Theory and Applications of Self-Tuning Regulators

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1. INTRODUCTION.

Linear stochastic control theory gives the potential to formulate and solve regulation problems for industrial processes in a fairly realistic manner. This has also been demonstrated in several applications, [1]. The use of the theory does, however, require mathematical models of the process and its disturbances. Models of the process dynamics can sometimes be obtained from physical laws. Modeling of the disturbances will almost always require experimental data from the process. To apply the theory it is thus necessary to perform plant experiments and to make a system identification.

Since the dynamic characteristics of the process and the disturbances may change with time, it is necessary to repeat the identification regularly to maintain the quality of the regulation. This imposes heavy restrictions on the operation of the system and requires persons with significant theoretical skills to keep the system regulating well.

From a practical point of view it is therefore highly desirable to investigate the possibilities to obtain control algorithms which can adapt to changes in the dynamics of the process and the disturbances. There are many ways to formulate control problems which lead to such algorithms. The main difficulty is to pose a problem which reflects the practical problem sufficiently well and whose solution gives a regulator of reasonable complexity.

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The changes in the process and its environment are very slow in many cases. A possible formulation is then to consider the problem of controlling a system with constant but unknown parameters. Such a problem can be solved at least for linear stochastic systems. Examples of such solutions are found in [2], [5]. The solutions obtained are not practical because it is necessary to introduce the conditional probability density of the parameters as a state. Even the solution of very simple problems will require computations which far exceed the capacity of computers available today.

It can then be attempted to analyse more modest problems. For systems with constant but unknown parameters one possibility is to construct control algorithms, which do not require knowledge of the system parameters and which converge to the optimal regulator that could be designed if the system parameters were known. Such algorithms are called <u>self-tuning</u> or <u>self-optimizing</u> regulators. The approach to such regulators is partly heuristic in the sense that it is necessary to develop enough insight into the problem to propose candidates for the algorithms. Once the algorithms are obtained there are, however, interesting and important problems of analysing them. Mathematically the problem changes from an optimization problem to an analysis problem.

There are many possibilities to generate the desired control algorithms. One possibility is to analyse the properties of solutions to simple optimal control problems that can be generated numerically. See [2]. Another possibility is to exploit on-line identification methods. Such a scheme based on least squares identification was proposed in [10]. Similar algorithms have also been investigated in [15] and [17].

This paper surveys the properties of a class of self-tuning regulators. In Section 2 a simple example is used to explain and motivate the algorithm. Some properties of the algorithm are analysed in Section 3. Limitations of the algorithm are discussed in Section 4 where some extensions also are given. Practical applications of the algorithm to control of industrial processes in paper and steel industry are briefly discussed in Section 5.

2. AN ALGORITHM.

A self-tuning control algorithm will now be given. The main ideas are illustrated using a simple example. Consider the system

$$y(t+1) + a\, y(t) = b\, u(t) + e(t+1) + c\, e(t) \qquad (2.1)$$

where u is the input, y the output and {e(t)} a sequence of independent, equally distributed, random variables. The number c is assumed to be less than one. Let the criterion be to minimize the variance of the output i.e.

$$\min V = \min Ey^2 = \min E \frac{1}{t} \sum_{k=1}^{t} y^2(k) \qquad (2.2)$$

It is easy to show [1] that the control law

$$u(t) = \frac{a - c}{b}\, y(t) \qquad (2.3)$$

is a minimum variance strategy, and that the output of the system (2.1) with the feedback (2.3) becomes

$$y(t) = e(t) \qquad (2.4)$$

Notice that the control law (2.3), which represents a proportional regulator, can be characterized by one parameter only.

A self-tuning regulator for the system (2.1) can be described as follows:

ALGORITHM (Self-TUning REgulator).

Step 1. (Parameter Estimation).

At each time t, fit the parameter α in the model

$$\hat{y}(k+1) + \alpha\, y(k) = u(k), \quad k = 1,\dots,t-1 \qquad (2.5)$$

by least squares, i.e. such that the criterion

$$\sum_{k=1}^{t} \varepsilon^2(k) \qquad (2.6)$$

where

$$\varepsilon(k) = y(k) - \hat{y}(k) \qquad (2.7)$$

is minimal. The estimated obtained is denoted α_t to indicate that it is a function of time.

