Multivariable Adaptive Control

Johansson, Rolf

1983

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Johansson, R. (1983). *Multivariable Adaptive Control*. [Doctoral Thesis (monograph), Department of Automatic Control]. Department of Automatic Control, Lund Institute of Technology (LTH).

Total number of authors:
1

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

Multivariable Adaptive Control

Rolf Johansson

Lund 1983

MULTIVARIABLE ADAPTIVE CONTROL

av

ROLF JOHANSSON

CI, MK

Lunds Nation

Akademisk avhandling som för avläggande av teknologie doktorsexamen vid Tekniska Fakulteten vid Universitetet i Lund kommer att offentligen försvaras i sal M:A, Lunds Tekniska Högskola, tisdagen den 31 maj 1983, kl. 10.15.

NYTTFO BLAD FÖR DOKUMENT OBS! Det går att skriva ovanpå den blå texten.

TYP AV DOKUMENT
☐ Ansökan ☐ Tidskriftsartikel
☒ Doktorsavhandling ☐ Reserapport ☐ Konferensuppsats
☐ Examensarbete ☐ Delrapport ☐ ....
☐ Kompendium ☐ Slutrapport

DOKUMENTBETECKNING/CODEN
CODEN:LUTFD2/(TFRT-1024)/ /1-207/(1983)

AVDELNING/INSTITUTION
Institutionen för Reglerteknik, LTH, Box 725, 220 07 Lund 7; Tel.046-107000

FÖRFATTARE
Johansson, Rolf

DOKUMENTTITEL OCH UNDERTITEL
Multivariable Adaptive Control

SAMMANFATTNING
This thesis treats the problem of direct adaptive control of linear multivariable systems. The parametrization problem of adaptive control is discussed extensively. A pole placement problem and a model matching problem are formulated and interpreted in terms of model reference control. The problem is solved via a discussion on system invariants of multivariable systems as presented by Pernebo. The attention is then directed towards problems of identification and two different estimation schemes are formulated. Parameter convergence is guaranteed provided some conditions on a priori information are satisfied. The requested prior knowledge is formulated in terms of the non-invertible system zeros for the suggested prediction error identification algorithm. The second parameter adjustment law is shown to converge when a certain approximative of the left structure matrix i.e. the system invariant is known. This relaxes a result by Elliott et al where the interactor is required to be known.

The important question of stability of adaptive systems is also treated. The major result is a method for construction of Lyapunov functions for a class of single input adaptive systems. Stability in the sense of Lyapunov and exponential convergence are shown.

NYCKELORD
Adaptive Systems, Feedback Control, Linear Systems, Lyapunov Functions, Stability

DOKUMENTTITEL OCH UNDERTITEL — SVENSK ÖVERSÄTTNING AV UTLÄNDSK ORIGINALTITEL
Flervariabel Adaptiv Reglering

TILLÄMPNINGSOMRÅDE
Adaptiv reglering syftar till god reglering av processer varom man har ofullständig kännedom och där således normala dimensioneringsmetoder ej kan användas utan vidare. Det aktuella bidraget består i några förslag på beräkningsscheman för reglering av vissa knapphändigt kända system med flera in- och utsignaler. Stabilitet visas också.

NYCKELORD
Adaptiva System, Återkoppling, Linjära System, Ljapunov-funktioner, Stabilitet

UTGIVNINGSDATUM	ANTAL SID (inkl bilagor)	SPRÅK
år 1983 mån April	207	☐ svenska ☒ engelska ☐ annat ....

ÖVRIGA BIBLIOGRAFISKA UPPGIFTER

ISSN	
ISBN	
PRIS	

Mottagarens accessionsnummer

Date 31/3 1983 Signature Rolf Johansson

Printed in Sweden
Studentlitteratur
Lund 1983

(3)

70%

LUND INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AUTOMATIC CONTROL
Box 725
S 220 07 Lund 7 Sweden

Document name
Doctoral Dissertation

Date of issue
April 1983

Document number
CODEN:LUTFD2/(TFRT-1024)/1-207/(1983)

Author(s)
Rolf Johansson

Supervisor
Karl Johan Åström, Per Hagander

Sponsoring organization

Title and subtitle
Multivariable Adaptive Control

Abstract

This thesis treats the problem of direct adaptive control of linear multivariable systems. The parametrization problem of adaptive control is discussed extensively. A pole placement problem and a model matching problem are formulated and interpreted interms of model reference control. The problem is solved via a discussion on system invariants of multivariable systems as presented by Pernebo. The attention is then directed towards problems of identification and two different estimation schemes are formulated. Parameter convergence is guaranteed provided some conditions on a priori information are satisfied. The requested prior knowledge is formulated in terms of the non-invertible system zeros for the suggested prediction error identification algorithm. The second parameter adjustment law is shown to converge when a certain approximative of the left structure matrix i.e. the system invariant is known. This relaxes a result by Elliott et al where the interactor is required to be known.

The important question of stability of adaptive systems is also treated. The major result is a method for construction of Lyapunov functions for a class of single input adaptive systems. Stability in the sense of Lyapunov and exponential convergence are shown.

Key words
Adaptive Systems, Feedback Control, Linear Systems, Lyapunov Function, Stability

Classification system and/or index terms (if any)

Supplementary bibliographical information

ISSN and key title

ISBN

Language
English

Number of pages
207

Recipient's notes

Security classification

Distribution: The report may be ordered from the Department of Automatic Control or borrowed through the University Library 2, Box 1010, S-221 03 Lund, Sweden, Telex: 33248 lubbis lund.

Dedicated
To
Nils Johansson
Torup

PREFACE

The present work is concerned with multivariable adaptive control. The first few chapters deal with parametrizations of linear multivariable systems for direct adaptive control.

The problem was suggested by Professor Karl Johan Åström who proposed a systematic investigation of adaptive control for linear, multivariable systems. A lot of inspiration was also provided by Lars Pernebo in a course on algebraic control theory in 1980.

After this work was finished, I turned my attention to the problem of stability. An outline of the present contribution was made during a visit to Qing-hua University, Beijing, China in 1982.

I would like to thank my supervisors Karl Johan Åström and Per Hagander for valuable criticism of the manuscript. I am also indebted to Tore Hägglund, Lars Pernebo and Björn Wittenmark without whom the convergence of the material to the present state would not have taken place. Some financial support has been provided by STU Contract 82-8430. Finally, I would like to thank Leif Andersson for some of the excellent software facilities at the department.

Lund, March 1982

Rolf Johansson

TABLE OF CONTENTS

1
S
H

9 EPILOGUE

10 REFERENCES

APPENDICES

1 P V

1 INTRODUCTION

This thesis deals with multivariable direct adaptive control. The problems of closed-loop specification, parametrization, identification and stability are given special attention.

A large literature has appeared on the adaptive control problem in the past ten years. It has become a challenge to extend any new result of adaptive control to the multivariable case. Various extensions of SISO methods have been formulated by a number of authors. Some of the more important contributions have been provided by Borisson [Bor 2], Goodwin, Ramadge and Caines [GRC] and Elliott and Wolowich [E&W 1]. Keviczky, Hetthessy [K&H] and Koivo [Koi] have also contributed to the area. Comparisons of the contributions of the present paper with earlier work has largely been limited to the work of these authors.

The problems in SISO adaptive control relate to the time varying and non-linear properties. The formulation of a multi-input, multi-output (MIMO) direct adaptive control problem give rise to the same problems. There are also a number of technical difficulties which do not appear in the SISO case. Some of these problems are well-known topics from multivariable linear systems theory of which some are listed below.

D1: An input-output model of the control object may e.g. be described in terms of a left matrix fractional description

1

S

V

$$A_l^*(q^{-1})y(t) = B_l^*(q^{-1})u(t) \tag{1.1a}$$

or with a right matrix fractional description

$$\begin{cases} A_r^*(q^{-1})\xi(t) = u(t) \\ y(t) = B_r^*(q^{-1})\xi(t) \end{cases} \tag{1.1b}$$

where A_l^*, A_r^*, B_l^* and B_r^* are polynomial matrices and q^{-1} is the backward shift operator. The inputs and outputs are denoted by u and y, resp..

The representation (1.1a) is 'linear in parameters' and identification becomes easy. Design rules are on the other hand easier to formulate when starting from a right matrix fractional description. Both properties are certainly of interest in adaptive control. The problem of choice disappears in the SISO case since the two descriptions then coincide.

D2: Polynomial matrices do not commute.

D3: Treatment of cross-couplings and system zeros.

D4: The control laws are nonlinear expressions of the parameters of the control object.

The standard method for parametrization in SISO direct adaptive control is to express the control object by a non-minimal ARMA-model. The controller parameters then appear linearly except for the gain b_0 or estimates of b_0

which tend to appear nonlinearly in either the control law or in the identification algorithms. This method has no straightforward MIMO generalization and the nonlinearity properties with respect to the control object parameters are accentuated. A matter of special concern is

D5: The invertibility properties of B_0 where B_0 is the MIMO counterpart of the gain b_0.

The matrix B_0 relates the design and identification objectives in MIMO adaptive control in the same way as the gain b_0 of the SISO-case - unfortunately in a much more obscure fashion than in the SISO case.

Borisson (see [Bor 2]) considers multivariable extensions of the SISO adaptive control of ARMA-models like

$$A^*(q^{-1})y(t) = B^*(q^{-1})u(t-k) \quad ; \det B^*(0) \neq 0 \qquad (1.2)$$

where A^*, B^* are polynomial matrices. The drastic specialization to systems with equal time delay in all inputs is a consequence of the semi-explicit identification technique. Some non-minimal versions of the A^*- and B^*-matrices are identified and the control law is based on an easily solved Diophantine equation which in this approach gives a control law which is linear in the parameter estimates provided that $B_0=B^*(0)$ is known. The estimated parameters are in this case not asymptotically equal to the desired controller parameters but have to be premultiplied by the inverse of B_0. The asymptotical accuracy of the controller parameters is thus determined by the accuracy of the estimate of the inverse of $B^*(0)$. Extensions to more general systems while keeping the same identification methods would require more complex and nonlinear manipulations of the estimated parameter matrices.

A new step forward was presented in the article by Goodwin, Ramadge and Caines [GRC] where the following class of discrete-time systems with equal numbers of inputs and outputs was considered.

$$\begin{aligned} &A^*(q^{-1})y(t) = B^*(q^{-1})u(t) \; ; \\ &A^* \text{ diagonal and } \det B^*(z) \neq 0 \text{ for } z \in \{z : |z| \leq 1\} \end{aligned} \tag{1.3}$$

The joint conditions on the A^*- and B^*-matrices however rule out a large number of systems without any finite non-minimum phase zeros but including unstable poles. Transfer function matrices factorized to satisfy the diagonality criterion become overparametrized. This complicates the formulation of general design rules.

The articles [Bor 2] and [GRC] were formulated as generalizations of SISO-results. Elliott and Wolovich [E&W 1] approached the problem from the point of view of multivariable theory. They gave their results in terms of the notion of interactor which was formulated by Wolovich and Falb [W&F]. Elliott arrives at the conclusion that the interactor must be known <u>exactly</u> in order to guarantee parameter convergence and stability. This design method gives a better multivariable adaptive servo design than the previously suggested schemes. The assumption on exact knowledge of the full interactor is however more restrictive than the assumptions in [Bor 2] and [GRC].

The following presentation gives solutions to a number of the problems formulated above. The problem statement for this thesis is to make a study of the parameterization problems (D4, D5) for implicit adaptive control of linear multivariable systems. Careful attention will also be given to the problems of cross-couplings and systems zeros (D3). Different requirements on necessary a priori knowledge and their relations to stability concepts will also be investigated.

The thesis is presented in a typical way for adaptive control literature where the outline essentially coincides with the flowchart of the algorithm. First, algebraic manipulation of linear systems (ch.2-5) followed by identification (ch.6), some examples (ch.7) and finally stability considerations (ch.8). The interface of representation between the algebraic tools and the analytic tools of identification is defined in §5.8.

There are several reasons for the emphasis on an algebraic formulation. First, the problems of an algebraic nature (D1-D5) tend to be stressed when multivariable systems are considered. Another reason is to leave the designer with as much freedom as possible.

A third reason is an attempt to join the discussion on invariant parts of transfer functions under causal, stabilizing controllers. This debate has played an important role in the literature on multivariable control over the last few years. The presentation hereby follows the formulations of Pernebo who introduced the concept of 'left structure matrix' to describe these invariants (see [Per 2,3]). The 'interactor' by Wolovich and Falb [W&F] is a similar concept of less generality.

An issue not treated below is the question of parameter tuning versus adaptive control with true parameter tracking. It has been desirable to limit the class of control objects to time invariant ones in order to be able to use results from linear systems theory. This is in a sense sad since much of the raison d'etre of adaptive control consists in parameter tracking of slowly time varying systems. The given results apply only to tuning but may of course be seen as a prototype of a tracking system.

The major results given in the thesis may be summarized as follows:

I Although the left structure matrix has to be included in the closed-loop adaptive system, it does not have to be known exactly.

a) In order to formulate a stability proof of the same kind as given in [GRC], [E&W 1] and [G&C] it is sufficient to know the left structure matrix within certain limits (see (6.37)).

b) If some excitation persistency and boundedness condition are prevalent, it is sufficient to know a certain diagonal matrix containing the system zeros inside the unit circle for a discrete-time system formulated in the backward operator. This matrix - here defined as 'the internal structure matrix' - together with polynomial degrees represent sufficient a priori knowledge to obtain parameter convergence for implicit adaptive control.

c) Knowledge of system zeros in terms of the internal structure matrix and sufficient polynomial degrees are shown to represent the minimal and necessary a priori knowledge to guarantee a solution to the problem of direct adaptive control (§5.9).

II The stability proof of [GRC] may be strengthened to stability in the sense of Lyapunov and exponential convergence for a certain class of adaptive systems. A method for construction of Lyapunov functions for discrete-time, single-input adaptive systems is given.

The study of the parametrization problem closely follows that given in [Joh 1]. It gives a unified parametrization formalism for both discrete-time and continuous-time systems. The idea is to start from the right m.f.d (corresponding to a controllable state space formulation - see [Kai], sec.6.4)

$$\begin{cases} A^*(q^{-1})\xi(t) = u(t) \\ y(t) = B^*(q^{-1})\xi(t) \end{cases} \qquad (1.4)$$

When a right m.f.d. (1.4) is coupled together with a controller described by a left m.f.d. (corresponding to an observable state space formulation)

$$R^*(q^{-1})u(t) = - S^*(q^{-1})y(t) + T^*(q^{-1})u(t) \qquad (1.5)$$

and the closed-loop behaviour is prescribed, it turns out - analogous with the SISO-case - that the description of the control object (1.4) under certain circumstances may be reformulated in terms of the left m.f.d. for the controller with the polynomial matrices R^*, S^* and T^*. The design problem is then solved via the right m.f.d. and the identification problem via the left m.f.d..

Although the very flavour of adaptive control is to reduce necessary a priori knowledge, it seems necessary to assume some of the basic concepts to be known to the reader. The central fields of algebraic multivariable theory, adaptive control theory, identification and stability theory are assumed to be known.

The notation follows essentially [AWW] and [Ega] with some deviations in order to allow for multivariable [Per 1] and identification [GLS] notations. A reference list of notation for the convenience of the reader is given in appendix 1:1. The layout of the chapters is as follows:

Chapter 2 contains a review of the important algebraic properties of generalized polynomials as developed by Pernebo [Per 1].

Chapter 3 gives a review of linear pole-placement controllers and model matching for systems with known parameters.

Possibilities to use linear pole-placement design and model matching as a basis for implicit adaptive control is discussed in chapter 4. Some new pole placement design schemes suitable for adaptive controllers are presented.

A new parametric model for direct adaptive control is proposed in chapter 5. The model structure is chosen so that only linear operations are required to form the adaptive control law from the to-be parameter estimates. The relation between the proposed parametric models and the pole placement methods of the previous chapter is investigated.

Chapter 6 contains a description of identification properties when using two different parameter error identification principles. Some standard type parameter convergence proofs are given for the proposed algorithms.

Examples of parametrization of continuous time and discrete time systems are given in chapter 7. Some simulations are also presented.

Chapter 8 contains a discussion on different stability concepts and their relation to adaptive control. A method for construction of Lyapunov functions is given for a class of SIMO discrete time adaptive systems. A new proof of stability in the sense of Lyapunov and is given in §8.2. Exponential convergence properties are also shown.

2 ALGEBRAIC THEORETICAL BACKGROUND

2.1 Introduction

This chapter reviews some results of Pernebo ([Per 1], II:2) on algebraic properties of generalized polynomial matrices. The aim is to provide mathematical tools for the further analysis of linear multivariable systems. The important notion of left structure matrix is presented in §2.6 where also the new concept of 'internal structure matrix' is introduced.

2.2 Generalized Polynomial Matrices

Let R and C denote the fields of real and complex numbers respectively. Let polynomials with coefficients in R be denoted R[z]. The rational functions with real coefficients are denoted R(z) and let the class of rational function matrices of dimensions nxm be denoted $R^{nxm}(z)$.

The notions of 'poles' and 'zeros' are introduced as known from complex analysis. Consider the sets Z_- and Z_+ where

$$Z_- \cup Z_+ = C \cup \{\infty\} \qquad \text{and} \qquad Z_- \cap Z_+ = \phi$$

The concepts of zeros and poles are easily extended to be defined also in the case where the 'infinity point' is included. Let the set of rational functions in the complex variable z with no poles in a subset Z_- of the complex numbers C ∪ {∞} be called Z-generalized polynomials and let this set be denoted by

$$R_Z[z]$$

The set of <u>Z-generalized polynomial matrices</u>

$$R_Z^{nxm}[z]$$

consist of matrices whose elements are rational functions without poles in the region $Z_- \subset C \cup \{\infty\}$. Polynomial matrices of dimensions nxm are denoted $R^{nxm}[z]$.

The Z-generalized polynomials, $R_Z[z]$, together with the operations multiplication and addition constitute a ring. A non-zero element p in $R_Z[z]$ may be uniquely factorized as p_+p_- where p_+ is a unit in $R_Z[z]$ and p_- is a polynomial with all zeros in Z_-. The degree of p_- is called the Z-degree of p. The ring $R_Z[z]$ is an integral domain and forms together with the 'metric' of Z-degree a Euclidean domain (cf. [Per 1], Thm.II:2.1).

A rational function is said to be <u>Z-stable</u>, if there are no poles in Z_-. A matrix of rational functions is Z-stable, if the elements have no poles in Z_-. A matrix $M \in R_Z^{nxn}[z]$ is <u>Z-unimodular</u>, if there is a matrix $N \in R_Z^{nxn}[z]$ such that $MN = I$.

<u>Remark 2.1</u>

Pernebo uses the prefix notation 'Λ-' to indicate the properties obtained when avoiding poles in a prohibited region Λ. Polynomials etc. in this presentation are however given in the variable z and the prohibited areas are called

Z_-. The prefix notation should, according to this notation standard, be 'Z_--'. For convenience the simpler notation of 'Z-' will however be used here.

2.3 System description

Consider a linear, time-invariant, causal, finite dimensional, dynamical system S_O. Let it be characterized by the transfer operator G_O. The input-output relation is

$$y(t) = G_O(z)u_O(t) \tag{2.1}$$

where $G_O(z)$ is assumed to be an nxm matrix of rational functions in the variable z. For a discrete time system z denotes the forward shift operator q. For continuous time systems z denotes the differential operator p.

Let the system input vector u_O of dimension mx1 be decomposed as

$$u_O = \begin{pmatrix} u \\ d \end{pmatrix} \quad \text{and} \quad d = \begin{pmatrix} v \\ w \end{pmatrix} \tag{2.2}$$

where u is an m_u x 1 vector of control variables and d is an m_d x 1 vector of disturbance inputs to the system. The disturbance vector d is further decomposed into v and w where v denotes measured disturbances, which may be used for feedforward compensation. The disturbance vector w represents the non-measurable inputs to the system. Furthermore, let y be decomposed as

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{2.3}$$

where y_1 denotes the n_1 vector of outputs to be controlled while y_2 denotes the n_2 vector of additional outputs.

The transfer operator $G_o(z)$ may be partitioned as

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = G_o(z)\; u_o(t) = \begin{bmatrix} G_{1u}(z) & G_{1d}(z) \\ G_{2u}(z) & G_{2d}(z) \end{bmatrix} \begin{bmatrix} u(t) \\ d(t) \end{bmatrix}$$

or

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{1u} & G_{1v} & G_{1w} \\ G_{2u} & G_{2v} & G_{2w} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{2.4}$$

2.4 Fractional Representations

A transfer operator matrix $G(z)$ of dimensions $n{\times}m$ may be decomposed into fractional representations. A system description is then

$$A_y(z)\; y(t) = B_u(z)\; u(t) \quad \Rightarrow \quad G(z) = A_y^{-1}(z)\; B(z) \tag{2.5a}$$

This factorization is called a <u>left matrix fractional description</u> (left m.f.d.). Another description is given by a <u>right matrix fractional description</u> (right m.f.d.).

$$\begin{cases} A_\xi(z)\; \xi(t) = u(t) \\ y(t) = B_\xi(z)\; \xi(t) \end{cases} \quad \Rightarrow \quad G(z) = B_\xi(z)\; A_\xi^{-1}(z) \tag{2.5b}$$

9 S H

The matrices A_y, B_u, A_ξ and B_ξ are Z-generalized polynomial matrices. It is required that the matrices $A_y \in R_Z^{nxn}[z]$ and $A_\xi \in R_Z^{mxm}[z]$ are full rank Z-generalized polynomial matrices.

Consider a m.f.d (A,B) of the type (2.5a) or (2.5b) and define

$$M_l(z) = \begin{bmatrix} A_y(z) & B_u(z) \end{bmatrix} \quad ; \quad M_r(z) = \begin{bmatrix} A_\xi(z) \\ \\ B_\xi(z) \end{bmatrix} \tag{2.6}$$

A $z=z_0 \in Z_-$ and such that $M_l(z_0)$ (or $M_r(z_0)$) looses rank is called a common Z-zero of A and B.

If a m.f.d. has a common zero of A and B, then it is possible to find another representation which does not contain any common left (or right) divisor such that M_l (or M_r) looses rank in Z_- (cf.[Kai], p.370). A left m.f.d. with this property is called a <u>left Z-coprime factorization</u> (l.Z.c.f) and is unique up to multiplication from the left by a Z-unimodular matrix. Similarly, a <u>right Z-coprime factorization</u> (r.Z.c.f) is unique up to multiplication from the right by a Z-unimodular matrix. The A-matrices can thus be made triangular or block triangular by using the non uniqueness of the factorization.

2.5 Stability and Causality

A discrete time transfer operator G(q) is asymptotically stable, if it has no poles outside the unit disc. Analogously, a continuous time transfer operator G(p) is asymptotically stable if it has no poles in the closed right half plane. The unstable regions are

$$Z_- = \left\{z: z\in C \text{ and } \mathrm{Re}\ z\geq 0\right\} \quad ; \text{(Continuous Time Case)}$$

$$Z_- = \left\{z: z\in C \text{ and } |z|\geq 1\right\} \quad ; \text{(Discrete Time Case)} \tag{2.7}$$

If stronger stability concepts are required, this will be called practical stability. In general an unstable region Z_- is defined, in which it is undesirable to have poles of a transfer operator G(z). The stable region will be denoted by Z_+. Thus we have

$$Z_- \cup Z_+ = C \cup \{\infty\} \quad \text{and} \quad Z_- \cup Z_+ = \phi$$

The transfer operators are called proper if

$$\lim_{z\to\infty} \|G(z)\| < \infty \tag{2.8}$$

In the discrete time case this means that the system is causal, i.e. the output does not depend on future values of the inputs. For a continuous time system it means that there are no pure differentiators.

Properness may be guaranteed by avoiding poles at infinity. Only proper systems will be treated in the sequel and the following conditions are therefore imposed on the instability region Z_-.

- Z_- contains the regions of asymptotic instability
- Z_- is symmetric with respect to the real axis
- Z_- may not contain all points on the real axis
- Z_- contains the 'infinity point' (2.9)

2
P
V

Properness and stability may then be treated by the same formalism. The fractional representations with polynomial entries of z corresponding to the differential operator p or the forward shift operator q are thus not allowed, since all non-constant polynomials have poles at infinity. This will be avoided by a transformation of the variable z such that Z_- including infinity is mapped into a bounded region Z^*_-.

Consider the one-to-one transformation

$$z^* = \frac{b}{z - a} \tag{2.10}$$

where $a \in Z_+ \cap R$ and $b > 0$. Infinity will then be mapped into the origin, and the unstable region Z^*_- will be a bounded subset of C when a is an internal point of Z_+. The positive constant b may be chosen arbitrarily. In this presentation b is however always chosen such that $z^*=1$ corresponds to $z=1$ for discrete time and $z=0$ for continuous time systems. This means that $z^*=1$ corresponds to the static properties of both discrete time and continuous time systems. For discrete time systems we will choose $a=0$. This means that z^* corresponds to the backward shift operator q^{-1}. For continuous time systems the parameter a is chosen as a negative real number.

A matrix M^* will denote a Z^*-generalized polynomial matrix expressed in the variable z^*. In particular, all polynomial matrices in z^* are Z^*-generalized polynomial matrices. According to standard multivariable theory on matrix fraction descriptions (cf.[Kai], pp.367-370) it can now be stated that there exist polynomial, fractional representations (A_y^*, B_u^*) and (A_ξ^*, B_ξ^*), which are l.Z^*.c.f and r.Z^*.c.f. respectively.

2.6 The Structure Matrix

Difficulties of control in SISO-systems are largely determined by the zeros of the involved transfer functions. Similar problems arise in the MIMO case and the difficult part of the transfer function can be described by a number of 'structure matrices'. The structure matrices (left, internal, right) act as system invariants and determine the properties of servo behaviour, information transmission and input reconstructors. The presentation is here confined to the algebraic description and the interpretations will be elaborated in natural contexts of coming chapters.

Consider a Z^*-generalized polynomial matrix $M^* \in R_{Z^*}^{n\times m}[z^*]$ of rank r. The matrix M^* may be factorized as

$$M^*(z^*) = L^*(z^*)\, S^*(z^*)\, R^*(z^*) \tag{2.11}$$

where $L^* \in R_{Z^*}^{n\times n}[z^*]$, $S^* \in R^{n\times m}[z^*]$ and $R^* \in R_{Z^*}^{m\times m}[z^*]$. Here L^* is left Z^*-invertible, R^* is right Z^*-invertible and the polynomial matrix S^* contains all the Z^*-zeros of M^* and has no zeros outside Z^*. The matrix S^* is called the Z^*-Smith form of M^* (cf.([Ros], 2:3) and ([Per 1], II:2)).

2

$$S^* = \begin{pmatrix} A^* & 0_{n,m-n} \end{pmatrix}, \qquad n < m$$

S

V

$$S^* = \Lambda^* \qquad , \qquad n = m$$

$$S^* = \begin{bmatrix} \Lambda^* \\ O_{n-m,m} \end{bmatrix} \qquad , \qquad n > m \qquad (2.12)$$

where Λ^* is $\mathrm{diag}(s_1^*,\ldots\ldots,s_r^*,0,\ldots,0)$. The matrix Λ^* is a diagonal, polynomial matrix in the variable z^* and contains the invariant polynomials s_i^* on the principal diagonal. Each non-zero polynomial s_i^* is monic and divides s_{i+1}^* for $i=1,2,\ldots\ldots,r-1$. The polynomials s_i^* have all their zeros in $\underline{Z}^*$. The Z^*-Smith form is unique but L^* and R^* are not unique.

Other decompositions will be used in this presentation. The Smith-form is a particular decomposition of the form

$$M^* = M_L^* M_S^* M_R^* \quad ; \quad M_S^* = \begin{bmatrix} M_\Lambda^* & O_{n,m-n} \end{bmatrix} \text{ etc.} \qquad (2.13)$$

with the same dimensions as L^*, S^*, R^* and Λ^*. Let the divisibility conditions of the invariant polynomials s_i^* be relaxed so that any ordering of the invariant polynomials on the first r positions on the principal diagonal is sufficient. Let furthermore the condition of the monic, invariant polynomials s_i^* be replaced by the requirement that

$$(M_S^*)_{ii}(1) = 1 \qquad \text{for} \quad 1 \leq i \leq r \qquad (2.14)$$

for the class of decompositions given by (2.13). This requirement is possible to satisfy in the ordinary stability case except when M^* looses rank for $z^*=1$. The condition is of importance for the static behaviour of a system. A decomposition of this kind will be used in lemma 4.1 and onwards.

A number of concepts which will be used extensively in the rest of the presentation are now mentioned. Let M^* be some numerator matrix of a m.f.d. and let the factorization (2.13) hold together with the condition (2.14).

<u>Definition 2.1</u>: M^*_S is an internal Z^*-structure matrix of M^*.

The concept 'internal structure matrix' is introduced since it will be shown to - loosely speaking - represent the <u>information transmission</u> properties of the system. This concept is new and not found in Pernebo's presentation.

The M^*_S-matrix is obviously not unique. The arbitrariness will later on (4.7) be exploited to find an M^*_S with special properties.

<u>Remark 2.2</u>: $M^*_{LS}=M^*_L M^*_S$ is a representative of the left Z^*-structure matrix of M^*.

A Z^*-generalized polynomial matrix M^* may be factorized (cf.[Per 1], II.2.3) as a left structure matrix and a right invertible matrix. The matrix M^*_R of (2.13) is one such right invertible matrix.

Pernebo has shown that the left structure matrix of a system determines the <u>servo properties</u> of the system ([Per 1], II:5).

<u>Remark 2.3</u>: $M^*_{SR} = M^*_S M^*_R$ is a right Z^*-structure matrix of M^*.

The right structure matrix of a system determines the properties of an <u>input reconstructor</u> acting on the output ([Per 1], II.7).

Diagonalization of M^*_{LS} by multiplication from the right by a Z^*-generalized matrix N^* can always be achieved when M^*_{LS} is square. The diagonalizing matrix N^* is however not in general Z^*-unimodular, which implies that $\det(M^* N^*)$ in general will contain additional Z^*-zeros compared to $\det(M^*)$. Resulting, diagonal, Z^*-generalized polynomial matrices will be denoted by subscript 'D' e.g. M^*_D.

3 MULTIVARIABLE POLE PLACEMENT DESIGN

3.1 Introduction

Pole-placement controllers for systems with known parameters will be reviewed in this chapter. The reader is referred to ([Per 1], II) for a full presentation. This section has a preparatory character and gives a review of some algebraic synthesis concepts.

Some general control objectives and constraints on design are given by assumptions 3:1-3. Pole placement and model matching and their limitations will be discussed. Some implications for controller design for known control objects are given in terms of the left structure matrix.

