(k) \\ \vdots \\ v_i(k) \\ \vdots \\ v_m(k) \end{pmatrix} = \begin{pmatrix} \underline{\varphi}_1^T(T_s) \underline{v}(k-1) + \underline{g}_1^T(T_s) \underline{u}(k-1) \\ \vdots \\ \underline{\varphi}_i^T(T_s) \underline{v}(k-1) + \underline{g}_i^T(T_s) \underline{u}(k-1) \\ \vdots \\ \underline{\varphi}_m^T(T_s) \underline{v}(k-1) + \underline{g}_m^T(T_s) \underline{u}(k-1) \end{pmatrix} \quad /2/$$

as this shall conform with /1/, one finds the following identities

* In this paper, vectors are denoted by underlining the respective variable.

$$\underline{y} \equiv \underline{v} (k)$$

$$\underline{x}_1^T = [v_1 (k-1), v_2 (k-1) \dots v_m (k-1), u_1 (k-1), u_2 (k-1) \dots u_m (k-1)]$$

$$\underline{a}_1^T = [\varphi_{11} \ \varphi_{12} \ \dots \ \varphi_{im} \ g_{11} \ g_{12} \ \dots \ g_{im}]$$

- *Differential Equation Form*

The vector-matrix form containing a set of first order differential equations reads:

$$\begin{pmatrix} \dot{v}_1(t) \\ \vdots \\ \dot{v}_i(t) \\ \vdots \\ \dot{v}_m(t) \end{pmatrix} = \begin{pmatrix} \underline{b}_1^T(t) \underline{v}(t) + \underline{d}_1^T(t) \underline{u}(t) \\ \vdots \\ \underline{b}_i^T(t) \underline{v}(t) + \underline{d}_i^T(t) \underline{u}(t) \\ \vdots \\ \underline{b}_m^T(t) \underline{v}(t) + \underline{d}_m^T(t) \underline{u}(t) \end{pmatrix} \quad /3/$$

going also in this case confirm with /1/, the following identities result:

$$\underline{Y} = \underline{\dot{V}}$$

$$\underline{X}_i^T = [V_1(t), V_2(t), \dots V_m(t), u_1(t), u_2(t) \dots u_m(t)]$$

$$\underline{a}_i^T = [b_{i1} \ b_{i2} \ b_{i3} \ \dots \ b_{im}, d_{i1} \ d_{i2} \ \dots \ d_{im}]$$

- *Relation between Difference and Differential Form*

Sometimes it is more suitable to have a difference model, sometimes a differential model. On the other hand, it is convenient to have available the matrices Φ, G and B, D . For the linear, time invariant case with a constant sampling interval exists a birelation between these two pairs of matrices.

$$B = 1/T_s \ln \Phi$$

$$D = (\Phi - I)^{-1} B \cdot G \quad /4/$$

where

$$B = \begin{pmatrix} \underline{b}_1^T \\ \vdots \\ \underline{b}_i^T \\ \vdots \\ \underline{b}_m^T \end{pmatrix}; \quad D = \begin{pmatrix} \underline{d}_1^T \\ \vdots \\ \underline{d}_i^T \\ \vdots \\ \underline{d}_m^T \end{pmatrix}; \quad \Phi = \begin{pmatrix} \varphi_1^T \\ \vdots \\ \varphi_i^T \\ \vdots \\ \varphi_m^T \end{pmatrix}; \quad G = \begin{pmatrix} \underline{g}_1^T \\ \vdots \\ \underline{g}_i^T \\ \vdots \\ \underline{g}_m^T \end{pmatrix}$$

The matrix B can be calculated using series expansions:

$$B = \frac{\Phi^*}{T_s} - \frac{\Phi^{*2}}{2 T_s} + \frac{\Phi^{*3}}{3 T_s} - \frac{\Phi^{*4}}{4 T_s} + \dots \quad /5/$$

with $\Phi^* = \Phi - I$, /5/ converges surely, when the elements of the matrix $(\Phi - I)$ are in the range -1 to +1, except the elements are all equal.

The other way around reads:

$$\begin{aligned} \Phi &= e^{BT_s} \\ G &= B^{-1} (\Phi - I) D \end{aligned} \tag{6/}$$

The calculation is done by series expansion:

$$\Phi = I + BT_s + \frac{1}{2!} B^2 T_s^2 + \frac{1}{3!} B^3 T_s^3 + \dots + \frac{1}{k!} B^k T_s^k + \dots$$

2.1.2 A Recursive On-Line Identification Method for Multivariable Processes

Since the methods treated in this work shall be applicable to some practical cases, they have to be as simple as possible. The best known technique to find the parameters of an unknown process is the least square technique. In this work the so-called "equation error approach" will be investigated (ref. 6, 7). This approach is simple and general and entails modelling the dynamics of the process by equation /1/. Each row in this vector-matrix-equation is treated by a separate estimation procedure running concurrently. An equation error can be defined:

$$e_i = \underline{x}_i^T \hat{\underline{a}}_i - y_i \tag{7/}$$

$\hat{\underline{a}}_i$ is the estimate of the i'th parameter row vector \underline{a}_i .

Substituting the i'th element from /1/ in /7/ gives:

$$e_i = \underline{x}_i^T (\hat{\underline{a}}_i - \underline{a}_i) \tag{8/}$$

The process can be identified by choosing these estimates $\hat{\underline{a}}_{ij}$ which minimize the criterion function:

$$(I_2)_i = \sum_{l=1}^k [\underline{x}_i^T \hat{\underline{a}}_i - y_{il}]^2 \tag{9/}$$

The problem is a *linear least-squares fitting problem*.

This equation error approach can be applied whenever the mathematical models of the process can be manipulated into equation /1/.

The minimisation of /9/ requires that all partial derivatives of (I_2) with respect to each of the parameter estimates should be set simultaneously to zero:

$$\nabla_{\hat{\underline{a}}_i} (I_2)_i = \left(\sum_{l=1}^k \underline{x}_{il} \underline{x}_{il}^T \right) \hat{\underline{a}}_i - \sum_{l=1}^k \underline{x}_{il} y_{il} = 0 \tag{10/}$$

where \underline{x}_i^T is the transpose of \underline{x}_i and k is the sampling instance.

The solution is:

$$\hat{\underline{a}}_{ik} = \underline{P}_{ik} \underline{B}_{ik} \tag{11/}$$

where

$$P_{iK} = \left(\sum_{l=1}^k \underline{x}_{il} \underline{x}_{il}^T \right)^{-1} \quad /12/$$

is an nxn time variable weighting matrix and

$$B_{iK} = \sum_{l=1}^k \underline{x}_{il} y_{il} \quad /13/$$

is an nxl vector.

For the parameter estimation a recursive form of equation /11/ is needed in which the estimator vector \underline{a}_i at the k-th instant is a linear sum of the estimate at the (k-1)-th instant plus a corrective term based on the data at the k-th instant:

$$P_{i,k}^{-1} = P_{i,k-1}^{-1} + [\underline{x}_{ik} \underline{x}_{ik}^T]$$

and

/14/

$$B_{iK} = B_{i, k-1} + \underline{x}_{ik} y_{ik}$$

After some rearrangements of equations /11/ and /14/ and considering a stochastic interpretation of the problem, a non-stationary recursive least-squares estimator can be obtained (Ref. 6).

A "statistical model" of the parameter variations can be written as:

$$\underline{a}_k = \phi(k, k-1) \underline{a}_{k-1} + \underline{q}_{k-1} \quad /15/$$

where ϕ is an nxn transition matrix and \underline{q}_{k-1} is an n vector of random disturbance elements, having zero mean and covariance matrix $E(\underline{q}_i \underline{q}_j^T) = Q \delta_{ij}$ (δ is the Kronecker delta function).

In the light of these above considerations it is possible to construct a "dynamic equation error" algorithm, which in its most useful case, when $\phi = I$ (random walk model of parameter variations) reads as follows: (the following equations are ment for the i'th row in equation /1/. To ease the writing, the i is cancelled):

$$\hat{\underline{a}}_k = \hat{\underline{a}}_{k-1} - P_{k/k-1} \underline{x}_k [1 + \underline{x}_k^T P_{k/k-1} \underline{x}_k]^{-1} \cdot (\underline{x}_k^T \hat{\underline{a}}_{k-1} - y_k)$$

$$P_{k/k-1} = P_{k-1} + E \quad /16/$$

$$P_k = P_{k/k-1} - P_{k/k-1} \underline{x}_k [1 + \underline{x}_k^T P_{k/k-1} \underline{x}_k]^{-1} \underline{x}_k^T P_{k/k-1}$$

The time variable weighting matrix P_k is a strictly decreasing function of the sample number. Large corrections are possible at the start, less correction is needed, when the estimation progresses and the estimates converge. The great advantage of the recursive algorithm /16/ is that it provides for a minimum of computer storage and that there is no need for direct matrix inversion since the expression $(1 + \underline{x}^T P_{k-1} \underline{x}_k)$ is a scalar.

E is a positive, definite matrix analogous in its effect to the covariance matrix Q in the regression case. Individual elements can be limited to different degrees, specifying different expected rates of parameter variations. If necessary, E can be modified later experimentally.

In computational terms, the E-matrix limits the lower bound on the elements of matrix P, preventing the elements from becoming too small and allowing for continuous correction of \hat{a} . Starting values for P and \hat{a} have to be specified.

