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# On Event-Based Sampling for LQG-Optimal Control

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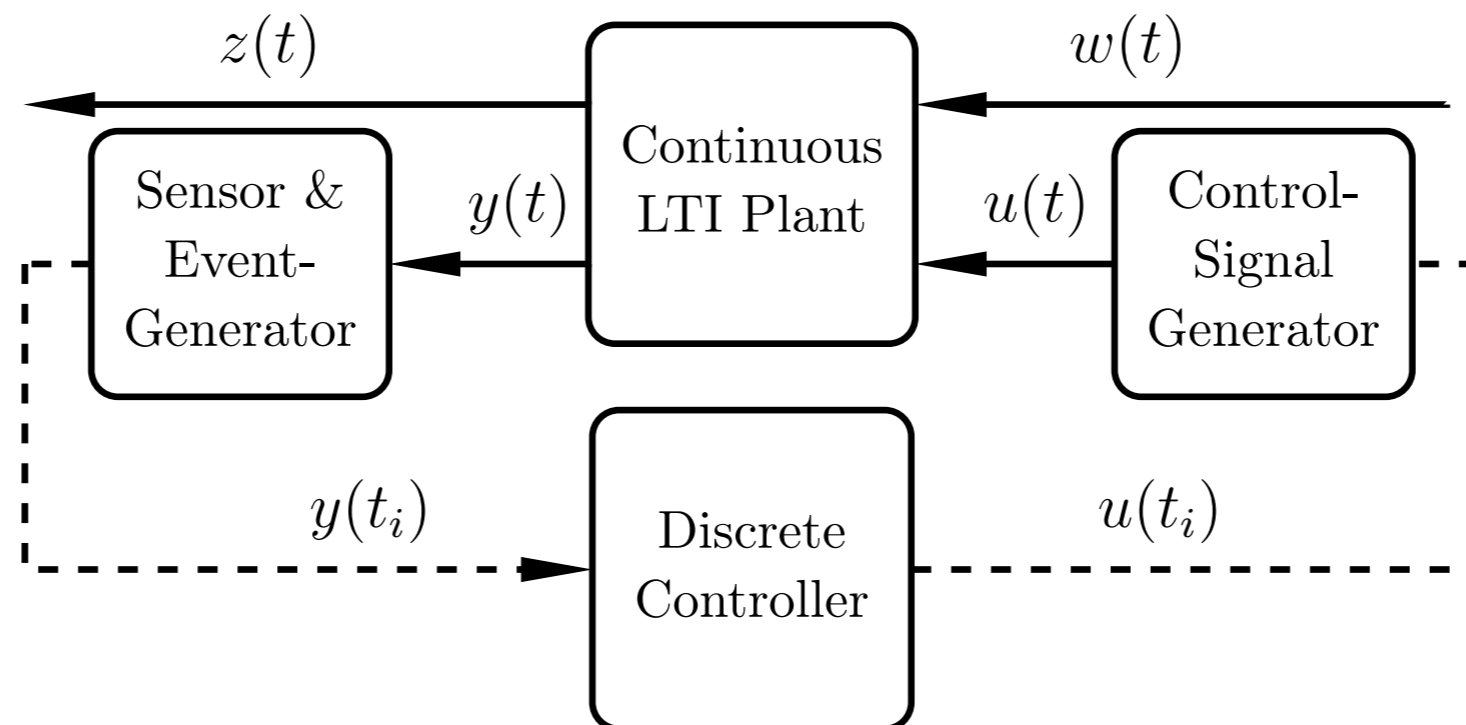
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## INTRODUCTION

**Event-based control** is the concept of sampling and actuating based on system signals rather than a periodic timer, with the goal of more resource-efficient control. We consider the classic LQG formulation, with an added penalty  $\rho$  on the average sampling rate  $f$ :

$$\text{Minimize } J = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[ \int_0^T z(t)^\top z(t) dt \right] + \rho f.$$

For this objective, the optimal controller structure is available [1], and we consider the remaining problem of optimizing the sampling policy [2].