It will be stated that the left structure matrix has to be included in feasible servo designs. Further, reference models describing a desired system behaviour to be followed perfectly has to include a representative of the left structure matrix. The relevance of different representations of the Z^{*}-zeros for adaptive control is treated in chapters 4 and 5.

3.2 General Design Constraints

Some general assumptions for pole placement design will now be made.

Assumption 3:1

Consider a control object of the type presented in §2.3 i.e. a time-invariant, finite-dimensional and causal linear system. Assume also that the control object is stabilizable from the control input u. The transfer operator from u to the outputs y is furthermore assumed to be strictly proper.

Assumption 3:2

Consider a linear, causal and finite dimensional control law of the type

$$R_u^*(z^*)u(t) = - S_y^*(z^*)y(t) + T_{uc}^*(z^*)u_c(t) \tag{3.1}$$

from the outputs y and the command signal u_c to the control input u.

Assumption 3:3

The closed loop system is required to:

° be stable

° be able to asymptotically follow a reference signal

° be able to reject disturbances (3.2)

The stability requirement of assumption 3:3 implies also internal stability. It then follows that the closed-loop system must not have any uncontrollable or unobservable Z^*-unstable modes. It is also necessary that R_u^* and S_y^* are relatively left Z^*-prime.

All pole placement problems do not have causal solutions. The conditions for existence are conveniently expressed using the left structure matrix (cf.[Per 1], II:7). Some hints of the implications for servo design are given in the following paragraph.

3.3 Servo Design Constraints

Consider a r.Z^*.c.f. representation of a control object.

$$\begin{cases} A^*_\xi \xi = u \\ y = B^*_\xi \xi \end{cases} \tag{3.3}$$

Choose a linear controller in a l.Z^*.c.f. form

$$R^*_u u = - S^*_y y + T^* u_c \tag{3.4}$$

The closed loop system is given by

$$y = B^*_\xi \left(R^*_u A^*_\xi + S^*_y B^*_\xi \right)^{-1} T^* u_c \tag{3.5}$$

Any attempt to cancel the Z^*-zeros of the control object will result in unobservable unstable modes. This is a serious difficulty of pole placement and model matching. The usual pole-placement algorithms do not avoid any cancellations of unstable zeros in a control object unless all zeros that should remain in the transfer function are explicitly specified.

An admissible controller will therefore result in a closed loop system of the type

$$y = B^*_{LS} F^*_u u_c \tag{3.6}$$

where B^*_{LS} - the left structure matrix - represents the part of the B^*-matrix which is non-invertible from the right by a Z^*-stable compensator and thus invariant. The matrix F^*_u is free to choose as any Z^*-generalized polynomial matrix. The properties of the left structure matrix in servo design are investigated in ([Per 1], II:5) and in [Per 2]. The left structure matrix is necessary to include in pole placement design in general and thus also for adaptive design based on pole placement.

Consider for a moment the Z^*-zeros and how they relate to the left structure matrix. An explicit knowledge of the Z^*-zeros of the control object $G^*=B^*A^{*-1}$ may be represented in several ways. Assume for simplicity that B^* is a quadratic full-rank polynomial matrix. The simplest quantity which contains all the Z^*-zeros is then

$$\det B^*(z^*) \tag{3.7}$$

The Z^*-zeros of a system can also be described by factorizations like (2.11) or by the Z^*-Smith form (2.13). Here the S^*-matrix contains the zeros on the diagonal and all non-diagonal elements are zero.

$$B^* = L^* S^* R^* \tag{3.8}$$

The S^*-matrix is an entity of less complexity than B^*_{LS}.

Consider now the matrix

$$B^{*}_{LS} \tag{3.9}$$

which can be represented by the product of the L^{*}- and S^{*}-matrices of (2.11). The inclusion of L^{*} in the left structure matrix - despite of its invertibility properties - comes from the fact that the regulator identities in (3.3)-(3.5) only operate from the right. The left structure matrix B^{*}_{LS} thus contains not only information about the system Z^{*}-zeros but also of the cross-couplings.

3.4 Reference Models

A major design objective is the ability to follow a given reference value. It is in many cases desirable to impose some limitations on the closed-loop behaviour e.g. in terms of a bandwidth limitation. A convenient way to describe such properties is to assume that the reference signal is generated from a command signal u_c by a specified model on the form

$$A^{*}_{M}(z^{*})\ y^{m}_{1}(t) = B^{*}_{M}(z^{*})\ u_{c}(t) \tag{3.10}$$

It was on the other hand made plausible in §3.3 that a left structure matrix $B^{*}_{LS}(z^{*})$ is incorporated in all feasible closed-loop transfer operators. A straightforward implication is that any reference model to be followed perfectly by a system with the characteristics of the control object (3.3) must contain some representative of this system invariant.

Let therefore the class of reference models be restricted to models of the form

$$\begin{cases} A_M^*(z^*)\, y_1^m(t) = B_M^*(z^*)\, u_c(t) \\ y_1^M(t) = B_{LS}^*(z^*) T_r^*(z^*)\, y_1^m(t) \end{cases} \qquad (3.11)$$

where u_c is the command signal, y_1^m is the reference signal and y_1^M is the model output to be tracked perfectly. The compensator T_r^* is free to choose as long as it is stable. The matrices A_M^* and B_M^* should be polynomial and relatively left Z^*-prime. The matrix $A_M^*(z^*)$ is required to be Z^*-unimodular since the reference model has to be stable. The factorization between A_M^* and B_M^* will be chosen such that

$$A_M^*(0) = I \qquad (3.12)$$

which does not impose any more restrictions, since the transfer operators are required to be of full rank and proper. Triangularization of A_M^* by elementary row operations on A_M^* and B_M^* may also be achieved.

4 MULTIVARIABLE ADAPTIVE CONTROL STRATEGIES

4.1 Introduction

The presentation has sofar contained reviews of what can be achieved when a linear control object is fully known. The case of unknown parameters will now be discussed. Two MIMO control problems will be formulated and new design algorithms (4.17), (4.21) will be given. The algorithms are of moderate intrinsic interest for MIMO controller design with known parameters. They will however be shown (ch.5) to have excellent properties for adaptive control. A specialization of the earlier presented structure matrices will be made to fit the adaptive control formulation.

It was made plausible in §3.5 that any model to be followed perfectly by the closed-loop system must include a representation of some system invariant. Elliott&Wolovich [E&W 1] and Goodwin&Chan [G&C] have discussed this problem. They have given solutions in those cases where the so called interactor (see [W&F]) is explicitly included in the reference model. Exact knowledge of the interactor is unfortunately a very restrictive requirement.

The model matching problem is presented below with a general problem formulation in terms of the left structure matrix. A foundation for a solution of the adaptive control problem is given in this chapter. It will be shown later that this allows a solution which does <u>not require explicit knowledge of the left structure matrix.</u>

3.
P.
H

A somewhat simpler problem will first be treated. The MIMO adaptive pole placement problem will be given a new formulation in §4.2. Only explicit pole placement strategies implemented by direct adaptive control will be covered. The more complex MIMO minimum-variance strategies will thus not be treated.

A similar problem has been addressed by Elliott et al [EWD]. They have shown that prior knowledge of the controllability indices is sufficient to guarantee that there is a solution to the MIMO adaptive pole placement problem. The controllability indices reflect the infinite zero properties of the control object in the sense that they give an upper bound on the infinite zero multiplicities (see [Kai], p.406). The solution in this presentation aims at a characterization of the required a priori knowledge in terms of the system zeros (including the infinite zeros with respect to z) described by the matrix B_S^* to be presented.

In short, the following chapter provides a basis for solution of one MIMO adaptive pole placement regulator problem and one model matching problem. The two problems have a notational framework in common. This is directed towards a specification of required a priori information. Three new assumptions, 4:1-3, will be introduced.

4.2 Adaptive Control Objectives

Consider the control object

$$A^*(z^*)\xi(t) = u(t)$$

$$y(t) = \begin{bmatrix} y_1(t) \\ \\ y_2(t) \end{bmatrix} = \begin{bmatrix} B_1^*(z^*) \\ \\ B_2^*(z^*) \end{bmatrix} \xi(t) = B^*(z^*)\xi(t) \qquad (4.1)$$

where y_1 are the measured outputs to be controlled and y_2 are other measured outputs. It is assumed that A^* and B^* are relatively right Z^*-prime, polynomial matrices. A more complete problem set-up involving different kinds of disturbances, feedforwards etc. is formulated in appendix 4:3.

Two more assumptions will be made except for those already given by assumptions 3:1-3 namely

Assumption 4:1

Let the number of independent control inputs be equal to the number of full rank, controlled outputs. □

This specialization only imposes mild restrictions. The rank condition does obviously never allow the number of full rank, controlled outputs to exceed the number of linearly independent control inputs. Any remaining control inputs will be formally referred to as a part of the disturbance input vector v for which no controller parametrizations will

3 S H

be derived. See appendix 4:3 for a full problem formulation!

Assumption 4:2

The open-loop transfer operator is such that

$$\det B_1^*(1) \neq 0 \qquad \square$$

This assumption is a necessary requirement on the dynamics of y_1 in order to assure a solution with independent set points for each of the components of y_1 (cf.(2.14) etc.). Assume now a controller of the type (3.3) i.e.

$$R_u^*(z^*)u(t) = - S_y^*(z^*)y(t) + T_{uc}^*(z^*)u_c(t) \tag{4.2}$$

or

$$R_u^* u = - S_y^* y + T_{ym}^* y_1^m \tag{4.2'}$$

when the control law is expressed w.r.t. a reference value y_1^m instead of a command signal u_c (cf.§3.4). This gives the following closed-loop transfer operator to the controlled outputs y_1

$$y_1(t) = B_1^* \left[R_u^* A^* + S_y^* B^* \right]^{-1} T_{ym}^* y_1^m(t) \tag{4.3}$$

Two different design objectives (DO:1, DO:2) will be studied. The two problems will be referred to as the pole placement case (DO:1) and the model matching case (DO:2), respectively.

Consider a control object (4.1) and a controller (4.2) satisfying assumptions 3:1-3 and 4:1-2. The design objectives are:

DO:1

POLE PLACEMENT: The closed-loop system should be stable with certain closed-loop poles and with a unit static gain from reference value y_1^m to the controlled outputs y_1. There are no specifications on transient cross-coupling properties of the transfer operator from the reference signal y_1^m to the outputs y_1.

DO:2

MODEL MATCHING: The servo case should satisfy not only the requirements for the pole placement case but should also have completely specified transient input-output properties. The transfer operator from the reference signals y_1^m to the controlled outputs y_1 is specifically required to be diagonal.

Regulator design with respect to measurable disturbance inputs v can also be treated. A problem statement including disturbances is found in App.4:3. The performance of the adaptive feedforward control will not be considered here but in the chapter of parametric models (§5.3 and §5.7) because of the non-attractive present form of the transfer operator G_{vy}^*. Some new identities (4.17a-c), which are also useful for a simplification of G_{vy}^*, will first be introduced.

3
P
V

The case of regulator design with respect to non-measurable disturbances w will not be treated since the models will not be linear in parameters. The disturbances are however still kept in the system description for purposes of analysis.

The transfer operator properties obtained in (A4.3.10) guarantee stability and boundedness with respect to bounded disturbances w since the closed loop system has stable transfer operators(cf.(A4.3.10)). It is however not possible to neglect w in the identification algorithms. The accuracy of parameter estimates obtained from an identification algorithm is highly dependent of influence from unknown inputs to the system.

4.3 The Reference Model Revisited

An interpretation in terms of a reference model will now be given for the design objectives of the previous paragraph. The main reason to make such a formulation is to facilitate comparisons with earlier contributions in the area of adaptive control.

It was made plausible in §3.4 that a left structure matrix $B^{*}_{LS}(z^{*})$ must be incorporated in any reference model to be followed perfectly by a system with the characteristics of the control object (4.1). The class of reference models was therefore in §3.4 restricted to models of the form

$$\begin{cases} A_M^*(z^*)\, y_1^m(t) = B_M^*(z^*)\, u_c(t) \\ y_1^M(t) = B_{LS}^*(z^*)\, T_r^*(z^*)\, y_1^m(t) \end{cases} \tag{3.11}$$

The polynomial matrices A_M^* and B_M^* should be relatively left Z^*-prime. The matrix $A_M^*(z^*)$ is required to be Z^*-unimodular since the reference model has to be stable.

The requirement that B_{LS}^* must be incorporated in the model is no particular restriction for the investigation of cross coupling and static gain properties when B_{LS}^* is known. If the full left structure matrix B_{LS}^* of the system is known, then it is also possible to find a suitable compensator T_r^* to satisfy desired specifications on static gains etc..

Knowledge of cross-couplings and zeros does however become an awkward problem in the adaptive context. The matrix B_{LS}^* is then unknown and it is not evident how cross-couplings and gain properties should be evaluated. The formulation of a reasonable reference model satisfying (3.11) therefore leads to considerable technical difficulties when B_{LS}^* is unknown. On the other hand, a requirement to know B_{LS}^* is very restrictive and should be avoided - in particular if it is only needed in order to formulate a reference model. It would be desirable to have B_{LS}^* incorporated in the reference model without actually knowing it. The problem is solved by certain choices of the compensator T_r^* so that

35V

$$B_{LS}^*(z^*)T_r^*(z^*) \tag{4.4}$$

becomes less complicated than B_{LS}^* itself. The pole placement problem with a unit static gain specification would then involve the question of finding a stable T_r^* such that

$$B_{LS}^*(1)T_r^*(1) = I \tag{4.5}$$

The model matching problem would similarly include the problem of finding a stable T_r^* such that

$$B_{LS}^*(z^*)T_r^*(z^*) = B_D^*(z^*) \quad \text{and} \quad B_D^*(1) = I \tag{4.6}$$

for some feasible, polynomial matrix B_D^* which is chosen diagonal in order to avoid cross couplings and to fulfil specifications on input-output properties.

A problem formulation in terms of model reference adaptive control would then include the problem to find T_r^* adaptively so that B_{LS}^* does not have to be known. Further efforts are therefore directed towards a characterization of this compensator.

Remark 4.1

'Truely' diagonal transfer operators may be achieved also by diagonalizing the open loop system and applying ordinary SISO control design methods. This design scheme obviously requires the knowledge of the diagonalizing matrix and/or its inverse. Although these matrices might be found in an adaptive regime, the controller design problem remains to be solved. This case will not be elaborated.

4.4 A New Structure Matrix Decomposition

The left structure matrix B^*_{LS} has sofar deliberately been described in very general terms. The problem formulation in §4.2 will soon be investigated and it seems desirable to be more specific without imposing new restrictions. Some attention will therefore be directed towards certain algebraic characterizations of this system invariant.

Most authors of MIMO adaptive control literature seem to favour different triangular system invariant matrices like the interactor or some Hermite form (see [E&W 1], [G&C1]). Another approach is taken in linear systems literature where the Smith-form decompositions seem to prevail.

Unfortunately, none of these common representations seem to serve the stated adaptive purposes. Therefore a new matrix factorization of the transfer operator relating u and y_1 will be presented in order to facilitate the analysis below. This factorization is a particular choice among all possible choices of factorizations of the control object. It does not impose any additional restrictions as will be seen in the following lemma.

Lemma 4.1

A square (nxn) transfer operator G^* of full rank with $\det(G^*(1))\neq 0$ corresponding to a strictly proper transfer operator G may be decomposed into a relatively r.Z^*.c.f. (A^*,B^*). The factorization is such that A^* contains all the Z^*-poles of G^* and B^* contains all Z^*-zeros of G^* and such that

$$G^{*} = B^{*} A^{*-1} = B_{L}^{*} B_{S}^{*} B_{R}^{*} A^{*-1} \qquad (4.7)$$

where

A^{*} is a square, full rank polynomial matrix

B_{L}^{*} and B_{R}^{*} are polynomial Z^{*}-unimodular matrices

B_{S}^{*} is a diagonal polynomial matrix (4.8)

satisfying

$$A^{*}(0) = I$$

$$B_{S}^{*}(1) = I$$

$$B_{R}^{*}(0) = \left[b_{ij}\right]_{R} \text{ is upper right triangular and invertible}$$

$$(b_{ii})_{R} = 1 \quad \text{for} \quad 1 \leq i \leq n \qquad (4.9)$$

The polynomial matrix B_{S}^{*} contains all zeros of G^{*} in Z_{-}^{*} but has no zeros in Z_{+}^{*}. The stability region considered is assumed to be given by (2.7).

□

The proof is given in appendix 4:1 in the form of a construction of one such decomposition and is based on the Z^{*}-Smith form.

<u>Remark 4.2</u>

It is necessary to demand $\det(G^*(1)) \neq 0$ for the lemma to hold. The critical point is that the monic polynomials of the Z^*-Smith form may not rescaled such that $s_i^*(1)=1$. This is essentially a static, full rank condition which is necessary to impose in order to satisfy static gain requirements for y_1 (cf.Ass.4:1-2, (2.14)).

It follows from lemma 4.1 that the transfer operator G_1^* from u to y_1 of (4.1) can be decomposed as

$$G_1^* = B_1^* A^{*-1} \tag{4.10}$$

with the properties

$$A_u^*(0) = I$$

and

$$B_1^* = B_L^* B_S^* B_R^* \tag{4.11}$$

A representative of the left structure matrix is given by

$$B_{LS}^* := B_L^* B_S^* \tag{4.12}$$

The diagonal matrix B_S^* containing all the non-invertible zeros of the transfer operator between u and y_1 will in next chapter be shown to be a way to characterize necessary a priori information for MIMO adaptive control. This matrix will be referred to as the <u>internal structure matrix</u> in the sequel.

4.5 A Special Pole Placement Solution

Consider now the pole placement problem DO:1 which was formulated in §4.2. Let a polynomial denominator matrix A_M^* describe the desired poles to be included in the closed-loop system. This kind of specification on closed-loop poles has already been touched in the context of reference models in §3.4 and §4.3. Some technical assumptions will now be made. The polynomial matrix A_M^* is required to be Z^*-unimodular and is also scaled such that

$$A_M^*(0) = I \tag{3.13}$$

Let now T_1^* be a unimodular (or Z^*-unimodular) matrix which makes A_M^* lower triangular and such that

$$T_1^*(0) = I \quad \Rightarrow \quad T_1^*(0)A_M^*(0) = I \tag{4.13}$$

The product $T_1^*A_M^*$ may in principle be rational as long as Z^*-unimodular. The proofs below are however formulated for polynomial T_1^* and A_M^* and a unimodular T_1^*. Extensions are trivial.

An appropriate choice of A_M^* may be an alternative to introduction of T_1^*. The inclusion of T_1^* is essentially given here in order to allow comparisons to design rules for certain SISO adaptive control algorithms (cf.e.g.[Ega]).

A final assumption will now be made.

<u>Assumption 4:3</u>

Consider the polynomial, internal structure matrix

$$B_S^* = \begin{bmatrix} b_1^*(z^*) & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & b_n^*(z^*) \end{bmatrix} \tag{4.14}$$

The off-diagonal elements of the lower triangular matrix

$$T_1^* A_M^* = \begin{bmatrix} p_{11}^*(z^*) & & & 0 \\ \vdots & \ddots & & \\ \vdots & & \ddots & \\ p_{n1}^*(z^*) & \dots & & p_{nn}^*(z^*) \end{bmatrix} \tag{4.15}$$

should be chosen such that each polynomial p_{ij}^* of a certain column j in $T_1^* A_M^*$ has the polynomial b_j^* of the B_S^*-matrix as a factor so that

$$p_{ij}^*(z^*) = b_j^*(z^*) p'^*_{ij}(z^*) \quad \text{for} \quad 1<i<n \quad \text{and } j<i \tag{4.16}$$

□

This condition does not impose any restrictions on pole placement per se but it cripples some attempts to specify cross-couplings.

The following lemma forms a fundament for the rest of the presentation. The proof is given in appendix 4:2.

Lemma 4.2

There exist polynomial matrix solutions R_u^*, S_y^*, T_2^*, T_3^* and a diagonal B_D^* satisfying the following equations for given A_M^*, A^*, B_L^*, B_S^*, B_R^*, B^* and T_1^* with the properties given by the expressions of (4.7)-(4.9) and (4.15-16) under assumptions 3:1-3, 4:1-3. The matrix T_2^* is a polynomial Z^*-unimodular matrix.

$$T_2^*T_1^*A_M^* B_L^*B_S^* = B_S^* T_1^*A_M^*$$

$$R_u^*A^* + S_y^*B^* = T_1^*A_M^*B_R^* \quad ; \qquad R_u^*(0) = B_R^*(0)$$

$$T_2^*T_1^*A_M^*B_D^* = B_S^*T_3^* \qquad (4.17a-c)$$

Remark 4.3

Notice that the solution for a polynomial T_2^* in some cases requires a trivial change in the factorization of lemma 4.1. This case is covered in the proof. A polynomial solution is obtained by a transfer of a Z^*-unimodular polynomial from the columns of B_L^* to the rows of B_R^*. The matrix B_L^* is thus made rational instead of polynomial but is still of course Z^*-unimodular. The matrix B_S^* is not affected.

The following theorem gives a solution to the pole placement problem DO:1. It gives also a basis for MIMO adaptive control.

Theorem 4.1

Consider the control object (4.1) under assumptions 3:1-3, 4:1-3 with B_1^* decomposed according to (4.7)-(4.9). Let R_u^*, S_y^* and T_{ym}^* of (4.2)' be chosen from the polynomial solutions R_u^*, S_y^* and T_2^* to the equations

$$T_2^* T_1^* A_M^* B_L^* B_S^* = B_S^* T_1^* A_M^*$$

$$R_u^* A^* + S_y^* B^* = T_1^* A_M^* B_R^* \quad ; \quad R_u^*(0) = B_R^*(0)$$

$$T_{ym}^* = T_2^* T_1^* A_M^* \qquad \text{(Pole Placement)} \qquad (4.18a-c)$$

The closed-loop system then represents a solution to the pole placement problem DO:1.

□

Proof:

The existence of the identities (4.17a-c) is shown by lemma 4.2. The closed loop system is obtained via the expressions (4.1), (4.2)' and (4.18a-c) as

$$\begin{cases} T_1^* A_M^* B_R^* \xi(t) = T_{ym}^* y_1^m(t) \\ \\ y(t) = B^* \xi(t) \end{cases}$$

and the transfer operator (4.3) to y_1 is

$$G^*_{y_1 y_1 m}(z^*) = B^*_1\left[R^*_u A^* + S^* B^*_y\right]^{-1} T^*_2 T^*_1 A^*_M = \qquad (4.19)$$

$$= B^*_L B^*_S B^*_R\left[T^*_1 A^*_M B^*_R\right]^{-1} T^*_2 T^*_1 A^*_M = \left[T^*_2 T^*_1 A^*_M\right]^{-1} B^*_S\left[T^*_2 T^*_1 A^*_M\right]$$

It is easy to see that the transfer operator (4.19) is related to B^*_S by a similarity transformation. The closed-loop system contains the desired poles described by A^*_M and also those of B^*_R i.e. some of the control object zeros appear as closed-loop poles. The static gain requirement of the pole placement problem DO:1 in §4.2 i.e. (4.5) is also satisfied since

$$G^*_{y_1 y_1 m}(1) =$$

$$= \left[T^*_2(1) T^*_1(1) A^*_M(1)\right]^{-1} B^*_S(1)\left[T^*_2(1) T^*_1(1) A^*_M(1)\right] = I \qquad (4.20)$$

The transient cross coupling properties may however not be investigated without knowledge of T^*_2.

Remark 4.4

The solution via (4.18) has been shown to include the poles of A^*_M in the closed-loop transfer operators. Certain other closed-loop poles do however also appear namely the control object zeros of B^*_R. There are also possibly poles from T^*_1 and U^* of (A4.2.4) depending on a special choice of A_M as well as possible poles of z^*. The characterization of (4.18) as a pole placement solution is therefore in a more precise sense somewhat misleading. The presentation will however not deviate from the established abuse of language in the field of adaptive control.

A reference model which exhibits the same input-output properties as that of the pole placement solution (4.18) to DO:1 may also be given. The compensator T_r^* would then be

$$T_r^*(z^*) = \left[T_1^*(z^*)A_M^*(z^*)\right]^{-1} T_2^*(z^*) \left[T_1^*(z^*)A_M^*(z^*)\right]$$

A much simpler choice of T_r^* would however satisfy the requirements of (4.5) and the design objective DO:1 in §4.2 since only pole placement and static gain properties are required. It may be argued that any choice of a reference model is somewhat artificial in the pole placement case. Any feasible model specifies namely a lot more than what is required in the design objective.

4.6 The New Model Matching Solution

The solution to the model matching problem DO:2 is similar to the pole placement case. The following theorem holds.

Theorem 4.2

The transfer operators of the closed-loop system (4.3) and the reference model (3.11), (4.6) under assumptions 3:1-3, 4:1-3 coincide if R_u^*, S_y^* and T_{ym}^* of the controller (4.2)' are chosen from the following polynomial solutions R_u^*, S_y^*, T_2^* and T_3^* for diagonal matrices B_S^* and B_D^*. The polynomial matrix T_2^* is Z^*-unimodular.

4PH

$$T_2^* T_1^* A_M^* \; B_L^* B_S^* = B_S^* \; T_1^* A_M^*$$

$$R_u^* A^* + S_y^* B^* = T_1^* A_M^* B_R^*$$

$$T_2^* T_1^* A_M^* B_D^* = B_S^* T_3^*$$

$$T_{ym}^* = T_3^* \qquad \text{(Model Matching)}$$

$$(4.21a\text{–}d)$$

<u>Proof</u>

The existence of the identities (4.21a-d) follows from lemma 4.2. The following calculations show that the model-matching problem DO:2 has a solution through the identities (4.21a-d). The transfer operator from y_1^m to y_1 is

$$G^*_{y_1 y_1^m}(z^*) = B_L^* B_S^* B_R^* \left(T_1^* A_M^* B_R^*\right)^{-1} T_3^* =$$

$$= \left(\left(T_2^* T_1^* A_M^*\right)^{-1} B_S^* \left(T_1^* A_M^*\right)\right)\left(T_1^* A_M^*\right)^{-1} T_3^* =$$

$$= \left(T_2^* T_1^* A_M^*\right)^{-1} B_S^* T_3^* = \left(T_2^* T_1^* A_M^*\right)^{-1}\left(T_2^* T_1^* A_M^*\right) B_D^* = B_D^* \qquad (4.22)$$

These matrix manipulations are allowed since T_2^*, T_1^* and A_M^* are Z^*-unimodular matrices. It is also now seen that the closed-loop system contains the poles described by A_M^*. The input-output transfer operator is furthermore diagonal as prescribed in in the model matching problem formulation DO:2 of §4.2. □

An interpretation in terms of the reference models of §4.3 can also be given. The diagonal matrix B_D^* may be chosen considerably less complicated than B_{LS}^* and is reasonable to include in a model which should be followed perfectly by the closed-loop system. A compensator T_r^* which diagonalizes B_{LS}^* from the right is obtained from (4.21) and (4.22) as

$$T_r^*(z^*) = \left[T_1^*(z^*)A_M^*(z^*)\right]^{-1} T_3^*(z^*) \tag{4.23}$$

A reference model of the type (4.6) with the compensator of (4.23) would then exhibit the same input-output behaviour as the solution to (4.21). The compensator T_r^* is of course not necessary to know in order to erect a model. The sufficient knowledge consists of one B_D^* such that there are solutions to (4.17).

4.7 An Example

A simplified example showing the decompositions of (4.7) and the matrix identities of (4.17) is now given. The matrix calculations are made in unknown parameters but with known polynomial degrees. Consider the control object

$$y = \begin{pmatrix} b_1 q^{-3} & b_2 q^{-1} \\ 0 & b_3 q^{-2} \end{pmatrix} u \overset{\Delta}{=} B^*(q^{-1})u \tag{4.25}$$

Y

S

H

where it is assumed that b_1, b_2 and b_3 are unknown non-zero constants. It is obvious that the control object transfer function has the poles at the origin and an A^*-matrix equal to identity matrix. Notice that the B_0-matrix is singular.

Let the objective be to find a control scheme which gives a diagonal transfer function with unit gain from the reference value to the output. Assume also that the poles should be kept as poles at the origin.

A decomposition of $G^* = B^*$ of the type (4.7) is possible since $\det(B^*(1)) \neq 0$. The full rank condition on B^* is satisfied as long as b_1 and b_3 are non-zero. Elementary row and column operations on B^* now give

$$B^*(q^{-1}) = B_L^*(q^{-1})B_S^*(q^{-1})B_R^*(q^{-1}) =$$

$$= \begin{bmatrix} 0 & b_2 \\ -\frac{b_1 b_3}{b_2} & b_3 q^{-1} \end{bmatrix} \begin{bmatrix} q^{-4} & 0 \\ 0 & q^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{b_1}{b_2} q^{-2} & 1 \end{bmatrix} \tag{4.26}$$

The condition on dead-beat poles is satisfied by the choice

$$T_1^* A_M^* = I \tag{4.27}$$

A solution to (4.17b) i.e.

$$R_u^* A^* + S_y^* B^* = T_1^* A_M^* B_R^* \tag{4.28}$$

is given by

$$R_u^* = \begin{bmatrix} 1 & 0 \\ \frac{b_1}{b_2} q^{-2} & 1 \end{bmatrix} \quad \text{and} \quad S_y^* = 0 \tag{4.29}$$

The matrix T_2^* of (4.17a) is the inverse of B_L^*

$$T_2^* = \begin{bmatrix} \frac{1}{b_1} q^{-1} & -\frac{b_2}{b_1 b_3} \\ \frac{1}{b_2} & 0 \end{bmatrix} \tag{4.30}$$

A feasible B_D^* which diagonalizes B^* is given by

$$B_D^* = \begin{bmatrix} q^{-3} & 0 \\ 0 & q^{-4} \end{bmatrix} \tag{4.31}$$

The corresponding T_3^* of (4.17c) is then given by

$$T_3^* = \begin{bmatrix} \frac{1}{b_1} & -\frac{b_2}{b_1 b_3} \\ \frac{1}{b_2} q^{-2} & 0 \end{bmatrix} \tag{4.32}$$

Application of the control law (4.2)'

$$R_u^*(q^{-1})u(t) = T_3^*(q^{-1})y^m(t) \tag{4.33}$$

4
p
V

given by (4.23) - or explicitly

$$u_1(t) = \frac{1}{b_1} y_1^m(t) - \frac{b_2}{b_1 b_3} y_2^m(t)$$

$$u_2(t) = \frac{1}{b_2} y_1^m(t-2) - \frac{b_1}{b_2} u_1(t-2) \tag{4.34}$$

gives the 'closed loop' system

$$y = B^* R^{*-1} T_3^* y^m = B_D^* y^m \tag{4.35}$$

The alternative control law of (4.18)

$$R_u^*(q^{-1}) u(t) = T_2^*(q^{-1}) y^m \tag{4.36}$$

which does not promise a diagonal transfer function but at least unit static gains, result in the transfer function

$$y(t) = T_2^{*-1} B_S^* T_2^* y^m(t) =$$

$$= \begin{bmatrix} q^{-1} & 0 \\ \frac{b_2}{b_3}\left(-q^{-5} + q^{-2}\right) & q^{-4} \end{bmatrix} y^m(t) \tag{4.37}$$

which is statically diagonal.