Step 2. (Control).

At each time t, choose the control

$$u(t) = \alpha_t y(t) \qquad (2.8)$$

where α_t is the estimate obtained in step 1.

□

Motivation.

There are several ways to arrive at the algorithm STURE given above. It can be obtained as a solution to a stochastic control problem based on the assumption that the problem can be separated into one identification problem and one control problem, which are solved separately. The algorithm is thus not a dual control in the sense of Feldbaum. The algorithm can also be obtained from the model reference principle. See e.g. [11].

Extensions.

The algorithm given for the simple example can be extended in many ways. A generalization to systems of n:th order is given in [3] and a multivariable version is given in [16].

3. ANALYSIS.

The properties of a closed loop system controlled by a self-tuning regulator will now be discussed. Since the closed loop system is nonlinear, timevarying and stochastic, the analysis is not trivial. In this section the major results will be stated for the simple example. For details we refer to the references [3], [12] and [13].

It is fairly obvious that the regulator will perform well if it is applied to a system (2.1) with b = 1 and c = 0, because in this case the least squares estimate α_t will be an unbiased estimate of a. The regulator (2.8) will thus converge to a minimum variance regulator if the parameter estimate α_t converges. It is surprising, however, that the regulator will also converge to the minimum variance regulator if $c \neq 0$ as will be demonstrated below.

The least squares estimate is given by the normal equation

$$\frac{1}{t}\sum_{k=1}^{t} y(k+1)y(k) + \alpha_{t+1}\frac{1}{t}\sum_{k=1}^{t} y^2(k) = \frac{1}{t}\sum_{k=1}^{t} y(k)u(k)$$

Assuming that the estimate α_t converges towards a value which gives a stable closed loop system, then it is straightforward to show that

$$\frac{1}{t}\sum_{k=1}^{t} (\alpha_{t+1} - \alpha_k)y^2(k) \rightarrow 0$$

Thus the closed loop system has the property

$$\lim_{t\to\infty} \frac{1}{t}\sum_{k=1}^{t} y(t+1)y(t) = 0 \tag{3.1}$$

Furthermore, assuming that the system to be controlled is governed by (2.1), the output of the closed loop system obtained in the limit is given by

$$y(t) + [a - \alpha b]y(t) = e(t) + c\, e(t-1) \tag{3.2}$$

The covariance of $\{y(t)\}$ at lag 1 is then given by

$$E\, y(t+1)y(t) = -f(\alpha) = \frac{(c-a+\alpha b)(1-ac+\alpha bc)}{1-(a-\alpha b)^2} \tag{3.3}$$

The condition (3.1) gives

$$f(\alpha) = 0$$

This is a second order equation for α which has the solutions

$$\alpha = \alpha_1 = \frac{a - c}{b} \tag{3.4}$$

$$\alpha = \alpha_2 = \frac{a - 1/c}{b} \tag{3.5}$$

The corresponding poles of the closed loop system are $\lambda_1 = c$ and $\lambda_2 = 1/c$ respectively. Since c was assumed less than one, only the value α_1 corresponds to a stable closed loop system. Notice that α_1 corresponds to the gain of the minimum variance regulator (2.3).

Hence, if the parameter estimate α_t converges to a value which gives a stable closed loop system, then the closed loop system obtained must be such that (2.7) holds. This means that the algorithm can be thought of as a regulator which attempts to bring the covariance of the output at lag one, i.e. $r_y(1)$, to zero in the same way as an integrating regulator brings the integral of the control error to zero.

If the system to be controlled is actually governed by (2.1), then the self-tuning regulator will converge to a minimum variance regulator if it converges at all. These properties of the self-tuning regulator are easy to extend to arbitrary n:th order systems. This is done in [4]. Simulations of the regulator are given in [4], [15] and [18].

As was already pointed out by Kalman [10], the regulator has strong stabilizing properties; because if the output assumes large values, then the terms containing u and y in (2.1) will dominate the stochastic terms, and the estimate α will become close to a/b. This will bring the closed loop poles close to the origin. Hence, if the values of the output becomes very large, they will quickly be reduced to the level of the disturbances again. A rigorous analysis is provided in [13].