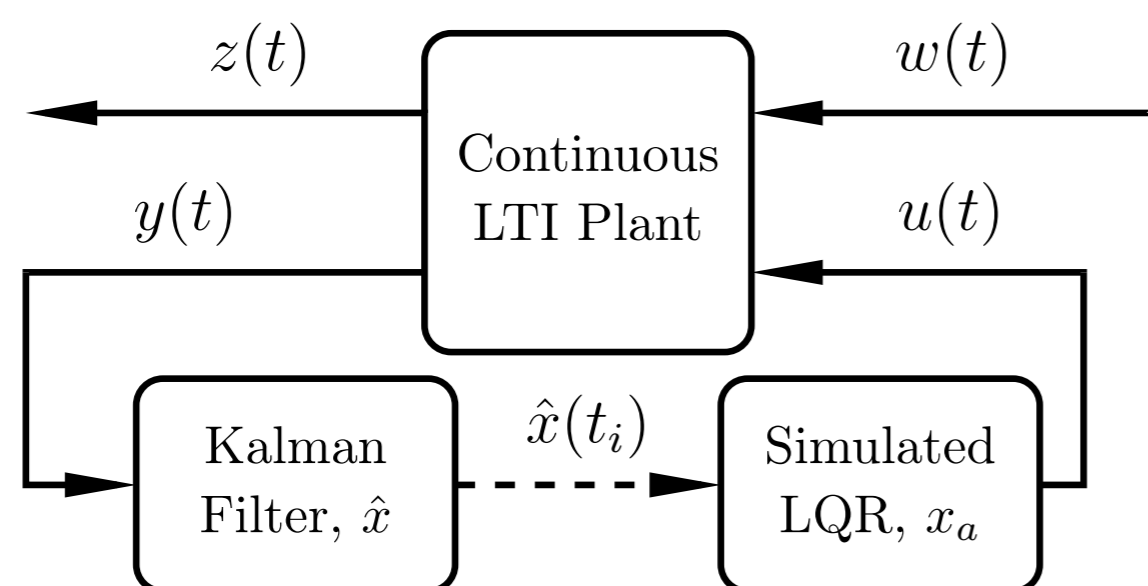


### Our Contributions:

- Identifying an equivalent free boundary PDE formulation of the optimal sampling problem
- Deriving a numerical scheme to approximately compute the optimal sampling policy for general LTI systems, and observing that sampling thresholds can be non-convex
- Deriving tight bounds on the improvement over periodic sampling in the special case of multidimensional integrators ( $A = 0$ ), see [2].

## OPTIMAL CONTROLLER STRUCTURE

The optimal controller structure from [1] can intuitively be represented as:



When sampling we assign  $x_a(t_i) = \hat{x}(t_i)$ , and the error  $\tilde{x} = \hat{x} - x_a$  is reset to zero. The error dynamics are:

$$\text{Dynamics: } \begin{cases} d\tilde{x} = A\tilde{x}dt + d\epsilon, \\ \tilde{x}(t_i) = 0, \end{cases} \quad \text{Noise: } \begin{cases} \mathbb{E}[d\epsilon] = 0, \\ \mathbb{E}[d\epsilon d\epsilon^\top] = R dt \succ 0. \end{cases}$$

This fundamental reset system determines the closed-loop cost

$$J = \gamma_0 + \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[ \int_0^T \tilde{x}(t)^\top Q \tilde{x}(t) dt \right] + \rho f,$$

where  $\gamma_0$  is the optimal continuous-time LQG-cost.