4.8 Conclusions

This chapter has considered two MIMO control problems using certain polynomial matrix-techniques. Three restrictions have been imposed compared to the general problem description which involves assumptions 3:1-3. Assumption 4:1-2 are new as well as the condition (4.16) of ass.4:3 on the matrix $T_1^* A_M^*$. The intention has not been to suggest any new MIMO design methods but to provide a basis for direct adaptive control. The emphasis to describe the properties of a closed-loop system has motivated the introduction of the concepts of B_{LS}^*, B_D^* and T_r^*. The matrix B_S^* has sofar only played a technical role but will soon (ch.5-6) be shown to determine some properties of identification. The relevance for adaptive control will be shown in next chapter.

5 PARAMETRIC MODELS FOR ESTIMATION

5.1 Introduction

The purpose of this section is to derive estimation models for implementation of the solutions in the previous chapter by implicit adaptive control. The concept of implicit adaptive control is presented extensively e.g. in [Ega], [A&W 1] or [A&W 2] where also further references are given. The idea is to estimate the controller parameters directly rather than to make a detour via identification of the control object followed by control law computations.

A typical way to solve the problem in the SISO-case of both self-tuning regulators and model reference adaptive control is to describe a minimum-phase control object in terms of the control law by a non-minimal degree ARMA-model

$$y(t+k) = b_0\left[u(t) + r_1 u(t-1) + \ldots + s_0 y(t) + \ldots\right] \qquad (5.1)$$

The estimation algorithm aims at finding successively better estimates of the control law parameters r_i and s_i by output prediction error identification. The time delay k of the system and an estimate β_0 of some accuracy of the gain b_0 are usually required to be known in advance in order to obtain convergence.

Another way to formulate the problem was introduced by Johnson and Tse [J&T] who stated the input matching principle.

The latter approach will be followed here. The SISO assumption about a known time delay will be shown to have its MIMO counterpart in the diagonal matrix B_S^*, which contains the non-invertible zeros of the control object. This result considerably relaxes the conditions on required prior knowledge discussed in [E&W 1] or [G&C] in terms of interactor etc..

In the special case of a discrete-time MIMO control object without finite non-minimum phase zeros, it holds that B_S^* contains pure time delays in the diagonal entries. The correspondence with the SISO-case is here obvious.

The gain b_0 may be given natural interpretations for MIMO systems (see e.g. [Bor 2], [GRC]). The MIMO counterpart B_0 in previous formulations however causes technical problems concerning the invertibility when used in both gradient method estimation algorithms [GRC] or in the control law [Bor 2]. Most authors have therefore been forced to restrict the class of feasible control objects to such with an invertible B_0.

A hint to see the lack of representability of B_0 for the gain properties of a control object may be obtained from the decomposition in theorem 4.1 of the B^*-matrix

$$B^* = B_L^* B_S^* B_R^* \quad ; \quad \det B_L^*(0) \neq 0 \ , \ \det B_R^*(0) \neq 0 \tag{5.2}$$

where it is easily conceived of that the gain properties relate to both B_L^* and B_R^*.

A solution in the general case is given below in terms of the matrix T_2^* introduced in the previous section. It will be shown in chapter 7 that T_2^* actually plays a role similar to the inverted b_0 of the SISO-case.

It is highly desirable - for the sake of computation as well as for stability proofs - to express the control laws in terms of identified parameters such that addition and multiplication are the only operations required for the computation of the control law from the parameter estimates. An estimation based on output prediction involves - as discussed above - the a priori knowledge of an approximate gain matrix inverse which is not trivial to know and which has to be estimated one way or the other. A remedy is to be given below.

The following presentation is therefore kept as an attempt to avoid all explicitly nonlinear formulations which would imply division by parameter estimates etc.. The objective is to find expressions that are linear in the desired parameters as well as in measured input-output data.

The first goal of this chapter is then to find a representation - linear in parameters - of the type

$$u_i = \vartheta_i^T \phi_i \tag{5.3}$$

for each control variable u_i. The ϕ-vector denotes some known data vector and ϑ denotes the controller parameters associated with a pole placement algorithm. The design properties of the pole placement controller should be fulfilled by using the control input of (5.3).

The adaptive problem is however precisely that the parameter vector ϑ_i is not known. Thus the second problem of this chapter is to find equations, which allow estimation of ϑ_i. Let '¯' denote the application of some linear operator on input-output data. Assume that some computable sequence $\{[\bar{u}(t),\bar{\phi}(t)]\}$ satisfies the same equation i.e.

$$\bar{u}_i(t) = \vartheta_i^T\bar{\phi}_i(t) \tag{5.4}$$

Then it is possible to estimate ϑ_i by standard vector space methods. Expression of the type (5.4) - linear in input-output data and desired controller parameters and furthermore identifiable - will be called linear, identifiable parametric models (LIP-models) below. The objective of the reparametrization is then to develop some function of lowest possible complexity which realizes (5.4).

5.2 Model Transformations

Consider a r.Z*.c.f description of a MIMO control object

$$A^*(z^*)\xi(t) = u(t)$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} B_1^*(z^*) \\ B_2^*(z^*) \end{bmatrix}\xi(t) = B^*(z^*)\xi(t) \qquad (4.1),\ (5.5)$$

which is assumed to be square and full-rank between the input u and the controlled outputs y_1 and also in other respects satisfy the assumptions (3.1)-(3.4), 4:1-2. No disturbances are considered at this moment. Extensions with measurable and non-measurable disturbances are formulated in §§5.3,5.7 below where also adaptive feedforward is treated.

The matrices A^* and B^* are not identifiable from linear expressions since the partial state $\xi(t)$ is not measurable. The r.Z^*.c.f is from this point of view no good starting point even for explicit adaptive control. A reformulation is necessary. Consider the controlled outputs y_1

$$y_1(t) = B_1^*(z^*)\xi(t) \tag{5.6}$$

Using the identities of lemma 4.2 this may be formulated as

$$y_1(t) = B_1^*\left(T_1^*A_M^*B_R^*\right)^{-1}\left(R_u^*A^* + S_y^*B^*\right)\xi(t) =$$

$$= B_L^*B_S^*\left(T_1^*A_M^*\right)^{-1}\left(R_u^*u(t) + S_y^*y(t)\right) \tag{5.7}$$

where B_1^* has been decomposed according to lemma 4.1. Let now the invertible filter operator $T_2^*T_1^*A_M^*$ be applied on the output y_1

$$T_2^*T_1^*A_M^*y_1(t) = T_2^*T_1^*A_M^*\,B_L^*B_S^*\left(T_1^*A_M^*\right)^{-1}\left(R_u^*u(t) + S_y^*y(t)\right) \tag{5.8}$$

Recalling lemma 4.2 it is possible to simplify the operator

$$T_2^*T_1^*A_M^*\,B_L^*B_S^*\left(T_1^*A_M^*\right)^{-1} = B_S^* \tag{4.17a}$$

and the following parametric model appears

$$T_2^* T_1^* A_M^* y_1(t) = B_S^* \left[R_u^* u(t) + S_y^* y(t) \right] \tag{5.9}$$

Define

$$y_f(t) = T_1^* A_M^* y_1(t) \tag{5.10}$$

Then

$$T_2^* y_f(t) = B_S^* \left[u(t) + R_0 u(t) + R^* u(t) + S^* y(t) \right] \tag{5.11}$$

where

$$R_0 = B_R^*(0) - I$$

$$R^* = R_u^* - R_0 - I$$

$$S^* = S_y^* \tag{5.12}$$

The parameters of interest in order to state the pole placement control law (4.2) for each control input u_i consist of the parameters of row i of

$$T_2^*,\ R_0,\ R^* \text{ and } S^* \tag{5.13}$$

since B_S^* is diagonal. Moreover, the parameters of (5.11) can be identified since the expressions are linear in the desired parameters and input-output data. Notice that y_f can be computed since $T_1^* A_M^*$ is specified by the designer. The diagonal matrix B_S^* is then assumed known.

The parameters of each row are linearly related by an expression involving u, y and $T^*_1 A^*_M y_1$ since B^*_S is diagonal. This property will be used to derive LIP-models for the control inputs.

Remark 5.1

The property that the matrix B^*_S is diagonal is crucial for the existence of expressions, which are 'pure' in the controller parameters. Any non-diagonal matrix in the place of B^*_S would result in sums etc. of the parameters.

Hence a linear, identifiable parametric model is derived for the controller. This holds for the disturbance-free case since B^*_S is assumed to be a known operator and since R_O is an upper triangular matrix with zeros on the diagonal. This imposes the mild restriction that when computing a control input u it is necessary to compute u_{i+1} before u_i. Despite of this, each u_i is still a linear expression of inputs and outputs to the system. This restriction is removed, when $B^*_R(0)$ happens to be diagonal. Then it will be possible to compute each control input u_i without any regards to the computations of u_j for $i \neq j$ at the same time instant.

A similar formulation for the model matching case is obtained by rephrasing the calculations (5.6)-(5.12) on the output error

$$e = y_1(t) - y^M_1(t) = y_1(t) - B^*_D y^m_1(t) \tag{5.14}$$

Filtration by $T_2^* T_1^* A_M^*$ and substitution of

$$T_2^* T_1^* A_M^* B_D^* = B_5^* T_3^* \tag{4.17c}$$

give the result

$$T_2^* e_f = B_5^* \left[u + R_0 u + R^* u + S^* y - T_3^* y_1^m \right] \tag{5.16}$$

where

$$e_f(t) = T_1^* A_M^* e(t) \tag{5.17}$$

This model is satisfactory for identification when no disturbance acts on the system. An equation for estimation of the controller parameters of each control input may now easily be formulated from (5.11) or (5.16) as

$$\bar{u}_i(t) = \theta_{2i}^T \varphi_{2i}(t) - \theta_{1i}^T \bar{\varphi}_{1i}(t)$$

where '‾' means filtering with b_i^*. The parameter vectors θ_{2i} and θ_{1i} contain the parameters of row i of T_2^* and R_0, R^*, S^*, T_3 ,resp.. The φ_i's are the corresponding data vectors. Estimation of parameters from a linear model is now straightforward and will be covered in chapter 7.

Example 5.1

The example of §4.7 can be brought to an estimation scheme for direct adaptive control via (5.16) which in this case specializes to

$$T_2^*(q^{-1})\,e = B_S^*\left[u + R^*(q^{-1})u - T_3^*(q^{-1})y^m\right]$$

or

$$u_1(t-4) = \frac{1}{b_1}e_1(t-1) - \frac{b_2}{b_1b_3}e_2(t) + \frac{1}{b_1}y_1^m(t-4) - \frac{b_2}{b_1b_3}y_2^m(t-4)$$

$$u_2(t-1) = \frac{1}{b_2}e_1(t) - \frac{b_1}{b_2}u_1(t-3) + \frac{1}{b_2}y_1^m(t-3)$$

The two relations form linear models for estimation of all the necessary parameters of the control law

$$u_1(t) = \frac{1}{b_1}y_1^m(t) - \frac{b_2}{b_1b_3}y_2^m(t)$$

$$u_2(t) = \frac{1}{b_2}y_1^m(t-2) - \frac{b_1}{b_2}u_1(t-2)$$

An elaborated example with simulations is found in §7.4.

□

The next few paragraphs are however not devoted to estimation but to extensions of the problem formulation. In order to formulate a more general case it is necessary to investigate the error propagation in the system. This is done in next paragraph.

5.3 Disturbance Considerations

Some attention will now be given the impacts of disturbances. The presentation therefore returns to the full problem formulated in appendix 4:3.

The transfer operator to the controlled outputs in (A4.3.9) in presence of disturbances may be written (use eq.(4.18), (4.23) and (A4.3.1)).

$$y_1(t) = B^*_{1u}(z^*)\xi_u(t) + B^*_{1v}(z^*)\xi_v(t) + B^*_{1w}(z^*)\xi_w(t) =$$

$$= B^*_{1u}\left(T^*_1 A^*_M B^*_R\right)^{-1}\left(R^* A^*_{uu} + S^* B^*_{y.u}\right)\xi_u(t) +$$

$$+ B^*_{1v}\xi_v(t) + B^*_{1w}\xi_w(t) =$$

$$= B^*_L B^*_S\left(T^*_1 A^*_M\right)^{-1}\left(R^*_u u + S^*_y y\right) + H^*_d d \tag{5.18}$$

where

$$H^*_d d = - B^*_L B^*_S\left(T^*_1 A^*_M\right)^{-1}\left[\left(R^* A^*_{uv} + S^* B^*_{y.v}\right)\xi_v + \left(R^* A^*_{uw} + S^* B^*_{y.w}\right)\xi_w\right]$$

$$+ B^*_{1v}\xi_v + B^*_{1w}\xi_w \tag{5.19}$$

Recall that $T^*_1 A^*_M$ was chosen according to (4.15)-(4.16) such that it is possible to find a Z^*-unimodular polynomial T^*_2 satisfying the relation

5
P
H

$$T_2^* T_1^* A_M^* B_L^* B_S^* = B_S^* T_1^* A_M^* \qquad (4.17a)$$

If the expression (5.18) is multiplied on both sides with the Z^*-unimodular polynomial matrix

$$T_2^* T_1^* A_M^* \qquad (5.20)$$

it is possible to simplify the right hand side to

$$T_2^* T_1^* A_M^* y_1 = B_S^* \left[R_u^* u + S_y^* y \right] + T_2^* T_1^* A_M^* H_d^* d \qquad (5.21)$$

Consider now the disturbance term (5.19) in the proposed parametric model (5.21). Utilize the identity (4.17a)

$$T_2^* T_1^* A_M^* H_d^* d =$$

$$= \left[\left[T_2^* T_1^* A_M^* B_{1v}^* - B_S^* \left[R_u^* A_{uv}^* + S_y^* B_{.v}^* \right] \right] \right] \xi_v +$$

$$+ \left[\left[T_2^* T_1^* A_M^* B_{1w}^* - B_S^* \left[R_u^* A_{uw}^* + S_y^* B_{.v}^* \right] \right] \right] \xi_w \qquad (5.22)$$

An expression for ξ_v is obtained from appendix 4:3. This notation is introduced in (5.22) which give

$$F_v'^* = T_2^* T_1^* A_M^* B_{1v}^* \quad \text{and} \quad F_w'^* = T_2^* T_1^* A_M^* B_{1w}^*$$

$$T_v'^* = \left[R_u^* A_{uv}^* + S_y^* B_{.v}^* \right] \quad \text{and} \quad T_w'^* = \left[R_u^* A_{uw}^* + S_y^* B_{.w}^* \right]$$

If this notation is introduced above (5.23)

$$T_2^* T_1^* A_M^* H_d^* d = \left(F'^*_v - B^*_S T'^*_v\right) A^{*-1}_{vv} v + H'^*_w w \tag{5.24}$$

where

$$H'^*_w = -\left(F'^*_v - B^*_S T'^*_v\right) A^{*-1}_{vv} A^*_{vw} A^{*-1}_{ww} + \left(F'^*_w - B^*_S T'^*_w\right) A^{*-1}_{ww} \tag{5.25}$$

It is necessary to cancel the denominators introduced by the inverted A^*_{vv} in (5.24) in order to obtain a polynomial expression for the terms corresponding to the disturbance inputs v. This may be done by introduction of a new diagonal Z^*-unimodular matrix U^* chosen such that the diagonal entries consist of the l.c.d. of each row of the matrices

$$\left(F'^*_v A^{*-1}_{vv} \; , \; B^*_S T'^*_v A^{*-1}_{vv}\right) \tag{5.26}$$

and scaled such that

$$U^*(0) = I \tag{5.27}$$

If (5.21) is multiplied by U^* from the left, a polynomial matrix will be the result - except for the w-dependent terms. This result is presented with a somewhat abbreviated notation in (5.29).

Introduce the following shorter notation

$$y_f = T_1^* A_M^* y_1$$

5
S
H

$$e = y_1 - y_1^M$$

$$e_F = T_1^* A_M^* e$$

$$T_F^* = U^* T_2^* \qquad (5.28a\text{-}g)$$

$$R^* = U^* R_u^* - B_O$$

$$S^* = U^* S_y^*$$

$$F_v^* = U^* F_v'^* A_{vv}^{*-1} \quad , \; T_v^* = U^* T_v'^* A_{vv}^{*-1} \quad \text{and} \quad H_w^* = U^* H_w'^*$$

The full parametric model for the pole placement case of (4.18a-c) is given by (5.29), where B_S^* is the known (diagonal) internal structure matrix and R_O is guaranteed to be an upper triangular constant matrix with zeros on the diagonal.

Although a formal dissimilarity between the formulations of (5.21) and (5.29) holds, the following may be stated.

<u>Theorem 5.1</u>

The parametric models (5.21) and (5.29) represent the same transfer operators from (u,v) to y_1.

$$T_F^* y_F = B_S^* \left[u + R_O u + R^* u + S^* y - T_v^* v \right] + F_v^* v + H_w^* w \qquad (5.29)$$

□

Proof

Since (5.21) and (5.29) differ only by a common Z^*-unimodular factor

$$U^*$$

multiplied from the left, this common factor may be cancelled whereby the identities coincide. The common Z^*-unimodular factors may be interpreted as decoupled stable modes, which do not appear in the resulting closed loop transfer operators.

Remark 5.2

A non-Z^*-unimodular T_2^* can always diagonalize the structure matrix from the left such that B_0 becomes diagonal. However the new Z^*-zeros would introduce unstable modes in the closed loop system as is seen in equation (5.29) and is thus not allowed.

5.4 Parametric Models - Model Matching

Let the output error be defined as $e = y_1 - y_1^M$ (cf.(5.28b)). If we form the output error equation of the system we obtain

$$T_f^* T_1^* A_M^* e =$$

$$= B_S^* \left(u + R_0 u + R^* u + S^* y - T_v^* v \right) + F_v^* v - T_f^* T_1^* A_M^* B_D^* y^m + H_w^* w$$

$$= B_S^*\left(u + R_0 u + R^* u + S^* y - T_v^* v - T_m^* y_1^m \right) + F_v^* v + H_w^* w$$

where (5.30)

$$T_m^* = U^* T_3^* \qquad (5.31)$$

where the polynomial matrix $T_2^* T_1^* A_M^* B_D^*$ has been refactorized as $B_S^* T_3^*$ (cf. 4.17c). Note that all the requirements on B_D^* are stated by the conditions of (4.17a-c). Then T_m^* may be estimated directly.

A parametric model for the model matching case is

$$T_f^* e_f = B_S^*\left(u + R_0 u + R^* u + S^* y - T_v^* v - T_m^* y^m \right) +$$

$$+ F_v^* v + H_w^* w \qquad (5.32)$$

The quantities to estimate which are of interest for the controller are

R_0, R^*, S^*, T_v^* and T_m^*

while

T_f^* and F_v^*

also should be represented in the estimation algorithm in order to guarantee existence of solutions. The terms of $H_w^* w$ will however not be represented in the estimation algorithms.

5.5 Another Interpretation Of The Parametric Models

It is possible to make the following observations in the case when $w=0$ and where all outputs are controlled outputs, i.e. where $y_1 = y_0$.

The parametric model (5.29) may be rearranged into the form

$$\left[T_f^* T_1^* A_M^* - B_S^* S^*\right] y = B_S^*\left[I + R_0 + R^*\right] u + \left[F_v^* - B_S^* T_v^*\right] v \tag{5.33}$$

which is immediately recognized as a left m.f.d. representation of the open loop system.

Let this be expressed in the left m.f.d. notation as

$$A_y^* y = B_u^* u + C_v^* v \tag{5.34}$$

Similarly (5.32) can be interpreted as a left m.f.d. representation of the open loop system

$$\left[T_f^* T_1^* A_M^* - B_S^* S^*\right] e = B_S^*\left[I + R_0 + R^*\right] u$$

$$+ \left[F_v^* - B_S^* T_v^*\right] v - B_S^*\left[T_m^* - S^* B_D^*\right] y_1^m \tag{5.35}$$

which might be denoted in the left m.f.d fashion as

$$A_e^* e = B_u^* u + C_v^* v - B_{ym}^* y_1^m \tag{5.36}$$

In the case where all outputs are controlled outputs these arguments may be summed up as

5 S V

Theorem 5.2

The pole placement identities (4.17a-c) perform a transformation from a right m.f.d. to a left m.f.d. of the open loop system. □

Remark 5.3

The parametric model is independent of whether the system operates in closed or open loop. This is an important property useful for identification.

The parametric models have been derived only by using the polynomial matrix identities and open loop properties of the control object. The control inputs and outputs have nowhere been specified in the calculations.

Remark 5.4

This property will be used in a following presentation to show that parameter identification can be performed in both open and closed loop modes.

Remark 5.5

Notice that the resolution of a known A_y^* associated with the proposed pole placement solution i.e.

$$A_y^* = T_f^*\left[T_1^* A_M^*\right] - B_S^* S^* \tag{5.37}$$

for known $T_1^* A_M^*$ and B_S^* is associated with the general polynomial matrix equation

$$XA + BY = C \qquad (5.38)$$

where A, B and C are known matrices while X and Y are the desired solutions. Although a linear equation this is usually considered hard to solve ([Par 1], II:2.5).

5.6 The Pole Placement Control Law

The correct control law to apply for the pole placement controller with feedforward from v and y_1^m is the following

$$u = - R_0 u - R^* u - S^* y + T_v^* v + T_{ym}^* y_1^m$$

where

$$T_{ym}^* = T_f^* T_1^* A_M^* \qquad \text{(Pole Placement)} \qquad (5.39)$$

or

$$u = - R_0 u - R^* u - S^* y + T_v^* v + T_{ym}^* y_1^m$$

$$T_{ym}^* = T_m^* \qquad \text{(Model Matching)} \qquad (5.40)$$

Remark 5.6

The terms F_v^* and T_v^* should be incorporated in the identification algorithm, since influence of unknown inputs to the system will always corrupt identification.

It is however not necessary to incorporate the feedforward term $T_v^* v$ in the control law. When knowing both F_v^* and T_v^* from the on-line identification it is also possible to compute some other suitable feedforward. It would also be possible to have a fixed, chosen feedforward etc..

The application of the control law (5.39) gives the closed loop system (cf.(4.19))

$$T_f^* T_1^* A_M^* y_1 = B_S^* T_f^* T_1^* A_M^* y_1^m + F_v^* v + H_w^* w \qquad (5.41)$$

for the pole placement case - and

$$T_f^* T_1^* A_M^* e = F_v^* v + H_w^* w \qquad (5.42)$$

for the model matching case(cf.(4.22)).

The pole placement solution has the prescribed static gain properties since

$$B_S^*(1) = I \qquad (4.9)$$

Remark 5.7

The pole placement control law (A4.3.4) may be restituted from (5.37) by collecting terms and by cancellation of the common factor U^*. Alternatively it might be considered to be a control law derived with another Z^*-unimodular T_1^*.

5.7 Feedforward Properties

It is possible to ignore or modify the feedforward suggested by the terms F^*_v and T^*_v although these factors are important to include in the identification algorithm. When a control law described by (5.39) is applied then the influence of v on the output is given by

$$T^*_f T^*_1 A^*_M e = F^*_v v \tag{5.43}$$

It remains to show that the application of the feedforward term $T^*_v v$ really is such that the influence of v on the output is in some sense small. The approach taken here is to show that no feedforward terms influence the terms of F^*_v of powers less than that of the corresponding element of B^*_S. It is on the other hand possible to eliminate all terms of F^*_v of powers greater than that of the corresponding element of B^*_S. This result is similar to that of §3.4. Then F^*_v represents a necessary correlation matrix for the input disturbance v with respect to the control input channel. It is possible - since B^*_S is diagonal - to make the decomposition of C^*_v in (5.36) into F^*_v and T^*_v row by row such that the highest power of the elements in each row of F^*_v is lower than the highest power of the corresponding diagonal element of B^*_S. This is done by standard use of the division algorithm for polynomials row by row. Let this be formulated in the following theorem.

Theorem 5.3

There exists a polynomial matrix solution

$$\begin{pmatrix} F_v^* & T_v^* \end{pmatrix} \tag{5.44}$$

satisfying the relation - where B_S^* is diagonal

$$C_v^* = F_v^* - B_S^* T_v^* \tag{5.45}$$

and such that the highest power of each row of F_v^* is lower than the power of the corresponding entry in the diagonal matrix B_S^*.

The proof is given in appendix 5:1.

It is now straightforward to see that the proposed control law (5.39) or (5.40) realizes the choice of T_{ff}^* in the control law (A4.3.4) with least degrees of the corresponding F_v^*. The disturbance influence on the output is given by

$$G_{vy_1}^* = B_{1u}^* \left[R_u^* A_{uu}^* + S_y^* B_{\cdot u}^* \right]^{-1} T_{ff}^* + \left[U^* T_2^* T_1^* A_M^* \right]^{-1} H_v^* \tag{5.46}$$

which is the transfer operator from v (cf.(A4.3.10)). If the polynomial identities (4.17) are used to simplify the first term above, it follows that

$$G_{vy_1}^* = \left[U^* T_2^* T_1^* A_M^* \right]^{-1} \left[B_S^* U^* T_{ff}^* + F_v^* - B_S^* T_v^* \right] \tag{5.47}$$

The lowest possible input polynomial degree

$$U^* T^*_{ff} = T^*_v \qquad (5.48)$$

since F^*_v is impossible to manipulate by choices of T^*_{ff}. Finally - since (A4.3.4) and (5.39)-(5.40) differ by the common factor U^* that appears in (5.48) it is clear that the feedforward of (5.39)-(5.40) satisfy the optimality condition of a minimal degree input polynomial with respect to v.

Example 5.2

A common choice of a feedforward is such that the static influence from v on the output is eliminated i.e.

$$G^*_{vy_1}(1) = 0$$

This may be achieved if the component T^*_v of (5.39) or (5.40) is replaced by the feedforward component

$$T^*_v - F^*_v(1)$$

which is verified by direct substitution into (5.46).

5.8 Control Input LIP-Models

Consider a parametric model of the type (5.29). Denote the elements of the matrices T^*_f, R_0, R^*, S^*, T^*_v and F^*_v by their lower case equivalents such that

$$T_f^* = \left[(t_f^*)_{ij}\right]$$

$$R_O = \left[(r_O)_{ij}\right] \quad \text{etc.} \tag{5.49}$$

Let the notation

$$(t_f^*)_{i.} \quad \text{and} \quad (s^*)_{.j} \tag{5.50}$$

mean the i:th row and j:th column of T_f^* and S^* respectively.

Each row i of (5.29) may then be expressed as

$$(t_f^*)_{i.} y_f =$$

$$= (b_i^*)\left[u + (r_O)_{i.} u + (r^*)_{i.} u + (s^*)_{i.} y - (t_v^*)_{i.} v\right] +$$

$$+ (f_v^*)_{i.} v \tag{5.51}$$

It is possible to commute any two multiplied polynomials since all the polynomials are time-independent by assumption. Let the following notation be introduced for any quantity x_i

$$(\bar{x}_i)_j = b_j^* x_i \tag{5.52}$$

or when i=j

$$\bar{x}_i = b_i^* x_i$$

Overbar '‾' denotes that the quantity x has been filtered by

b^*_i. It is now straightforward to obtain

$$(t^*_f)_{i.} y_f =$$

$$= \bar{u}_i + (r_0)_{i.}(\bar{u})_i + (r^*)_{i.}(\bar{u})_i + (s^*)_{i.}(\bar{y})_i -$$

$$- (t^*_v)_{i.}(\bar{v})_i + (f^*_v)_{i.} v \tag{5.53}$$

which may be rearranged into

$$\bar{u}_i = - \left[(r_0)_{i.}(\bar{u})_i + (r^*)_{i.}(\bar{u})_{i.} + (s^*)_{i.}(\bar{y})_i - (t^*_v)_{i.}(\bar{v})_i \right]$$

$$+ (t^*_f)_{i.} y_f - (f^*_v)_{i.} v \tag{5.54}$$

Let $(\theta_1)_i$ be a vector containing the coefficients of the polynomials in $(r_0)_{i.}$, $(r^*)_{i.}$, $(s^*)_{i.}$ and $(t^*_v)_{i.}$. Let analogously $(\theta_2)_i$ and $(\theta_v)_i$ denote the two vectors of coefficients of the polynomials in $(t^*_f)_{i.}$ and $(f^*_v)_{i.}$ respectively. Then define the vector $(\varphi_1)_i$, of the same dimensions as $(\theta_1)_i$, and containing outputs and inputs y, u and v corresponding to the polynomials represented by $(\theta_1)_i$. Let similarly $(\varphi_2)_i$ and $(\varphi_v)_i$ contain y_f and v up to sufficient powers of z^* - corresponding to the dimensions of $(\theta_2)_i$ and $(\theta_v)_i$ respectively.

Now the following LIP-model for each control input u_i in the square case (5.29) may be formulated.

$$\bar{u}_i = -(\theta_1)_i^T(\bar{\varphi}_1)_i + (\theta_2)_i^T(\varphi_2)_i - (\theta_v)_i^T(\varphi_v)_i \tag{5.55}$$

These models give possibilities for identification since $\bar{u}_i$ and $(\bar{\varphi}_1)_i$ may be computed and $(\varphi_2)_i$ and $(\varphi_v)_i$ are known.

Define $(\varphi_2^m)_i$ as a vector of the same kind as $(\varphi_2)_i$ where the entries contain components of

$$y_f^m = T_1^* A_M^* y_1^m = T_1^* B_M^* u_c \tag{5.56}$$

instead of the y_f. Now the control input (5.39) associated with the pole placement could be written as

$$u_i = -(\theta_1)_i^T(\varphi_1) + (\theta_2)_i^T(\varphi_2^m) \tag{5.57}$$

Obviously the θ_v-dependent component is omitted since this corresponds to unknown future disturbances v. An exception is formulated in example 5.2.

