Dynamic Models for Heated Air-Temperature to Roomair-Temperature

Jensen, Lars

1974

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Jensen, L. (1974). *Dynamic Models for Heated Air-Temperature to Roomair-Temperature*. (Research Reports TFRT-3076). Department of Automatic Control, Lund Institute of Technology (LTH).

Total number of authors:
1

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

THE LUND INSTITUTE OF TECHNOLOGY

DEPARTMENT OF
BUILDING SIENCE

DIVISION OF
AUTOMATIC CONTROL

REPORT 1974:3

Dynamic modells for heated air-temperature to roomair-temperature

L H Jensen

DYNAMIC MODELS FOR HEATED AIR-TEMPERATURE TO ROOMAIR-TEMPERATURE

L.H. Jensen

This work has been supported by Grant D698 from the Swedish Council for Building Research to the Department of Building Science and the Division of Automatic Control, Lund Institute of Technology, Lund, Sweden.

Abstract

Different methods to derive dynamic models for the input output system heated airtemperature to roomair-temperature for a fullscale test room are compared. One method is to use construction data and a simple first order heatbalance equation. The other method is to identify model parameters from experimental data. The parameters computed from construction data differs about 30% from the identified parameters for a first order model.

Table of contents

1 Introduction 1

2 The experiment room and experiments 2

3 A model based on construction data 6

4 Models from experiment data 9

4.1 Least squares model and method 9

4.2 Maximum likelihood model and method 10

4.3 Result from least square identification 11

4.4 Result from maximum likelihood identification 13

5 Comparison and conclusion 15

6 References 17

1 Introduction

The main purpose with this is to make a comparison of dynamic models for the input output system heated air-temperature to room air-temperature for a fullscale testroom derived in different ways either by using construction data and a simple heatbalance equation or using data from specially made experiments and identification programs.

These experiments and the fullscale test room are described in section 2. The model based on construction data and a simple heatbalance equation is described in section 3. In section 4 experiment data is used to derive models with the least squares method and the maximum likelihood method. The comparison is done in section 5 between the different types of models.

2 Experiments

The experiment room has been developed by the department of Building Science, Lund Institute of Technology, Adamson, 1969. The room has a length of 4.5 m, a width of 3.6 m and a height of 3.0 m. It is connected to outdoor air through a window and a front wall. The other three walls of the room divide the room from another room. All the walls, the ceiling and the floor are built with only one half of the normal thickness. This is done so that the masses connected to the room air should be the same as if the room was surrounded by similar rooms under the same circumstances. More details are given in table 2.1.

The room air is heated by an electrical air heater. A fan blows the room air through the heater with a capacity of 505 m^3/h. The room volume is 48.6 m^3.

The temperature measurements were made with termocouples and a data logger, which punches the measurements on a papertape every minute.

The effect to the heater was controlled according to a PRBS sequence (Pseudo Random Binary Signal), which consists of only two states. These were implemented as on and off of the heater (In the experiment 1 and 2 the on effect was 1 kW resp. 2 kW).

The two used experiments are shown in figure 2.1 and 2.2.

Further details are given in Jensen, 1973.

Table 2.1 Summary of building data from the testroom at the department of Building Science, Lund Institute of Technology.

Surface	Materials	Thickness m	Heat cond. λ W/^{o}C m	Heat cap S Wh/^{o}C m^3
Floor 16.2 m^2	Reinforced concrete	0.125	1.51	560
	Mineral wool (Gullfiber S200,200kg/m^3)	0.20	0.0406	48.8
Ceiling 16.2 m^2	Reinforced concrete	0.125	1.51	560
	Mineral wool (Gullfiber 3004,16kg/m^3)	0.20	0.0406	3.90
Partition walls 2·13.4 m^2	Light concrete	0.075	0.151	125
	Mineral wool 16kg/m^3	0.10	0.0406	3.90
Corridor wall 8.7 m^2	Light concrete	0.075	0.151	125
	Mineral wool 16kg/m^3	0.10	0.0406	3.90
Door to corridor 1.5 m^2	Masonite	0.004	0.13	465
	Air	0.04	0.0256	0.30
	Masonite	0.004	0.13	465
Door window 0.5 m^2	Machine glass P3	0.002	0.81	605
Facade wall 3.1 m^2	Light concrete	0.25	0.151	125
Window 7.6 m^2	Machine glass	0.003	0.81	605
	Air	0.009	0.0256	0.30
	Machine glass	0.003	0.81	605