## OPTIMAL SAMPLING PROBLEM

**Problem:** Find the optimal trigger threshold in the  $\tilde{x}$ -space, from which we reset  $\tilde{x}$  to zero and pay  $\rho$ . This threshold can be expressed in the *relative value function*  $V(\tilde{x})$  of the optimization problem, where it is optimal to sample when

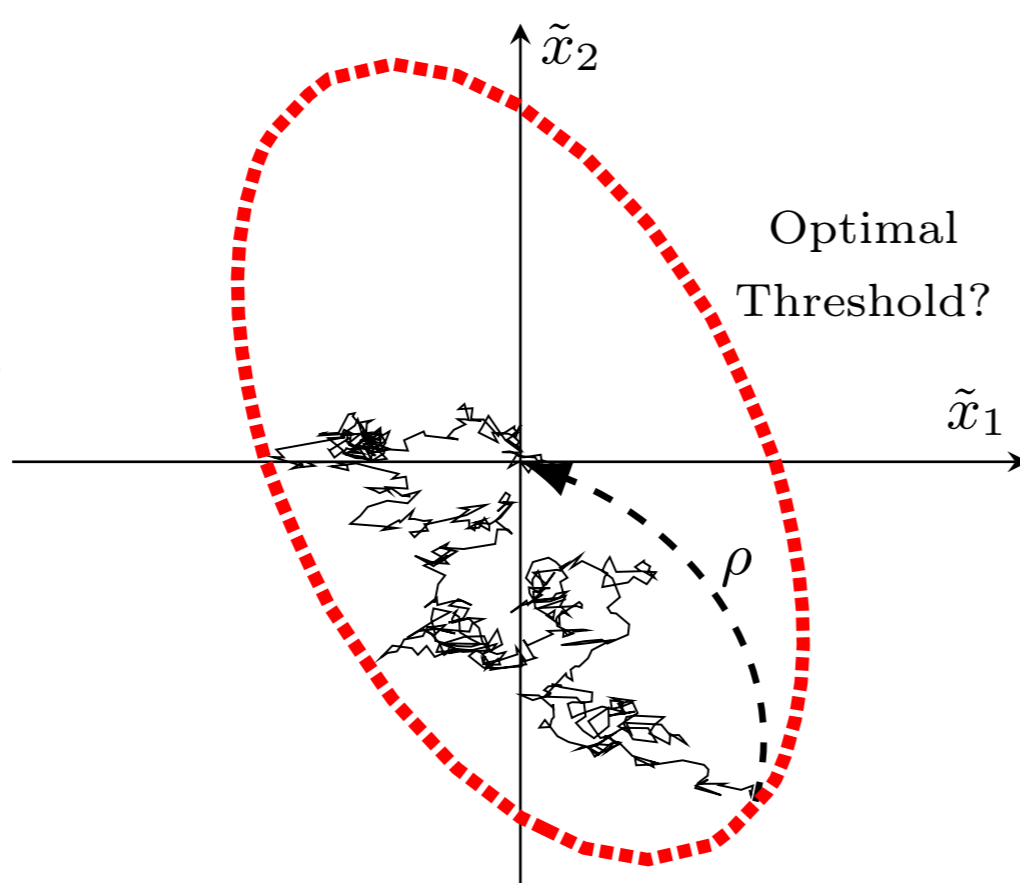
$$V(\tilde{x}) = 0.$$

$V$  satisfies a free boundary PDE:

$$\tilde{x}^\top Q \tilde{x} + \tilde{x}^\top A^\top \nabla V + \frac{1}{2} \text{Tr}(R \nabla^2 V) - J = 0, \quad V(\tilde{x}) \leq 0, \quad \forall \tilde{x}$$