The adaptive control law in which the θ:s are substituted by their estimated counterparts $\hat{\theta}$ becomes

$$u_i = -(\hat{\theta}_1)_i^T(\varphi_1) + (\hat{\theta}_2)_i^T(\varphi_2^m) \tag{5.58}$$

In the model matching case $\bar{\varphi}_2$ has to be replaced by a vector $\bar{\varphi}_2^e$ where y_f in the elements has been replaced by

$$e_f = y_f - y_f^M \tag{5.59}$$

It is also necessary to introduce a parameter vector θ_3 corresponding to the parameters of T_m^* in (5.40) which is essentially T_3^* (cf.(5.31)). The corresponding reference values are given by the data vector φ_3.

The estimation model is

$$\bar{u}_i = -(\theta_1)_i^T(\bar{\varphi}_1) + (\theta_2)_i^T(\varphi_2^e) - (\theta_v)_i^T(\varphi_v) - (\theta_3)_i^T(\bar{\varphi}_3) \tag{5.60}$$

The associated adaptive control law is

$$u_i = -(\hat{\theta}_1)_i^T(\varphi_1) - (\hat{\theta}_3)_i^T(\varphi_3) \tag{5.61}$$

5.9 On Uniqueness

Two questions on uniqueness naturally arise namely

1) Does B_S^* really represent the minimal a priori knowledge for these kinds of algorithms? Could e.g $\det(B_S^*)$ be sufficient?

2) Do the estimation models really give the expected solutions provided the identification works properly? Are there possibly many solutions? Do possible other solutions give the same input-output characteristics?

The first problem deals with the question whether the condition of a known B_S^* can be relaxed further. It would be a challenge to assume the zeros to be known in the form of e.g. $\det(B_S^*)$ without any structural information about the matrix. In fact, a simple counterexample can be given which discourages further efforts regarding extensions of the proposed algorithm.

6 P H

Example 5.2

Consider two independent, SISO, minimum phase, discrete time systems G_1 and G_2 with time delays k_1 and k_2 resp.. Let $\det(B_S^*) = q^{-(k_1+k_2)}$ be known to the designer and let the design objective be described as a separate control of the two objects. The designer will fail to make a correct parametrization since the two resulting SISO adaptive control problems require separate knowledge of k_1 and k_2.

No definite answer to the second question is given. Some partial answers may however be formulated. The uniqueness of a certain given parametrization may be decided by the criteria (i) - (iii) below.

Consider the control object (5.5) without v-dependence. The versions of the estimation models (5.55), (5.60) have one-to-one correspondence to the expressions (5.11), (5.16). Substitution of (5.5) into (5.11), (5.16) give the result that solutions to the estimation models satisfy the polynomial matrix equation

$$\left(T_2^*\left(T_1^* A^* B_M^*\right) - B_S^*\left(R_u^* A^* + S_y^* B^*\right)\right)\xi = 0 \tag{5.62}$$

with respect to the unknown polynomial matrices T_2^*, R_u^* and S_y^*. The matrix T_3^* may be excluded from this investigation immediately since ξ and y_1^m are not linearly related unless all parameters have converged to the correct ones.

The polynomial matrix equation (5.62) is a special case of the equation

$$XA_0 + B_0Y = C_0 \tag{5.63}$$

which has solutions X, Y for given A_0, B_0 and C_0. All solutions to this equation may be written as

$$\begin{cases} X = X_p - B_0K \\ Y = Y_p + KA_0 \end{cases} \tag{5.64}$$

where X_p, Y_p is a particular solution to (5.63) and K is an arbitrary matrix which decides the influence of the homogenous solution.

When this applies to the solution of (5.62) it holds that any polynomial matrices

$$T_2^{*\prime} = T_2^* - B_S^*K_1^*$$

$$\left[R_u^*A^* + S_y^*B^*\right]' = \left[R_u^*A^* + S_y^*B^*\right] + K_1^*T_1^*A^*B_M^* \tag{5.65a-b}$$

for some polynomial matrix K_1^* would satisfy the equations. This multitude of solutions provides no uniqueness at all. It is however possible to give criteria to determine the uniqueness when some apriori information on the element degrees of the desired matrix solutions is available.

6
S
H

i) An independence of contributions from the homogenous equation is definite when for all rows i of T_2^* it holds that

$$\max \text{ degree } (T_2^*)_{i.} < \text{degree } (b_i^*) \tag{5.66}$$

This is seen from (5.65a) where each row of the contribution from the homogenous solution has b_i^* as a factor. The only way to avoid the condition (5.66) with high degree powers of z^* is to fix the higher degree coefficients of T_2^*.

ii) Since the right hand side of (5.62) is zero, it follows that row operations can be performed by some matrix K_2^* and give new feasible solutions

$$T_2^{*\prime} = K_2^* T_2^*$$

$$\left(R_u^* A^* + S_y^* B^*\right)^{\prime\prime} = K_3^*\left(R_u^* A^* + S_y^* B^*\right)$$

$$K_2^* B_S^* = B_S^* K_3^* \tag{5.67}$$

for K_2^* and K_3^* satisfying the third relation. The matrix K_3^* must however also satisfy the upper triangularity condition of $R_u^*(0)$ (cf.(4.9)).

It follows that a T_2^* of given polynomial degrees has a unique solution if (i) the independence of the homogenous solution is established and (ii) the condition (5.67) only can be fulfilled with K_2^* and K_3^* as an identity matrix.

If uniqueness cannot be established for T_2^* by the methods (i) and (ii) it is necessary to check if all solutions are suitable solutions. The most important issue to check is if there are any solutions with Z^*-zeros of T_2^*. Such solutions would not give asymptotically stabilizing controllers.

iii) A general test is to check the formal determinant of T_2^* i.e.

$$\det T_2^* \tag{5.68}$$

expressed in known polynomial degrees but in unknown parameters. If this quantity is a constant, it follows that there are no unstable solutions to the identification.

The issue of separation between R_u^* and S_y^* is not important from an algebraic point of view since any solution to

$$R_u^* A^* + S_y^* B^* = T_1^* A_M^* B_R^* \tag{4.17a}$$

gives a suitable controller. It is however safer for identification numerics etc. if there is only one solution.

A lengthy method may be suggested to give a unique solution. Start with an expression involving particular and homogenous solutions. All polynomial solutions may be written as

6

p

V

$$\begin{cases} R^* = R_p^* + K_4^* R_H^* \\ S^* = S_p^* + K_4^* S_H^* \end{cases} \tag{5.69}$$

where R_p^*, S_p^* is a particular solution to (4.17a), K_4^* is an arbitrary polynomial matrix and R_H^*, S_H^* is a coprime solution to the homogenous equation i.e.

$$R_H^* A^* + S_H^* B^* = 0 \tag{5.70}$$

The idea is to find a unique solution based on minimization of element degrees of R_u^* by manipulations with the homogenous solution. Different ways to achieve this may be found in e.g. ([Per 1], II.8.3). These procedures give in some sense minimal degrees of the elements of the matrix R^*. This solves the problem in the SISO case. The MIMO case is different and parameter solutions are not guaranteed to be unique since lower degree terms may sometimes still be contributed from the homogenous solution. The most convenient way to obtain the desired uniqueness is then to clamp the values of a sufficient number of the parameters that could be affected. When uniqueness of R^* is established this also follows for S^*.

6 IDENTIFICATION

6.1 Introduction

The main concern of this section is to confirm the results of previous chapter by application of some standard identification results to the actual problem.

A review of some common recursive identification methods (§6.2) is given. More detailed versions and further references are found in [SLG], [Jaz] and [G&P]. The application to the identification of parameters of the parametric models of ch.5 is treated in §6.3. Some parameter convergence results are also given. Two identification principles are used. They are here called prediction error identification (PEI) and feedback error identification (FEI). The former updates the parameter estimates by a function of the prediction error of the current parameter estimates. The latter makes corrections based on evaluation of the performance of an adaptive controller using the current parameter estimates.

Parametrizations of continuous time processes have been derived together with the discrete-time systems. No attempt has been made here to proceed with a unified description of identification for the two classes. In fact, only discrete-time identification has been elaborated below.

All recursive methods taken into account here have the common feature of finding successively better fitting

65V

estimates $\hat{\theta}$ of a linear model

$$\zeta(t) = \theta^T \bar{\phi}(t) \tag{6.1}$$

for known data sequences $\{\zeta(i)\}_{i=0}^{t}$ and $\{\bar{\phi}(i)\}_{i=0}^{t}$.

Several other principles of identification may of course be considered as a basis for adaptive control. Such alternatives are however not treated here. No methods like correlation analysis, impulse or frequency response analysis are included below. Neither are nonlinear parameter models treated.

The reader who gets lost notationally is kindly referred to the notation dictionnary of appendix 1:1 for references and explanations.

6.2 Identification Methods

This paragraph is a review of some commonly used identification methods. No new methods are introduced but the presentation is given with some detail since the properties of parameter convergence are of considerable importance for the closed-loop system behaviour.

Gradient Methods

The gradient methods make use of some quantity $\varepsilon = -\tilde{\theta}^T\bar{\phi}$ which is the inner product between the parameter error $\tilde{\theta}$ and a known data vector $\bar{\phi}$. The updates of the estimated parameters $\hat{\theta}$ are formed as

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \gamma(t,\|\bar{\phi}\|)\bar{\phi}(t)\varepsilon(t)$$

$$\varepsilon(t) = -\tilde{\theta}^T(t-1)\bar{\phi}(t) \qquad (6.2)$$

where γ is a scalar function. Different methods to obtain ε from the data will be shown in §6.3.

The Euclidean norm of the parameter error develops as

$$\|\tilde{\theta}(t)\|^2 - \|\tilde{\theta}(t-1)\|^2 = \left[-2\gamma(t,\|\phi\|)+\gamma^2(t,\|\bar{\phi}\|)\|\bar{\phi}(t)\|^2\right]\varepsilon^2(t) \qquad (6.3)$$

It is readily seen that any $\gamma(t,\|\bar{\phi}\|)$ such that

$$0 < \gamma(t,\|\phi(t)\|) < \frac{2}{\|\bar{\phi}(t)\|^2} \qquad (6.4)$$

guarantees that the magnitude of the parameter error decreases whenever ε is different from zero. The optimal γ is

$$\gamma_{opt} = \frac{1}{\|\bar{\phi}(t)\|^2} \qquad (6.5)$$

A special choice of γ that satisfies (6.5) and

$$\lim_{t\to\infty} \sum_{i=0}^{t} \gamma(i) = \infty \quad \text{but} \quad \lim_{t\to\infty} \sum_{i=0}^{t} \gamma^2(i) < \infty \qquad (6.6)$$

is usually referred to as stochastic approximation (SA). Notice that the restriction (6.6) requires

$$\gamma(t) \to 0 \quad \text{as} \quad t \to \infty \qquad (6.7)$$

and is therefore not suitable for tracking of time varying parameters. The method is used extensively and has been used in [Ega] to show global stability.

In the notations of classical stability theory it holds that

$$V(\tilde{\theta}(t)) = \|\tilde{\theta}(t)\|^2 \qquad (6.8)$$

is a discrete time Lyapunov function for the parameter estimation (6.2) under the restriction (6.4). The estimates of θ then converge but not necessarily so that the parameter errors go to zero. The magnitude of the parameter errors however decreases as long as the indicated error ε is non-zero.

Least Squares Estimation

The recursive least squares method estimates $\hat{\theta}(t)$ of the parameters of the linear relation

$$\xi(t) = \theta^T \bar{\phi}(t) \qquad (6.1)$$

for known sequences of ξ and $\bar{\phi}$. The algorithm minimizes the criterion

$$V(\tilde{\theta}(t),t) = \min_{\hat{\theta}} \left\{ \sum_{i=0}^{t} \lambda^{t-i} \left[\xi(i) - \hat{\theta}^T \bar{\phi}(i) \right]^2 \right\} \qquad (6.9)$$

where $\lambda \in [0,1]$ is the so called forgetting factor. A detailed presentation is found in e.g. [SLG] or [Jaz].

The algorithm is

$$P(t) = \frac{1}{\lambda}\left[P(t-1) - \frac{P(t-1)\bar{\phi}(t)\bar{\phi}^T(t)P(t-1)}{\lambda + \bar{\phi}^T(t)P(t-1)\bar{\phi}(t)}\right]$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + P(t)\bar{\phi}(t)\varepsilon(t) \tag{6.10}$$

for $\varepsilon(t) = -\tilde{\theta}^T(t-1)\bar{\phi}(t)$. The cost function

$$V(\tilde{\theta}(t),t) =$$

$$= \min_{\hat{\theta}}\left(\sum_{i=0}^{t} \lambda^{t-i}\left[\zeta(i) - \hat{\theta}^T\bar{\phi}(i)\right]^2\right) = \tilde{\theta}^T(t)P^{-1}(t)\tilde{\theta}(t)$$

decreases as (6.11)

$$V(\tilde{\theta}(t),t) - V(\tilde{\theta}(t-1),t-1) =$$

$$= -\frac{\lambda\varepsilon^2(t)}{\lambda + \bar{\phi}^T(t)P(t-1)\bar{\phi}(t)} \tag{6.12}$$

6.3 Parameter Error Detection Principles

Sofar the problem of how to form parameter errors on the form

$$\varepsilon(t) = -\tilde{\theta}^T(t-1)\bar{\phi}(t) \tag{6.13}$$

has not been touched. Most of the identification and adaptive control literature hereby uses prediction error identification, which is described in e.g. [SLG].

T

Another method is presented here as a feedback error identification algorithm (FEI). The idea is to use the information obtained when recursively updated controller parameters result in an output error of the closed-loop system behaviour. This approach has been used for adaptive control of continuous time processes by e.g. Parks [Par]. A discrete time adaptive controller using this method is found in e.g ([GRC], (SISO II)).

The two methods may be formulated to coincide when the time delay of the system is equal to one.

Prediction Error Identification

The method uses the parametric model

$$\zeta(t) = \theta^T\bar{\phi}(t) \tag{6.1}$$

where $\{\zeta(i)\}_{i=0}^{t}$ and $\{\bar{\phi}(i)\}_{i=0}^{t}$ are known sequences of input-output data. A prediction of the quantity $\zeta(t)$ using the current parameter estimates $\hat{\theta}(t-1)$ gives the estimate

$$\hat{\zeta}(t) = \hat{\theta}^T(t-1)\bar{\phi}(t) \tag{6.14}$$

The prediction error is given by

$$\varepsilon(t) = -\hat{\zeta}(t) + \zeta(t) = -\tilde{\theta}^T(t-1)\bar{\phi}(t) \tag{6.15}$$

which may be exploited to feed the algorithms of §6.2 since ε satisfies (6.13)

In the adaptive literature this method is used for output estimation of a process (see e.g. the self-tuning regulators in [L&W]). The output PEI however leads to a question of how the estimated parameters should be used to form the control law since the estimated parameters differ by a factor b_0 from the desired ones (cf.(5.1) or [Egal). The factor b_0 represents in some sense the gain of the system. Several solutions to this problem has been suggested (see e.g. [L&W]) but most schemes hereby end up by presupposing the knowledge of some constant β_0 such that

$$0 < \frac{b_0}{\beta_0} < 2 \tag{6.16}$$

The problems of this approach are accentuated when attempts are made to make straightforward generalizations to multivariable systems. Borisson is for example led to drastic specializations in order to avoid awkward on-line matrix manipulations (see [Bor 1], ch.5). Even so he is forced to include an approximate inverse of a gain matrix in the control law. The control law is then not even asymptotically correct. That approach does however still work due to robustness qualities of adaptive control.

A solution to this problem is proposed here. The prediction error methods are applied to estimate the parameters of different parametrizations (5.60)-(5.61)

$$\bar{u}_i(t) = -\theta_{1i}^T\bar{\varphi}_{1i}(t) + \theta_{2i}^T\varphi_{2i}(t) = -\begin{pmatrix}\theta_{1i}^T & \theta_{2i}^T\end{pmatrix}\begin{bmatrix}\bar{\varphi}_{1i}(t) \\ -\varphi_{2i}(t)\end{bmatrix} \tag{6.17}$$

for each control input u_i to be tuned. The parameters of the feedback portion of the the control law are those of θ_{1i}. The parameters of T_v^* and T_3^* may also be included as shown in §§5.4-5.8 and (5.60)-(5.61).

Some advantages compared to earlier contributions based on PEI can be stated.

First, the identification does not have to be supplemented by some a priori known, good enough gain matrix B_0 in order to give an asymptotically stabilizing controller.

Second, the parameter estimates are asymptotically the correct controller parameters as opposed to those of Borisson.

Third, the model structures are more general in this presentation. No conditions on certain time delay structures or invertibility conditions of B_0 are required here. This is already discussed in §5.1.

Fourth, the controllers are separated identification-wise. This makes it possible to 'decentralize' identification loop by loop - a property of considerable practical significance. An implementation is made possible with modest hardware and data communication requirements.

Another good property is common to the given method and the output prediction methods of other authors. The identification does not explicitely require any particular control law but works as such for any input-output data of the system. The convergence rate is of course affected by different choices of control law as commented below. In this formulation there is evidently a formal separation between the parameter tuning and the adaptive control law.

A number of problems are also worth mentioning. A problem often encountered in the literature [SLG] and equally applicable to the suggested method is the following. Although the parameter error is guaranteed to decrease in the sense of the criterion of the identification method, as long as $\varepsilon \neq 0$, this property does actually not give very much information about the convergence rate of the parameter estimates as a function of time. It is also difficult to relate the parameter convergence to the the output behaviour of the closed-loop system when the parameter estimates are used in the controller.

Different remedies have been suggested. Conditions on excitation persistency ([GLS], [SLG]), a priori known gain estimates within certain limits [Ega], dual controllers and cautious controllers (see [Wit] for further references) have been proposed with various success and are equally applicable to this approach.

The difficulty to relate the prediction error to the closed-loop output performance of a system which uses the current parameter estimates for an adaptive control law, also accounts for a lot of the problems encountered in attempts to give stability proofs for adaptive systems (see ch.8).

The drawbacks of non-identifiability will be shown by means of two examples. A survey of identifiability in a closed-loop system is found in [SLG].

Example 6.1

Assume that the parameters of the model of ex. 7.1 are to be estimated. Let the certainty equivalence control law be used. Assume that the parameters have not been updated since time (t-k). The prediction error is then

$$\varepsilon(t) = -\tilde{\theta}_1^T \bar{\varphi}_1(t) + \left(\widetilde{\frac{1}{b_0}}\right) e_f(t) = \left(\widehat{\frac{1}{b_0}}\right) e_f(t) \qquad (6.18)$$

where the time arguments of the parameters have been left out. There is obviously a risk of deadlock between identification and the certainty equivalence control law in this case. When

$$\hat{\theta}_2 = \left(\widehat{\frac{1}{b_0}}\right) = 0 \qquad (6.19)$$

it follows that the prediction fits and no prediction error is indicated. A non-desirable stable convergence point has been reached. Different ad hoc methods to prevent this might be suggested. One way out is to correct $\hat{\theta}_2$ when close to zero if some bound like (6.16) is known. This is however restrictive since a priori knowledge is presupposed. Another - less restrictive - method is to use some other control law when $\hat{\theta}_2$ is small. The modification

$$u = -\hat{\theta}_1^T(t)\varphi_1(t) + v \quad ; \quad v = O(\|\varphi_1\|) \qquad (6.20)$$

where v is any additional quantity of sufficient magnitude – e.g. $v = e_f$ – would for example solve the problem. A simple calculation based on example 7.1 shows that this shakes the PEI alive.

Example 6.2

The general question of identifiability of systems in closed-loop has been investigated and presented in [SLG]. Some consequences may be drawn also for this case.

Assume that the identification is used to find the desired controller parameters of a SISO system that is already stabilized by some other controller

$$u(t) = -\theta_0^T \varphi_1(t) \tag{6.21}$$

Let the LS-method be used with $\lambda=1$ and assume that the identification has been running since time $t=0$. At time t the cost criterion has developed such that

$$V(\tilde{\theta}(t)) = \tilde{\theta}^T(t) P^{-1}(t) \tilde{\theta}(t) \tag{6.22}$$

where

$$P^{-1}(t) = \sum_{i=0}^{t} \bar{\varphi}(i)\bar{\varphi}^T(i) = M\left[\sum_{i=0}^{t} \bar{\varphi}_1(i)\bar{\varphi}_1^T(i)\right]M^T \tag{6.23}$$

since

7

P

H

$$\bar{\varphi}(i) = \begin{pmatrix} \bar{\varphi}_1(i) \\ \\ -\varphi_2(i) \end{pmatrix} = \begin{bmatrix} I \\ \\ b_0(\theta_0-\theta_1)^T \end{bmatrix} \bar{\varphi}_1(i) \stackrel{\Delta}{=} M\bar{\varphi}_1(i) \tag{6.24}$$

A parameter error such that

$$\tilde{\theta} \in \mathrm{Ker}(M^T) \tag{6.25}$$

can not be detected by this algorithm unless some excitation signal like noise etc. compensates for the rank deficit of M. The problem does not occur in the case of an adaptive regulator since

$$\tilde{\theta}^T M = \tilde{\theta}^T \begin{pmatrix} I \\ \\ b_0\tilde{\theta}_1^T \end{pmatrix} = b_0\hat{\theta}_2\tilde{\theta}_1^T \neq 0 \quad \text{if} \quad \|\tilde{\theta}_1\| \neq 0,\ \hat{\theta}_2 \neq 0 \tag{6.26}$$

which is another formulation of ex.6.1.

The contribution of the suggested algorithm consists in a possibility for estimation of the controller parameters of a linear multivariable system without any necessary a priori estimates of gains etc.. Only system time delays and non-invertible zeros of the internal structure matrix and polynomial degrees have to be known. Admittedly, the identifiability of parameters is not assured unless some condition on excitation persistency is satisfied.

Feedback Error Identification

As shown in example 7.1 it holds that a SISO, minimum-phase system with time delay k can be formulated as

$$e(t) = \frac{b_0 q^{-k}}{T_1^*(q^{-1})A_M^*(q^{-1})}\left[u(t) + \theta_1^T\varphi_1(t)\right] \tag{7.1)'}$$

The definition of

$$e_f(t) = T_1^*(q^{-1})A_M^*(q^{-1})e(t) \tag{5.17}$$

and the application of the certainty equivalence pole placement control law

$$u(t) = -\hat{\theta}_1^T(t)\varphi_1(t) \tag{6.27}$$

gives

$$e_f(t) = -b_0\hat{\tilde{\theta}}_1^T(t-k)\varphi_1(t-k) \tag{6.28}$$

The filtered error $e_f(t)$ could obviously be used for identification. A slightly modified version of (6.2) like

$$\hat{\theta}_1(t) = \hat{\theta}_1(t-k) + \gamma(\|\varphi_1(t-k)\|)\varphi_1(t-k)e_f(t)$$

where

$$\gamma(\|\varphi_1(t-k)\|) = \frac{1}{\beta_0\|\varphi_1(t-k)\|^2} \tag{6.29}$$

has been used by many authors (e.g [GRC], SISO II) in the context of adaptive control. The gain b_0 is assumed positive and β_0 should satisfy (6.16). This algorithm has the advantage from an analysis point of view that the error - filtered by a stable and stably invertible filter - directly relates to the parameter convergence.

7 S H

The strong implications for stability proofs have been exploited in [GRC]. A still stronger global stability proof showing exponential stability for some cases is given in §8.2.

A parameter error Lyapunov function is given by

$$V(\tilde{\theta}_1(t)) = \sum_{i=0}^{k-1} \|\tilde{\theta}_1(t-i)\|^2 \tag{6.30}$$

which decreases as

$$V(\tilde{\theta}_1(t)) - V(\tilde{\theta}_1(t-1)) =$$

$$= \|\tilde{\theta}_1(t)\|^2 - \|\tilde{\theta}_1(t-k)\|^2 = \left(\gamma^2 \bar{\varphi}_1^T \bar{\varphi}_1 - 2\beta_0 \gamma\right) e_f^2 =$$

$$= \frac{1}{\beta_0}\left(-\frac{2}{b_0} + \frac{1}{\beta_0}\right)\frac{e_f^2(t)}{\|\bar{\varphi}_1(t)\|^2} \tag{6.31}$$

when γ is chosen according to (6.5). No updating is performed for $\|\bar{\varphi}_1\|=0$.

Example 6.3

More natural algorithms using the output error may be given. The following recursive scheme

$$\hat{\theta}_1(t) = \frac{1}{k}\sum_{i=1}^{k}\hat{\theta}_1(t-i) + \frac{1}{k\beta_0}\,\frac{1}{\|\varphi_1(t-k)\|^2}\varphi_1(t-k)e_f(t) \tag{6.32}$$

for $\|\varphi_1(t-k)\| \neq 0$ shows convergence since the Lyapunov function

$$V(\tilde{\theta}_1(t)) = \sum_{i=0}^{k-1} \alpha_i \|\tilde{\theta}_1(t-i)\|^2 \; ; \quad \alpha_i = \frac{k-i}{k}$$

decreases as (6.33)

$$V(\tilde{\theta}_1(t)) - V(\tilde{\theta}_1(t-1)) \le \frac{1}{k\beta_0}\left(-\frac{2}{b_0} + \frac{1}{\beta_0}\right)\frac{e_f^2(t)}{\|\varphi_1(t-k)\|^2} \qquad (6.34)$$

and has the advantage to (6.29) that a smoothing between the k 'parameter families' $\hat{\theta}_1(t),\ldots,\hat{\theta}_1(t-k+1)$ is included. The condition (6.16) is required to be satisfied also in this case.

□

Extensions to multivariable FEI could also be considered. The vector of filtered errors is then

$$\begin{bmatrix} e_{f1}(t) \\ \vdots \\ \vdots \\ e_{fn}(t) \end{bmatrix} = T_2^{*-1}(q^{-1})B_S^*(q^{-1})\begin{bmatrix} -\tilde{\theta}_{11}^T(t)\varphi_{11}(t) \\ \vdots \\ \vdots \\ -\tilde{\theta}_{1n}^T(t)\varphi_{1n}(t) \end{bmatrix} \qquad (6.35)$$

or in a more compact notation

$$e_f(t) = T_2^{*-1}(q^{-1})\bar{v}(t) \quad \text{where } v_i(t) = -\tilde{\theta}_{1i}^T(t)\varphi_{1i}(t)$$

$$\bar{v}_i(t) = b_i^*(z^*)\; v_i(t)$$

The matrix B_S^* contains only time delays in the case where there are no finite non-minimum phase zeros. The analogy of a known upper bound β_0 of the gain b_0 for the SISO-case

7 p. V

corresponds to a polynomial matrix τ_2^* which is some good enough approximation of the matrix T_2^*.

When τ_2^* is such that

$$\tau_2^*(q^{-1})T_2^{*-1}(q^{-1}) = I \qquad \text{or}$$

$$\tau_2^*(q^{-1})T_2^{*-1}(q^{-1}) = \Lambda \; ; \; \Lambda \text{ diagonal and } 0 < \lambda_i < 2 \tag{6.36}$$

it is possible to make trivial extensions of the SISO-method to the multivariable case.

A less restrictive condition equivalent to (6.16) for parameter estimation of multivariable adaptive controllers and which assures parameter convergence can however be given.

Theorem 6.1

Assume that T_2^* is known to such an extent that the estimate τ_2^* may be formulated as

$$\tau_2^* = 2\left(I + H^*\right)^{-1} T_2^* \tag{6.37}$$

for some Z^*-generalized polynomial matrices H^* and $(I+H^*)^{-1}$ where H^* is also positive real. Assume also that the MIMO CE control law is used. The knowledge of B_S^* and τ_2^* then represents sufficient a priori information for formulation of a convergent parameter adjustment law.

□

The proof and the algorithm are given in appendix 6:1.

The condition of a known B^*_S and the knowledge of a sufficiently accurate estimate T^*_2 may be compared with an approximate of the interactor (cf.[E&W 1]). The result of theorem 6.1 thus formally relaxes the condition of an exactly known interactor.

Moreover, the result links the proposed algorithms with the stability proof techniques ([GRC], SISO II)) developed for MIMO systems.

6.4 Conclusions

FEI requires prior knowledge of the internal structure matrix B^*_S and an approximate τ^*_2 of T^*_2 - all together an approximate of B^*_{LS}. There is a good identifiability of the parameters without any sensitivity to overparametrization.

Theorem 6.1 may be interpreted as a robustness result for algorithms with included system invariants - like the interactor - in the reference model. The implicit form of (6.37) is however not a sound basis for a construction of an algorithm.

A more practical way is provided by the PEI-methods. When no τ^*_2 with the quality of (6.36) is known, then it is still possible to identify both T^*_2 and the controller parameters by PEI provided their internal structure matrix is known.

7
S
V

Less prior knowledge is requested but the identifiability properties are worse than those of FEI. Parameter uniqueness is a desirable property in order to secure parameter convergence. This condition is less important for FEI where any parameter set which does not give any output errors is a suitable solution.

A comparison with some earlier results in the area has here been restricted to the works of Borisson [Bor 2], Goodwin et al.[GRC] and Elliott and Wolovich [E&W 1]. Comments on the contributions of these authors have already been provided and many details have been touched in the text. A few general comments remain to be made.

The model classes considered in [Bor 2], [GRC] have been extended in this presentation to cases with singular B_0 and non-diagonal A^*-matrices

The results in [E&W 1] requiring an exactly known interactor have been relaxed to an approximate knowledge for FEI-type algorithms and to the knowledge of B_S^* - the system zeros - for PEI-type algorithms.

The given results have sofar related to earlier work in terms of algebraic requirements and parameter convergence. Necessary conditions have been imposed to assure good asymptotic properties. No closed-loop stability proofs have yet been given.

7 EXAMPLES

7.1 Introduction

This chapter gives a number of illustrating examples to the proposed algorithms. References ((ref.)) are given to each step of the algorithm checklist in app.1:2. The first example considers a general, 'minimum-phase', discrete-time SISO system for which an input matching parametrization is derived. Example 2 shows the algorithm manipulations for a simple continuous time system with unknown parameters. A MIMO continuous time system is decomposed according to (4.7) in example 3. A MIMO discrete time example in unknown parameters is given in §7.5. Some simulations are also given.

7.2 Example 1

Consider a SISO-system of the configuration

$$y(t) = b_0 q^{-k} \frac{B^*(q^{-1})}{A^*(q^{-1})} u(t) =$$

$$= b_0 q^{-k} \frac{1 + b_1 q^{-1} + \ldots\ldots + b_m q^{-m}}{1 + a_1 q^{-1} + \ldots\ldots\ldots + a_n q^{-n}} u(t) \qquad \text{(ii), (7.1)}$$

where B^* has all zeros outside the unit circle.