2.1.3 Considerations on Excitation Signals and Data Filtering

- Excitation Signals

Process identification is to a great extent dependent on the kind of input signals which excite the unknown process. If the process remains inactive due to a lack of input stimulus, the measurement matrix can become singular. The elements of the measurement vector \underline{x} shall not be linearly dependent or strongly correlated. The number of discrete frequency components in a periodic signal should exceed $d > m/2$ where m is the order of the model equation. In this work periodic rectangular and pseudo random binary noise signals (PRBNS) have been used.

- Data Filtering

Digital lowpass-, bandpass-, and time derivative filters, designed by the bilinear z-transform, have been used.

If the process parameters of a linearized process model shall be estimated on-line, the process input- and output data have to be brought into such a form that they fit into this linearized model. This means the mean value of the data has to be subtracted from the signal itself, in order to get rid of biased measurements. Furthermore this difference will be normalized to this mean value. Then the equations describing the process are dimensionless. If the process is described in difference equation form, then these normalized differences are the input signals to the on-line identification algorithm. In case the process is given in differential equation form, this normalized difference-signals are passed through a set of digital transformation filters to obtain the first time derivative of these normalized difference signals. These transformation filters are composed by the method of "multiple filters" (Ref. 5).

2.1.4 Simulation Results

Various simulations have been performed in order to test out the on-line estimator. As a basis for all simulation tests a known model of the HBWR plant at a certain operating condition was used. It serves as a comparison for the results. Figure 2.1 shows the general simulation and test scheme.

The following partial HBWR-model was considered. In the discrete version:

$$p_2(k) = \varphi_{11} p_2(k-1) + \varphi_{12} p_3(k-1) + g_{11} p_1(k-1) + g_{12} \text{VB 282}(k-1) \quad /17a/$$

$$p_3(k) = \varphi_{21} p_2(k-1) + \varphi_{22} p_3(k-1) + g_{21} p_1(k-1) + g_{22} \text{VB 282}(k-1) \quad /17b/$$

where

- p_1 = the primary pressure
- p_2 = the secondary pressure
- p_3 = the tertiary pressure
- VB 282 opening of steam outlet valve

and in the continuous version:

$$\dot{p}_2 = b_{11} p_2 + b_{12} p_3 + d_{11} p_1 + d_{12} \text{VB 282} \quad /18a/$$

$$\dot{p}_3 = b_{21} p_2 + b_{22} p_3 + d_{21} p_1 + d_{22} \text{VB 282} \quad /18b/$$

If /17/ and /18/ are modelled into equation /1/, one gets for:

e.g.: /17a/: $y_1 = p_2(k)$; $\underline{x}_1^T = [p_2(k-1), p_3(k-1), p_1(k-1), \text{VB 282}(k-1)]$

$$\underline{a}_1^T = [\varphi_{11}, \varphi_{12}; g_{11}, g_{12}]$$

/18a/: $y_1 = p_2$; $\underline{x}^T = [p_2, p_3, p_1, \text{VB 282}]$

$$\underline{a}_1^T = [b_{11}, b_{12}; d_{11}, d_{12}]$$

Figure 2.2 shows the resulting estimated parameters in case of parameter tracking for the discrete model.

The parameters φ_{12} , g_{11} and g_{21} were changed along ramps. The estimated ramps of the parameters g_{11} and g_{21} are almost identical with the actual ramps, while the estimated ramp in φ_{12} is lacking a bit behind the actual ramp. It also can be seen that changes in some parameters influence the others during these changes. The starting values for the P matrix was $10^4 \times I$ and the elements for E were chosen due to the parameter changes as follows:

for equation /17a/

$$E1 = \begin{pmatrix} 10^{-4} & & 0 \\ & 10^{-1} & \\ 0 & & 10^{-1} \\ & & & 10^{-4} \end{pmatrix}$$

and for equation /17b/

$$E2 = \begin{pmatrix} 10^{-4} & & 0 \\ & 10^{-4} & \\ 0 & & 10^{-1} \\ & & & 10^{-4} \end{pmatrix}$$

As an example one element of the matrix P1 is plotted in Figure 2.3

2.1.5 Measurements on the HBWR Plant

- Test of the On-Line Identifier with HBWR Plant Data

To test the on-line estimator with real process data, an experiment with PRBS-sequences applied to the HBWR plant was performed to collect data. The control elements (steam outlet valve, valve in subcooling circuit and rods) were perturbed simultaneously with uncorrelated PRBS-signals. These signals together with responses of a number of plant variables were logged in time intervals of 2 seconds.

The model equation /17/ was assumed:

$$P_{\text{NORM}} = \frac{P_D - \bar{P}_D}{\bar{P}_A}$$

where

P_{NORM} = normalized pressure

P_D = pressure measurement from differential meter

- \bar{P}_D = mean value of P_D over the observation interval
 \bar{P}_A = mean value of the pressure measurements from absolute meter over the observation interval

Test runs with precalculated mean values were made, and runs, where a digital filter combination was used to obtain P_{NORM} . For the latter case the estimated parameters are given in Figure 2.4. The initial P-matrix was $10^4 \times I$ and E was $10^{-4} \times I$. The estimates show a slow convergence and some of them are also biased. This is a result of the noisy data.

Comparison of the Results with the Model obtained by the Maximum Likelihood Method

The estimated parameters for model /17/, obtained by method /16/ supplied to real plant data were compared with parameters obtained off-line with the Maximum Likelihood Method.

	φ_{11}	φ_{12}	g_{11}	g_{12}	φ_{21}	φ_{22}	g_{21}	g_{22}
On-Line	.845	.027	.145	-.008	.306	.67	-.06	-.04
Off-Line	.8194	.04312	.1311	-.00043	.035	.952	.00265	-.0189

Parameters with absolute high numerical values are in good agreement. Those with lower values show less agreement. The estimator has then obviously difficulties to find these small values.

2.1.6 Conclusions

The "dynamic least square" algorithm is very similar to the Kalman filter. It is able to track parameter variations, is most flexible; needs due to the recursive character little computer space, and is relatively simple to handle. The dimension of the parameter matrices Φ , G , respectively B , D which can be handled is restricted by the computer speed. If the process transients are relatively slow, then the sampling interval can be increased. In this way there is then more time available, necessary for estimating parameter matrices of bigger dimensions.

The identification programmes are written in Fortran. The computation time can be down-scaled substantially when the routines are coded in Assembler.

List of Symbols

- y measurement vector
 x_i the i'th measurement vector
 a_i the i'th process parameter vector
 $\underline{v}(k)$ state variable vector of the discrete model at time k
 Φ state transition matrix
 G state transmission matrix
 B coefficient matrix

D	driving matrix
ϕ_i	i'th row vector of Φ
g_i	i'th row vector of G
b_i	i'th row vector of B
d_i	i'th row vector of D
\dot{y}	first time derivative of the state vector of the continuous model
I	identity matrix
e_i	equation error for the i'th row in equation /1/
\hat{a}_i	estimated parameter vector for the i'th row in equation /1/
P_k	time variable weighting matrix
T_s	sampling interval

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- Fig. 2.2 Parameter Tracking for the Discrete Process Model
- Fig. 2.3 Matrix Elements of P1 as a Function of the Sampling Number
- Fig. 2.4 Parameter Estimation from HBWR Plant Data with Prefiltering

2.2 Maximum Likelihood Identification of the HBWR Dynamics

A linear model with constant parameters has been fitted to the input - output samples from experiments, performed at the HBWR.

Analysis of several runs has been performed and is reported in detail elsewhere (14). In the present paper some problems connected to these identifications are discussed and three models of the control rod influence on nuclear power are presented and compared.

The available reactor information is a sequence of measured variables of the input $[u(t), t = 1, \dots, N]$, generated by the computer, and the output $[y(t), t=1, \dots, N]$. The sampling interval is normalized to 1.

In this application the Maximum Likelihood technique uses a model of the form

$$A^*(q^{-1})y(t) = B^*(q^{-1})u(t) + \lambda C^*(q^{-1})e(t)$$

where

q is the forward shift operator

$$A^*(q^{-1}) = 1 + a_1q^{-1} + \dots + a_nq^{-n}$$

$$B^*(q^{-1}) = b_0 + b_1q^{-1} + \dots + b_nq^{-n}$$

$$C^*(q^{-1}) = 1 + c_1q^{-1} + \dots + c_nq^{-n}$$

A^* , B^* and C^* have no common factor. The roots of the polynomials $p^{DA^*}(p^{-1})$ are situated inside the unit circle.

The disturbances $e(t)$ are assumed stationary, independent stochastic variables with zero mean and unit variance.

Thus the noise sequence is correlated, and generally a least square method will give biased parameter estimates. The Maximum Likelihood estimates, however, are consistent, asymptotically normal and efficient under mild conditions.

The method has been developed by Åström (15). I. Gustavsson has written the programmes (1) used in these identifications. The method has been applied to other industrial applications (12) as well.

2.2.1 The Experiments

The results from the following two experiments are discussed.

- Experiment 1 EP 708, run 84
- Experiment 2 EP 710, run 5

The power regulator was decoupled during the experiments. The experimental conditions were as follows.

	Exp. 1	Exp. 2
Power at start (MW)	8.1	9.5
Subcool. (MW)		1.35
Input (1 step up and down)	rod 11	rods 13, 15, 17
No of scans	780	1019

2.2.2 Input Characteristics

The inputs are chosen pseudo random sequences, but of different amplitude and frequency. Because of the nature of the dynamics, with very small and very large time constants, the choice of input signals is crucial. Problems of that nature are discussed e.g. in (16, 12).

A large input amplitude is desirable because of the disturbances. On the other hand, process restrictions and nonlinear effects make it necessary to limit the amplitude.

The Maximum Likelihood method does not impose any restriction on the input signal, except that it should be persistently exciting, e.g. a PRBS (15).