### Conditions on the Free Boundary:

$$\text{Dirichlet: } V(\tilde{x}) = 0, \quad \text{Neumann: } \nabla V = 0.$$



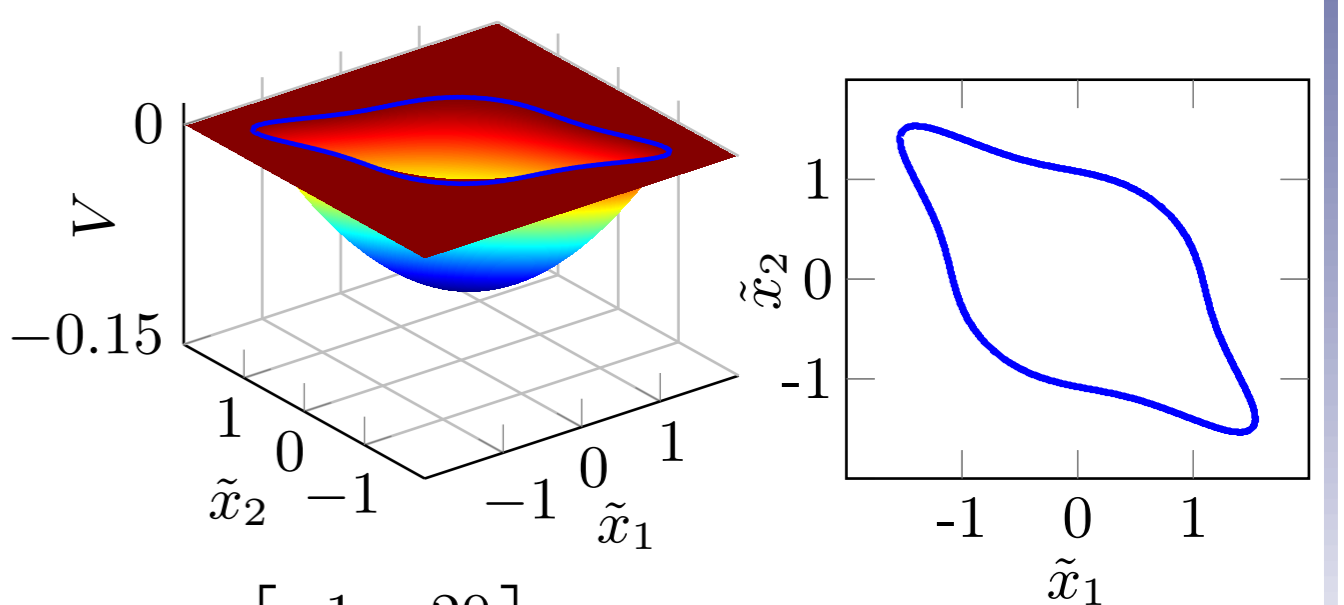
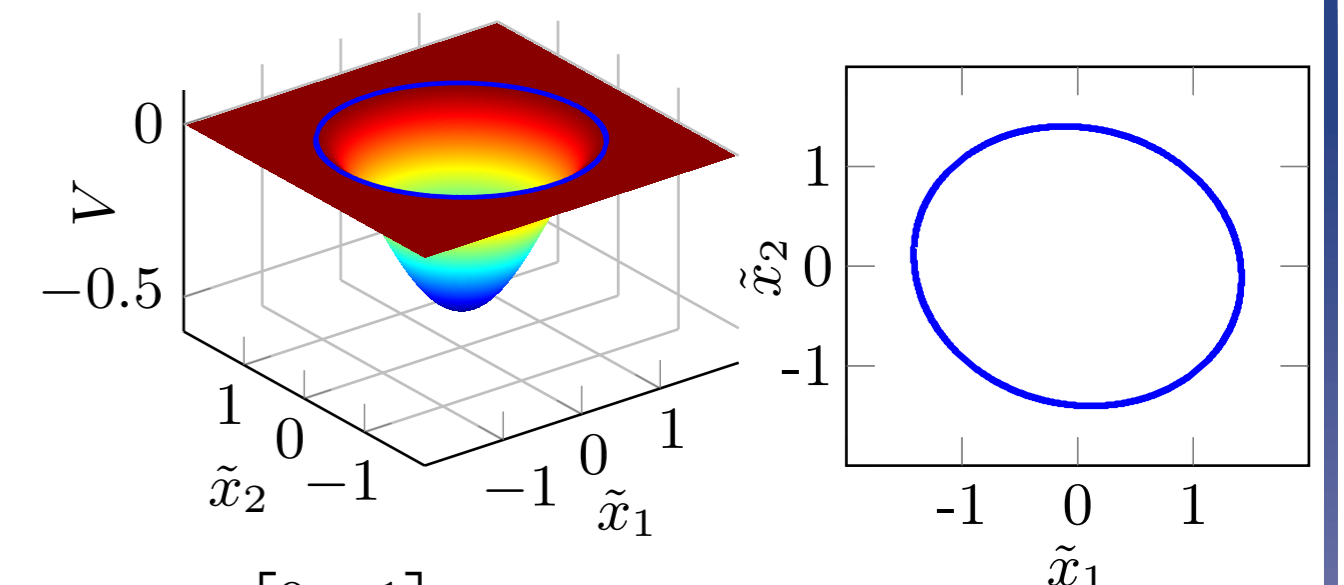
## NUMERICAL SOLUTION