The decomposition of (4.7) gives

$$B^*_L = b_0 \quad ; \quad B^*_S = q^{-k}; \quad B^*_R = B^*; \quad A^* = A^* \qquad \text{(iii)},(7.2)$$

The model has to include q^{-k} ((iii)) and turns out as

$$y^M(t) = q^{-k}\, y^m(t) = q^{-k} \frac{B^*_M(q^{-1})}{A^*_M(q^{-1})} u_c(t) \qquad \text{(iv)}, (7.3)$$

The equation (4.17a) appear as

$$R^*_u A^* + b_0 q^{-k} S^* B^* = T^*_1 A^*_M B^* \qquad \text{(vi)}, (7.4)$$

and

$$T^*_2 = T^*_3 = \frac{1}{b_0} \qquad \text{(vi)}, (7.5)$$

where it is required that $T^*_1(0)A^*_M(0) = 1$, gives the parametric model (5.11)

$$\frac{1}{b_0}\left[T^*_1(q^{-1})A^*_M(q^{-1})\right]y(t) =$$

$$= q^{-k}\left\{ u + R^*(q^{-1})u + S^*(q^{-1})y(t) \right\} \qquad \text{(vii)}, (7.6)$$

or possibly with an included y^m on the form of (5.16). Now rename the quantities

$$T^*_2(q^{-1}) = \frac{1}{b_0} = \theta_2$$

$$T_1^*(q^{-1})A_M^*(q^{-1})y(t) = \varphi_2(t)$$

$$R^*(q^{-1})u(t) + S^*(q^{-1})y(t) = \theta_1^T\varphi_1(t) \qquad \text{(viii), (7.7)}$$

The identification is done with the expression

$$u(t-k) = -\theta_1^T\varphi_1(t-k) + \theta_2^T\varphi_2(t) \qquad \text{(viii), (7.8)}$$

The control law is

$$u(t) = -\theta_1^T\varphi_1(t) + \theta_2^T\varphi_2^m(t) \qquad \text{(x), (7.9)}$$

where

$$\varphi_2^m(t) = T_1^*(q^{-1})A_M^*(q^{-1})y^m(t) \qquad (7.10)$$

and θ_i are substituted by estimates $\hat{\theta}_i$ in the adaptive control law.

The parametrization is easily transfered via (7.6) into e.g. Egardts unified formulation for adaptive control schemes (including self-tuners and model reference adaptive systems) with output matching.

7.3 Example 2

Consider a system with the state-space formulation

$$\begin{cases} \dot{x}(t) = k_1 x(t) + u(t) \\ y(t) = k_2 x(t) \end{cases} \qquad \text{(i), (7.11)}$$

where k_1 and k_2 are unknown, real constants such that $k_2 \neq 0$. Assume that y is measurable.

An output feedback should be designed such that the closed loop transfer operator becomes

$$\frac{3}{p+3} \qquad \text{(iv), (7.12)}$$

The system may be rewritten

$$\begin{cases} \dfrac{p-k_1}{p+1} x = \dfrac{1}{p+1} u \\ y = k_2 x \end{cases} \qquad \text{(ii), (7.13)}$$

Introduce the transformed variable z^*

$$z^* = \frac{1}{p+1} \qquad (7.14)$$

which gives the system description

$$\begin{cases} \left[1 - (k_1 + 1) z^* \right] x = z^* u \\ y = k_2 x \end{cases} \qquad \text{(ii), (7.15)}$$

Define

$$A^*(z^*) = 1 - (1+k_1)z^* \overset{\Delta}{=} \left[1 - \alpha_1 z^* \right]$$

$$B^*(z^*) = k_2 z^* \overset{\Delta}{=} \beta_1 z^* \qquad \text{(ii), (7.16)}$$

Also make the decomposition of (4.7)

$$B_L^* = \beta_1 \qquad B_S^* = z^* \qquad \text{and} \qquad B_R^* = 1 \qquad \text{(iii), (7.17)}$$

Introduce the new internal variable ξ such that

$$\begin{cases} A^*(z^*)\ \xi(t) = u \\ \\ y(t) = B^*(z^*)\ \xi(t) \end{cases} \qquad \text{(ii), (7.18)}$$

that is

$$\begin{cases} \left[1 - \alpha_1 z^* \right] \xi = u \\ \\ y = \beta_1 z^* \xi \end{cases} \qquad \text{(ii), (7.19)}$$

The reference model is

$$\frac{B_S^*\ B_M^*}{A_M^*} = \frac{3}{p+3} = \frac{3z^*}{1 + 2z^*} \qquad \text{(iv), (7.20)}$$

This equality holds for

$$A_M^* = 1 + 2z^* \quad \text{and} \quad B_M^* = 3 \qquad (7.21)$$

The reference model transfer operator is realizable since the model has the same structure polynomial as the control object. Let T_1^* arbitrarily be defined as

$$T_1^* = 1 \qquad \text{(v), (7.22)}$$

The polynomial equation (4.17)

$$R_u^* A^* + S_y^* B^* = T_1^* A_M^* B_R^* \qquad \text{(vi), (7.23)}$$

becomes

$$\left(1 - \alpha_1 z^*\right) R^* + \beta_1 z^* S^* = 1 + 2z^* \qquad (7.24)$$

which is satisfied for

$$R^* = 1 \quad \text{and} \quad S^* = \frac{2 + \alpha_1}{\beta_1} \overset{\Delta}{=} s_0 \qquad (7.25)$$

The control object is

$$y = \beta_1 z^* \xi = \beta_1 z^* \frac{1}{1+2z^*}\left[R^* u + S^* y\right] = \frac{\beta_1 z^*}{1+2z^*}\left[u + s_0 y\right]$$

$$= \frac{\beta_1 z^*}{1+2z^*}\left[u + s_0 y\right] \qquad \text{(vii), (7.26)}$$

Define

$$y_f = T_1^* A_M^* y = \left[1+2z^*\right] y \qquad (7.27)$$

The quantity T_2^* is found as

$$T_2^* = \frac{1}{\beta_1} \qquad (7.28)$$

and

$$T_2^* T_1^* A_M^* y = \frac{1}{\beta_1} y_f = z^*\left[u + s_0 y\right] \qquad \text{(vii), (7.29)}$$

Define

$$\bar{u} = z^* u \qquad \text{etc.} \tag{7.30}$$

and

$$\theta_1 = s_0 \qquad \text{and} \qquad \theta_2 = \frac{1}{\beta_1} \tag{7.31}$$

The model for parameter estimation is then

$$\bar{u} = -\theta_1 \bar{y} + \theta_2 y_f \qquad \text{(viii), (7.32)}$$

and the correct control law is

$$u = -\theta_1 y + \theta_2 y_f^m \qquad \text{(x), (7.33)}$$

where

$$y_f^m = T_1^* A_M^* y^m = T_1^* B_M^* u_c = 3u_c \tag{7.34}$$

The correct parameters are

$$\theta_1 = s_0 = \frac{3+k_1}{k_1} \qquad \text{and} \qquad \theta_2 = \frac{1}{k_1} \tag{7.35}$$

which give the closed loop system

$$\dot{x} = k_1 x + \left(-\frac{3+k_1}{k_2}(k_2 x) + \frac{1}{k_2}3u_c\right) = -3x + \frac{3}{k_2}u_c \tag{7.36}$$

The transfer operator to y becomes

$$y = \frac{3}{p+3}u_c \tag{7.37}$$

as required.

7.4 Example 3

The decompositions (4.7) and the solutions to (4.17) are given below for a MIMO continuous time system in known parameters. The system has unstable poles and the controller design is complicated by non-minimum phase properties.

Consider the transfer operator G_0

$$G_0(p) = \begin{bmatrix} \dfrac{1}{p-2} & \dfrac{1}{p^2-4} \\[2ex] \dfrac{p-3}{(p+1)(p-2)} & \dfrac{p^2-2p-1}{(p+1)(p^2-4)} \end{bmatrix} \qquad \text{(i), (7.38)}$$

Choose $z^*=1/(p+1)$ for the transform and express G^*_0

$$G^*_0(z^*) = \begin{bmatrix} \dfrac{z^*}{1-3z^*} & \dfrac{z^{*2}}{(1+z^*)(1-3z^*)} \\[2ex] \dfrac{z^*(1-4z^*)}{1-3z^*} & \dfrac{(1-4z^*+2z^*)z^*}{(1+z^*)(1-3z^*)} \end{bmatrix} \qquad \text{(ii), (7.39)}$$

A r.Z^*.c.f. is obtained as

$$A^*(z^*) = \begin{bmatrix} (1-3z^*) & -z^* \\ 0 & (1+z^*) \end{bmatrix}$$

$$B^*(z^*) = \begin{bmatrix} z^* & 0 \\ z^*(1-4z^*) & z^*(1-2z^*) \end{bmatrix} \qquad \text{(ii), (7.40)}$$

The B^*-matrix is easily decomposed into $B^*_R=I$ and

$$B_L^* = \begin{bmatrix} 1 & 0 \\ 1-4z^* & -1 \end{bmatrix} \; ; \; B_S^* = \begin{bmatrix} z^* & 0 \\ 0 & -z^*(1-2z^*) \end{bmatrix}$$

(iii), (7.41)

If $T_1^* A_M^* = I$, it follows from (4.17) that

$$R_u^*(z^*) = \begin{bmatrix} 1-z^{*2} & z^* - z^{*2} \\ -z^* & 1-z^* \end{bmatrix}$$

$$S_y^*(z^*) = \begin{bmatrix} 3+z^* - 3z^{*2} & 0 \\ 1-3z^* & 0 \end{bmatrix}$$

and (vi), (7.42)

$$T_2^*(z^*) = \begin{bmatrix} 1 & 0 \\ (1-4z^*) & -1 \end{bmatrix}$$

(vi), (7.43)

and with omitted arguments

$$T_3^* = \begin{bmatrix} (2z^* - 1) & 0 \\ (1-4z^*) & -1 \end{bmatrix} \; ; \; B_D^* = \begin{bmatrix} -z^*(1-2z^*) & 0 \\ 0 & -z^*(1-2z^*) \end{bmatrix}$$

The pole placement design (4.18) gives (vi), (7.44)

$$B_L^* B_S^* T_2^* = \begin{bmatrix} z^* & 0 \\ z^*(2-10z^* + 8z^{*2}) & -z^*(1-2z^*) \end{bmatrix} \qquad (7.45)$$

as a closed-loop transfer operator. This is triangular but

8 P H

at least statically diagonal($z^*=1$) as promised in chapter 4. The servo case (4.21) result in the transfer operator B^*_D due to the given choice of $T^*_1 A^*_M$.

Only a brief comment will be made of how parameter identification for this example would be approached. The continuous time case is not elaborated with respect to identification in this thesis but the estimation model can still be given. Start with the estimation model description of §5.8

$$\theta_2^T \varphi_2 = B_S^* \left(u + \theta_1^T \varphi_1 \right) \qquad \text{(vii)}, \ (7.46)$$

Since B_S^* is diagonal we obtain

$$\bar{u} = -\theta_1^T \bar{\varphi}_1 + \theta_2^T \varphi_2 \qquad \text{(viii)}, \ (7.47)$$

without parameter cross couplings. The control law is

$$u = -\theta_1^T \varphi_1 + \theta_2^T \varphi_2^m \qquad \text{(x)}, \ (7.48)$$

where

$$\bar{\varphi}_2^m = \varphi_2^M \qquad (7.49)$$

and φ_2^m is the vector corresponding to φ_2 containing y^m. The control law for u_1 in the considered example is

$$u_1(t) = + z^{*2} u_1 - z^* u_2 + z^{*2} u_2 - 3y_1 - z^* y_1 + 3z^{*2} y_1$$
$$- y_1^m + 2z^* y_1^m \qquad \text{(x)}, \ (7.50)$$

and the LIP-model for u_1 is

$$
\begin{aligned}
z^* u_1 = {} & z^{*3} u_1 - z^{*2} u_2 + z^{*3} u_2 - 3z^* y_1 - z^{*2} y_1 + 3z^{*3} y_1 \\
& - z^* y_1^m + 2z^{*2} y_1^m + \\
& + e \qquad\qquad (viii), \quad (7.51)
\end{aligned}
$$

where the first row is the components of R^* and S^*. The second row of both (7.50) and (7.51) contain the parameters of T_3^* and the third row of (7.51) contain the component of T_2^*. The same kind of relation is valid for u_2^* where however $-z^*(1-2z^*)$ is the filtering component.

7.5 Example 4

Consider the 2x2 system

$$
y(t) = \begin{pmatrix} b_1 q^{-3} & b_2 q^{-1} \\ 0 & b_3 q^{-2} \end{pmatrix} \begin{bmatrix} 1 & a_3 q^{-1} \\ a_2 q^{-1} & 1 - a_1 q^{-1} \end{bmatrix}^{-1} u(t) \qquad (7.52)
$$

Some interesting cross-coupling features appear when b_2 is non-zero. The most striking property is the singularity of B_0 which has already been considered in an example of chapter 4. The matrices A^* and B^* of (7.52) together form an open-loop transfer operator

8 S H

$$G_O^*(q^{-1}) = \begin{bmatrix} -a_2b_2q^{-2}+b_1q^{-3}-a_1b_1q^{-4} & b_2q^{-1}-a_3b_1q^{-4} \\ -a_2b_3q^{-3} & b_3q^{-2} \end{bmatrix} \frac{1}{\Delta(q^{-1})}$$

where (7.53)

$$\Delta(q^{-1}) = \det A^*(q^{-1}) = 1 - a_1q^{-1} - a_2a_3q^{-2} \tag{7.54}$$

Assume a dead-beat controller with a diagonal servo transfer operator to be the design objective.

A decomposition of B^* of the type (4.7) is given by elementary row and column operations on B^* and is identical with that of the simplified example.

$$B^*(q^{-1}) = \begin{bmatrix} 0 & b_2 \\ -\frac{b_1b_3}{b_2} & b_3q^{-1} \end{bmatrix} \begin{bmatrix} q^{-4} & 0 \\ 0 & q^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{b_1}{b_2}q^{-2} & 1 \end{bmatrix} = B_L^* B_S^* B_R^*$$

A feasible B_D^* is given by (7.55)

$$B_D^* = \begin{bmatrix} q^{-3} & 0 \\ 0 & q^{-4} \end{bmatrix} \qquad \text{(iii), (7.56)}$$

The dead-beat pole specification gives (7.57)

$$T_1^*A_M^* = I \qquad \text{(v), (7.58)}$$

The solutions to (4.17) now appear as

$$T_2^* = \begin{bmatrix} \frac{1}{b_1} q^{-1} & -\frac{b_2}{b_1 b_3} \\ \frac{1}{b_2} & 0 \end{bmatrix} \qquad \text{(vi), (7.59)}$$

The corresponding T_3^* is

$$T_3^* = \begin{bmatrix} \frac{1}{b_1} & -\frac{b_2}{b_1 b_3} \\ \frac{1}{b_2} q^{-2} & 0 \end{bmatrix} \qquad \text{(vi), (7.60)}$$

A solution to

$$R_u^* A^* + S_y^* B^* = T_1^* A_M^* B_R^* \qquad (4.17)$$

has a different solution than in the simplified example. The necessary recalculations give

$$R_u^* = \begin{bmatrix} 1 + \frac{a_3}{b_2} b_1 q^{-3} & 0 \\ -a_2 q^{-1} + \frac{b_1}{b_2} q^{-2} + a_1 \frac{b_1}{b_2} q^{-3} + a_2 a_3 \frac{b_1}{b_2} q^{-4} + a_3 (b_1/b_2)^2 q^{-5} & 1 \end{bmatrix}$$

and the S_y^*-matrix is (vi), (7.61)

$$\begin{bmatrix} -\frac{a_3}{b_2} & -\frac{a_3^2 b_1}{b_2 b_3} q^{-2} \\ \frac{a_1}{b_2} + \frac{a_2 a_3}{b_2} q^{-1} - \frac{b_1 a_3}{(b_2)^2} q^{-2} & \frac{b_1 a_3}{b_2 b_3} q^{-2} + \frac{b_1 a_2 a_3^2}{b_2 b_3} q^{-3} - \left(\frac{b_1}{b_2}\right)^2 \frac{1}{b_3} a_3^2 q^{-4} \end{bmatrix}$$

The estimation model of (5.16) is

$$T_2^*(q^{-1})\, e = B_S^*\left[u + R^*(q^{-1})u + S^*(q\;)y - T_3^*(q^{-1})y^m\right]$$

and give the LIP-model for u_1 (7.63)

$$u_1(t-4) = -\theta_{11}u_1(t-7) - \theta_{12}y_1(t-4) - \theta_{13}y_2(t-6) +$$

$$+ \theta_{31}y_1^m(t-4) + \theta_{32}y_2^m(t-4) +$$

$$+ \theta_{21}e_1(t) + \theta_{22}e_2(t) \qquad \text{(ix), (7.64)}$$

expressed in the coefficients of R_u^*, S_y^*, T_3^* and T_2^*. An equivalent estimation model for u_2 is obtained from the second row of the mentioned matrices. The correct control law for u_1 would be

$$u_1(t) = -\theta_{11}u_1(t-3) - \theta_{12}y_1(t) - \theta_{13}y_2(t-2) +$$

$$+ \theta_{31}y_1^m(t) + \theta_{32}y_2^m(t) \qquad \text{(x), (7.65)}$$

where parameter estimates enter in the case of adaptive control. Simulations of this example have been done using the program package SIMNON (cf.[ElmJ). The given example shows a 'cold-start' where all parameter estimates of θ_1 and θ_3 start at zero. The parameters have have been estimated with the PEI-scheme using LS and the certainty equivalence dead-beat control law was applied during all of the simulation. The reference values to be followed are shown by fig.7.1

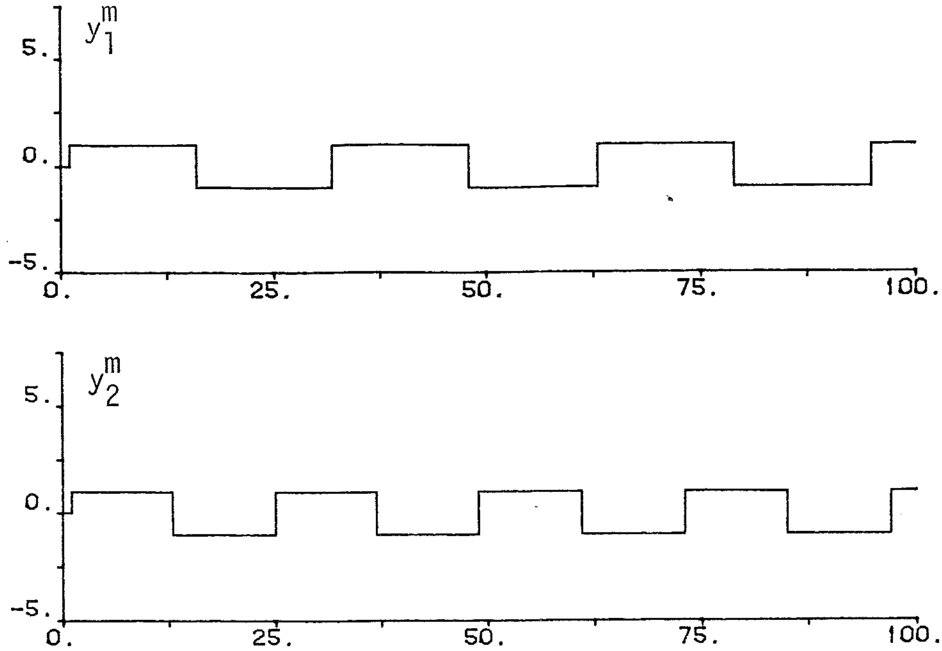


Fig.7.1: Reference values y_1^m and y_2^m

The performance of the adaptive regulator is shown in fig.7.2 and fig.7.3 where the outputs and inputs, resp., are represented. The parameters of the control object were chosen as

$$\left(b_1, \ b_2, \ b_3, \ a_1, \ a_2, \ a_3\right) = \left(1, \ 2, \ 3, -2, \ 1, \ -1\right)$$

in the actual example. The A^*-parameters then result in double integrator (i.e. z=1) pole locations of the open-loop system. The sampling time was chosen to 1.

8

S.

V

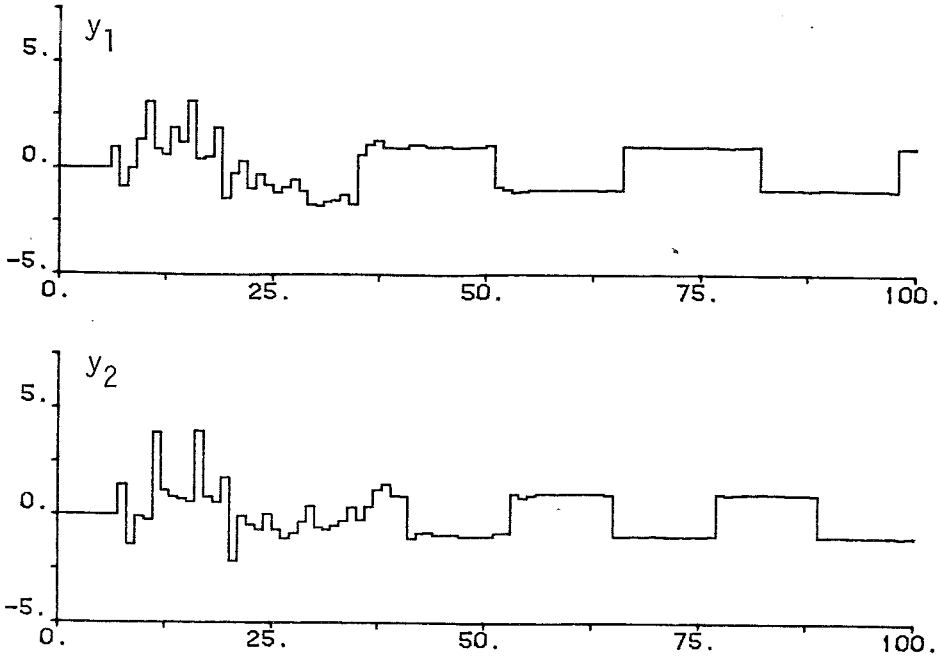


Fig.7.2: Outputs y_1 and y_2

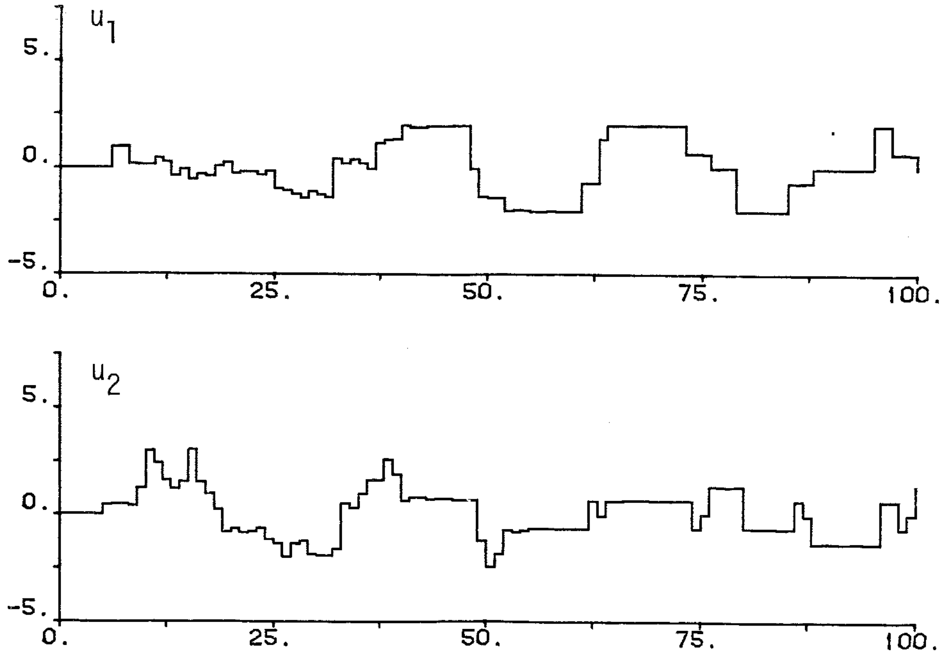


Fig.7.3: Control inputs u_1 and u_2

8 STABILITY

8.1 Introduction

The common problems concerning stability investigations of adaptive control algorithms relate to their time varying and nonlinear properties. Parameter convergence is usually easy to establish. The difficult step in most proofs is to deduct closed-loop stability from the parameter convergence results.

The convergence proofs in adaptive control theory use many different stability concepts and techniques for example

- Asymptotic Properties
- Local Stability
- Minimization of cost function
- Bounded input, bounded output (BIBO)
- Hyperstability
- L^{∞}-stability
- Stability in the sense of Lyapunov
- Asymptotic stability
- Exponential stability

It is not the intention to survey the interrelations between these properties. There is consensus in the literature on the meaning of most concepts. The definitions of L^{∞}-stability - or rather l^{∞}-stability for discrete time systems (cf.[D&V]) - are however exceptions. The concept of L^{∞}-stability used in e.g. [Ega] is the same as

BIBO-stability. Another definition of L^{∞}-stability is found in [Vid] where the BIBO-condition has to be supplemented with a norm restriction such that

$$\|y\|_{\infty} \leq k_1 \|u_c\|_{\infty} + k_2 \tag{8.1}$$

for bounded positive constants k_1 and k_2.

Most efforts in the last few years have been directed towards BIBO-type proofs. The stability proofs in [GRC] and [Ega] are the prototypes. The proofs start by assumptions on bounded initial parameter estimates and a bounded command signal sequence. These properties lead to a limited maximal growth rate of the data vector $\|\varphi\|$ so that

$$\|\varphi(t)\| \leq K_1 \|\varphi(t-1)\| + K_2 \tag{8.2}$$

is satisfied for some bounded positive numbers K_1 and K_2. The proofs then show by contradiction that $\|\varphi\|$ cannot grow forever and the obtained boundedness result is exploited to show that the output error decreases. MIMO-extensions are also found in [GRC].

The problem of global asymptotic stability has been given a lot of attention since the works by Parks [Par]. A Lyapunov function comprising both output errors and parameter errors was given for a class of continuous time systems. The celebrated 'positive real condition' on the process transfer functions is however crucial for the proof to hold.

There are however fewer results on stability for discrete time adaptive control based on stronger stability concepts such as l^{∞}-stability (including (8.1)) or stability in the sense of Lyapunov or still better exponential stability.

The stability properties of the algorithms proposed in this thesis may be developed along the same outlines. A certain hierarchy of proof qualities may be given as follows:

First, the PEI algorithm given in ch.6 is essentially a pure reformulation of the identification problem. This approach does not imply the use of any particular control law and no conclusions on stability may be drawn from the parameter convergence results per se. No assumptions on application of a certain control law is made. If the CE control law is used it follows that correct asymptotic properties are obtained when the parameters have converged.

Second, the scheme using FEI gives less freedom of choice since a certainty equivalence control law is assumed already in the problem formulation. The parameter error Lyapunov function is then discounted by some function of the output error and not by a prediction error. There is thus a closer connection between the control and output performances and the identification properties than in the case of PEI. The identification algorithm may be said to decode the output errors in terms of parameter errors. The bounded-input, bounded-output (BIBO) type stability proof given for certain discrete-time systems in [GRC] (SISO II and the MIMO extension) is quite applicable to the MIMO-case. The proof holds directly if the condition (6.36), (A6.1.11) is supplemented by assumptions on bounded initial parameter estimates and a bounded sequence of reference values.

Third, the proof in [GRC] may under certain circumstances be considerably strengthened. A new stability proof below shows global stability in the sense of Lyapunov for a class of adaptive systems. Exponential convergence of the data vector $\|\varphi_1\|$ and the output error is shown under an extra condition.

8.2 A Lyapunov Function

A proof of uniform global stability will now be given for a class of discrete-time adaptive systems. Below are considered only control objects with one input of the type

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} B_1(q) \\ B_2(q) \end{bmatrix} \frac{1}{A(q)} u(t) \tag{8.3}$$

with deg $A(q) = n_A$. The adaptive control laws are assumed to be derived w.r.t. the scalar output y_1 while other measured outputs are included in the vector y_2. The polynomials $A(q)$ and $B_1(q)$ are assumed to have no common factors. The transfer operator of (4.3) is required to be strictly proper and $B_1(q)$ is assumed to have all zeros within the unit circle. A m.f.d. on the form (4.1) of the same control object in reciprocal polynomials is

$$A^*(q^{-1})\xi(t) = u(t)$$

$$y(t) = \begin{bmatrix} B_1^*(q^{-1}) \\ B_2^*(q^{-1}) \end{bmatrix} \xi(t) = B^*(q^{-1})\xi(t) \tag{8.4}$$

where the partial state ξ is a scalar, A^* and B_1^* are polynomials and B_2^* is a polynomial vector. The transfer operator from u to the controlled output y_1 is the open-loop, minimum-phase discrete time control object

$$y_1(t) = \frac{B_1^*(q^{-1})}{A^*(q^{-1})}u(t) = b_0 q^{-k}\frac{B_R^*(q^{-1})}{A^*(q^{-1})}u(t) \tag{8.5}$$

where $A^*(0)=1$ and $B_R^*(0)=1$. The time delay from u to y_1 is then given by k and the numerator polynomial B_R^* is stably invertible. Consider a general, linear controller including a command signal u_c

$$R_u^*(q^{-1})u(t) = - S_y^*(q^{-1})y(t) + T^*(q^{-1})u_c(t) \tag{8.6}$$

for model reference adaptive control or for a pole placement regulator. Assume a desired, stable pole polynomial $T_1^*A_M^*$ to be chosen such that $T_1^*(0)A_M^*(0)=1$. A solution is found via the Diophantine equation

$$R_u^* A^* + S_y^* B^* = T_1^* A_M^* B_R^* \quad ; \quad R_u^*(0) = 1 \tag{8.7}$$

and gives a closed-loop system where the unknown, stably invertible numerator polynomial B_R^* is cancelled and where $T_1^*A_M^*$ is the desired pole polynomial. A parametrization for direct adaptive control of y_1 in the regulator case - T^* and u_c are excluded - is given in terms of the desired pole placement control law so that

$$y_1(t) = B_1^*(q^{-1})\xi(t) = \tag{8.8}$$

$$= \frac{b_0 q^{-k} B_R^*(q^{-1})}{T_1^*(q^{-1})A_M^*(q^{-1})B_R^*(q^{-1})}\left[R_u^*(q^{-1})u(t) + S_y^*(q^{-1})y(t)\right] =$$

$$= \frac{b_0 q^{-k} B_R^*(q^{-1})}{T_1^*(q^{-1})A_M^*(q^{-1})B_R^*(q^{-1})}\left[u(t) + \theta_1^T\varphi_1(t)\right] \tag{8.9}$$

where θ_1 contains the parameters of R_u^* and S_y^* and φ_1 is a data vector of u and y such that

$$\varphi_1(t) = \left[u(t-1) \; \ldots \; y_1(t) \; y_1(t-1) \; \ldots \; y_{21}(t) \; \ldots\right] \tag{8.10}$$

where $y_{2i}(t)$ is component i of y_2. The adaptive control law is then

$$u(t) = -\hat{\theta}_1^T(t)\varphi_1(t) \tag{8.11}$$

where $\hat{\theta}_1$ is an estimate of θ obtained from some estimation algorithm. The closed-loop system is then obtained as

$$y_1(t) = \frac{b_0 q^{-k}}{T_1^*(q^{-1})A_M^*(q^{-1})}\left[-\tilde{\theta}_1^T(t)\varphi_1(t)\right] \tag{8.12}$$

where $\tilde{\theta}$ is the parameter error vector. The stable factors B_R^* of the numerator and denominator are then cancelled in the input-output behaviour.