In all cases, the sampling time was 2 seconds. A pseudo random sequence can be characterized by its amplitude, its basic interval T , period $M T_P$ and maximum length of constant sign $r T_P$.

The autospectrum of a PRBS has the property, that it has decreased less than 3 dB for frequencies smaller than $\frac{2.8}{T_P}$ rad/sec.

This means that T_P should be about half of the smallest time constant. Practical experiences show, that time constants about 3 - 5 times smaller than T_P could be found, if the gain of these frequencies is large enough.

The period $M \cdot T_P$ should be about five times longer than the largest time constant. Another

intuitive rule is based on the argument, that the estimates of the gain and the longest time constants are improved, if the process is allowed to approach a new steady state during the experiment. Then $r \cdot T_p$ should be larger than $3 \times T_{max}$.

The discussion is summarized in the following table.

	Amplitude	T_p , sec	$r T_p$, sec	$M T_p$, sec	T_{min} , sec	T_{max} , sec
Exp 1	1	10	240		~2 - 4	~480
Exp 2	~3	2	18	1980	~0.4-0.7	~36

The table indicates, that it should be possible to find higher frequencies in experiment 2 than in experiment 1. This is true, and a 0.8 sec. time constant was found. On the other hand, the simulations show clearly that the low frequency behaviour of exp. 1 is significantly better than that of exp. 2.

2.2.3 Identification Results

It is assumed, that the rod input is directly coupled to the nuclear power output. This depends on the neutron kinetics, and for the sampling interval 2 seconds the kinetics is considered instant.

Models of different orders were adjusted to the experimental data. The order decision was based on a number of comparisons. The loss functions for different models were compared by statistical F-test quantities of a 5% risk level. Static gains, Bode plots, poles and zeroes, parameter variances and finally simulations were used to find the most relevant models. The order of the dynamics never exceeded four. The results are shown in Table I.

In Figures 2.5 and 2.6 simulations of the models are shown. The curves are

- a) input $u(t)$
- b) output $y(t)$ (nuclear power (MW))
- c) output of deterministic model

$$y_d(t) = \frac{B^*(q^{-1})}{A^*(q^{-1})} u(t)$$

2.2.4 Comparisons between the Models

The different models of exp. 1 and 2 have been compared in several ways in (14). Some simulations, that demonstrate the behaviour, are shown here. The model, achieved from exp. 2, is simulated with the input from exp. 1 and vice versa. The deterministic model outputs are shown as the curves d in Figures 2.5 and 2.6.

Both in Figures 2.5 and 2.6 a difference in the low frequency behaviour between the curves c and d is demonstrated. The discrepancy is most clear in Figure 2.5.

In both models there is a dipole close to the unit circle which cannot be removed. This causes the static gain to be calculated with bad accuracy. The pole to the unit circle gives about the same

Table I
Identification Results, Showing Relations between Rod and Nuclear Power Output.
The σ Limits are Shown at the Figures

	Experiment 1	Experiment 2	From state model
a_1	-2.108 ± 0.165	-2.730 ± 0.047	-2.795
a_2	1.364 ± 0.316	2.630 ± 0.092	2.607
a_3	-0.254 ± 0.152	-0.948 ± 0.050	-0.814
a_4		0.051 ± 0.006	0.0028
b_0	0.148 ± 0.006	0.138 ± 0.001	0.138
b_1	-0.298 ± 0.029	-0.364 ± 0.001	-0.383
b_2	0.187 ± 0.052	0.332 ± 0.012	0.355
b_3	-0.037 ± 0.025	-0.105 ± 0.006	-0.110
c_1	-0.954 ± 0.163	-1.624 ± 0.057	
c_2	0.210 ± 0.126	0.685 ± 0.059	
c_3	-0.218 ± 0.056	0	
c_4		0	
λ	0.028	0.0271	

behaviour as an integrator. The reason is, that the sampling interval, 2 sec., is very short in comparison with the largest time constant of the system.

Presently experiments from the series EP 714 are analysed. The influence of different sampling intervals is investigated.

In that experiment the basic input interval length is 12 seconds. Then it is possible to select every 2nd, 3rd or 6th input-output sample in order to get the sampling times 4, 6 and 12 seconds.

2.2.5 Comparison to a State Model

The identification results have been compared to a 11:th order linear state model, developed in Halden (13). Input-output data were generated with the rod input from experiment 2 applied to the model. No disturbances were added.

A deterministic 11th order model should of course give a parametric model of the same order. In this case, however, already a fourth order model gave satisfactory results. For a fifth order model no significant decrease of the loss function was achieved. Numerical difficulties arose for higher order models because of the reason, that the prediction error could not be made significantly smaller. The model is shown in Table I. It has also been simulated with the inputs from experiments 1 and 2. The deterministic outputs from the fourth order model are shown in Figures 2.5.e and 2.6.e.

A model, where every state variable is corrupted by independent white noise showed a bad agreement with the stochastic part of the experimental results, as was expected. Only the noise amplitude was acceptable.

2.2.6 Conclusions

All identification results show, that the prediction error can be made satisfactorily low with a stochastic controller based on a fourth order model. The noise description must be more sophisticated than just independent noise, added to the state variables.

The choice of sampling interval and input character is crucial for the model accuracy. Because of the large difference between the small and the large time constants in the reactor, it is difficult to get a model by identification which is accurate in the whole frequency range.

3. ADJOINT PROBLEMS

3.1 Prefiltering of Sampled Input Data to the Process Computer

A basic problem in process parameter identification and estimation is the "prefiltering" of raw, digitized process data. Below are listed the three main problem areas in this field where the use of these data could benefit by an appropriate prefiltering technique before use by the process control system.

- Additional noise contributions for the basic input and output signals of the system can adversely affect the accuracy of estimated parameters. Therefore, a successful analysis of this kind requires a good noise rejection technique under the following points of view, (a) achievement of the optimum "signal-to-noise" ratio, but (b) without disturbing the frequency transmission characteristics of the process itself.
- For the same type of analysis sometimes only the first time derivatives of the signals of the plant are needed.
- Frequently, data with the dc-component removed are necessary for unbiased parameter estimation (17).

In the first case mainly digital low-pass filters have to be designed whereas in the second and third case, time derivative and band-pass filters are needed, respectively. In the following chapter a design procedure is described which allows to effectively accomplish these tasks.

3.1.1 Design of Fading-Memory Digital (FMD-) Filters by the "Bilinear z-transform"

To develop digital filters the so-called "bilinear z-transform" (1,2,18, 19) has been used. This method allows one to convert a given continuous filter transfer function $H(s)$ in the Laplace domain into a discrete transfer function $H(z)$ by the following substitution of the Laplacian variable,

$$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} \quad /19/$$

in which z denotes the discrete operator, defined as $z = \exp(j\omega T)$ where T is the sampling interval in sec.-units. The transformation obtained exhibits the same frequency response characteristics as the

continuous filter, except for a non-linear warping of the frequency scale. Compensation for this warping is made by replacing all characteristic corner frequencies ω_i of the continuous filter by so-called "pseudo frequencies" ν_i in the discrete z-domain using the relationship

$$\nu_i = \frac{2}{T} \tan \frac{\omega_i T}{2} \quad i = 1, 2, \dots, I \quad /20/$$

Since the bilinear transformation used is purely algebraic in form, rational transfer functions in the s-domain are represented in polynomial or factored form in the z-domain. Negative powers of z, however, indicate how many sampling intervals a signal is delayed. Therefore, the digital filter output Y(k) at time k can be expressed as

$$Y(k) = \sum_{n=1}^N a_n Y(k-n) + \sum_{m=0}^M b_m X(k-m) \quad /21/$$

where X(k), X(k-1) are the filter inputs at time k, k-1, etc. and the coefficients a_n and b_m are functions of the pseudo frequencies ν_i and T. The numbers of additive terms in each sum of equation /21/, N and M, are dependent both on the type (low-, high-pass, etc.) and on the order of the filter in the s-domain.

The great advantages of this procedure are (a) the elimination of the aliasing effect which is inherent in the "standard z-transform" (1), and (b) the fact that the algorithms obtained are directly given in a recursive difference equation. Since the weighting coefficients a_n and b_m can be interpreted to serve as fading parameters to control the fading rate of the shape function (= stress function (20)) of the filter's memory, these filters are sometimes referred as Fading Memory Digital (FDM-) Filters (21, 22, 20).

Table II contains a comprehensive summary of different filter types designed in this work.

Figure 3.1a shows a typical time record of an original process output signal and represents the output signal from a calibrated ion-chamber located below the core of the HBWR. This signal is the response to a perturbation of the reactivity and the primary pressure by a pseudo random binary driving function. In Figure 3.1b this record was submitted to a low-pass filter type of first order to reject high frequency noise contributions. In order to eliminate the dc-level and to smooth the record simultaneously, the original signal was band-filtered. The resulting function is shown in Figure 3.1c. Figure 3.1c shows the output of a first time derivative filter combined with a low-pass filter of second order (Derivative filters must always be combined with a smoothing filter since the derivative operator's enhances the high frequency noise).

Summarizing the experience with the bilinear design method it can be stated that sufficient success in the prefiltering of process raw data was obtained. In the case of an on-line application, all the necessary computations according to equation /21/ have to be done in between two subsequent samples. Prefiltering is restricted by the sampling interval and the computer speed. Therefore, the digital filter algorithm should be implemented in assembly language in the cases of on-line routine applications.