### Numerical scheme:

- Finite-difference approximation
- Simulate dynamic version of PDE
- Enforce  $V(\tilde{x}) \leq 0$
- Run until stationarity

### Note:

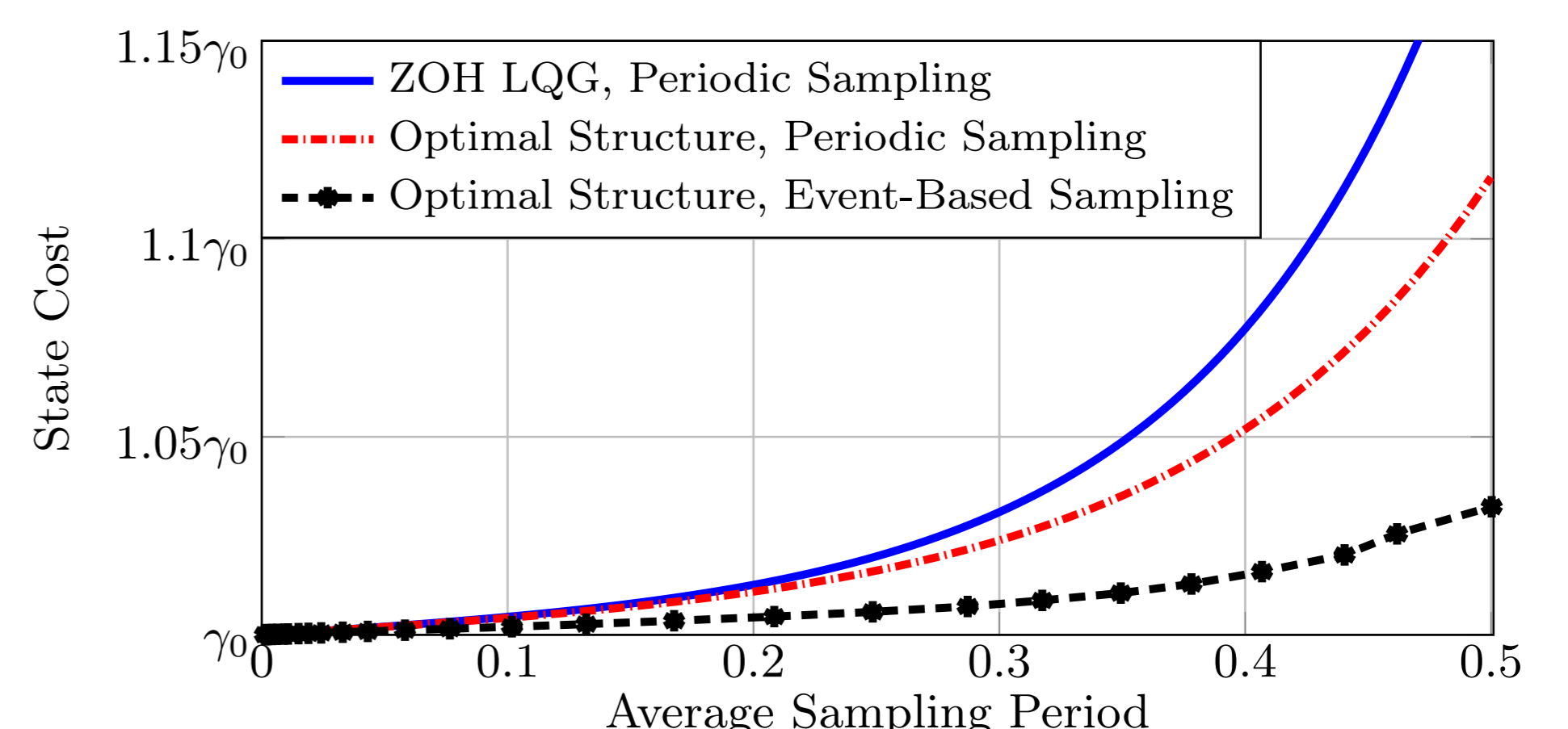
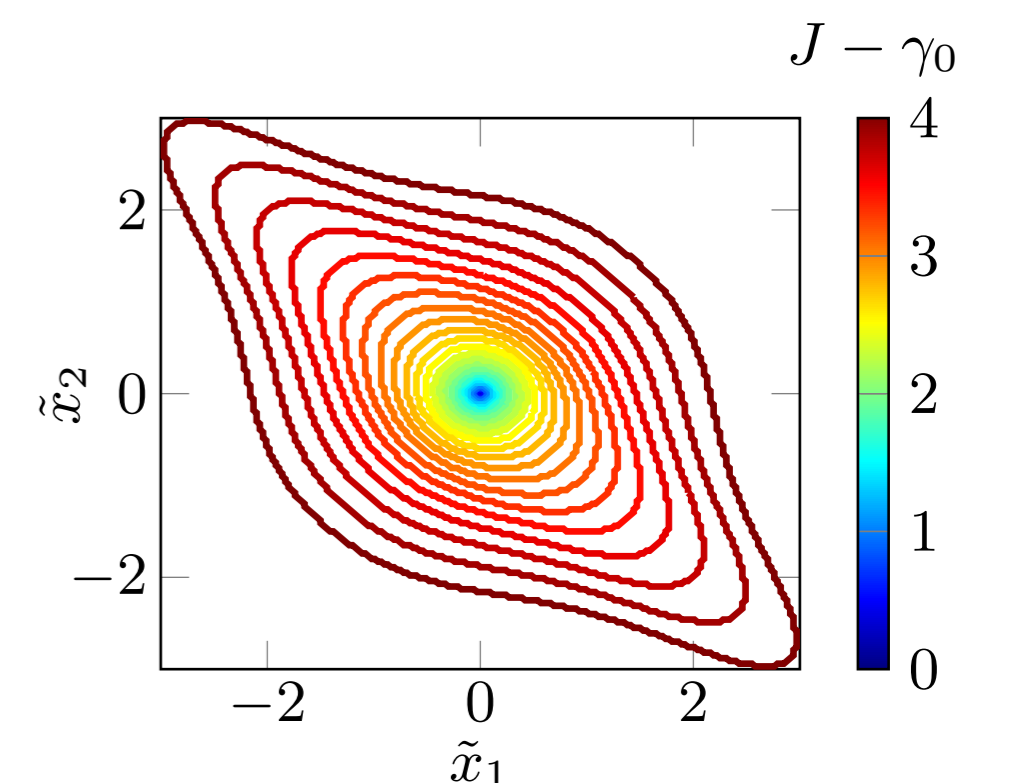
The optimal thresholds are not necessarily convex. Typically, this is the case for systems with strong cross-coupling between the states.



## COMPARISON TO PERIODIC SAMPLING

By numerically computing the optimal sampling policy, we can compare the closed-loop performance using periodic- and event-based sampling for different average sampling periods. Example:

$$A = \begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}, \quad R = Q = I.$$



## REFERENCES

- [1] L. Mirkin. "Intermittent Redesign of Analog Controllers via the Youla Parameter". *IEEE Trans. Automatic Control*, vol 62, no. 4, 2017.
- [2] M. Thelander Andrén, B. Bernhardsson, A. Cervin, K. Soltesz. "On Event-Based Sampling for LQG-Optimal Control". In: *56th IEEE Conference on Decision and Control*, 2017.