Proofs of BIBO-type stability are only concerned with input-output characteristics. Attempts to make proofs of asymptotic stability must however also take the internal behaviour into account - e.g. in terms of a state-space description. A natural choice is to relate an internal description to the partial state ξ of (8.4) (cf.[Kai], sec.6.4). The state-vector of the controllable canonical state-space description of the control object is given by

$$\begin{bmatrix} x_1(t) & \dots & x_{n_A}(t) \end{bmatrix} \tag{8.13}$$

where $x_i(t)=\xi(t-i)$. A reformulation suitable for the adaptive control context with an adaptive output feedback control law (8.11) will now be given. A state-space representation $x(t)$ of (8.3) with the control law (8.11) on the controllable canonical form

$$x(t+1) =$$

$$= \begin{bmatrix} -p_1 & \dots & -p_{n_p} & 0 \dots 0 \\ 1 & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 \quad 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} -\tilde{\theta}_1^T(t)\varphi_1(t) \end{bmatrix}$$

$$y_1(t) = \begin{bmatrix} 0 & \dots & b_0 & b'_1 & \dots & b'_m & 0 & \dots 0 \end{bmatrix} x(t) \tag{8.14}$$

where the p_i's are the unknown coefficients of

$$P^*(q^{-1}) = T_1^*(q^{-1})A_M^*(q^{-1})B_R^*(q^{-1}) =$$

$$= 1 + p_1 q^{-1} + \dots + p_{n_p} q^{-n_p} \qquad (8.15)$$

for a sufficient dimension n to be determined below (cf.(8.21)). The b_i's are given by the coefficients of the B_1^*-polynomial and and $x_i(t)=\xi(t-i)$. Introduce the following shorter notation for the state-space formulation (8.14)

$$x(t+1) = F\ x(t) + G\ (-\tilde{\theta}_1^T(t)\varphi_1(t))$$

$$y_1(t) = H\ x(t) \qquad (8.14)'$$

Another state-space formulation is given by the φ_1-vectors of input-output data. This choice gives a non-minimal state-space vector which is observable since all u and y are measured data but on the other hand not controllable from u. It was however shown in the pole placement design calculations that no unstable modes are introduced. The uncontrollability of φ_1 is therefore certainly benign and the difference between x and φ is of no particular concern in pole placement designs for known systems. There are however some consequences in the case of adaptive control and the difference may be interpreted via the formulation (8.14). The input shows dependence of the φ_1-vector and the stable, uncontrollable modes may be susceptible to excitation due to e.g. the time variant properties. A formulation where these problems are avoided will therefore be given some attention. Consider the case where the data vector φ_1 may be expressed as a linear combination of the states x_i i.e. let there be some matrix M such that

$$\varphi_1(t) = M\, x(t) \tag{8.16}$$

The condition (8.16) is valid for strictly proper transfer operators and is thus no constraint for the considered class of systems. The control object (8.3) is not affected by restrictions of (8.16) since the sequences

$$\{u(s)\}_{-\infty}^{t-1}\ ,\ \{y_1(s)\}_{-\infty}^{t}\ ,\ \{y_{21}(s)\}_{-\infty}^{t}\ \text{etc.} \tag{8.17}$$

may be described in terms of components of

$$\{\xi(s)\}_{-\infty}^{t-1} \tag{8.18}$$

via

$$y_1(t) = B_1^*(q^{-1})\xi(t)\ ;\ u(t) = A^*(q^{-1})\xi(t)\quad \text{etc.} \tag{8.19}$$

The system (8.14) may under the assumption of (8.16) be recognized as a state-space formulation on a controllable form with an input $\hat{\theta}_1^T\varphi_1$ without impacts from anything but the system states and the parameter errors. It is now possible to determine the dimension n of the adaptive closed-loop system state representation (8.14) without parameter error dynamics. Assume that the solutions to the Diophantine equations (8.7) are of degrees

$$\deg R_u^* = n_R \le k-1;\quad \deg \left[S_y^* B^*\right] = n_{SB} \tag{8.20}$$

9
P
H

Then it follows that

$$n = \max\left[n_P, \max(n_{SB}, n_R + n_A) + k\right] \tag{8.21}$$

relating to the maximal time delay acting on ξ due to P^* or $q^{-k}\varphi_1$. Notice that the obtained degree n exceeds the dimension n_A of the control object state vector. The controller states hereby get a representation.

<u>Example 8.1</u>

The SISO example 7.1 is not restricted by (8.16) since the φ_1-vector consists of u and y such that

$$\varphi_1(t) = \left[u(t-1) \;\ldots\; u(t-k+1) \quad y(t) \;\ldots\right]^T \tag{8.22}$$

which result in the M-matrix of (8.16) on the form

$$M = \begin{bmatrix} 1 & a_1 & a_2 & \ldots a_{n_P} & 0\ldots \\ 0 & 1 & a_1 & \ldots & . \\ : & & & & : \\ 0 & \ldots & b_0 & b_0 \; b_1 & \ldots \\ 0 & \ldots & 0 & b_0 & \ldots \\ : & & & & : \end{bmatrix} \tag{8.23}$$

where

9

S

V

$$y(t) = b_0 q^{-k} B_R^*(q^{-1})\xi(t) = b_0 x_k(t) + b_0 b_1 x_{k+1}(t) + \ldots$$

$$(8.24)$$

$$u(t-1) = A^*(q^{-1})\xi(t-1) = x_1(t) + a_1 x_2(t) + \ldots$$

□

A number of assumptions are now to be stated in order to give the sufficient conditions for existence of a Lyapunov function. The assumptions 8:2 and 8:5-6 are the restrictive conditions and relate to a priori knowledge and control object properties, respectively. The remaining assumptions are simply given for the problem formulation and do not impose any restrictions.

<u>Assumption 8:1</u>

Assume a linear control object of the type (8.3), without feedthrough so that (8.16) holds

$$\varphi_1(t) = M\, x(t) \tag{8.16}$$

<u>Assumption 8:2</u>

The time delay k and an approximate estimate β_0 of the gain b_0 should be known such that

$$0 < \frac{b_0}{\beta_0} < 2 \tag{8.25}$$

Assumption 8:3

The control law is the certainty equivalence control law i.e.

$$u(t) = -\hat{\theta}_1^T(t)\varphi_1(t) \tag{8.26}$$

and is assumed to start at initial time t_0.

Assumption 8:4

Assume the following identification algorithm to be used for parameter estimation from time t_0+k where t_0 is the initial time

$$\hat{\theta}(t) = \hat{\theta}(t-k) + \gamma(\|\varphi_1(t-k)\|)\varphi_1(t-k)e_f(t)$$

$$\gamma(\|\varphi_1(t)\|) = \begin{cases} \dfrac{1}{\beta_0\|\varphi_1(t)\|^2} & ; \quad \|\varphi_1(t)\| \geq a^2 \\[2ex] \dfrac{1}{\beta_0 a^2} & ; \quad \|\varphi_1(t)\| < a^2 \end{cases} \tag{8.27}$$

$$e_f(t) = T_1^*(q^{-1})A_M^*(q^{-1})e(t) \; ; \quad e(t) = y_1(t)$$

Assumption 8:5

Introduce the real weighting matrix

$$Q = \begin{bmatrix} 1 & & & & 0 \\ & \varrho & & & \\ & & \cdot & & \\ & & & \cdot & \\ 0 & & & & \varrho^{n-1} \end{bmatrix} \quad \text{where} \quad 0 < \varrho < 1 \tag{8.28}$$

and the nx1 vector

$$p = \begin{bmatrix} p_1 & \ldots & p_{n_p} & 0 & \ldots 0 \end{bmatrix}^T \tag{8.29}$$

which relate to the parameters of (8.15). Consider now only systems where the closed-loop pole polynomial is restricted such that

$$\|Q^{-1}p\|^2 = \sum_{i=1}^{n} p_i^2 \varrho^{-2(i-1)} < \frac{1}{2}\left(1 - \varrho^2\right) \tag{8.30}$$

This condition restricts the model reference poles of $T_1^* A_M^*$ and the control object zeros of B_R^* to be well-damped.

<u>Assumption 8:6</u>

A somewhat restricted alternative of assumption 8:5 is

$$0 < \varrho^2 + 2\|Q^{-1}p\|^2 < 1 - \delta \quad \text{for some } \delta > 0 \tag{8.31}$$

and will be used to show exponential convergence.

□

Now introduce the full state vector for the adaptive system consisting of control object, controller and parameter estimator.

9
P
V

$$X(t) \overset{\Delta}{=} \left[\tilde{\theta}_1^T(t-k+1) \; \ldots \; \tilde{\theta}_1^T(t) \; x^T(t)\right]^T =$$

$$\overset{\Delta}{=} \left[z^T(t) \; x^T(t)\right]^T \tag{8.32}$$

It is easy to see that $X(t)$ represents the full system state since (8.14) and (8.27) together form the autonomous system

$$X(t+1) = \begin{bmatrix} O & O & \ldots\ldots & O & I & O \\ I & O & \ldots\ldots\ldots & & O & O \\ O & I & O & \ldots\ldots & O & O \\ \vdots & & \ddots & \ddots & \vdots & \vdots \\ O & O & \ldots O & I & O & O \\ O & O & \ldots\ldots\ldots & & O & F \end{bmatrix} X(t) + \begin{bmatrix} L(X(t)) \\ O \\ \vdots \\ O \\ G \end{bmatrix} v(X(t))$$

where F and G are defined by (8.14)' and

$$L(X(t)) \overset{\Delta}{=} \gamma(\|\varphi_1(t)\|) b_0 \varphi_1(t)$$

$$v(X(t)) \overset{\Delta}{=} -\tilde{\theta}_1^T(t)\varphi_1(t) \tag{8.33}$$

Introduce also the functions

$$V_\theta(z(t)) \overset{\Delta}{=} \sum_{i=0}^{k-1} \|\tilde{\theta}_1(t+i)\|^2$$

and

$$v_x(x(t)) \stackrel{\Delta}{=} \ln\left[1 + x^T(t)Q^TQx(t)\right] \tag{8.34}$$

and

$$V(X(t)) \stackrel{\Delta}{=} v_\theta(z(t)) + Kv_x(x(t)) \tag{8.35}$$

The function V will be shown to be a Lyapunov function for an adaptive system under assumptions 8:1-5 for certain positive constants K. The suggestion of a logarithmic Lyapunov function is very natural for adaptive systems where the parameter errors enter in a multiplicative way. A closer investigation gives

Theorem 8.1

There is a bounded positive constant K such that the function

$$V(X(t)) = v_\theta(z(t)) + Kv_x(x(t)) \tag{8.35}$$

is a Lyapunov function for an adaptive system under assumptions 8:1-5. The difference

$$V(X(t+1)) - V(X(t)) \tag{8.36}$$

is negative semidefinite for $\|X\| \neq 0$ and assures uniform global stability in the sense of Lyapunov for the system. Furthermore, (8.36) is strictly negative for $\|x\| \neq 0$. □

Proof: See appendix 8:1. The upper bound on K is given by (A8.1.26).

The adaptive system consisting of both parameter estimates and state vectors is stable in the sense of Lyapunov but not necessarily asymptotically stable. There may be a persisting, non-zero parameter error even when $\|x\|$ is zero.

The immediate conclusion of theorem 8.1 is that $\|x\|$ and therefore also $\|\varphi_1\|$ and e converge to zero. It can in fact be shown that all solutions are such that $\|x\|$ and e decrease exponentially for $\|\varphi_1\|^2 \geq a^2$ if (8.30) of assumption 8:5 is restricted by δ to assumption 8:6 i.e.

$$0 < \varrho^2 + 2\|Q^{-1}p\|^2 < 1 - \delta \text{ for some } \delta > 0 \tag{8.31}$$

Theorem 8.2

An adaptive system under assumptions 8:1-6 with an associated Lyapunov function V given by (8.35) is characterized by exponential convergence w.r.t. $\|Qx(\tau)\|$ for all τ in a time interval $[\tau_0, \tau_1]$ with $\|\varphi_1(\tau)\|^2 \geq a^2$ so that

$$\sup_{V(X(\tau_0)) \leq C_0} \|Qx(\tau)\|^2 \leq C(\tau_0)\exp\left[-\delta(\tau-\tau_0)\right] ; \; C(\tau_0) = \exp\left[C_0/K\right] \tag{8.37}$$

Proof: See appendix 8:2. The upper bound on K is given by (A8.1.26).

The theorem states that the algorithm exhibits exponential convergence of $\|Qx\|$ whenever φ_1 has a certain magnitude. The constant a^2 in assumption 8:4 is free to choose arbitrarily small and the limiting case $a^2=0$ with

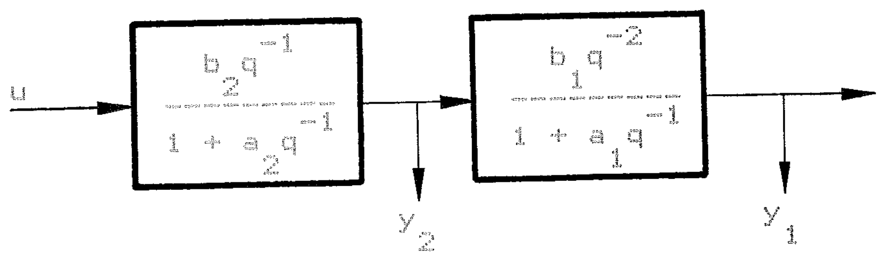
$$\gamma(\|\varphi_1\|) = \begin{cases} \dfrac{1}{\beta_0\|\varphi_1\|^2} & ; \quad \|\varphi_1\|\neq 0 \\ 0 & ; \quad \|\varphi_1\|=0 \end{cases}$$

would then result in global exponential convergence. A more reasonable choice of the constant 'a' still shows a strictly negative value of (8.36) and thus convergence although not necessarily with an exponential decay for small magnitudes of φ_1.

The following example is chosen to demonstrate the applicability of the given theorems.

Example 8.2

Consider the following open-loop system



Determine an adaptive dead-beat control strategy for y_1.

Starting with the transfer function

$$\begin{bmatrix} y_1(t) \\ \\ y_2(t) \end{bmatrix} = \begin{bmatrix} b_1 b_2 q^{-3} \\ \\ b_2\left(1+a_1 q^{-1}\right)q^{-1} \end{bmatrix} \frac{1}{\left(1+a_1 q^{-1}\right)\left(1+a_2 q^{-1}\right)} u(t) \tag{8.38}$$

and a right coprime m.f.d., it is straightforward to find the control law

$$u(t) = -s_1 y_1(t) - \left(s_2 + s_3 q^{-1}\right) y_2(t) \tag{8.39}$$

with

$$s_1 = -\frac{a_1^3}{b_0}, \quad s_2 = -\frac{1}{b_2}\left(a_1 + a_2\right), \quad s_3 = \frac{a_1^2}{b_2} \tag{8.40}$$

where $b_0 = b_1 b_2$. A state-space representation of the system

$$y_1(t) = \frac{b_0 q^{-3}}{T_1^*(q^{-1}) A_M^*(q^{-1})}\left(u(t) + \theta_1^T \varphi_1(t)\right) = b_0 q^{-3}\left(u + \theta_1^T \varphi_1\right) \tag{8.41}$$

with the control law $u(t) = -\hat{\theta}_1^T(t)\varphi_1(t)$ is

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \\ x_5(t+1) \\ x_6(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \left[\tilde{\theta}_1^T(t)\varphi_1(t)\right]$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -b_0 & 0 & 0 & 0 \\ -b_2 & -b_2a_1 & 0 & 0 & 0 & 0 \end{bmatrix} x(t) \tag{8.42}$$

where

$$\theta_1 = \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^T \tag{8.43}$$

and

$$\varphi_1(t) = \begin{bmatrix} y_1(t) & y_2(t) & y_2(t-1) \end{bmatrix}^T \tag{8.44}$$

The matrix relating $\varphi_1(t)$ and x of (8.16) is

$$M = \begin{bmatrix} 0 & 0 & -b_0 & 0 & 0 & 0 \\ -b_2 & -b_2a_1 & 0 & 0 & 0 & 0 \\ 0 & -b_2 & -b_2a_1 & 0 & 0 & 0 \end{bmatrix} \tag{8.45}$$

An upper bound for λ_{max} of $Q^{-T}M^TM\,Q^{-1}$ is given by

$$\lambda_{max} < \sum_{i=1}^{6} \lambda_i = \operatorname{tr} Q^{-T}M^TMQ =$$

$$= \frac{b_1^2b_2^2}{\varrho^6} + \frac{b_2^2}{\varrho^2}\left(1+\frac{1}{\varrho^2}\right) + \frac{b_2^2a_1^2}{\varrho^4}\left(1+\frac{1}{\varrho^2}\right) \tag{8.46}$$

For any choice of the weighting parameter ϱ in the range

$$0 < \varrho < 1 \tag{8.28}$$

and a parameter estimator (8.27)

$$\hat{\theta}_1(t) = \hat{\theta}_1(t-3) + \gamma(\|\varphi_1(t-3)\|)\varphi_1(t-3)e_f(t)$$

with $e_f = e = y_1$ and

$$\gamma(\|\varphi_1\|) = \begin{cases} \dfrac{1}{\beta_0\|\varphi_1\|^2} & ; \quad \|\varphi_1\| \geq 1 \\[2ex] \dfrac{1}{\beta_0} & ; \quad \|\varphi_1\| < 1 \end{cases} \tag{8.47}$$

there is a Lyapunov function of the type (8.35) since $\|p\|=0$. A Lyapunov function for this example is e.g.

$$V(X(t)) = \sum_{i=0}^{2}\|\tilde{\theta}_1(t+i)\|^2 +$$

$$+ \frac{b_0^2}{\beta_0^2}\left(2\frac{\beta_0}{b_0} - 1\right)\frac{1}{3\lambda_{max}}\ln\left[1 + \|Qx(t)\|^2\right] \tag{8.48}$$

where β_0 is some estimate of b_0 satisfying assumption 8:2.

8.3 Summary

A method for construction of a Lyapunov function for a class of adaptive systems has been given. The Lyapunov function is decrescent and shows global stability in the sense of Lyapunov for the adaptive system. Convergence for the state vector is shown and the proof may be strengthened to exponential convergence. The considered cases require an a priori estimate β_0 of the gain b_0 in order to assure parameter convergence. The restriction imposed by assumption 8:5 implies that only fairly fast control object zeros of B_R^*

and reference model poles of $T_1^* A_M^*$ could be considered $(\|p\|^2 < 0.5)$.

Although usually inferior to prediction error identification, feedback error identification has been used here since it relates parameter errors and control input errors in a more transparent way than PEI methods do. This is exploited in the proof. An adaptive scheme which utilizes LS-estimation would therefore be expected to exhibit tighter bounds on the errors than what is promised by the given Lyapunov functions.

The suggested Lyapunov functions are rather conservative and are chosen to cover all 'worst' cases of growth of the state vector due to bad parameter estimates.

9 EPILOGUE

This review contains a short summary of what has been solved, what remains unsolved and what new problems that have been created. The full problem formulations are given in the introductory chapter and are therefore superfluous to repeat.

The thesis has considered the problem of direct adaptive control of linear multivariable systems. Special parametrizations of linear multivariable systems suitable have been developed. The necessary a priori information in order to formulate convergent parameter estimation schemes is formulated in terms of the non-invertible system zeros. A prediction error identification algorithm is given and parameter convergence is established.

The case where the system output errors are used for parameter updating (FEI) is examined in greater detail than the properties of prediction error identification. A condition on prior knowledge is given in order to guarantee convergence of parameters in MIMO adaptive systems based on FEI.

A method for construction of Lyapunov functions has been presented in chapter 8. The method covers a regulator case with one input. Stability in the sense of Lyapunov and exponential convergence are shown.

A number of problems have been touched but not treated in this presentation and may be challenges for further research. The case of continuous time adaptive control has not been elaborated. Stochastic considerations and influences from time-varying parameters have not been treated. The problem of closed-loop stability under prediction error identification with LS-estimation has not been investigated. A solution to the latter problem would be most valuable in order to obtain a better understanding of adaptive control as well as features like dual and cautious control.

The suggested methods have all been given with certain assumptions on a priori knowledge. It can be argued that the condition of a known B_S^* is fairly implicit and it is natural to request methods to obtain the desired prerequisite. Reliable software for formula manipulations in unknown parameters would e.g. be valuable. Another problem is to validate certain linear systems assumptions on the control object.

10 REFERENCES

[Ast]

Aström, K.J.(1970): Introduction To .Stochastic Control Theory, Academic Press, New York, USA.

[ABLW]

Aström, K.J., Borisson, U., Ljung, L., Wittenmark, B.(1975): Theory And Applications Of Adaptive Regulators Based On Recursive Parameter Estimation, IFAC 6th World Conference, Boston, USA, 1975.

[AWW]

Aström, K.J., Westerberg, B., Wittenmark, B.: Self-Tuning Controllers Based On Pole-Placement Design, Dept.Of Automatic Control, Lund Inst.Of Technology, Sweden, CODEN:LUTFD2/(TFRT-3148)/1-052/(1978).

[A&W 1]

Aström, K.J., Wittenmark, B.(1971): Problems Of Identification And Control, J. Of Math.Analysis And Applications, Vol-34, pp.90-113, 1971.

[A&W 2]

Aström, K.J., Wittenmark, B.(1973): On Self Tuning Regulators, Automatica, Vol.9, pp.185-199, 1973.

[Bor 1]

Borisson, U.(1975): Self Tuning Regulators - Industrial Application And Multivariable Theory, Report 7513, Dept.Of Autom.Control, Lund Inst.Of Techn., Sweden.

[Bor 2]

Borisson, U.(1979): Self-Tuning Regulators For A Class Of Multivariable Systems, Automatica, Vol.15, pp.209-217, 1979.

[D&V]

Desoer, C.A., Vidyasagar, M.(1975): Feedback Systems: Input-Output Properties, Academic Press, New York, USA.

[Ega]

Egardt, B.(1979): Stability of Adaptive Controllers, Springer Verlag, New York, USA.

[E&W 1]

Elliott, H., Wolowich, W.(1981): A Parameter Adaptive Control Structure For Linear Multivariable Systems, IEEE Trans.Autom.Control, Vol.AC-26, 1981.

[E&W 2]

Elliott, H., Wolowich, W.(1982): Adaptive Control Of Stably Invertible Discrete Multivariable Systems: The Pole-Placement Approach, Report #FB82-DELENG-1, Colorado State University, 1982.

[EWD]

Elliott, H., Wolowich, W., Das, M.(1982): Arbitrary Adaptive Pole Placement For Linear Multivariable Systems I-II, Reports #FB82-DELENG-2, #MA82-DELENG-1 Colorado State University, 1982.

[Elm]

Elmqvist, H.(1975): SIMNON, An Interactive Simulation Program For Nonlinear Systems, Report TFRT-3091, Dept.Of Autom.Control, Lund Inst.Of Techn., Lund, Sweden.

[GLS]

Gustavsson, I., Ljung, L., Söderström, T.(1977): Identification of Processes In Closed Loop - Identifiability And Accuracy Aspects, Automatica, Vol.13, pp.59-75, 1977.

10 p. H

[G&C]

Goodwin, G.C., Chan, S.W.(1981): Deterministic And Stochastic Adaptive Control With Purely Deterministic Disturbances, Techn.Report EE8109, University of Newcastle, Australia.

[G&L]

Goodwin, G.C., Long, R.S.(1980): Generalizations Of Results On Multivariable Adaptive Control, IEEE Trans.Autom.Control, Vol.AC-25, pp.1241-1245, 1980.

[G&P]

Goodwin, G.C., Payne, R.L.(1977): Dynamic System Identification: Experiment Design And Data Analysis, Academic Press, New York, USA.

[GRC]

Goodwin, G.C., Ramadge, P.J., Caines, P.E.(1980): Discrete Time Multivariable Control, IEEE Trans.Autom.Control, Vol. AC-25, pp.449-456, 1980.

[Hah]

Hahn, W. (1967): Stability Of Motion, Springer Verlag, New York, USA.

[Jaz]

Jazwinski, A.H.(1970): Stochastic Processes And Filtering Theory, Academic Press, New York, USA.

[Joh 1]

Johansson, R. (1982): Parametric Models Of Linear Multivariable Systems For Adaptive Controllers, Dept.Of Autom.Control, Lund Inst. Of Techn., Sweden, CODEN:LUTFD2/(TFRT-7233)/1-95/(1982).

[Joh 2]

Johansson, R. (1982): Parametric Models Of Linear Multivariable Systems For Adaptive Controllers, Proc.21st IEEE CDC, Orlando, USA, 1982.

[J&T]

Johnson Jr., C.R., Tse, E.(1978): Adaptive Implementation Of One Step Ahead Optimal Control Via Input Matching, IEEE Trans.Autom.Control, Vol.AC-27, pp.865-872, Oct.1978.

[Kai]

Kailath, T. (1980): Linear Systems, Prentice-Hall, Englewood Cliffs, N.J., USA.

[Kan]

Kant, I. (1781): Kritik Der Reinen Vernunft, Königsberg, Prussia.

[K&H]

Keviczky, L., Hetthessy, J.(1977): Self-Tuning Minimum Variance Control Of MIMO Discrete Time System, Autom.Control Theory And Appl.5(1), pp.11-17, 1977.

[Koi]

Koivo, H.N.(1980): A Multivariable Self-Tuning Controller, Automatica, Vol.16, pp.351-356, 1980.

[Kre]

Kreisselmeier, G.(1982): On Adaptive State Regulation, IEEE Trans.Autom.Control, Vol. AC-27, pp.3-17, 1982.

[Lan]

Landau, Y.D.(1979): Adaptive Control - The Model Reference Approach, Marcel Dekker Inc., New York.

[LaS]

LaSalle, J.P.(1976): The Stability Of Dynamical Systems, SIAM, Philadelphia, USA.

[Lju]

Ljung, L.(1977): On Positive Real Transfer Functions And The Convergence Of Some Recursive Schemes, IEEE Trans.Autom.Control, AC-22, pp.539-550, 1977.

10
S.
H

[L&W]

Ljung, L., Wittenmark, B.(1974): Asymptotic Properties Of Self-Tuning Regulators, Report 7404, Div.Of Autom.Control, Lund Inst.Of Techn., Sweden.

[Mon 1]

Monopoli, R.V.(1973): The Kalman-Yakubovich Lemma In Adaptive Control Design, Vol.AC-18, pp.527-529, 1973.

[Mon 2]

Monopoli, R.V.(1974): Model Reference Adaptive Control With An Augmented Error Signal, IEEE Trans.Autom.Control, Vol.AC-19, pp.474-484, 1974.

[Mor]

Morse, A.S.(1980): Global Stability Of Parameter Adaptive Control Systems, IEEE Trans.Autom.Control, Vol. AC-25, pp.433-439, 1980.

[NLV]

Narendra, K.S., Lin, Y.H., Valavani, L.S.(1980): Stable Adaptive Controller Design - Part II: Proof Of Stability, IEEE Trans.Autom.Control, Vol. AC-25, pp.440-448, 1980.

[Par]

Parks, P.C.(1966): Liapunov Redesign Of Model Reference Adaptive Control Systems, IEEE Trans.Autom.Control, Vol. AC-11, pp.362-367, 1966.

[Per 1]

Pernebo, L.(1978): Algebraic Control Theory For Linear Multivariable Systems, Lund Inst. Of Technology, Lund, Sweden. CODEN:LUTFD2/(TFRT - 1016)/1-307/(1978)

[Per 2]

Pernebo, L.(1981): An Algebraic Theory For The Design Of Controllers For Linear Multivariable Systems - Part I: Structure Matrix And Feedforward Design. IEEE Trans.Autom.Control, Vol.AC-26, pp.171-182, 1981.

[Per 3]

Pernebo, L.(1981): An Algebraic Theory For The Design Of Controllers For Linear Multivariable Systems - Part II: Feedback Realizations And Feedback Design. IEEE Trans.Autom.Control, Vol.AC-26, pp.182-193, 1981.

[Per 4]

Pernebo, L.(1980): Algebraic Control Theory (Lecture Notes), Dept. Of Automatic Control, Lund Inst. Of Technology.

[Ros]

Rosenbrock, H.H.(1970): State Space And Multivariable Theory, Nelson, London

[Sha]

Shapiro, L.(1975): Introduction To Abstract Algebra, McGraw-Hill, New York, USA.

[SLG]

Söderström, T., Ljung, L., Gustavsson, I.(1974): A Comparative Study Of Recursive Identification Methods, Report 7427, Dept.of Autom.Control, Lund Inst Of Techn., Sweden.

[Vid]

Vidyasagar, M.(1978): Nonlinear Systems Analysis, Prentice-Hall, Englewood Cliffs, N.J., USA.

[Wit]

Wittenmark, B.(1975): Stochastic Adaptive Control Methods: A Survey, Int.J.Control, Vol.21, No.5, 1975.

10
P
V

APPENDIX 1:1

The following notation reference list give some of the most common notations of the text. The references are given to definitions and and other important key references.