Table II

Summary of Digital Filters Designed for Prefiltering Process Data
by means of the "bilinear z-transform" Method

Transfer Function in the Laplace Domain $H(s)$	Weighting Coefficients a_n and b_m for $ z /$ the digital filter algorithm in Equation /21/	Frequency Range
$\frac{\omega_H}{\omega_H + s}$ (low-pass filter of first order)	$a_1 = 1 - 2\beta$ $b_0 = \beta$ $b_1 = \beta$	$\beta = \nu_H/(2/T + \nu_H)$ $\omega < \omega_H$
$s \frac{\omega_H}{(\omega_H + s)(\omega_2 + s)}$ (band-pass filter of first order)	$a_1 = \beta_1$ $a_2 = \beta_2$ $b_0 = \beta_3$ $b_2 = -\beta_3$	$\beta_1 = 2a(1 - \nu_L \nu_H)$ $\beta_2 = a(\nu_L + \nu_H - \nu_L \nu_H - 1)$ $\beta_3 = a \nu_H$ $a = 1/(1 + \nu_L \nu_H + \nu_L + \nu_H)$ $\omega_L < \omega < \omega_H$
$s \frac{\omega_H^2}{(\omega_H + s)^2}$ (time derivative filter including low pass filter to suppress high frequency noise)	$a_1 = \beta_1$ $a_2 = \beta_2$ $b_0 = \beta_3$ $b_2 = -\beta_3$	$\beta_1 = 2(1 - 2\beta)$ $\beta_2 = -(1 - 2\beta)^2$ $\beta_3 = 2\beta^2/T$ $\beta = \nu_n/(\nu_n + 2/T)$ $\omega \ll \omega_H$
$\frac{1}{[(s/\omega_H)^2 + 2P(s/\omega_H) + 1]^{-2}}$ (fourth order low-pass filter P =peaking factor = 0.6)	$a_1 = -(2a\beta/a^2)$ $a_2 = -(2\beta\gamma + \beta^2)/a^2$ $a_3 = -(2\beta\gamma)/a^2$ $a_4 = -\gamma^2/a^2$ $a = 1 + 2Pa + a^2,$ $\gamma = 1 - 2Pa + a^2,$	$b_0 = 1/a^2$ $b_1 = 4 b_0 = b_3$ $b_2 = 6 b_0$ $b_4 = b_0$ $\beta = 2(1 - a^2)$ $a = \frac{2}{T} / \nu_H$ $\omega < \omega_H$
$\frac{1}{[(s/\omega_H)^2 + 2P(s/\omega_H) + 1]^{-2}}$ (sophisticated time derivative filter including a fourth order low-pass filter, $P=0.6$)	$a_1 = -(2a\beta/a^2)$ $a_2 = -(2a\beta + \beta^2)/a^2$ $a_3 = -(2\beta\gamma)/a^2$ $a_4 = -\gamma^2/a^2$ $a = x^2 + 2P\nu_H x + \nu_H^2;$ $\gamma = x^2 - 2P\nu_H x + \nu_H^2;$	$b_0 = x \nu_H^4$ $b_1 = 2b_0 = -b_3$ $b_2 = 0$ $b_4 = -b_0$ $\beta = 2(\nu_H^2 - x^2)$ $x = 2/T$ $\omega \ll \omega_H$

The computation of the pseudo frequencies ν_H and ν_L according to equation /20/.

3.2 Power Spectral Density Analysis of Sampled Process Data

When transfer functions of a system by means of perturbation experiments have to be measured or when reactor noise measurements are analyzed, the power spectral density function of process data has to be determined. The standard digital computer technique most commonly used for these purposes makes use of the Wiener-Khinchin relations, i.e. the first step in that computational procedure is the determination of the corresponding correlation function in the time domain. A subsequent Fourier-transformation using an appropriate lag window weighting function yields finally the desired "smoothed estimate" of the power spectral density function.

Due to its operational principle this indirect method possesses the following disadvantages. (a) it is not suited to be applied on-line without considerable amount of computational effort, (b) the equivalent frequency bandwidth resolution Δf of the power spectral density function is constant throughout the whole spectrum. However, in most practical situations one desires to work with a bandwidth resolution which should be relatively constant, i.e. $\Delta f/f_c = \text{const.}$ where f_c is the frequency under investigation, (c) the necessary use of any lag window weighting function to "smooth out" the truncation error when a correlation function of finite length is fourier-transformed sometimes gives difficulties in the interpretation of spectral densities due to the influence of the finite side lobes of the above mentioned frequency windows.

Based on the previously described digital filter work, a new approach of a power spectral density analyzer for sampled data has been developed to overcome the aforesaid adversities. The structure of its digital algorithm makes use of the heterodyne filtering technique (19, 23, 24, 25) which provides the transition of a certain spectral range of the actual signal to the frequency range of a low-pass filter. Hence, the problem of moving a band-pass filter with respect to the spectrum is replaced by moving the spectrum with respect to the filter. The digital low-pass filtering - as the most crucial part of the analyzer - is then carried out by taking advantage of the sophisticated fourth-order low-pass filter from Table II.

Figure 3.2 shows the block diagram for the computational procedure in the continuous case and Figure 3.3 presents a typical plot of the output of the analyzer for a white noise plus an isolated sine function as the input function (upper curve) and for a pure white noise input (lower curve).

Furthermore, the analyzer allows also to evaluate time-dependent spectra when the true time averaging procedure (averaging circuit in Fig. 3.2) is replaced by a first order fading memory (low-pass) filter from Table II. For more details the reader is referred to the original literature (19). An abbreviated version of this detailed paper is given in (25).

3.3 Special Problems in the Field of Reactor Power Noise Analysis using Self-Powered Neutron Detectors

For control and flux distribution measurements in large water cooled power reactors more and more self-powered neutron probes are utilized. The most commonly used detectors of this type of construction use vanadium, rhodium and cobalt as the emitter material.

Due to the nuclear interactions of the emitter materials with the reactor neutrons the cobalt detectors show a prompt response while the vanadium and rhodium detectors possess either a

prompt part and a delayed part in its dynamic response (28). The prompt part results primarily from Compton and photo-electrons emitted after self-absorption of the neutron capture gammas and the delayed part results from β -particles emitted by neutron activation products.

Therefore, for the analysis of random neutron flux fluctuations (reactor noise analysis) of power reactors the cobalt detectors are particularly well suited. A first systematic utilization of these detectors for reactor power noise investigations is described in (26). However, when using e.g. vanadium detectors - possessing the lowest burn-up rate compared with cobalt and rhodium detectors - only that higher frequency range $f > f_c$ of the spectrum of the flux fluctuations can be analyzed where the prompt part of their sensitivity becomes effective. For the determination of this critical frequency f_c the knowledge of the ratio of the prompt to the delayed sensitivity, W_p/W_d , is necessary. Until now, this ratio was determined by transient measurements to be "below 10%, usually around 5% (27). The following chapter passes in review a novel technique which is based upon methods of stochastic processes to determine the above mentioned ratio more precisely.

3.3.1 Determination of the Ratio of the Prompt to the Delayed Response of a Vanadium Self-Powered Neutron Detector

The method uses two detectors, the actual vanadium detector and a prompt response "reference" cobalt detector which are located close to each other in the reactor core. Due to the inherent fluctuations of the neutron flux (reactor noise) the mean output currents of both detectors are superimposed by small randomly fluctuating signals. If the mean currents are suppressed and only the fluctuating components are amplified, it can be shown (28) that these quantities can be represented by

$$U_V(t) = C_1 [W_p \varphi(t) + W_d \int_0^{\infty} H(\xi) \varphi(t - \xi) d\xi] \quad /22/$$

for the vanadium detector, and

$$U_C(t) = C_2 W_c \varphi(t) \quad /23/$$

for the cobalt detector

where

- C_1, C_2 = Constants consisting of the input resistances of the amplifiers and their gain factors
- W_p, W_d = Prompt and delayed detector sensitivity of the vanadium detector (in A/nv)
- W_c = Detector sensitivity of the cobalt "reference" detector (in A/nv)
- $\varphi(t)$ = Stochastic neutron flux fluctuations at the space point of the location of the detectors
- $H(t) = \lambda e^{-\lambda t}$ = Normalized impuls response function for the delayed part of the response of the vanadium detector due to the β -decay of V^{52} (in sec^{-1})
- $\lambda = \ln 2 / T_{1/2}$ = Decay constant of $V^{52} = 3.072 \cdot 10^{-3} \text{ sec}^{-1}$ ($T_{1/2} = 3.76 \text{ min}$).

For the analysis in the time domain, the cross correlation function between $U_V(t)$ and $U_C(t)$ normalized to the geometrical mean of the mean square values of these functions at zero time delay

(the so-called correlation coefficient) is of special interest. This quantity is given by (28):

$$R_{V,C}(0) = \frac{W_p/W_d + F}{[(W_p/W_d)^2 + 2 W_p/W_d F + 1]^{1/2}} \quad /24/$$

where

$$F = \int_0^{\infty} H(\xi) R_{\varphi\varphi}(\xi) d\xi \quad /25/$$

and $R_{\varphi\varphi}(\xi)$ is the auto-correlation function of $U_C(t)$ normalized to its mean square value.

By measuring the correlation coefficient $R_{V,C}(0)$ and by determining F through the procedure of equation /25/ the ratio W_p/W_d can be determined.

Figures 3.4 and 3.5 show the results of the corresponding measurements. The reactor power was 7 MW. The record length and the sampling rate were 18 hours and 5 seconds, respectively. The sampled data were evaluated on the GIER computer using the Halden Noise Programme system which allows to compute auto- and cross-correlation functions as well as the corresponding power spectral density functions. The measurement lag number was $M = 350$ resulting in a maximum delay time of $5 \text{ sec.} \times 350 = 1750 \text{ sec.}$

The value of F was obtained to be 0.472. Together with the measured correlation coefficient $R_{V,C}(0) = 0.518$ from Figure 3.4 the ratio of the prompt to the delayed response of the vanadium detector (commercial type, Model 5503, AB Atomenergi, Studsvik, Sweden) could be determined to be $W_p/W_d = (6.2 \pm 0.45)\%$.