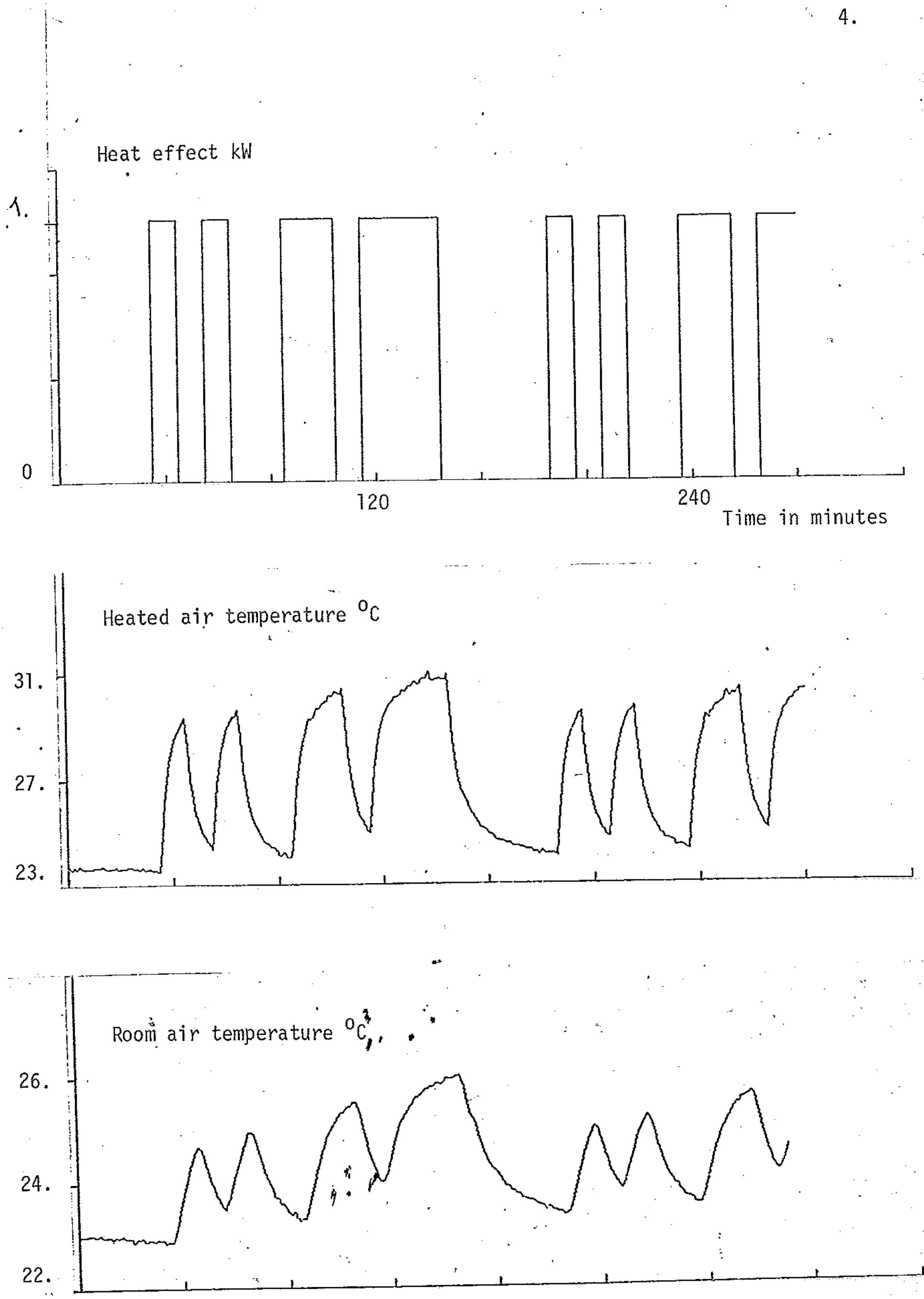

FIGURE 2.1 Experiment 1

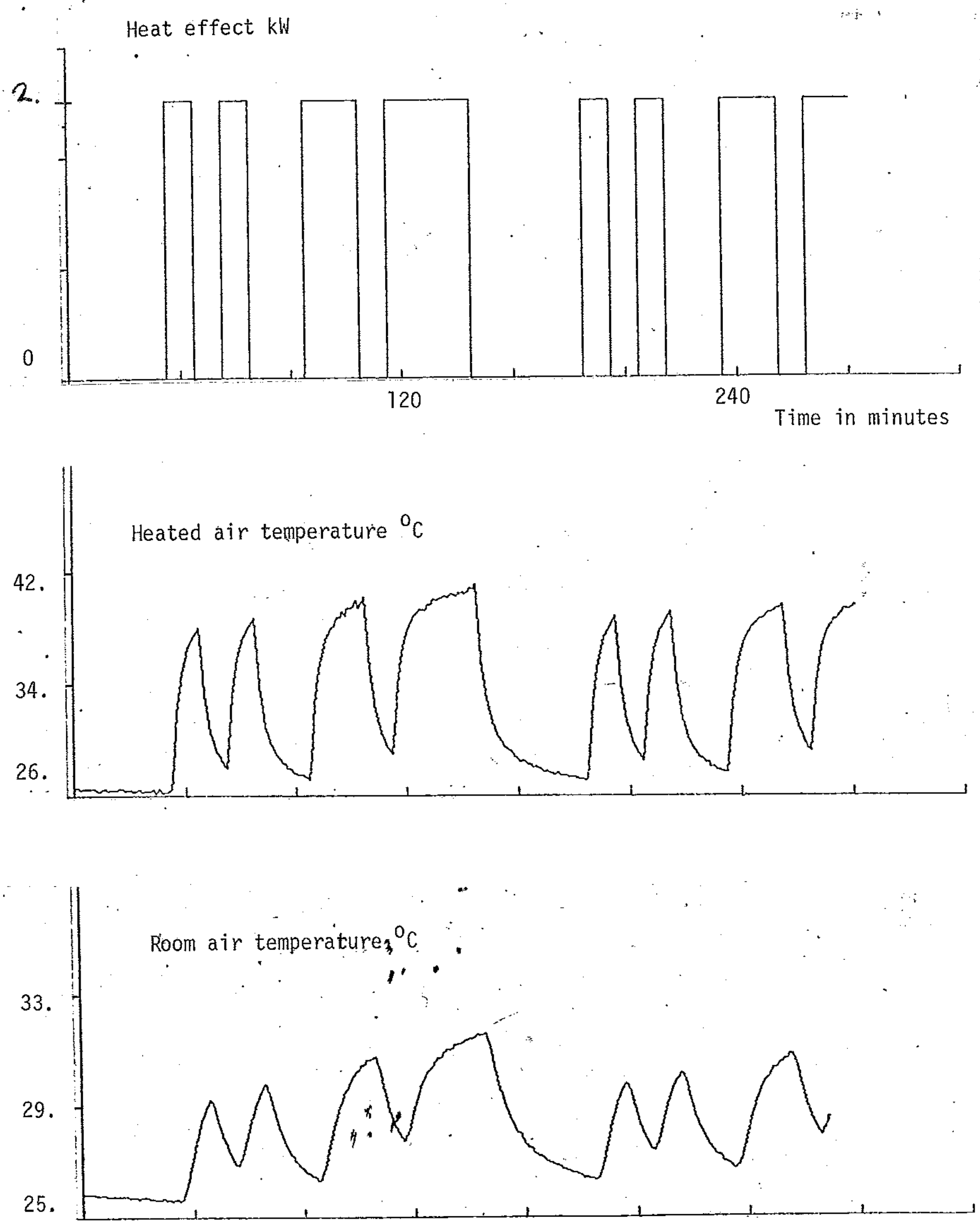

FIGURE 2.2 Experiment 2

3 A model based on construction data

A very simple first order model for the input-output system heated air temperature to room air temperature can easily be obtained if the following assumptions are made:

1. the room air is assumed to be totally mixed, implying that the air temperature is the same everywhere in the room
2. the heated air, the walls and the outdoor air temperature are assumed to be inputs and are not affected by the room air temperature
3. all construction data, like heat transfer coefficients etc, are assumed to be independent of the temperature

The word walls is understood to also include the ceiling and the floor.

The assumption that the temperature of the walls is not affected by the room air temperature may be a good approximation if the heat capacity of the walls is much larger than the heat capacity of the room air. Then the wall temperature will only change very slowly and with small amounts.

