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# RELATIVE FREQUENCIES OF BLUE STRAGGLERS IN GALACTIC GLOBULAR CLUSTERS: CONSTRAINTS FOR THE FORMATION MECHANISMS<sup>1</sup>

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#### **ABSTRACT**

We discuss the main properties of the Galactic globular cluster (GC) blue straggler stars (BSSs), as inferred from our new catalog containing nearly 3000 BSSs. The catalog has been extracted from the photometrically homogeneous V versus (B-V) color-magnitude diagrams (CMDs) of 56 GCs, based on Wide Field Planetary Camera 2 images of their central cores. In our analysis, we used consistent relative distances based on the same photometry and calibration. The number of BSSs has been normalized to obtain relative frequencies ( $F_{\rm BSS}$ ) and specific densities ( $N_{\rm S}$ ) using different stellar populations extracted from the CMD. The cluster  $F_{\rm BSS}$  is significantly smaller than the relative frequency of field BSSs. We find a significant anticorrelation between the BSS relative frequency in a cluster and its total absolute luminosity (mass). There is no statistically significant trend between the BSS frequency and the expected collision rate. The value of  $F_{\rm BSS}$  does not depend on other cluster parameters, apart from a mild dependence on the central density. Post–core-collapse clusters act like normal clusters as far as the BSS frequency is concerned. We also show that the BSS luminosity function for the most luminous clusters is significantly different, with a brighter peak and extending to brighter luminosities than in the less luminous clusters. These results imply that the efficiency of BSS production mechanisms and their relative importance vary with the cluster mass.

Subject headings: blue stragglers — globular clusters: general — Hertzsprung-Russell diagram — stars: luminosity function, mass function

#### 1. INTRODUCTION

Globular clusters (GCs) are important astrophysical laboratories for investigating the stellar dynamics and stellar evolution of low-mass stars (e.g., Meylan & Heggie 1997). In recent years, it became clear that we cannot study these two astrophysical processes independently if we want to understand GCs and properly address several long-standing problems concerning their stellar content.

Among the most puzzling products of the interplay between stellar evolution and dynamics are the blue straggler stars (BSSs). This group of stars was originally identified by Sandage (1953) in the cluster M3 as a bluer and brighter extension of the main-sequence turnoff stars. At present, the most popular mechanisms suggested to account for their origin are *primordial binary evolution* (McCrea 1964), i.e., mass transfer and/or co-

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alescence in primordial binary systems (Carney et al. 2001), and collisional merging, i.e., the collision of single and/or binary systems (Bailyn 1995). Unfortunately, current photometric investigations do not allow us to figure out the mechanism that triggers the formation of BSSs in GCs, and indeed it has been suggested that both primordial binary evolution and collisions are probably at work in different clusters (Ferraro, Fusi Pecci, & Bellazzini 1995; Piotto et al. 1999; Ferraro et al. 2003) or even within the same cluster (Ferraro et al. 1997). The observational scenario concerning BSS formation has been recently enriched by the results of a spectroscopic survey by Preston & Sneden (2000, hereafter PS00). On the basis of multiepoch radial velocity data of field blue metal-poor (BMP) stars, PS00 found that more than 60% of the stars in their sample are binaries. On the basis of empirical evidence, PS00 concluded that at least 50% of BMP stars are BSSs. Moreover, PS00 suggested that the BSSs in their sample must have formed via mass transfer in binaries. Finally, PS00 found that the specific frequency of BSSs in the local halo is an order of magnitude larger than in GCs. This discrepancy opens several new questions concerning the origin of field and cluster BSSs.

In an attempt to better understand the properties of BSSs in GCs, we took advantage of our homogeneous database of color-magnitude diagrams (CMDs) from Wide Field Planetary Camera 2 (WFPC2) images (Piotto et al. 2002) to select a sample of nearly 3000 BSSs in 56 GCs characterized by different morphological and dynamical properties. In this Letter, we exploit the new BSS catalog to investigate empirically whether the BSS population is related to any of the properties of the parent GC. Here we present the results we believe to be the most relevant and original. The entire BSS photometric catalog and further details on the BSS extraction will be published in a forthcoming paper (F. De Angeli et al. 2004, in preparation), and it will become available at the Padova Globular Cluster Group Web site.