Acronyms		Ref
FEI	Feedback error identification	§6.1,§6.3
l.c.f	Left coprime factorization	§2.4
LIP	Linear in parameters and identifiable	§5.8
LS	1) Left Structure (as a subscript)	§2.6
	2) Least Squares (as a prefix)	§6.2
m.f.d	Matrix fractional description	§2.4
MIMO	Multi-input, multi-output	
PEI	Prediction error identification	§6.1,§6.3
r.c.f	Right coprime factorization	§2.4
SA	Stochastic Approximation	(6.6)
SIMO	Single input, multi-output	
SISO	Single input, single ouput	

Qualifiers

$*$	Function in transformed variable z^*	§2.5
$-$	Filtering by component of B_S^*	(5.52)
$\sim$	Error in estimated parameter vector	§6.2
$\wedge$	Predicted value	§6.2
$\lVert\cdot\rVert$	Euclidean norm	
Z^*-	Generalized algebraic property w.r.t. Z_+^*	§2.2

Notations	Verbal Description	Ref
$A(z)$	Denominator matrix of m.f.d	§2.4
A^*	Denominator matrix of m.f.d. in variable z^* (Reciprocal pole polynomial in SISO-case)	
A^*_{uu}, A^*_{uv}, A^*_{uw}, A^*_{vv}, A^*_{vw}, A^*_{ww}		(A4.3.1)
A_ξ		(3.3)
A^*_M	Denominator matrix of a ref. model	(3.11)

10

S

V

$B(z)$	Numerator matrix of a m.f.d.	§2.4
B_o	Gain matrix (used by other authors)	ch.1
B^*	Numerator matrix of m.f.d. in var. z^*	§2.5
B_1^*	B^*-matrix w.r.t. y_1	(4.1)
B_2^*	B^*-matrix w.r.t. y_2	(4.1)
B_{1u}^*, B_{1v}^*, B_{1w}^*, B_{2u}^*, B_{2v}^*, B_{2w}^*		(A4.3.1)
$B_{.u}^*$		(A4.3.11)
B_ξ^*		(3.3)
B_D^*	Diagonalized left structure matrix	(4.6)
B_L^*	Left invertible divisor matrix of B^*	(4.7)
B_{LS}^*	Left structure matrix	(3.6),(3.11),(4.4)
B_M^*	Numerator matrix of reference model	(3.10)
B_R^*	Right invertible divisor matrix of B^*	(4.7)
B_S^*	Internal structure matrix	(4.7)
b_o	Gain in SISO control object	(7.1),(8.2)

b_1, b_2, b_3		(7.36)
b_{ij}		(4.9)
b_i^*	Component i of B_S^*	(4.14)
$C(\tau_0)$		(8.37)
C^*	Numerator matrix w.r.t. disturbance	
C_v^*		(5.34)
d	Disturbance vector	(2.2)
e	Output error vector	(5.14)
e_f	Filtered output errors ($e_f = T_1^* A_M^* e$)	(5.17)
F^*	Transfer operator	
F_u^*		(3.6)
$F_v^{*'}$, F_v^*		(5.23), (5.23)
$F_w^{*'}$, F_w^*		(5.23), (5.28)
G	Transfer operator in variable z	(2.1)
$G_0, G_{1u}, G_{1v}, G_{1w}, G_{2u}, G_{2v}, G_{2w}$		(2.4)

G^*	Transfer operator in variable z^*	
$G^*_{u_c y_0}$, $G^*_{vy_0}$, $G^*_{wy_0}$		(A4.3.10)
H^*	Transfer operator in variable z^*	
H^*_d		(5.19)
H^*_v		(A5.1.2)
H^*_w		(5.28g)
h	Magnitude of $\tilde{\theta}_1$-proj. along φ_1	(A8.2.1)
K		(A8.1.16)
K_L		(8.35)
K^*		(A5.1.5)
k	Time delay of discrete time SISO object	(7.1)
k_1, k_2		(7.9)
L^*	Left invertible matrix	(2.11)
M, M^*	Matrix	
N_i	Dimension reducer for vectors	(A6.1.1)

N^*		(A5.1.8)
P	Gain matrix in LS-estimation	(6.10)
p	Differential operator	§2.3
p_i	Coefficient of a pole polynomial	(8.7)
p^*_{ij}	Component of $T^*_1 A^*_M$	(4.15)
Q	Positive definite weighting matrix	(8.28)
q	Forward shift operator	§2.3
q^{-1}	Backward shift operator	§2.5
R^*	1)	(2.11)
	3)	(5.12)
R_o		(5.12)
R^*_u	1)	(3.1)
	2)	(4.2)
r_i	Coefficient of R^*-polynomial	(5.1)
S^*	1)	(2.11)
	3)	(5.12), (5.28f)

S_y^*	1)	(3.1)
	2)	(4.2)
s_i	Coefficient of S^*-polynomial	(2.14)
T^*		(3.3)
T_1^*		(4.13)
T_2^*		(4.17)
T_3^*		(4.17)
T_{ff}^*		(A4.3.7)
T_f^*		(5.28)
T_m^*		(5.31)
T_r^*		(4.4)
T_{uc}^*		(4.2)
T_{ym}^*		(4.2)'
T_v^*		(5.28g)
t	Time	

U^*	Unimodular or Z^*-unimodular matrix	(5.25-27)
u	Control input signal vector	(2.2)
u_c	Command signal vector	(3.1)
V	Lyapunov function (candidate)	(6.8)
v_x, v_θ	Terms of V	(8.34)
v	Measurable disturbance inputs	(2.2),(A4.3.1)
w	Non-measurable disturbancesn	(2.2),(A4.3.1)
y	System output vector	(2.1)
y_1	Controlled, measured outputs	(4.1)
y_2	Other measured outputs	(4.1)
y_f	Filtered output ($y_f=T_1^*A_M^*y_1$)	(5.10)
y_1^M	Desired output vector	(3.11)
y_1^m	Reference signal vector	(3.11)
Z_+	Stability region w.r.t. operator z	(2.7)
Z_-	Prohibited area of pole location w.r.t. z	(2.7)

Z^*_+	Stability region w.r.t. operator z^*	§2.5
Z^*_-	Prohibited area of pole location w.r.t. z^*	§2.5
z	Operator variable (p or q)	§2.3
z^*	Transformed operator variable	(2.10)
α	Angle between $\tilde{\theta}_1$ and φ_1	(A8.1.4)
α_1		(7.16)
β_0	A priori estimate of b_0	(6.16)
β_1		(7.16)
γ	Gain in estimation algorithm	(6.2)
δ		(8.31)
ε	Prediction error	(6.2)
ζ		(6.1)
θ	Parameter vector	(6.1)
θ_0		(6.21)
θ_1	Parameters ref. to R^*_u and S^*_y	§5.8

θ_2	Parameters ref. to T_2^*	§5.8
θ_3	Parameters ref. to T_3^*	§5.8
θ_v	Parameters ref. to T_v^*	§5.8
ϑ		(5.3)
Λ	Diagonal matrix	(6.36)
Λ^*	Part of Smith-form	(2.12)
λ	Forgetting factor	(6.9)
λ_{max}		(A8.1.10)
ν	Input error $(-\tilde{\theta}_1^T\varphi_1)$	(6.35)
Ξ	Parameter matrix of θ-vectors	(A6.1.3)
ξ	Partial state, Internal variable	(3.3),(4.1)
ϱ	Weighting coefficient	(8.28)
τ_0, τ_1	Boundaries of time interval	(8.37)
τ_2^*	A priori estimate of T_2^*	(6.36)
Φ	Data vector	(A6.1.1)
φ	Data vector	§5.8

φ_1	Data vector containing $\bar{u}$ and $\bar{y}$	§5.8
φ_2	Data vector containing y_f	§5.8
φ_3	Data vector containing y_1^m	§5.8
φ_v	Data vector containing v	§5.8
ϕ		(6.1)

APPENDIX 1:2

A MIMO Adaptive Control Algorithm Checklist

The full adaptive algorithm for the MIMO servo case is given with references to the main text.

(i) Start with a transfer operator representation in unknown parameters of the control object $G_O(z)$

REF:(4.1), App.4:3

(ii) Find a representation G_O^* and a right m.f.d. in the transformed variable z^*

$$G_O^*(z^*) = B^*(z^*)A^{*-1}(z^*)$$

REF:§2.5, App.4:3

(iii) Decide B_S^* and B_D^*

REF:Lemma 4.1-2, (4.7)-(4.9), (4.17)

(iv) Give specifications in terms of a reference model

$$\begin{cases} A^*_M y^m_1 = B^*_M u_c \\ \\ y^M_1 = B^*_D y^m_1 \end{cases}$$

REF: §4.3

(v) Find T^*_1 such that $T^*_1 A^*_M$ is lower triangular

REF: (4.15)-(4.16)

(vi) Find polynomial degrees of R^*_u, S^*_y, T^*_2 and T^*_3

$$\begin{cases} R^*_y A^* + S^*_y B^* = T^*_1 A^*_M B^*_R \\ \\ T^*_2 T^*_1 A^*_M B^*_L B^*_S = B^*_S T^*_1 A^*_M \\ \\ T^*_2 T^*_1 A^*_M B^*_D = B^*_S T^*_3 \end{cases}$$

REF: (4.17), Lemma 4.2

(vii) Describe the estimation model and check uniqueness

$$T_2^* e_f = B_S^* \left[u + R_0 u + R^* u + S^* y - T_3^* \right]$$

REF:(5.16), (5.11), §5.9

(viii) Find LIP-models for each control input u_i where '‾' denotes filtering by component i of B_S^*

$$\bar{u}_i = \theta_2^T \varphi_2 - \theta_1^T \bar{\varphi}_1 + \theta_3^T \bar{\varphi}_3$$

REF:§5.8

(ix) Identify parameters recursively with e.g. LS-estimation

REF:§7.2

(x) Control with

$$u_i = - \hat{\theta}^T \varphi_1 + \hat{\theta}_3^T \varphi_3$$

REF:§5.8, §5.6

11 S H

* The two last steps are performed in each sampling interval

APPENDIX 4:1

Lemma 4.1

A square (nxn) transfer function G^* of full rank with $\det(G^*(1)) \neq 0$ corresponding to a strictly proper transfer function G may be decomposed into a relatively r.Z^*.c.f (A^*, B^*). The factorization is such that A^* contains all the Z^*-poles of G^*, B^* contains all Z^*-zeros of G^* and such that

$$G^* = B^* A^{*-1} = B_L^* B_S^* B_R^* A^{*-1} \tag{4.7}$$

where

A^* is a square, full rank polynomial matrix

B_L^* and B_R^* are polynomial Z^*-unimodular matrices

B_S^* is a diagonal polynomial matrix (4.8)

satisfying

$$A^*(0) = I$$

$$B_S^*(1) = I \tag{4.8}$$

$B_R^*(0) = \left[b_{ij}\right]_R$ is upper right triangular and invertible

$$(b_{ii})_R = 1 \quad \text{for} \quad 1 \leq i \leq n \tag{4.9}$$

The polynomial matrix B_S^* contains all the zeros of G^* in Z_-^* but has no zeros in Z_+^*. The stability region is assumed to be given by (2.7).

Proof

Step 1

The quadratic transfer function G^* of full rank may be factorized into a relatively r.Z^*.c.f. (A_1^*, B_1^*) where A_1^* and B_1^* are polynomial matrices

$$G^* = B_1^* A_1^{*-1} \tag{A4.1.1}$$

as known from §2.4.

Step 2

Since the transfer function is strictly proper, there are no poles at $z=\infty$ i.e. $z^*=0$. Via full rank- and r.Z^*.c.f.-conditions it follows that A^* does not loose rank at $z^*=0$ and it is possible to form

$$A_2^*(z^*) = A_1^*(z^*)A_1^{*-1}(0)$$

$$B_2^*(z^*) = B_1^*(z^*)A_1^{*-1}(0) \tag{A4.1.2}$$

Then

$$A_2^*(0) = I \tag{A4.1.3}$$

Step 3

The polynomial matrix B_2^* has a unique Z^*-Smith form S^* such that

11

p

V

$$B_2^* = L_1^* \, S^* \, R_1^* \qquad (A4.1.4)$$

where L_1^* and R_1^* are Z^*-unimodular matrices. Then $R_1^*(0)$ is invertible. Define a lower left triangular constant matrix L and an upper right triangular constant matrix R such that

$$R_1^*(0) = L \, R \qquad (A4.1.5)$$

where both L and R are invertible, constant matrices. The decomposition becomes unique by choosing the diagonal elements of R equal to one i.e.

$$r_{ii} = 1 \qquad \text{for} \quad 1 \leq i \leq n \qquad (A4.1.6)$$

A decomposition of the type (A4.1.5) does not always have a straightforward solution since the important l_{ii}-elements may become zero. A reordering of the rows of $R_1^*(0)$ solves the problem. It is however necessary to rearrange the matrices L_1^* and S^* correspondingly. Introduce the permutation matrices P_1 and P_2 and define

$$L_2 R_2 = P_1 R_1^*(0)$$

$$S_2^* = P_2 S^* P_1^{-1}$$

$$L_3^* = L_1^* P_2^{-1} \qquad (A4.1.7)$$

where P_1 is chosen such that $P_1 R_1^*(0)$ may be decomposed on the form (A4.1.5) and P_2 is chosen such that S_2^* becomes diagonal. If $S^*=\mathrm{diag}(s_1^*,\ldots,s_n^*)$ then S_2^* may be written

$$S_2^* = \operatorname{diag}\left[s^*_{k_1}, \ldots, s^*_{k_n}\right] \tag{A4.1.8}$$

for some permutation $\{k_1, \ldots, k_n\}$ of the numbers $\{1, \ldots, n\}$.

Step 4

Define

$$R_3^*(z^*) = R_1^{*-1}(0)R_1^*(z^*) \tag{A4.1.9}$$

The decomposition of B_2^* may now be rewritten

$$B_2^* = L_3^* \, S_2^* \, L_2 R_2 R_3^* \tag{A4.1.10}$$

Step 5

Consider the constant matrix

$$L_2 = \begin{bmatrix} l_{11} & & 0 \\ \vdots & \ddots & \\ l_{n1} & \cdots & l_{nn} \end{bmatrix} \tag{A4.1.11}$$

which multiplied from the left by S_2^* gives

$$M^* \triangleq S_2^* L_2 = \begin{bmatrix} l_{11} s^*_{k_1} & & 0 \\ \vdots & \ddots & \\ l_{n1} s^*_{k_n} & \cdots & l_{nn} s^*_{k_n} \end{bmatrix} \tag{A4.1.12}$$

It is desirable to factor out the invariant polynomials to the right. This is not possible to do directly since it is

11

S.

V

not known whether

$$s^*_{k_i} | s^*_{k_j} \quad \text{for } i<j \qquad (A4.1.13)$$

A construction proof is given in the following steps.

Step 6

Consider the first column of M^*. Factor out the g.c.Z*.d of the column elements and assign

$$p^*_1 = \text{g.c.Z}^*\text{.d}(m^*_{i1}) \quad ; \quad 1 \leq i \leq n \qquad (A4.1.14)$$

The factor p^*_1 must correspond to the least k_i for which $l_{i1} \neq 0$ since

$$s^*_{k_i} | s^*_{k_j} \quad \text{for } k_i<k_j \qquad (A4.1.15)$$

This gives

$$p^*_1 = s^*_{k_i} \qquad (A4.1.16)$$

It is now possible to eliminate the elements in row i of all the columns 2,...,n corresponding to the common factor p^*_1 since row i of M^* has this common factor.

Step 7

This procedure may be repeated for each column. Factor out the g.c.Z*.d. of column j and assign this to p^*_j. Eliminate the elements of M^* of the row where p^*_J was found, from each of the remaining columns $j+1,\ldots,n$ of M^*. Notice that all elements of a row has a common factor originating from S^*_2.

This may be written

$$M^*R_4 = L^*_4 S^*_3 \tag{A4.1.17}$$

where R_4 is an upper right triangular, constant matrix with '1''s on the principal diagonal. The matrix S^*_3 is

$$S^*_3 = \mathrm{diag}\left[p^*_1,\ldots,p^*_n\right] \tag{A4.1.18}$$

Through this procedure of elimination it is obvious that the factors $\{p^*_i\}$ represent some new permutation of

$$s^*_{k_1},\ldots,s^*_{k_n} \tag{A4.1.19}$$

These matrix manipulations give the intermediate result

$$B^*_2 = L^*_3\ S^*_2 L_2\ R^*_2 R^*_3 = L^*_3 L^*_4\ S^*_3\ R^{-1}_4 R_2 R^*_3 \tag{A4.1.20}$$

where the S^*_3 matrix is diagonal and $R^{-1}_4 R_2$ is upper triangular.

Step_8

In order to obtain the requirements stated in the lemma it is necessary to scale the monic polynomials in the diagonal entries of S^*_3. Define

$$D = S_3^*(1)$$

$$L_5^* = L_4^* \; D$$

$$S_4^* = D^{-1} \; S_3^* \tag{A4.1.21}$$

The matrix D is invertible since B^* is of full rank and since S_3^* is a polynomial matrix that is assumed to have no zeros at $z^*=1$ (cf.(2.7)).

Step 7

The matrix decomposition in the statement of the lemma is now obtained by assigning

$$B_L^* = L_3^* \; L_5^*$$

$$B_S^* = S_4^*$$

$$B_R^* = R_4^{-1} R_2 \; R_3^* \tag{A4.1.22}$$

since R_2 and R_4 are upper right triangular matrices and $R_3^*(0) = I$. The diagonal elements of R_2 and R_4^{-1} are all '1'. This guarantees that $B_R^*(0)$ is upper triangular.

APPENDIX 4:2

Lemma 4.2

There are polynomial matrix solutions R_u^*, S_y^*, T_2^*, T_3^* and a diagonal B_D^* satisfying the following equations for given A_M^*, A^*, B_L^*, B_S^*, B_R^*, B^* and T_1^* with the properties given by the expressions of (4.7)-(4.9) and (4.15)-(4.16) under assumptions 3:1-3, 4:1-3 The matrix T_2^* is a polynomial Z^*-unimodular matrix.

$$T_2^*T_1^*A_M^* B_L^*B_S^* = B_S^* T_1^*A_M^*$$

$$R_u^*A^* + S_y^*B^* = T_1^*A_M^*B_R^* ; \qquad R_u^*(0) = B_R^*(0)$$

$$T_2^*T_1^*A_M^*B_D^* = B_S^*T_3^*$$

(4.17a-c)

Proof:

Step 1 Existence of T_2^*

The product of T_1^* and A_M^* is a lower triangular, polynomial, Z^*-unimodular matrix according to assumption 4:3 (c.f. (4.15-16)). The matrix T_2^* is then possible to express as

$$T_2^* = B_S^* T_1^*A_M^*B_S^{*-1} B_L^{*-1} \left[T_1^*A_M^*\right]^{-1} \qquad (A4.2.1)$$

In order to guarantee a polynomial matrix solution it is necessary to investigate the expressions closer.

The matrix

$$B_S^* T_1^* A_M^* B_S^{*-1} \tag{A4.2.2}$$

is a polynomial, Z^*-unimodular, lower triangular matrix since the matrices B_S^* and B_S^{*-1} cancel each other perfectly on the diagonal and since the specification on the off-diagonal entries assure that these remain polynomials. Each element below the diagonal of the above matrix may be written

$$b_i^* p_{ij}^* \frac{1}{b_j^*} = b_i^* p'^*_{ij} \tag{A4.2.3}$$

which is a polynomial.

The resulting T_2^* is a polynomial, Z^*-unimodular matrix since the matrix of (A4.2.2) is a polynomial matrix, since B_L^* is Z^*-unimodular (lemma 4.1), and since $T_1^* A_M^*$ is a polynomial, Z^*-unimodular matrix by assumption.

In order to guarantee a polynomial matrix solution T_2^* it is possible to use the non-uniqueness of the factorization of the polynomial matrix B^* for a given B_S^*.

Consider a tentative Z^*-generalized matrix $T_2'^*$ obtained as shown above. Find the diagonal polynomial matrix U^*, where the elements consist of the least common denominators of the columns of $T_2'^*$. Then $T_2'^* U^*$ is a polynomial matrix. Since $T_2'^*$

is Z^*-unimodular it is clear that U^* is Z^*-unimodular.

Since U^* is Z^*-unimodular and commutes with B_S^*, it is possible to write

$$B^* = B_L^* B_S^* B_R^* = \left[B_L^* U^{*-1}\right] B_S^* \left[U^* B_R^*\right] \tag{A4.2.4}$$

whereby a new factorization - with the desirable property of assuring a polynomial Z^*-unimodular matrix T_2^* - is obtained.

$$T_2^* = T_2'^* U^* \tag{A4.2.5}$$

The matrix $U^* B_R^*$ is then still guaranteed to be a polynomial Z^*-unimodular matrix.

Step 2 Existence of R_u^* and S^*

It is well known that the polynomial matrix equation (4.17b) has polynomial matrix solutions R_u^* and S_y^* (cf. e.g. [Per 1], II:2.5) when there are no common left factors of A_{uu}^* and $B_{\cdot u}^*$. All solutions may be written as

$$\begin{aligned} R_u^* &= R_P^* + K^* R_H^* \\ S^* &= S_P^* + K^* S_H^* \end{aligned} \tag{A4.2.6}$$

where $\left[R_P^* \; S_P^*\right]$ is a particular solution satisfying

$$R_P^* A^* + S_P^* B^* = T_1^* A_M^* B_R^* \quad , \tag{A4.2.7}$$

$\left(* \quad * \right)$

and $\begin{bmatrix} R_H & S_H \end{bmatrix}$ satisfies the homogenous equation

$$R_H^* A^* + S_H^* B^* = 0 \quad , \tag{A4.2.8}$$

and - finally - K^* is any polynomial matrix of appropriate dimensions.

<u>Step 3</u> Existence of T_3^* and B_D^*

The equation

$$T_2^* T_1^* A_M^* B_D^* = B_S^* T_3^* \tag{A4.2.9}$$

has a solution T_3^* for a diagonal B_D^* and given polynomial matrices A_M^*, B_S^*, T_1^* and T_2^*. The solution is

$$T_3^* = B_S^{*-1} T_2^* T_1^* A_M^* B_D^* \tag{A4.2.10}$$

A polynomial matrix solution T_3^* for a diagonal matrix B_D^* with a minimal number of Z^*-zeros is obtained by inspecting the least common denominators of the columns of the matrix

$$B_S^{*-1} T_2^* T_1^* A_M^* \tag{A4.2.11}$$

and assigning these l.c.d. as the corresponding elements in the diagonal entries of the diagonal matrix B_D^*. Then a suitable B_D^* is obtained. A polynomial matrix solution for T_3^* is also immediately obtained from (A4.2.10), when B_D^* has been determined, since T_2^*, T_1^* and A_M^* are polynomial matrices, and since all denominators of (A4.2.11) derive from B_S^{*-1}.

Step 4

The matrix

$$R_u^*(0) = B_R^*(0) \tag{4.17b}$$

should satisfy (4.17b)

$$R_u^*(0)A^*(0) + S_y^*(0)B^*(0) = T_1^*(0)A_M^*(0)B_R^*(0) \tag{A4.2.12}$$

Via (4.7), (4.13) and the properness conditions in assumption 3:1 it follows that

$$R_u^*(0) = B_R^*(0) \tag{A4.2.13}$$

The proof of the lemma is now finished. The existence of the desired matrix solutions has been shown.

APPENDIX 4:3

A4.3.1 Introduction

The control object representation in §4.2 does not contain any disturbances. A more complete problem set-up is therefore given in this appendix.

A4.3.2 The Control Object

In this formulation the control object will be presented by a r.Z^*.c.f. pair (A^*, B^*) and the controller by a l.Z^*.c.f. triple (R^*, S^*, T^*). All indices introduced in the main text regarding the type of factorization will be omitted when not explicitly needed.

A general fractional representation for a linear, multivariable system S_0 is given by a block triangular r.Z^*.c.f.

$$\left[\begin{array}{c|c|c} A^*_{uu}(z^*) & A^*_{uv}(z^*) & A^*_{uw}(z^*) \\ \hline 0 & A^*_{vv}(z^*) & A^*_{vw}(z^*) \\ \hline 0 & 0 & A^*_{ww}(z^*) \end{array}\right] \left[\begin{array}{c} \xi_u(t) \\ \hline \xi_v(t) \\ \hline \xi_w(t) \end{array}\right] = \left[\begin{array}{c} u(t) \\ \hline v(t) \\ \hline w(t) \end{array}\right]$$

$$\left[\begin{array}{c} y_1(t) \\ \hline y_2(t) \end{array}\right] = \left[\begin{array}{c|c|c} B^*_{1u}(z^*) & B^*_{1v}(z^*) & B^*_{1w}(z^*) \\ \hline B^*_{2u}(z^*) & B^*_{2v}(z^*) & B^*_{2w}(z^*) \end{array}\right] \left[\begin{array}{c} \xi_u(t) \\ \hline \xi_v(t) \\ \hline \xi_w(t) \end{array}\right]$$

where (A4.3.1)

- y_1 – Measured outputs to be controlled
- y_2 – Other measured outputs
- u – Control input
- v – Measured disturbance inputs
- w – Non-measurable disturbance inputs

Let the assumptions 3:1-3 formulated in the main text also be valid here. The assumptions 4:1-3 are also included and 4:1 is rephrased below

Assumption 4:1

Let the number of independent control inputs be equal to the number of full rank controlled outputs. □

A particular case, where the number of independent control inputs is equal to the number of full rank controlled outputs, will thus be emphasized below. The rank condition does obviously never allow the number of full rank controlled outputs to surpass the number of linearly independent control inputs.

This specialization only imposes the mild restriction that any remaining control inputs will be formally referred to as a part of the disturbance input vector v for which no controller parametrizations will be derived.

A few technical details regarding stabilizability also

12
D.
H

deserve a comment.

It is assumed that A^*_{uu}, A^*_{vv} and A^*_{ww} are quadratic polynomial matrices of full rank which means no restriction. It is also assumed that A^*_{vv} and A^*_{ww} are Z^*-unimodular matrices. This requirement is needed to guarantee the existence of a feedback stabilizing linear controller which may also be seen from (A4.3.9) below. The system is not stabilizable, if A^*_{vv} or A^*_{ww} are not Z^*-unimodular. Attempts to compensate for a v with unstable poles originating from zeros in Z^*_- of A^*_{vv} would give a design scheme with cancellations of common factors with Z^*-zeros. The case of an A^*_{vv} with zeros in Z^*_- will therefore not be considered further.

The system left structure matrix of (A4.3.1) with $\dot{y}_1$ as controlled outputs is precisely the left structure matrix of B^*_{1u}.

A decomposition of the transfer function G^*_{11}, according to lemma 4.1, is

$$G^*_{11} = B^*_{1u} A^{*-1}_{uu} \tag{A4.3.2}$$

where A^*_{uu} and B^*_{1u} are polynomial matrices in z^*

$$A^*_{uu}(0) = I$$

and

$$B^*_{1u} = B^*_L \, B^*_S \, B^*_R \tag{A4.3.3}$$

with the properties given in the lemma and remark 4.2.

A trade-off between the restrictions imposed above and the specifications of the control objective gives rise to the general design scheme, which is to be stated below.

A4.3.3 The Controller

Let the controller be given by

$$R_u^*(z^*)u(t) = -S_y^*(z^*)\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + T^*(z^*)\begin{bmatrix} u_c(t) \\ v(t) \end{bmatrix} \tag{A4.3.4}$$

The matrices R_u^* and S_y^* will always be chosen as polynomial matrices with respect to z^* but may in general be chosen as any kind of Z^*-generalized polynomial matrices. The matrix T^* is required to be a Z^*-generalized polynomial matrix.

Denote

$$u_1(t) = \begin{bmatrix} u_c(t) \\ v(t) \end{bmatrix} \tag{A4.3.5}$$

Let the $S_y^*(z^*)$-matrix be partitioned into

$$S_y^*(z^*) = \begin{bmatrix} S_1^*(z^*) & S_2^*(z^*) \end{bmatrix} \tag{A4.3.6}$$

12 where each submatrix corresponds to the observations $[y_1^T \; y_2^T]$.

S. Let the $T^*(z^*)$-matrix be partitioned into

H

$$T^*(z^*) = \left[\; T^*_{uc}(z^*) \;\; T^*_{ff}(z^*) \; \right] \tag{A4.3.7}$$

where T^*_{uc} and T^*_{ff} represent the feedforward compensations from the command signal u_c and the measurable disturbance inputs v respectively.

A4.3.4 The Closed Loop System

If the expression (A4.3.4) is substituted into (A4.3.1), the following is obtained

$$R^*_u(z^*)\left[A^*_{uu}(z^*) \;\; A^*_{uv}(z^*) \;\; A^*_{uw}(z^*)\right]\xi(t) + S^*_y(z^*)B^*(z^*)\xi(t) =$$

$$= T^*(z^*)u_1(t) = T^*_{uc}(z^*)u_c(t) + T^*_{ff}(z^*)v(t) \tag{A4.3.8}$$

The system equations for the closed loop system become

$$\left[\begin{array}{c|c|c} R^*_u A^*_{uu} + S^*_y B^*_{.u} & R^*_u A^*_{uv} + S^*_y B^*_{.v} & R^*_u A^*_{uw} + S^*_y B^*_{.w} \\ \hline 0 & A^*_{vv} & A^*_{vw} \\ \hline 0 & 0 & A^*_{ww} \end{array}\right] \left[\begin{array}{c} \xi_u \\ \hline \xi_v \\ \hline \xi_w \end{array}\right] =$$

$$= \left[\begin{array}{c|c|c} T^*_{uc} & T^*_{ff} & 0 \\ \hline 0 & I & 0 \\ \hline 0 & 0 & I \end{array}\right] \left[\begin{array}{c} u_c \\ \hline v \\ \hline w \end{array}\right]$$

$$\left[\begin{array}{c} y_1 \\ \hline y_2 \end{array}\right] = \left[\begin{array}{c|c|c} B^*_{1u} & B^*_{1v} & B^*_{1w} \\ \hline B^*_{2u} & B^*_{2v} & B^*_{2w} \end{array}\right] \left[\begin{array}{c} \xi_u \\ \hline \xi_v \\ \hline \xi_w \end{array}\right] \qquad \text{(A4.3.9)}$$

The transfer functions of the closed loop system are then obtained as

$$G^*_{u_c y_0} = B^*_{.u}\left[R^*_u A^*_{uu} + S^*_y B^*_{.u}\right]^{-1} T^*_{uc}$$

$$G^*_{vy_0} = B^*_{.u}\left[R^*_u A^*_{uu} + S^*_y B^*_{.u}\right]^{-1}\left(T^*_{ff} + \left[R^*_u A^*_{uv} + S^*_y B^*_{y.v}\right]A^{*-1}_{vv}\right) + B^*_{.v} A^{*-1}_{vv}$$

$$G^*_{wy_0} = B^*_{.u}\left[R^*_u A^*_{uu} + S^*_y B^*_{y.u}\right]^{-1}\left(\left[R^*_u A^*_{uv} + S^*_y B^*_{y.v}\right]A^{*-1}_{vv} A^*_{vw} - \left[R^*_u A^*_{uw} + S^*_y B^*_{y.w}\right]\right)A^{*-1}_{ww}$$

$$- B^*_{.v} A^{*-1}_{vv} A^*_{vw} A^{*-1}_{ww} + B^*_{.w} A^{*-1}_{ww} \qquad \text{(A4.3.10)}$$

where $B^*_{.u}$ etc. denotes

$$B^*_{.u} = \begin{bmatrix} B^*_{1u} \\ B^*_{2u} \end{bmatrix} \qquad \text{(A4.3.11)}$$

Since it has already been assumed that A^*_{vv} and A^*_{ww} are polynomial and Z^*-unimodular matrices, the closed loop system will be stable iff the polynomial matrix

12

P.