Then the following heat balance equation can be derived for the room air temperature x(t):

$$C \frac{d}{dt} x(t) = n \quad C\,(u_1(t) - x(t)) + A_2 h_2 (u_2(t) - x(t)) +$$

$$+ A_3 h_3 (u_3(t) - x(t))$$

$x(t)$	room air temperature
$u_1(t)$	heated air temperature
$u_2(t)$	average wall temperature
$u_3(t)$	outdoor air temperature

A_2	surface between room air and walls
A_3	surface between room air and outdoor air
C	room air heat capacity
h_2	heattransfercoefficient for surface A_2
h_3	heattransfercoefficient for surface A_3
n	the number of room air changes per time unit

If the heat balance equation is Laplace transformed one would get three transferfunctions between the three inputs and the output as:

$$G_1(s) = \frac{K_1}{sT + 1}$$

$$G_2(s) = \frac{K_2}{sT + 1}$$

$$G_3(s) = \frac{K_3}{sT + 1}$$

The relations between parameters in the heat balance equation the ones in the transferfunctions are:

$$T = C/(n \quad C + A_2\, h_2 + A_3\, h_3)$$

$$K_1 = n \cdot C/(n \quad C + A_2\, h_2 + A_3\, h_3)$$

$$K_2 = A_2 h_2/(n \quad C + A_2\, h_2 + A_3\, h_3)$$

$$K_3 = A_3 h_3/(n \quad C + A_2\, h_2 + A_3\, h_3)$$

The parameters A_2, A_3, C, h_2, h_3 and n are needed to compute the transfer function parameters C, K_1, K_2 and K_3. A_2, A_3, C and n can easily be found or computed, but the heattransfercoefficient h_2 and h_3 varies with the temperature. h_3 is given by the manufacturer for the whole window to 24. W/^oC and h_2 is used with five different values 1(1)5 W/^oC m^2.

The other parameters were

A_2 = 70 m^2
A_3 = 7.6 m^2
C = 19 Wh/^oC
n = 10 /h

Table 3.1 Timeconstant T and static gain K_1 as a function of the heattransfercoefficient h_2

h_2 (W/^oC m^2)	T (min)	K_1
1	4.01	0.668
2	3.22	0.536
3	2.64	0.448
4	2.31	0.386
5	2.02	0.337

4 Models from experiment data

In this section first a short description will be made of the least squares model and method (section 4.1) and the maximum likelihood model and method (section 4.2). Only the input heated air temperature is used for the identifications. First and second order models from two experiments and from each identification method are given in section 4.3 and section 4.4.

4.1 The least squares model and method

The model which is used to describe the physical system in discrete time is as follows:

$$y(t) + a_1\, y(t - 1) + \ldots\ldots + a_n\, y(t - n) =$$

$$= b_1\, u(t - 1) + b_2\, u(t - 2) + \ldots\ldots + b_n\, u(t - n) +$$

$$+ \lambda e(t)$$

Here is $(u(t), y(t), t = 1,N)$ the input-output sequence and $e(t)$ the model error. The model parameters are a_i and b_i, $i = 1,n$. The model order is n and the number of data is N.

The model parameters can easily be found by minimizing loss-function

$$V = \sum_{t=1}^{N} e^2(t)$$

with the least squares method. If the model error $e(t)$ is white noise, then the model parameters will be unbiased. Further details about the method are given in Åström, 1968.

The achieved discrete time models can easily be transformed to continuous time models.

4.2 The maximum likelihood model and method

The model which is used for the maximum likelihood estimation is as follows:

$$y(t) + a_1\, y(t-1) + \dots + a_n\, y(t-n) =$$

$$= b_1\, u(t-1) + b_2\, u(t-2) + \dots + b_n\, u(t-n) +$$

$$+ \, (e(t) + c_1\, e(t-1) + \dots + c_n\, e(t-n))$$

The maximum likelihood estimate is obtained by minimization of the lossfunction

$$V = \sum_{t=1}^{N} e^2(t)$$

Further details are given in Åström and Bohlin, 1965 and in Gustavsson, 1969.

4.3 Result from least square identification

Models of first and second order for two different experiments has been identified. Discrete time model parameters see table 4.1. Corresponding continuous time model parameters see table 4.2. The order of the models was found to be of second order.