An alternative possibility for the W_p/W_d offers the analysis in the frequency domain. For this, the ratio of the imaginary part (Quad-Power) to the real part (Co-Power) of the cross power spectral density $U_V(t)$ and $U_C(t)$ as function of frequency has to be determined. The following simple expression is obtained (28):

$$P(\omega) = \frac{\text{Quad-Power}}{\text{Co-Power}} = \frac{\omega/\lambda}{1 + W_p/W_d(1 + (\omega/\lambda)^2)} \quad /26/$$

Figure 3.6 shows the experimental measuring points which were fitted to the theoretical expression equation /26/ by a Gaussian least squared method. The best value for W_p/W_d turned out to be $(6.5 \pm 0.35)\%$.

It should be emphasize that the theoretical expressions in both the time domain (eqs. /6/ and /25/) and in the frequency domain (eq. /26/) contain the ratio W_p/W_d as the only free parameter. Furthermore, these expressions are independent of W_c , C_1 and C_2 , i.e. it is not necessary to know the sensitivity of the "reference" cobalt detector as well as the amplification factors of the amplifiers since these quantities cancel automatically in such a cross correlation technique.

Since the measurements are carried out without mechanical manipulations with the detectors and at constant reactor power the normal reactor operation is not disturbed. Therefore, the described technique offers the advantageous feature that the ratio W_p/W_d of permanently installed vanadium detectors can be determined under real environmental reactor conditions.

Moreover, the critical frequency f_c from which on the shape of the power spectral density of the neutron flux fluctuations measured with a vanadium detector is identical with that measured with a purely prompt responding detector (e.g. with a cobalt detector) resulted in

$$f_c \cong (W_d/W_p) \lambda/2\pi$$

(28). The numerical value of this critical frequency is about 0.008 Hz. In conclusion, this means that vanadium detectors can also be used as in-core neutron sensors in reactor noise measurements when one is only interested in the frequency range $f > f_c$ of the flux fluctuation. However, this is practically always the case.

For more details and more experimental results the reader is referred to the original paper given in (28).

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- Fig. 3.6 Ratio of Quad-Power to Co-Power of the Cross Power Spectral Density between the Fluctuating Output Signals of the Vanadium and Cobalt Detectors as Function of Frequency.

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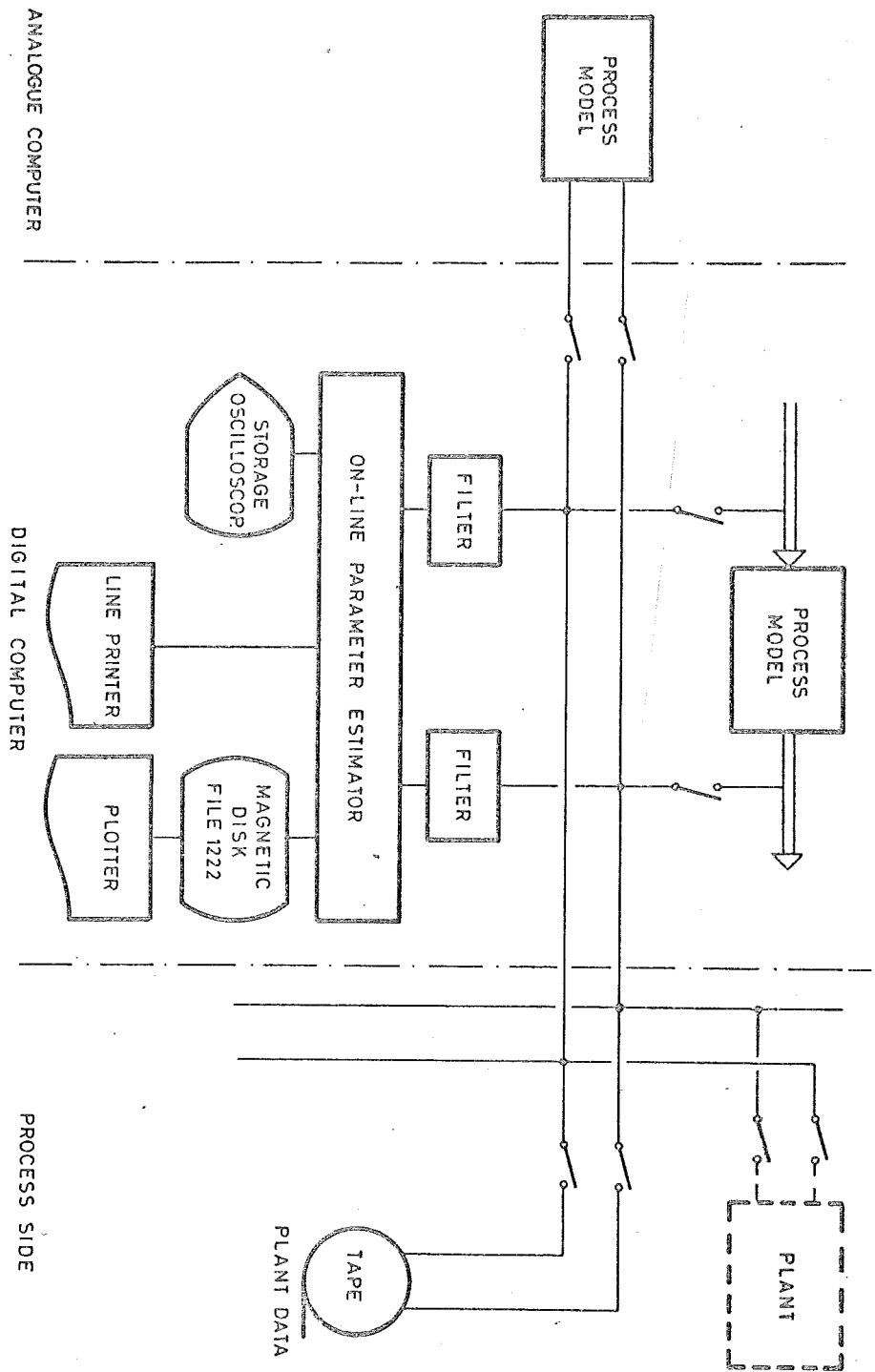