V

$$\left[R_u^*(z^*)A_{uu}^*(z^*) + S_y^*(z^*)B_{\cdot u}^*(z^*)\right] \tag{A4.3.12}$$

is Z^*-unimodular. The full expression

$$\det\left[R_u^*A_{uu}^* + S_y^*B_{.u}^*\right]\det\left[A_{vv}^*\right]\det\left[A_{ww}^*\right] \tag{A4.3.13}$$

gives some of the closed loop poles of the system. In addition there are other Z^*-stable poles which are not explicitly taken account of in B_L^* and T_{uc}^*. It is however possible to choose which poles to have by choosing A_M^*

A continuation is found in the main text in §§5.3-5.7.

APPENDIX 5:1

<u>Theorem 5.3</u>

There exists a polynomial matrix solution

$$\begin{bmatrix} F^*_v & T^*_v \end{bmatrix} \tag{5.33}$$

satisfying the relation – where B^*_S is diagonal

$$C^*_v = F^*_v - B^*_S T^*_v \tag{5.35}$$

and such that the highest power of each row of F^*_v is lower than the greatest power of the corresponding entry in the diagonal matrix B^*_S.

<u>Proof</u>

Consider eq. (A4.3.1) with a given choice of B^*_{1v} and A^*_{vv}. Any pair of matrices

$$\begin{cases} B'^*_{1v} = B^*_{1v} + B^*_{1u} K^* \\ A'^*_{uv} = A^*_{uv} + A^*_{uu} K^* \end{cases} \tag{A5.1.1}$$

where K^* is any stable Z^*-generalized polynomial matrix, may substitute A^*_{uv} and B^*_{1v}. The matrices A^*_{uu}, A^*_{vv} and $B^*_{\cdot u}$ are not changed by any such substitution. This means that column operations are performed on A^* and B^* of (A4.3.1) such that A^*_{uu}, A^*_{vv} and $B^*_{\cdot u}$ are left unaltered.

12

S

V

This fact may be exploited in the expressions for v of §5.3. All the v-dependent terms of (5.29) and (5.32) may be expressed as

$$H^*_v v \triangleq \left(F^*_v - B^*_S T^*_v\right) v \tag{A5.1.2}$$

The polynomial matrix H^*_v may be written (cf.§5.3)

$$H^*_v = U^*\left[T^*_2 T^*_1 A^*_M B^*_{1v} - B^*_S\left(R^*_u A^*_{uu} + S^*_y B^*_{\cdot v}\right)\right] A^{*-1}_{vv} \tag{A5.1.3}$$

Recall the relations (4.17a-b) to obtain

$$M^* \triangleq T^*_2 T^*_1 A^*_M B^*_{1u} = B^*_S T^*_1 A^*_M B^*_R = B^*_S\left[R^*_u A^*_{uu} + S^*_y B^*_{\cdot u}\right] \tag{A5.1.4}$$

Use this relation on

$$H^*_v = H^*_v + U^* M^* K^* A^{*-1}_{vv} - U^* M^* K^* A^{*-1}_{vv} \tag{A5.1.5}$$

where K^* is a Z^*-generalized polynomial matrix to be specified. Through substitution of M^* given by (A5.1.4) into (A5.1.5) it is confirmed that

$$H^*_v = U^*\left[T^*_2 T^*_1 A^*_M B'^*_{1v} - B^*_S\left(R^*_u A'^*_{uv} + S^*_y B'^*_{\cdot v}\right)\right] A^{*-1}_{vv} \tag{A5.1.6}$$

is independent of the choice of K^*. Consider now all the Z^*-generalized matrices F^*_v that can be achieved for given matrices A^*_{uu}, A^*_{vv} and $B^*_{\cdot u}$. These may be written

$$F^*_v = U^* T^*_2 T^*_1 A^*_M B'^*_{1v} A^{*-1}_{vv} = U^* T^*_2 T^*_1 A^*_M\left(B^*_{1v} + B^*_{1u} K^*\right) A^{*-1}_{vv} =$$

$$= U^* T_2^* T_1^* A_M^* B_{1v}^* A_{vv}^{*-1} + B_S^* U^* T_1^* A_M^* B_R^* K^* A_{vv}^{*-1} \qquad (A5.1.7)$$

The choice of U^* in §5.3 guarantees

$$N^* \stackrel{\Delta}{=} U^* T_2^* T_1^* A_M^* B_{1v}^* A_{vv}^{*-1} \qquad (A5.1.8)$$

to be a polynomial matrix. Now find polynomial matrices X^* and Y^* such that

$$N^* = X^* + B_S^* Y^* \qquad (A5.1.9)$$

where the polynomial elements of X^* should have the lowest possible degree with respect to z^*. This is achieved by an elementwise solution of the polynomial equations

$$n_{ij}^* = x_{ij}^* + b_i^* y_{ij}^* \qquad (A5.1.10)$$

The division algorithm for polynomials assures existence of a solution where each x_{ij}^* has lower degree than b_i^*.
Now choose

$$K^* = - B_R^{*-1} A_M^{*-1} T_1^{*-1} U^{*-1} Y^* A_{vv}^* \qquad (A5.1.11)$$

The final choice is

$$F_v^* = X^* \qquad (A5.1.12)$$

and

$$T_v^* = U^* \left(R_u^* A_{uv}^{*\prime} + S_y^* B_{\cdot v}^{*\prime} \right) A_{vv}^{*-1} =$$

$$= U^* \left(R_u^* A_{uv}^* + S_y^* B_{\cdot v}^* \right) A_{vv}^{*-1} + U^* \left(R_u^* A_{uu}^* + S_y^* B_{\cdot u}^* \right) K^* A_{vv}^{*-1} =$$

$$= U^* \left(R_u^* A_{uv}^* + S_y^* B_{\cdot v} \right) A_{vv}^{*-1} - Y^* \qquad \text{(A5.1.13)}$$

The choice of U^* in §5.3 guarantees the first term to be a polynomial matrix. The matrix Y^* is a polynomial matrix according to (A5.1.9) above.

The proof is now finished since polynomial matrices F_v^* and T_v^* with the prescribed properties and compatible with (A4.3.1) and the control law (A4.3.4) have been shown to exist.

APPENDIX 6:1

Theorem 6.1

Assume that T_2^* is known to such an extent that the estimate τ_2^* may be formulated as

$$\tau_2^* = 2\left[I + H^*\right]^{-1} T_2^* \tag{6.37}$$

for some Z^*-generalized polynomial matrices H^* and $(I+H^*)^{-1}$ where H^* is also positive real. Assume also that the MIMO CE control law is used. The knowledge of B_S^* and τ_2^* then represents sufficient a priori information for formulation of a convergent parameter adjustment law.

□

Let for formal convenience the data vector $\bar{\Phi}$ be defined as the vector containing the components of all $\bar{\varphi}_{1i}$. Then it holds that each $\bar{\varphi}_{1i}$ may be written as

$$\bar{\varphi}_{1i} = N_i \bar{\Phi} \tag{A6.1.1}$$

where the matrix N_i picks out the pertinent components of $\bar{\varphi}_{1i}$ from $\bar{\Phi}$. The parameter estimates $\hat{\theta}_{1i}$ may likewise be expanded to the size of $\bar{\Phi}$ by the transformation

$$\hat{\theta}'_{1i} = N_i^T \hat{\theta}_{1i} \quad \text{and} \quad N_i N_i^T = I \tag{A6.1.2}$$

Let $\hat{\Xi}$ be defined as

$$\hat{\Xi} = \left[\hat{\theta}'_{11}, \ldots, \hat{\theta}'_{1n}\right] \qquad (A6.1.3)$$

and similarly for $\tilde{\Xi}$ etc. In the case where B^*_S contain only time delays in the diagonal (i.e. when the control object does not have any finite non-minimum phase zeros), it holds that

$$\bar{\tilde{\Xi}}(t) = \left[\tilde{\theta}'(t-k_1), \ldots, \tilde{\theta}'(t-k_n)\right] \qquad (A6.1.4)$$

This denotation gives

$$\bar{\tilde{\Xi}}^T(t)\bar{\Phi}(t) = -\begin{bmatrix} v_1(t-k_1) \\ \vdots \\ v_n(t-k_n) \end{bmatrix} = -\begin{bmatrix} \bar{v}_1(t) \\ \vdots \\ \bar{v}_n(t) \end{bmatrix} = -\bar{v}(t) \qquad (A6.1.5)$$

The following updating algorithm now gives

$$\hat{\Xi}(t) = \bar{\hat{\Xi}}(t) + \gamma(\|\bar{\Phi}\|)\ \bar{\Phi}\ e'^T_f(t) \qquad (A6.1.6)$$

where

$$e'_f(t) = \tau^*_2(q^{-1})e_f(t) = \tau^*_2(q^{-1})T^{*-1}_2(q^{-1})\bar{v}(t) \qquad (A6.1.7)$$

for some matrix $\tau^*_2(q^{-1})$ to be determined and

$$\gamma(\|\bar{\Phi}\|) = \frac{1}{\|\bar{\Phi}(t)\|^2} \qquad (A6.1.8)$$

for $\|\bar{\Phi}\| \neq 0$ and $\gamma=0$ for $\|\bar{\Phi}\|=0$. Consider now the function

$$V(\tilde{\theta}_1(t)) = \sum_{i=0}^{n} \sum_{j=0}^{k_i-1} \|\tilde{\theta}_{1i}(t-j)\|^2 \qquad (A6.1.10)$$

Then it holds that

$$V(\tilde{\theta}_1(t)) - V(\tilde{\theta}_1(t-1)) =$$

$$= \mathrm{tr}\, \tilde{\Xi}^T(t)\tilde{\Xi}(t) - \mathrm{tr}\, \bar{\tilde{\Xi}}^T(t)\bar{\tilde{\Xi}}(t) =$$

$$= \gamma\, \mathrm{tr}\left(- \bar{\tilde{\Xi}}^T \overline{\Phi\nu}^T T_2^{*-T} \tau_2^{*T} - \tau_2^* T_2^{*-1} \bar{\nu}\Phi^T \bar{\tilde{\Xi}} + \tau_2^* T_2^{*-1} \overline{\nu\nu}^T T_2^{*-T} \tau_2^{*T}\right) =$$

$$= \gamma\, \bar{\nu}^T T_2^{*-T} \tau_2^{*T} \left(-2T_2^* \tau^{*-1} + I\right) \tau_2^* T_2^{*-1} \bar{\nu} \tag{A6.1.11}$$

which is negative for all $\bar{\nu}$ such that $\|\bar{\nu}\| \neq 0$ if

$$-2T_2^* \tau_2^* + I = -H^* \tag{A6.1.12}$$

for any stable, positive real transfer operator matrix H^* and a stable τ_2 satisfying

$$\tau_2^* = 2\left[I + H^*\right]^{-1} T_2^* \tag{6.37}$$

This is shown by a standard type 'positive real' argument - used extensively by many authors in the field (cf.[Lan]). A SISO version is found in e.g.([Ega], 2.4).

When (6.37) is satisfied, it holds that

$$V(\tilde{\theta}_1(t)) - V(\tilde{\theta}_1(t-1)) = -\gamma\, \bar{\nu}^T(t) P \bar{\nu}(t) \tag{A6.1.13}$$

for a stable transfer operator P^*

$$P^* = 4\left[I + H^*\right]^{-T} H^* \left[I + H^*\right]^{-1} \qquad \text{(A6.1.14)}$$

The RHS of (A6.1.13) is then non-positive definite which proves the claim.

The condition (6.37) reduces to (6.16) in the SISO-case when H^* specializes to a positive constant. The multivariable counterpart of (6.16) is the special case of a positive definite matrix $Q = H^*$. This condition is formulated as

$$\tau_2^* = 2\left[I + Q\right]^{-1} T_2^* \qquad (6.37)$$

for any pos.def. matrix Q. This easily transfers to

$$0 < \frac{b_0}{\beta_0} < 2 \qquad (6.16)$$

in the SISO-case where $T_2^*=1/b_0$.

APPENDIX 8:1

Theorem 8.1

There is a bounded positive constant K such that the function

$$V(X(t)) = v_\theta(z(t)) + Kv_x(x(t)) \qquad (8.35)$$

is a Lyapunov function for an adaptive system under assumptions 8:1-5. The difference

$$V(X(t)) - V(X(t-1)) \qquad (8.36)$$

is negative semidefinite for $\|X\| \neq 0$ and assures uniform global stability in the sense of Lyapunov for the system. Furthermore, (8.36) is strictly negative for $\|x\| \neq 0$.

Proof:

The proof is based on elementary Lyapunov function theory which is described in e.g.([Hah], §25), ([Vid], 5.2) or for discrete time systems ([LaS], ch.1).

The system operators included in (8.33) form continuous functions w.r.t X. The function V(X(t)) is radially unbounded and continuous w.r.t. $\|X\|$. Both v_x and v_θ are positive definite functions of x and z respectively, and V forms thus a feasible Lyapunov function candidate.

The output error is used for parameter estimation according to assumption 8:4 and the parameter adjustment law is the gradient algorithm

$$\hat{\theta}(t) = \hat{\theta}(t-k) + \gamma(\|\varphi_1(t-k)\|)\varphi_1(t-k)y_f(t)$$

with γ chosen according to (8.27) i.e.

$$\gamma(\|\varphi_1(t-k)\|) = \begin{cases} \dfrac{1}{\beta_O\|\varphi_1(t-k)\|^2} & ; \quad \|\varphi_1(t-k)\| \geq a^2 \\[2ex] \dfrac{1}{\beta_O a^2} & ; \quad \|\varphi_1(t-k)\| < a^2 \end{cases} \tag{A8.1.1}$$

for some chosen constant a. It is seen that no updating occurs for $\|\varphi_1(t-k)\|=0$. The filtered output y_f is obtained from (8.4), (8.5) and (8.7) as

$$y_f(t) \overset{\Delta}{=} T_1^*(q^{-1})A_M^*(q^{-1})y_1(t) =$$

$$= b_O q^{-k}\ T_1^*(q^{-1})A_M^*(q^{-1})B_R^*(q^{-1})\xi(t) =$$

$$= b_O q^{-k}\left[R_u^*(q^{-1})A^*(q^{-1}) + S_y^*(q^{-1})B^*(q^{-1})\right]\xi(t) =$$

$$= b_O q^{-k}\left[u(t) + \theta_1^T\varphi_1(t)\right] = -\ b_O q^{-k}\left[\tilde{\theta}_1(t)\varphi_1(t)\right] \tag{A8.1.2}$$

Notice that the dependence of $\xi(t)$ w.r.t. initial values etc. is fully covered by (A8.1.2) which incorporates control object (A^*, B^*) and regulator dynamics (R_u^*, S_y^*) in the φ_1-vector and the time delay k. It is also easy see that the formal cancellation of B_R^* between (8.9) and (8.12) does not represent any ignored dynamics. The further calculations will now be made for the two cases of γ in (A8.1.1). Define

$$\Phi_L = \left\{\varphi_1 : \|\varphi_1\| \geq a^2\right\} \qquad \text{and} \qquad \Phi_S = \left\{\varphi_1 : \|\varphi_1\| < a^2\right\}$$

Straightforward calculations now give for $\varphi_1(t) \in \Phi_L$

$$v_\theta(z(t+1)) - v_\theta(z(t)) = \|\tilde{\theta}_1(t+k)\|^2 - \|\tilde{\theta}_1(t)\|^2 =$$

$$= -\frac{1}{\beta_0^2}\left(2\frac{\beta_0}{b_0} - 1\right)\frac{y_f^2(t+k)}{\|\varphi_1(t)\|^2} = -\frac{b_0^2}{\beta_2^2}\left(2\frac{\beta_0}{b_0} - 1\right)\frac{\nu^2(X(t))}{\|\varphi_1(t)\|^2}$$

$$= -\frac{b_0^2}{\beta_0^2}\left(2\frac{\beta_0}{b_0} - 1\right)\|\tilde{\theta}_1(t)\|^2 \cos^2\alpha(t)$$

where

$$\cos\alpha(t) = \cos\left[\tilde{\theta}_1(t), \varphi_1(t)\right] \tag{A8.1.4}$$

The same calculations when $\varphi_1(t)$ is small i.e. $\varphi_1(t) \in \Phi_S$ give the result

$$v_\theta(z(t+1)) - v_\theta(z(t)) = \|\tilde{\theta}_1(t+k)\| - \|\tilde{\theta}_1(t)\| \leq$$

$$\leq -\frac{b_0^2}{\beta_0^2}\left(2\frac{\beta_0}{b_0} - 1\right)\frac{\nu^2(X(t))}{a^2} \tag{A8.1.5}$$

with the meaning of ν given by (8.33). Notice that no updating is performed when $\|\varphi_1\|=0$ and v_θ is then left unaltered. The function v_θ is thus decreasing with time when an error is indicated and if assumption 8:2 holds.

A bound for the growth of $v_x(x(t))$ is obtained from

$$v_x(x(t+1)) - v_x(x(t)) =$$

$$= \ln\left[1 + x^T(t+1)Q^TQx(t+1)\right] - \ln\left[1 + x^T(t)Q^TQx(t)\right] =$$

$$= \ln\left[\frac{1 + x^T(t+1)Q^TQx(t+1)}{1 + x^T(t)Q^TQx(t)}\right] <$$

$$< \ln\left[\frac{x^T(t+1)Q^TQx(t+1)}{x^T(t)Q^TQx(t)}\right] \qquad \text{(A8.1.6)}$$

when $x^T(t+1)Q^TQx(t+1) > x^T(t)Q^TQx(t)$. Otherwise it holds that

$$v_x(x(t+1)) - v_x(x(t)) \leq 0 \qquad \text{(A8.1.7)}$$

In order to develop (A8.1.6) further it is noticed that

$$x^T(t+1)Q^TQx(t+1) =$$

$$= \sum_{i=1}^{n} x_i^2(t+1)\rho^{2(i-1)} = x_1^2(t+1) + \sum_{i=2}^{n} x_i^2(t+1)\rho^{2(i-1)} \leq$$

$$\le x_1^2(t+1) + \sum_{i=1}^{n} x_i^2(t)\varrho^{2i} = x_1^2(t+1) + \varrho^2 x^T(t)Q^TQx(t) \tag{A8.1.8}$$

where the shift properties of the F-matrix of the canonical state-space representation (8.14)' have been exploited.

However

$$x_1^2(t+1) = \left[-p^T x(t) - \tilde{\theta}_1^T(t)\varphi_1(t)\right]^2 \le$$

$$\le 2|p^T x(t)|^2 + 2|\tilde{\theta}_1^T(t)\varphi_1(t)|^2 \le$$

$$\le 2\|Q^{-1}p\|^2\|Qx(t)\|^2 + 2\|\tilde{\theta}_1(t)\|^2\|\varphi_1(t)\|^2\cos^2\alpha(t) \tag{A8.1.9}$$

According to assumption 8:1

$$\|\varphi_1(t)\|^2 = \|Mx(t)\|^2 = x^T(t)Q^TQ^{-T}M^TMQ^{-1}Qx(t) \le$$

$$\le \lambda_{max}\|Qx(t)\|^2 \tag{A8.1.10}$$

where λ_{max} is the largest eigenvalue of the positive semi-definite matrix

$$Q^{-T}M^TMQ^{-1} \tag{A8.1.11}$$

Summing this up gives the upper bound

$$x^T(t+1)Q^TQx(t+1) \le$$

$$\le \left[\varrho^2+2\|Q^{-1}p\|^2\right]\|Qx(t)\|^2 +$$

$$+ 2\lambda_{max}\|\tilde{\theta}_1(t)\|^2\cos^2\alpha(t)\ \|Qx(t)\|^2 \tag{A8.1.12}$$

and

$$\ln\left[\frac{x^T(t+1)Q^TQx(t+1)}{x^T(t)Q^TQx(t)}\right] \le$$

$$\le \ln\left[\left[\varrho^2+ 2\|Q^{-1}p\|^2\right] + 2\lambda_{max}\|\tilde{\theta}_1(t)\|^2\cos^2\alpha(t)\right] \tag{A8.1.13}$$

If assumption 8:5 is fulfilled then

$$\varrho^2 + 2\|Q^{-1}p\|^2 < 1 \tag{A8.1.14}$$

and (A8.1.13) is bounded by

$$\ln\left[1 + 2\lambda_{max}\|\tilde{\theta}_1(t)\|^2\cos^2\alpha(t)\right] \le$$

$$\le 2\lambda_{max}\|\tilde{\theta}_1(t)\|^2\cos^2\alpha(t) \tag{A8.1.15}$$

For a positive constant κ_L (cf.Ass.8:2) such that

$$0 < \kappa_L \le \frac{b_0^2}{2\beta_0}\left[2\frac{\beta_0}{b_0} - 1\right]\frac{1}{2\lambda_{max}} \stackrel{\Delta}{=} K_L \tag{A8.1.16}$$

it is immediately clear that the function

$$V(X(t)) = v_\theta(z(t)) + \kappa_L v_x(x(t)) \tag{A8.1.17}$$

decreases everywhere in Φ_L except possibly where $\tilde{\theta}_1 \perp \varphi_1$. In that case when

$$\tilde{\theta}_1^T(t)\varphi_1(t) = 0 \tag{A8.1.18}$$

it holds that

$$V_\theta(z(t+1)) - V_\theta(z(t)) = 0 \tag{A8.1.19}$$

and

$$V_x(x(t+1)) - V_x(x(t)) =$$

$$= \ln\left[\frac{1 + x^T(t+1)Q^TQx(t+1)}{1 + x^T(t)Q^TQx(t)}\right] \le$$

$$\le \ln\left[\frac{1 + \left[\varrho^2 + 2\|Q^{-1}p\|^2\right]\|Qx(t)\|^2}{1 + \|Qx(t)\|^2}\right] \le$$

$$\le -\left[1 - \left[\varrho^2 + 2\|Q^{-1}p\|^2\right]\right]\frac{\|Qx(t)\|^2}{1 + \|Qx(t)\|^2} < 0 \tag{A8.1.20}$$

except at the stationary point $\|x\|=0$. This finishes the case of a large $\varphi_1(t)$ i.e. when $\varphi_1(t)\in\Phi_L$. In the case of small $\varphi_1(t)\le s$ i.e. $\varphi_1(t)\in\Phi_S$, it holds that (A8.1.9) may be developed as

$$x_1^2(t+1) \le 2\|Q^{-1}p\|^2\|Qx(t)\|^2 + 2v^2(X(t)) \tag{A8.1.21}$$

and the counterpart of (A8.1.12) gives

13
p.
V

$$\|Qx(t+1)\|^2 \le \left[\varrho^2 + 2\|Q^{-1}p\|^2\right]\|Qx(t)\|^2 + 2v^2(X(t)) \tag{A8.1.22}$$

The statement

$$v_x(x(t+1)) - v_x(x(t)) = \tag{A8.1.23}$$

$$= \ln\left(\frac{1 + \left[\varrho^2 + 2\|Q^{-1}p\|^2\right]\|Qx(t)\|^2 + 2v^2(X(t))}{1 + \|Qx(t)\|^2}\right) \le 2v^2(X(t))$$

is furthermore valid under assumptions 8:5. It is therefore possible to suggest the Lyapunov function candidate

$$V(X(t)) = v_\theta(z(t)) + \kappa_S v_x(x(t)) \tag{A8.1.24}$$

with

$$0 < \kappa_S \le \frac{b_0^2}{\beta_0^2}\left(2\frac{\beta_0}{b_0} - 1\right)\frac{1}{2a^2} \overset{\Delta}{=} K_S \tag{A8.1.25}$$

for the case when $\varphi_1(t) \in \Phi_S$. The growth of v_x in (A8.1.23) is then compensated by the decrescent v_θ of (A8.1.5). The case when $v=0$ is also covered by the choice (A8.1.24). The Lyapunov function is furthermore shown to be decrescent also when $\|x\| \neq 0$ and $v=0$ by an argument identical with (A8.1.20). A global Lyapunov function is then obtained as

$$V(X(t)) = v_\theta(z(t)) + Kv_x(x(t)) \tag{8.35}$$

with

$$0 < K \le \min\left[K_L\,,\ K_S\right] \tag{A8.1.26}$$

where K_L is the constant of (A1.1.16). Hereby it is established that V is non-increasing globally and the system in question is globally stable in the sense of Lyapunov. The stability is also uniform since no dependence of the initial time t_0 occurs. Furthermore, V decreases for all x except for x=0.

The initial value of V is defined by (8.35)

$$V(X(t_0)) = v_\theta(z(t_0)) + Kv_x(x(t_0)) \tag{A8.1.27}$$

where the parameter errors of the first k control inputs under assumption 8:3-4 define the initial value of

$$v_\theta(z(t_0)) = \sum_{i=0}^{k-1} \|\tilde{\theta}_i(t_0+i)\|^2 \tag{A8.1.28}$$

The function v_x at time t_0 represents the initial state of the control object and the regulator and may be expressed in terms of the partial state ξ (cf.(8.14)) as

$$v_x(x(t_0)) = \ln\left[1 + \sum_{i=0}^{n-1} \varrho^2 \xi^2(t_0-i-1)\right] \tag{A8.1.29}$$

□

13

S

V

APPENDIX 8:2

Theorem 8.2

An adaptive system under assumptions 8:1-6 with an associated Lyapunov function V given by (8.35) is characterized by exponential convergence w.r.t. $\|Qx(\tau)\|$ for all τ in a time interval $[\tau_0, \tau_1]$ with $\|\varphi_1(\tau)\|^2 \geq a^2$ so that

$$\sup_{V(X(\tau_0))\leq C_0} \|Qx(\tau)\|^2 \leq C(\tau_0)\exp\left[-\delta(\tau-\tau_0)\right] \;;\; C(\tau_0) < \exp\left[C_0/K\right] \tag{8.37}$$

Proof

Assume that the conditions of the theorem are satisfied and introduce the abbreviated notations

$$h^2(t) = \|\tilde{\theta}_1(t)\|^2\cos^2\alpha(t) \tag{A8.2.1}$$

where α is given by (A8.1.4). Recall also the notation for the upper bound K_L of K in (A8.1.16)

$$K_L = \frac{1}{2\lambda_{max}}\,\frac{b_0^2}{\beta_0^2}\left(2\frac{\beta_0}{b_0} - 1\right) \tag{A8.1.16'}$$

A direct derivation from (A8.1.12) of the growth rate of $\|Qx\|$ for any time $t\geq s$ shows that

$$\|Qx(t)\|^2 = \exp\left(\sum_{i=1}^{t-s} 2\ \ln\frac{\|Qx(s+i)\|}{\|Qx(s+i-1)\|}\right)\|Qx(s)\|^2 \leq$$

$$\leq \exp\left[\sum_{i=1}^{t-s} \ln\left[1 - \delta + 2\lambda_{max} h^2(s+i-1)\right]\right] \|Qx(s)\|^2 \leq$$

$$\leq \exp\left[\sum_{i=1}^{t-s} \left[- \delta + 2\lambda_{max} h^2(s+i-1)\right]\right] \|Qx(s)\|^2 \qquad (A8.2.2)$$

The following bound holds when $\|\varphi_1\| \geq |a|$ for all s,...,t

$$\sup_{n\geq 0} \exp\left[\sum_{i=0}^{n-1} \left[- \delta + 2\lambda_{max} h^2(s+i)\right]\right] \|Qx(s)\|^2 \leq$$

$$\leq \sup_{n\geq 0} \exp\left[- n\delta + v_\theta(z(s))/K_L\right] \|Qx(s)\|^2 \leq$$

$$\leq \exp\left[v_\theta(z(s))/K_L\right] \|Qx(s)\|^2 \overset{\Delta}{=} C(s) \qquad (A8.2.3)$$

The results of (A8.1.4), (A8.1.12) and (A8.2.2) have been used to obtain the result. Assume now that for some time τ_0

$$V(X(\tau_0)) = v_\theta(z(\tau_0)) + K \ln\left[1 + \|Qx(\tau_0)\|^2\right] \leq C_0 \qquad (A8.2.4)$$

Then

$$\|Qx(\tau_0)\|^2 \leq \exp\left[C_0/K\right] \exp\left[- v_\theta(z(\tau_0))/K\right] - 1 \qquad (A8.2.5)$$

An upper bound on $C(\tau_0)$ may be obtained from the Lyapunov function.

$$C(\tau_0) = \exp\left[v_\theta(z(\tau_0))/K_L\right] \|Qx(\tau_0)\|^2 < \exp\left[C_0/K\right] \qquad (A8.2.6)$$

An investigation of C for any τ such that $\|\varphi_1(\tau)\|^2 \geq a^2$ shows that

$$C(\tau+1) = \exp\left[v_\theta(z(\tau+1))/K_L\right]\|Qx(\tau+1)\|^2 =$$

$$= \exp\left[-2\lambda_{max}h^2(\tau)\right]\exp\left[v_\theta(z(\tau))/K_L\right]\|Qx(t+1)\|^2 \leq$$

$$\leq \exp\left[-2\lambda_{max}h^2(\tau)\right]\left[1-\delta+2\lambda_{max}h^2(\tau)\right]C(\tau) \qquad (A8.2.7)$$

Define the shorter notation

$$\sigma(t) = 1-\delta+2\lambda_{max}h^2(t) \quad ; \quad \sigma \geq 0 \qquad (A8.2.8)$$

The upper bound C may then be written

$$C(\tau+1) \leq f(\sigma(\tau))C(\tau) \qquad (A8.2.9)$$

with

$$f(\sigma) = \sigma\exp(-\sigma+1-\delta) \qquad (A8.2.10)$$

where f attains its maximum for $\sigma=1$ i.e.

$$\max_\sigma f(\sigma) = f(1) = \exp(-\delta) \qquad (A8.2.11)$$

Then it follows that (A8.2.9) gives a bound

$$C(\tau+1) \leq \exp(-\delta)\, C(\tau) \tag{A8.2.12}$$

and by recursion it follows for all τ in a a time interval $[\tau_0, \tau_1]$ with $\|\varphi_1(\tau)\|^2 \geq a^2$ that

$$C(\tau) \leq C(\tau_0) \exp\left[- \delta(\tau-\tau_0)\right] \tag{A8.2.13}$$

The theorem now follows directly from (A8.2.6) and (A8.2.13).