#### 2. THE DATABASE OF BSSs IN GCs

We have recently completed our *Hubble Space Telescope* (*HST*) WFPC2 snapshot project (GO 7470, GO 8118, GO 8723). By adding the data from our former GO 6095, and similar data from the archive (i.e., WFPC2 images collected in the cluster center, in the F439W and F555W bands), we have obtained a total of 74 CMDs. <sup>10</sup> Details on the data reduction are in Piotto et al. (2002). Here suffice it to say that all the data have been processed in the same way: instrumental photometry with DAOPHOT/ALLFRAME, charge transfer efficiency correction and calibration to the *B* and *V* standard systems following Dolphin (2000). Artificial star experiments have been run for all the clusters to estimate the completeness of star counts. Completeness corrections have been always applied when necessary. For the BSS sample, the completeness was typically larger than 90%.

BSSs are present in all 74 GCs of our catalog. However, 18 CMDs were too contaminated by background/foreground stars, or heavily affected by differential reddening, to allow us a reliable selection of BSSs. The final catalog includes 2798 BSS candidates in 56 GCs (actual stars, before completeness correction), i.e., about 5 times the number published in previous catalogs (Fusi Pecci, Ferraro, & Cacciari 1993; Sarajedini 1993).

The BSSs in our sample share one common feature: in the inner region mapped by our WFPC2 images, the radial distribution of BSSs is more centrally peaked than that of any other cluster population. In fact, we performed several Kolmogorov-Smirnov tests on the radial distributions of horizontal-branch (HB) stars, red giant branch (RGB) stars, and BSSs, and we found that the BSSs are more centrally concentrated at the level of 99.9%, apart from a few cases where the small number of stars prevents any statistically significant test.

### 3. THE RELATIVE FREQUENCY OF BSSs

In this section, we investigate whether the number of BSSs is correlated with any of the physical and morphological parameters of their parent GCs. In order to compare the number of BSSs in different GCs properly, we must adjust it to allow for how much of each cluster we sampled. To this end, a number of different specific frequencies (defined as the ratio between the number of BSSs and a reference population) have been used in the literature (see Ferraro et al. 1995). We estimated the specific frequencies by normalizing the number of BSSs to the HB ( $F_{\rm BSS}^{\rm HB}$ ) or the RGB ( $F_{\rm BSS}^{\rm RGB}$ ) stars. Interestingly, the results discussed below do not depend on which specific frequency we choose. Therefore, in the following we will adopt  $F_{\rm BSS}^{\rm HB}$  and will refer to it as  $F_{\rm BSS}$ . The numbers of BSSs and HB stars have been corrected for completeness (details in Piotto et al. 2002) before calculating  $F_{\rm BSS}$ .

The top panel of Figure 1 shows  $F_{\rm BSS}$  as a function of the integrated visual absolute magnitude of the cluster, from the integrated visual apparent magnitudes and the reddening values in the Harris catalog and the apparent distance moduli derived by Recio-Blanco et al. (2004) for the same clusters by following the procedure outlined by Zoccali et al. (2000). Even though the distribution of empirical data still shows a large scatter for  $-8 \le M_V \le -7.4$ , the data in this panel clearly show a correlation between  $F_{\rm BSS}$  and the integrated absolute magnitude  $M_V$ . In particular, the faintest clusters in our sample, namely, NGC 6717 and NGC 6838, present a BSS specific frequency that is more than a factor of 20 larger than that for the brightest clusters.

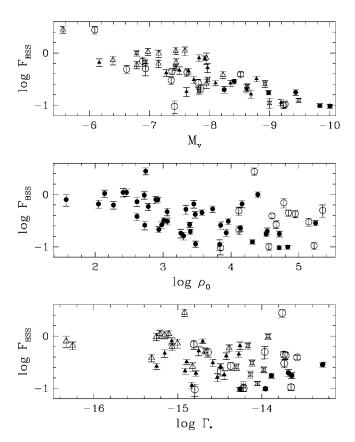


FIG. 1.—BSS relative frequency as a function of the integrated absolute magnitude of the cluster (top panel), the central density (middle panel), and the collision rate (bottom panel). Different symbols are used (in the top and bottom panels) for clusters with different central densities:  $\log \rho_0 < 2.8$ : open triangles;  $2.8 < \log \rho_0 < 3.6$ : filled triangles;  $3.6 < \log \rho_0 < 4.4$ : crosses;  $\log \rho_0 > 4.4$ : filled circles. In all panels, PCC clusters are open circles.