Fig. 2.1 Test and Simulation Scheme

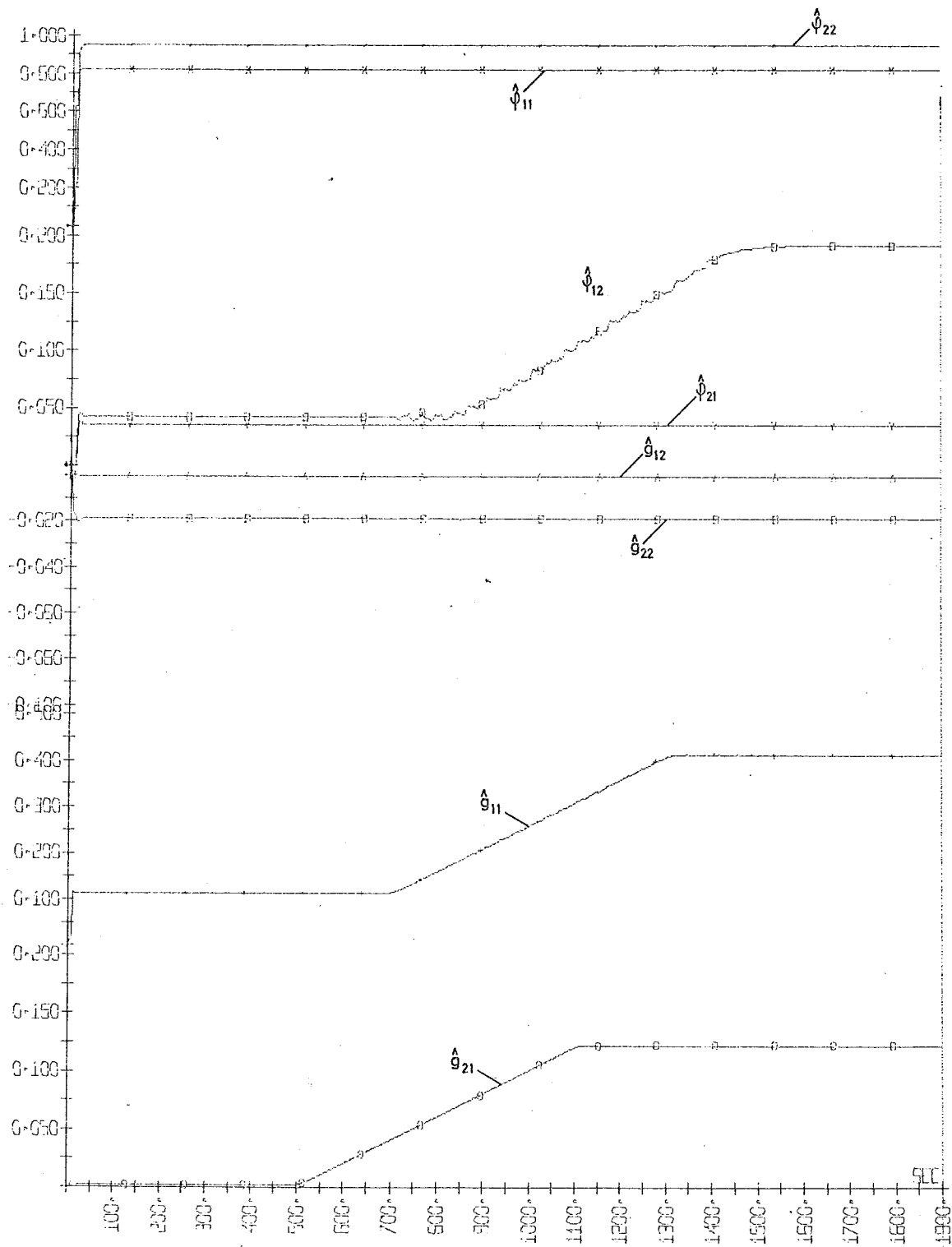


Fig. 2.2 Parameter Tracking for the Discrete Process Model

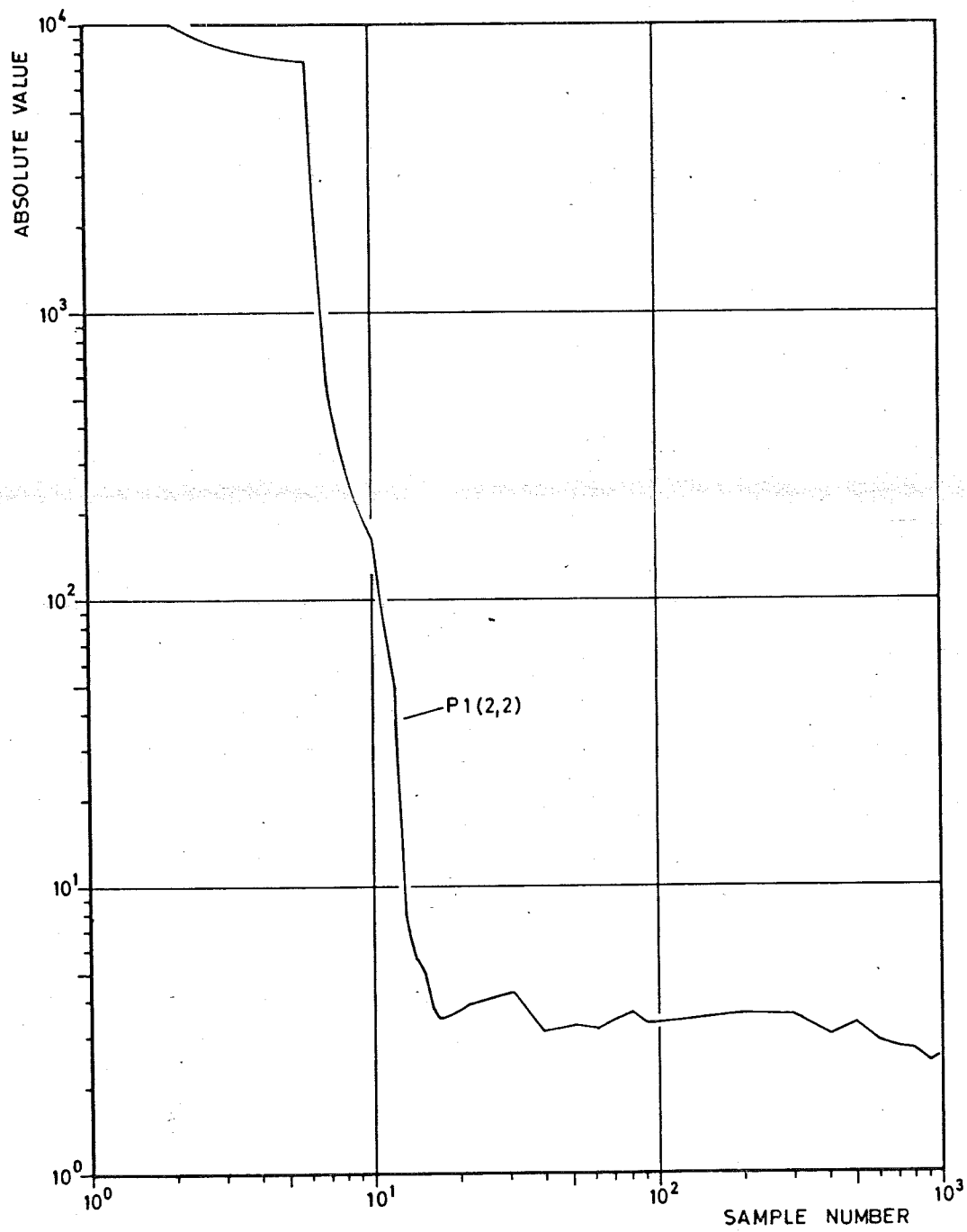


Fig. 2.3 Matrix Elements of P1 as a Function of the Sampling Number