The convergence of the algorithm is of course a key problem. This problem has been analysed in [12], [13], where necessary and sufficient conditions are given. The differential equation

$$\frac{d\alpha}{d\tau} = f(\alpha) \qquad (3.6)$$

where the function f is defined by (3.3), plays a crucial role in the stability analysis. It follows from the previous discussion that $\alpha = \alpha_1$ is a stationary solution. It is shown in [12] that the parameter estimates will in a certain sense be close to the trajectories of (3.6). One condition required for convergence of the estimates is that the solution $\alpha = \alpha_1$ is a stable solution to (3.6). In the particular example the estimate will converge if $0 < b < 2$, [13].

Since the inputs to the system are generated by feedback from the output, it may conceivably happen that there will be many values of α which give the same values to the loss function. To understand the behaviour of the self-tuning algorithm it is easy to show that α is identifiable. Conditions for identifiability of systems under closed loop experimental conditions are discussed in [9].

4. LIMITATIONS AND EXTENSIONS.

Consider a system described by the n:th order difference equation

$$A(q^{-1})\ y(t) = B(q^{-1})\ u(t) + C(q^{-1})\ e(t) \qquad (4.1)$$

where $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ are polynomials in the backward shift operator q^{-1} i.e.

$$A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_nq^{-n}$$

$$B(q^{-1}) = \qquad b_1q^{-1} + \ldots + b_nq^{-n}$$

$$C(q^{-1}) = 1 + c_1q^{-1} + \ldots + c_nq^{-n}$$

u is the input, y the output and $\{e(t)\}$ a sequence of uncorrelated, random variables. It has been found by simulation [18] that the self-tuning regulator will not necessarily converge when applied to the system (4.1) when the polynomial $B^*(x) = x^nB(x^{-1})$ has zeroes outside the unit circle, (non-minimum phase systems). There are also minimum phase systems for which the regulator does not converge,[13].

Several properties of the self-tuning regulator do also depend on a delicate balance of a bias in the least squares estimate and a modeling error. This balance is upset, if extra perturbation signals are introduced, or if the criterion (2.2) is changed e.g. to include weighting of the control signal. There are various modifications to the algorithm which can be introduced to overcome some of the difficulties. In [4] it was shown that the difficulties associated with non-minimum phase systems could sometimes be eliminated by modifying the control law.

In the algorithm given in section 2 no attempt is made to estimate the parameters of the polynomial C directly. These parameters will instead enter the procedure indirectly through the bias in the estimate of the parameter α of the model (2.5). In some cases e.g. when extra perturbation signals are introduced, or when the criterion includes penalty on the control actions,it is advantageous to estimate the parameters $c_1,\ldots,c_n$ too. This estimation problem is nonlinear. Approximative, linear estimation schemes may, however, sometimes be useful. One possibility is to substitute the estimation step of the algorithm by

Step 1 A. (Parameter Estimation)

At each time t the parameters of the model

$$\hat{y}(k+1) + A(q^{-1})\, y(k) = B(q^{-1})\, u(k) + C(q^{-1})\, \varepsilon(k) \qquad (4.2)$$

where ε is given by (2.7) and

$$A(q^{-1}) = \alpha_1 + \alpha_2 q^{-1} + \ldots + \alpha_n q^{-n+1}$$

$$B(q^{-1}) = \beta_1 + \beta_2 q^{-1} + \ldots + \beta_n q^{-n+1}$$

$$C(q^{-1}) = \gamma_1 + \gamma_2 q^{-1} + \ldots + \gamma_n q^{-n+1}$$

are estimated by least squares i.e. in such a way that the criterion (2.6) is minimal.