This result is in agreement with that found by PS00 on the basis of data collected from the literature for 30 GCs, although their plot tends to flatten for  $M_V < -7$ , while Figure 1 shows that  $F_{\rm BSS}$  decreases continuously with increasing cluster total luminosity, up to  $M_V < -9$ . Interestingly enough, post–core-collapse (PCC) clusters (*open circles*), with the exception of NGC 5946, which shows a small value of  $F_{\rm BSS}$ , behave as normal clusters.

The middle panel of Figure 1 shows that  $F_{\rm BSS}$  also depends on the cluster central density  $\rho_0$  (in units of  $L_{\odot}$  pc  $^{-3}$ ), although this correlation is statistically somewhat less significant than the previous one. (For internal consistency, we have recalculated the central densities using the equations suggested by Djorgovski 1993 but adopting our new distance moduli and the central surface brightness in the Harris catalog.) The  $F_{\rm BSS}$  for clusters with  $\log \rho_0 > 3.2$  shows a large dispersion and no correlation. For  $\log \rho_0 < 3.2$ , the BSS frequency increases with decreasing central density. Once again, the PCC clusters do not show any peculiar trend. We have also compared  $F_{\rm BSS}$  with the concentration parameter c and the half-mass relaxation time  $t_h$ . Here too there is no clear correlation, although GCs with  $\log t_h < 9$  have, on average, an  $F_{\rm BSS}$  3 times larger than clusters with a longer relaxation time.

In view of the proposed formation mechanisms for BSSs, it is interesting to check whether the  $F_{\rm BSS}$  depends on the expected frequency of stellar collisions. King, Surdin, & Rastorguev (2002) demonstrated that the rate of stellar collisions (per cluster and per year) in a King model GC is about  $\Gamma_c = 5 \times 10^{-15} (\Sigma_0^3 r_c)^{1/2}$ , where  $\Sigma_0$  is the central surface brightness in units

<sup>&</sup>lt;sup>10</sup> They can be found on the Padova Globular Cluster Group Web pages (http://dipastro.pd.astro.it/globulars).

of  $L_{\odot, V}$  pc<sup>-2</sup> (equivalent to  $\mu_v = 26.41$ ) and  $r_c$  is the core radius in units of parsecs (taken from Harris 1996). In order to calculate the probability  $\Gamma_*$  that a given star will have a collision in 1 yr, we have divided this collision rate by the total number of stars  $(N_{\rm star})$  in the cluster. This has been estimated by using the integrated visual absolute magnitude of the cluster, assuming M/L = 2 and a typical mass for the colliding stars of 0.4  $m_{\odot}$ . The bottom panel of Figure 1 plots  $F_{\rm BSS}$  as a function of the resulting  $\Gamma_*$ . There is no statistically significant correlation, although we note that, on average, the 11 clusters with the smallest collision probability ( $\Gamma_* < 10^{-15}$ ) have a BSS frequency 2–3 times higher than clusters with higher collision rates. It must be noted here that, according to the results by PS00, the BSS frequency of the GCs with the smallest collision probability in our sample is about 5 times smaller than the BSS frequency in the field, where collisions are so much rarer. We note that in the case of PCC clusters, the current values  $\Gamma_*$  and  $\rho_0$  may not be representative of the average dynamical environment in which currently observed blue stragglers have formed. However, Figure 1 shows that PCC clusters have BSS frequencies comparable to normal King model clusters, possibly indicating that their dynamical evolution has not affected in a significant way the BSS formation.

The anticorrelation between  $F_{\rm BSS}$  and total cluster luminosity, the lack of a statistically significant correlation with the collisional parameter, and the apparently higher relative frequency of BSSs where collision rates are very small are the most interesting results extracted from our catalog. These empirical facts are somehow puzzling. In fact, we would have expected more BSSs in clusters where the probability of collision is higher. We will discuss these results further in § 4.