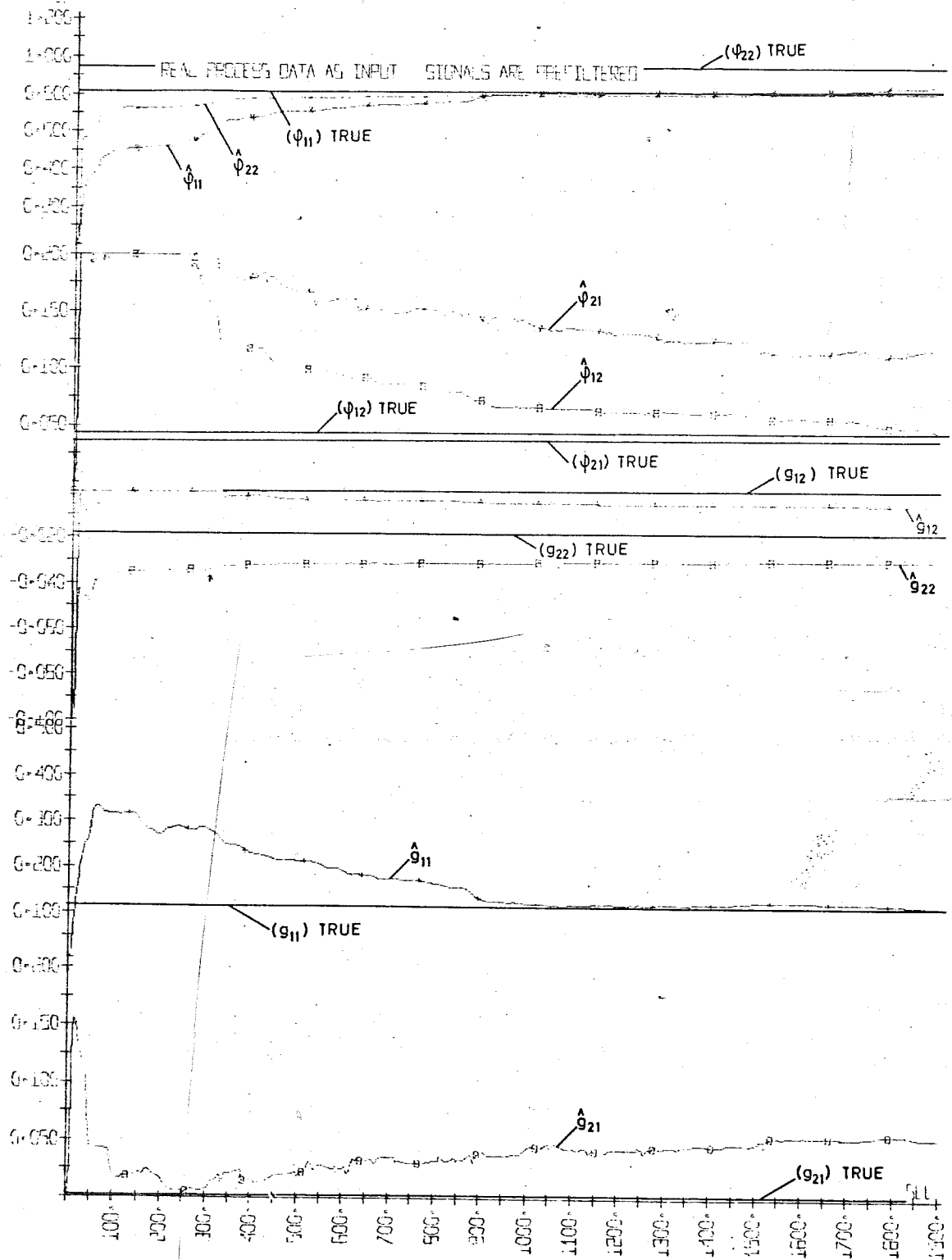
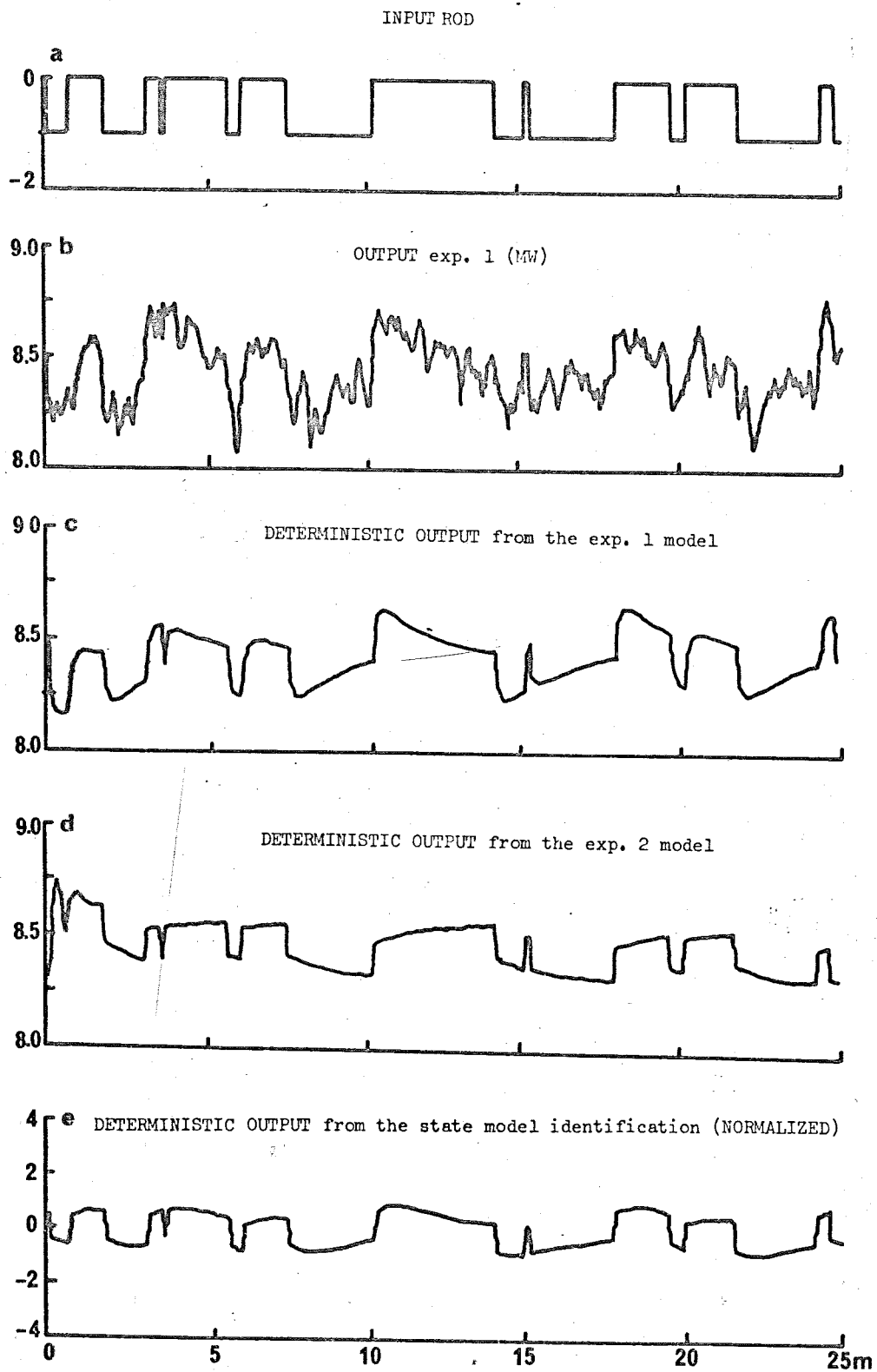
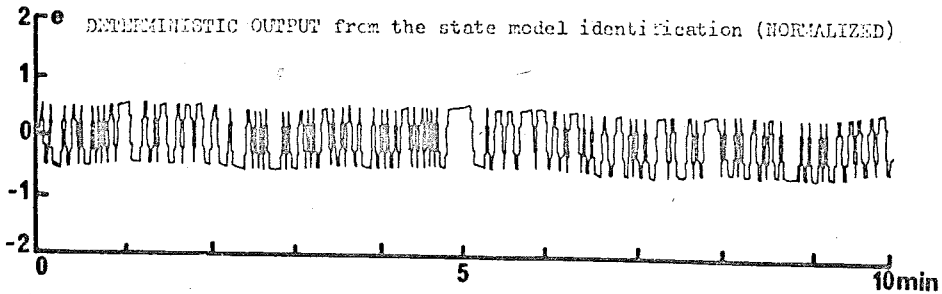
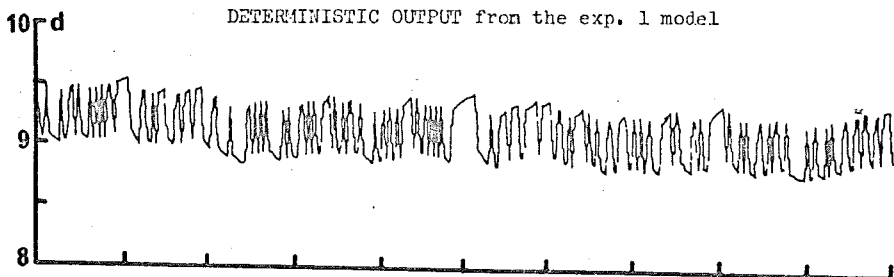
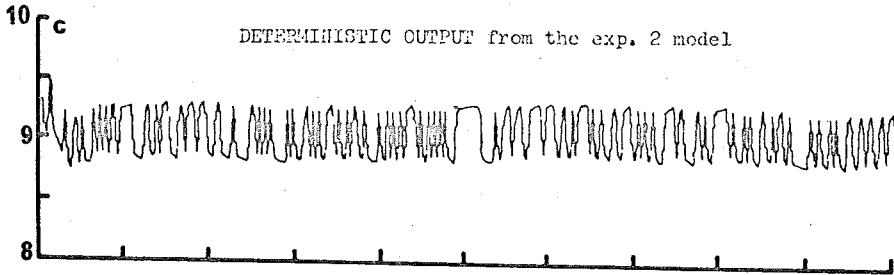
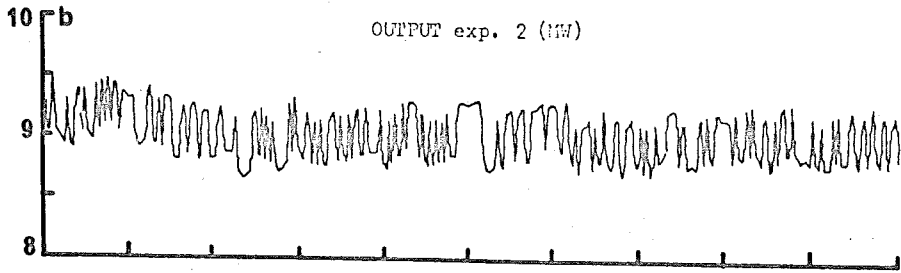
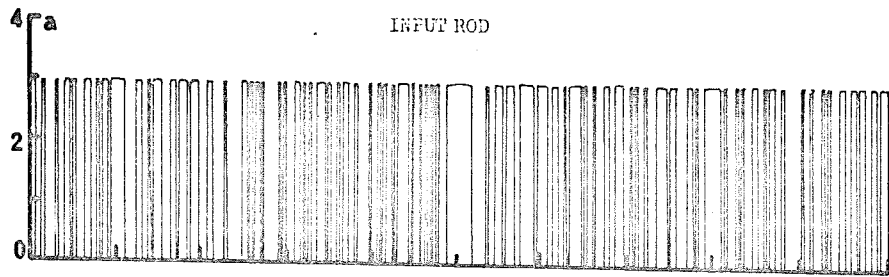