Table 4.1 Result from least square identification

Experiment	Model order	Model parameter
1	1	$a_1 = -0.7934$
		$b_1 = 0.0838$
		$V = 0.1998$
1	2	$a_1 = -1.2974$
		$a_2 = 0.3850$
		$b_1 = 0.1000$
		$b_2 = -0.0635$
		$V = 0.1252$
2	1	$a_1 = -0.7342$
		$b_1 = -0.0963$
		$V = 1.7368$
2	2	$a_1 = -1.5331$
		$a_2 = 0.5640$
		$b_1 = 0.0936$
		$b_2 = -0.0837$
		$V = 0.6191$

Table 4.2 Continuous time model parameters transformed from least square models

Experiment	Model order	Time constant	Static gain
1	1	4.3210	0.4056
1	2	1.2858	0.0858
		5.6558	0.3309
2	1	3.2365	0.3623
2	2	2.0429	0.2214
		12.0196	0.0990

4.4 Result from maximum likelihood identification

The same identifications have been done with this method. Only the discrete time model parameters a_1 and b_1 are given in table 4.3. Corresponding continuous time parameters - see table 4.4. The order of the models was found to be of second order.

Table 4.3 Result from maximum likelihood identification

Experiment	Model order	Model parameters
1	1	a_1 = -0.7925 b_1 = 0.0839 V = 0.1529
1	2	a_1 = -1.6983 a_2 = 0.7047 b_1 = 0.0970 b_2 = -0.0937 V = 0.0838
2	1	a_1 = -0.7389 b_1 = 0.0941 V = 0.9278
2	2	a_1 = -1.5703 a_2 = 0.5845 b_1 = 0.1087 b_2 = -0.1038 V = 0.5881

Table 4.4 Continuous time model parameters transformed from maximum likelihood models.

Experiment	Model order	Time constant	Static gain
1	1	4.2999	0.4043
1	2	3.0604	0.3329
		43.0510	0.1827
2	1	3.3048	0.3604
2	2	1.9990	0.2692
		27.2170	0.0759

5 Comparison and conclusion

The two earlier discussed methods to achieve dynamic models are here compared in this section. This is done with parameters from transfer functions of first order. Parameters from all first order transfer-functions are given in table 5.1.

Both the least squares method and the maximum likelihood method gives for first order models a larger timeconstant and a larger static gain for experiment 1. The explanation is that the heattransfercoefficient is increasing with the temperature difference. The temperature differences are larger in experiment 2 than in experiment 1 and thereby also the average heattransfercoefficient (compare the theoretic results for different heattransfercoefficient).

A reasonable value on the heattransfercoefficient h_2 is 1-2 $W/^oC\ m^2$. This gives a timeconstant about the identified, but the measured gain is smaller than the theoretically achieved.

One conclusion can be made that the theoretically achieved model parameters does not differ more than 30% for the first order models.

Another conclusion is that, although the identified models were found to be of second order, the difference to the first order models is not great in stepresponse and in lossfunction. This makes it possible to use a first order model based on construction data to simulate climate systems and to synthesize control systems for climate systems.

Table 5.1 Theoretical and identified first order model parameters

Method	Time constant	Static gain
theoretic h_2 = 1	4.01	0.67
" = 2	3.22	0.54
" = 3	2.64	0.45
" = 4	2.31	0.39
" = 5	2.02	0.34
LS exp 1	4.32	0.41
LS exp 2	3.24	0.36
ML exp 1	4.30	0.40
ML exp 2	3.30	0.36

6 References

Adamson, B, 1969, Program för studier av utomhusförhållandens inverkan på rumsklimatet. (Institutionen för byggnadskonstruktionslära, Lunds Tekniska Högskola) Arbetsrapport 1969:2. Lund (in Swedish)

Gustavsson, I, 1969, Parametric Identification on Multiple Input, Single Output Linear Dynamic Systems. (Lund Institute of Technology, Division of Automatic Control) Report 6907. Lund.

Jensen, L. H., 1973, Dynamiska modeller för ett rum, del 2. (Institutionen för byggnadskonstruktionslära och reglerteknik, Lunds Tekniska Högskola) Arbetsrapport 1973:7. Lund. (in Swedish)

Åström, K. J., & Bohlin, T., 1965, Numerical Identification of Linear Dynamic Systems from Normal Operating Records. (Proceedings of the IFAC Conference on Self-Adaptive Control Systems). Teddington.

Åström, K.J., 1968, Lectures on the Identification Problem - the Least Squares Method. (Lund Institute of Technology, Division of Automatic Control) Report 6806. Lund.