Since the estimation problem is linear in the parameters, it is easily solved. Introduce the vectors

$$\theta = \mathrm{col}[\alpha_1,\ldots,\alpha_n,\beta_1,\ldots,\beta_n,\gamma_1,\ldots,\gamma_n] \qquad (4.3)$$

$$\varphi(k) = [-y(k),\ldots,-y(k-n+1),u(k),\ldots,u(k-n+1),\varepsilon(k),\ldots,\varepsilon(k-n+1)] \qquad (4.4)$$

The prediction $\hat{y}$ defined by (4.2) can then be written as

$$\hat{y}(k+1) = \theta\varphi(k) \qquad (4.5)$$

and the criterion (2.6) becomes

$$V(\theta) = \sum_{k=1}^{t} [y(k) - \theta\varphi(k-1)]^2 \qquad (4.6)$$

The value θ_t of θ which minimizes this function is given by the normal equations.

$$\left[\sum_{k=1}^{t} \varphi^T(k-1)\varphi(k-1)\right]\theta_t = \sum_{k=1}^{t} \varphi^T(k-1)y(k) \qquad (4.7)$$

since

$$y(k) = \hat{y}(k) + \varepsilon(k) = \varphi(k-1)\theta_k + \varepsilon_k$$

it follows from the normal equations that

$$\frac{1}{t}\sum_{k=1}^{t} \varphi^T(k-1)\varepsilon(k) + \frac{1}{t}\sum_{k=1}^{t} \varphi^T(k-1)\varphi(k-1)[\theta_t-\theta_k] = 0$$

If the parameter estimates converge to such values that the polynomial $x^n(1+C(x^n))$ is stable, then the second term converges to zero and we get

$$\lim \frac{1}{t}\sum_{k=1}^{t} \varphi^T(k-1)\varepsilon(k) = 0$$

or

$$\begin{aligned} &E\ \varepsilon^o(t+\ell)y(t) = 0 && \ell = 1,\ldots,n \\ &E\ \varepsilon^o(t+\ell)u(t) = 0 && \ell = 1,\ldots,n \qquad (4.8)\\ &E\ \varepsilon^o(t+\ell)\varepsilon^o(t) = 0 && \ell = 1,\ldots,n \end{aligned}$$

where ε^o is the residual calculated from the limiting estimate.

Hence, if the parameter estimates converge, then the limiting parameter estimates are characterized by the property that the covariances (4.8) must vanish. The conditions (4.8) are generalizations of (3.1).

Assuming that the estimation procedure is applied to a system described by (4.1), the conditions (4.8) give nonlinear equations for the possible limits of the estimates. It is straightforward to show that one solution is given by $\alpha_i = a_i$, $\beta_i = b_i$ and $\gamma_i = c_i$, $i = 1,2,\ldots,n$.

For $n = 1$ it can also be shown that this is the only limiting point such that $x^n(1+C(x^{-1})$ is a stable polynomial if $(\Sigma_1 u^2(t))/t$ is stable. The problem of convergence is similar to the one for the simple algorithm.

The estimates obtained in this way are the same as those obtained by the method proposed in [20]. An alternative is to use a recursive version of the maximum likelihood method. When the parameter estimates are obtained, the control strategies can be determined in many different ways.

5. APPLICATIONS.

The algorithm outlined in Section 2 is easily implemented on a process computer. A program which handles systems of arbitrary order and includes tuning of feedforward parameters can be written using about 35 FORTRAN statements. Algorithms of this type have been successfully applied to control industrial processes. Applications to paper machine control are described in [6] and [8]. Control of an ore crusher is described in [7]. This application is of interest because of the extreme difficulty in modeling the process using physical principles only. It was implemented using tele-processing from a computer at Lund Institute of Technology to the plant in Kiruna covering a distance of about 1800 km.

In the applications the algorithm described in Section 2 is somewhat modified. An exponential discounting of past values is introduced in the criterion for estimation, (2.6), to allow for timevarying process characteristics. In the paper machine application [6] the regulator had 6 parameters which were updated using the algorithm (4 for the feedback and 2 for the feedforward). To control the ore crusher a regulator with 7 parameters was used.

The general conclusion that can be drawn from the applications is that self-tuning regulators can be useful for practical control problems. In paper machine applications it has been shown that the self-tuning regulator will perform just as well as a minimum variance regulator designed on the basis of plant experiment and system identification. The simulations shown in [3] are representative of what can be achieved in practice.

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