Fig. 2.4 Parameter Estimation from HBWR Plant Data with Prefiltering



2.5 Model Output Signals



2.6 Model Output Signals

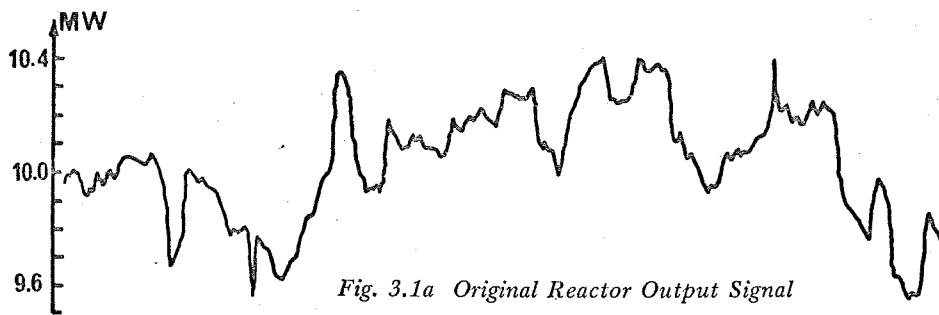


Fig. 3.1a Original Reactor Output Signal

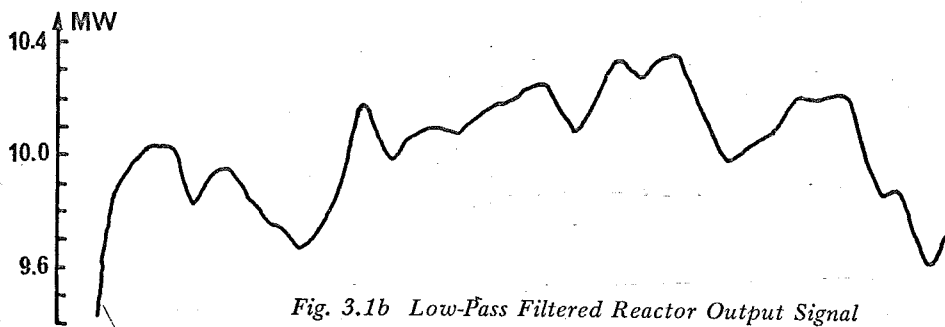


Fig. 3.1b Low-Pass Filtered Reactor Output Signal

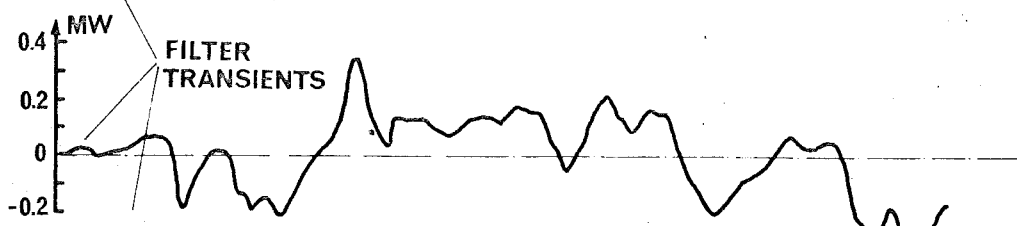


Fig. 3.1c Band-Pass Filtered Reactor Output Signal

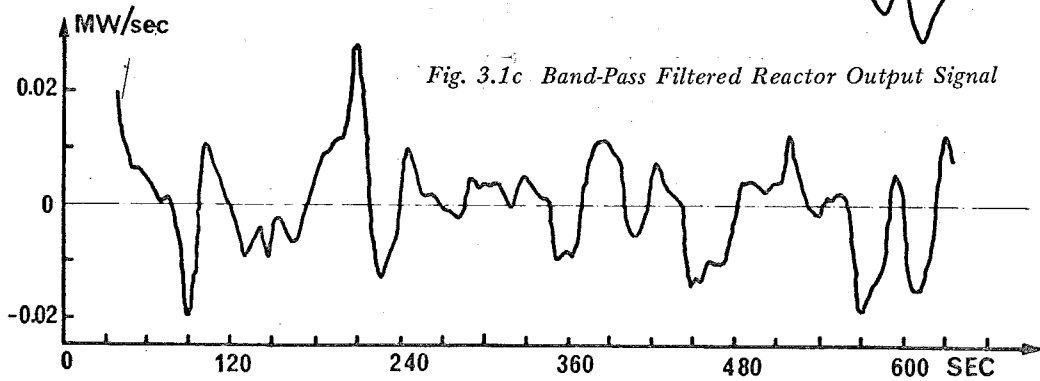


Fig. 3.1d First Time-Derivative Filtered Reactor Output Signal

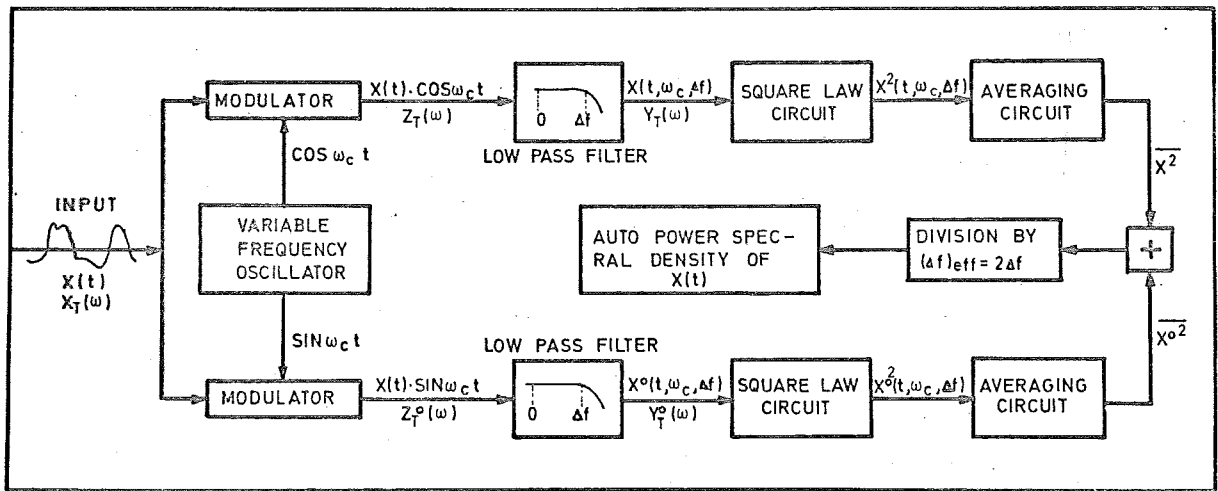


Fig. 3.2 Block Diagram of Heterodyne Power Spectral Density Analyser for Continuous Input Data

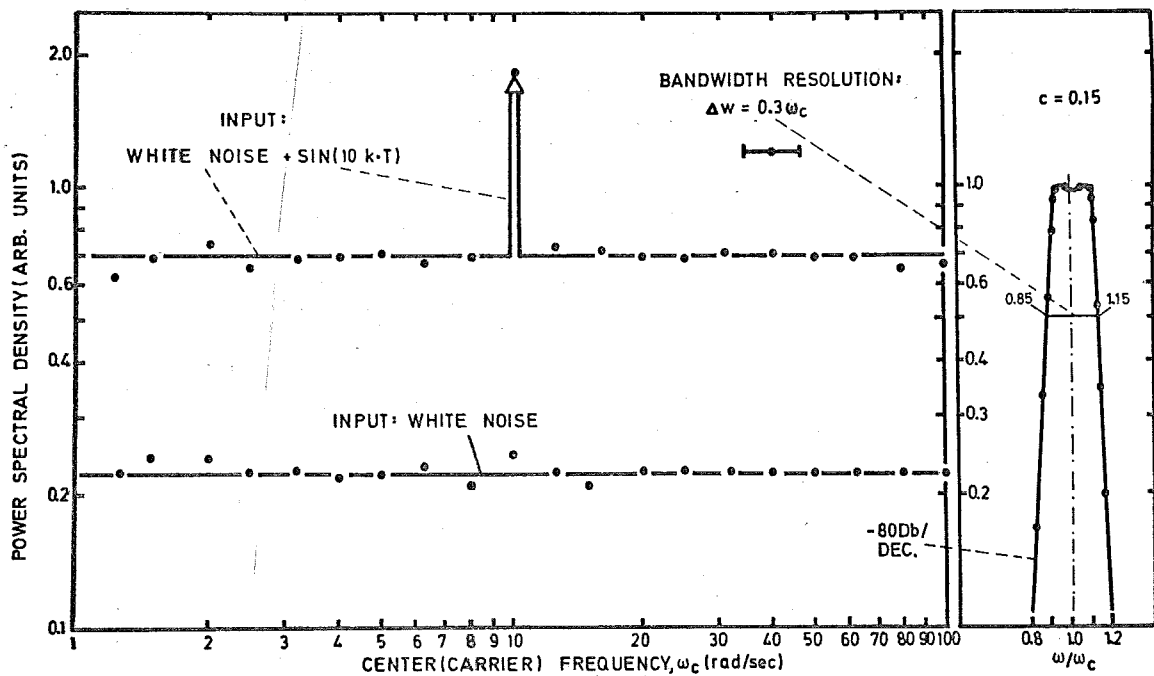


Fig. 3.3 Output of the Digital Heterodyne Power Spectral Density Analyser. The lower curve is the result for a white noise input and the upper curve for white noise added from isolated sine function

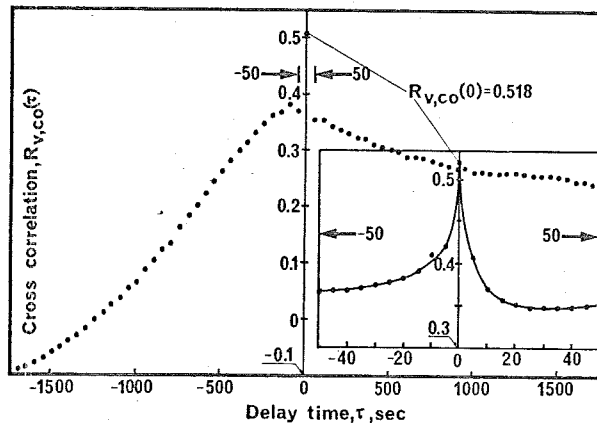


Fig. 3.4 Normalized Cross Correlation Function between the Fluctuating Output of the Vanadium and Cobalt Detectors

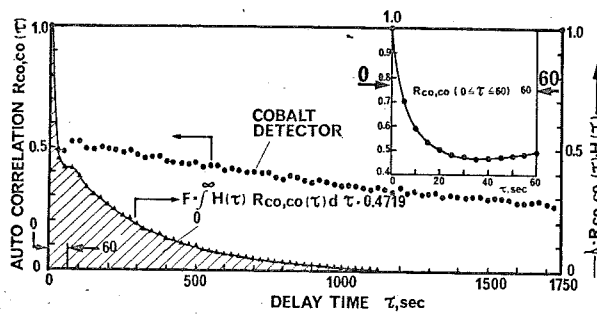


Fig. 3.5 Normalized Auto-Correlation Function of the Fluctuating Output of the Cobalt Detector

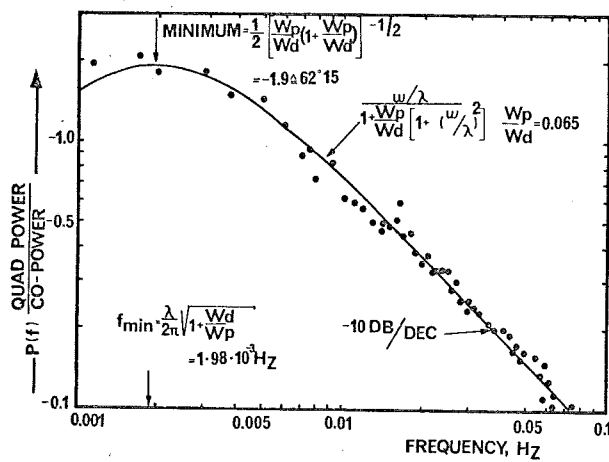


Fig. 3.6 Ratio of Quad-Power to Co-Power of the Cross Power Spectral Density between the Fluctuating Output Signals of the Vanadium and Cobalt Detectors as Function of Frequency