The error bars plotted in Figure 1 account for Poisson sampling errors and the uncertainty in the completeness corrections. Even if we assume an upper limit of 0.2 mag for the uncertainties in the individual distance moduli, the correlation shown in Figure 1 represents a robust empirical result, and the reasons are manifold. Unlike other data available in the literature, (1) our data set is photometrically homogeneous, (2) the star counts included the 2 innermost arcminutes, and (3) the star counts of BSSs and of the reference populations have been corrected for incompleteness. As a whole, the present data set is only very marginally affected by the thorny statistical problems affecting previous estimates of BSS specific frequencies (see discussion in Ferraro et al. 1995).

## 4. DISCUSSION

The statistically significant anticorrelation of the BSS relative frequency with the integrated luminosity and the dependence on the expected collision rate discussed in  $\S$  3 are noteworthy, and we will concentrate on them. These observational facts are complemented by the finding by PS00 that field BSSs have a frequency  $F_{\rm BSS}=4.0$ , an order of magnitude larger than the BSS frequency of the bulk of the GCs.

Figure 2 shows the same results of Figure 1, in a different way that may be more enlightening. Here we look at the number of BSSs, HB stars, and RGB stars relative to the total flux in the same region. We call this quantity  $N_s$ ; it is defined by

$$\log N_{\rm S} = \log \left( \frac{N}{F_{\rm HST}/10^{-0.4V_{\rm tot}}} \frac{1}{10^{-0.4M_{\rm V}}} \right),\,$$

where N is the total number of BSSs (or HB or RGB stars), corrected for completeness,  $F_{HST}$  is the total flux from all the

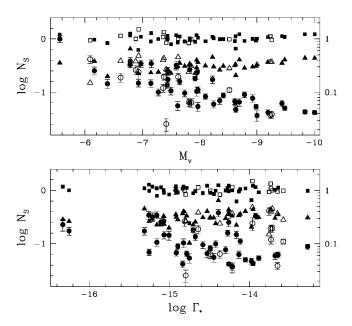


FIG. 2.—Number of BSSs (*circles*), HB stars (*triangles*), and RGB stars (*squares*) per absolute visual flux unit as a function of the integrated cluster magnitude (*top panel*) and of the collision rate (*bottom panel*). Open circles represent PCC clusters.

stars that we measured in the region,  $V_{\rm tot}$  is the integrated apparent magnitude of the cluster, and  $M_{\rm V}$  is its integrated absolute magnitude. (Note that our CMDs typically extend well below the turnoff, so that the contribution from the fainter stars is negligible.)

The first factor in the parentheses can be understood as follows: the quantity  $F_{HST}/10^{-0.4V_{\rm tot}}$  is the fraction of the total cluster flux that is sampled by the HST field. If the BSSs were distributed like the flux, then  $N/(F_{HST}/10^{-0.4V_{\rm tot}})$  would be the number of BSSs we would expect if we could observe the whole cluster. In fact, as the BSSs are more concentrated to the center than is the flux, the quantity above is still a reasonable approximation to the total number of BSSs to be expected. The defect in the approximation increases the scatter in  $N_S$ , but it does not introduce any systematic effects, since (as we have verified) there is no correlation between the fraction of flux included and  $M_V$ . The second factor in the parentheses is just our previous normalization to the size of the cluster but now in luminosity units.

Interestingly enough, Figure 2 confirms that the HB and RGB stars are very good tracers of the cluster population, as their absolute density remains constant over more than 4 mag in cluster total luminosity. This fact removes the risk that the results of Figure 1 might be due to some anomalous gradient in the distribution of HB and RGB stars (see Djorgovski, Piotto, & Capaccioli 1993). Figure 2 confirms that the density of BSSs decreases with increasing total cluster mass and that there is no correlation between the density of BSSs and the collisional parameter. However, we note that, given the small size of the error bars, the dispersion of the BSS density is much larger than the dispersion of the HB and RGB star densities and that, as noticed in Figure 1, clusters with  $\Gamma_* < 10^{-15}$  have a 2–3 times larger BSS density than clusters with higher collision rates. The lack of an overall dependence of  $F_{BSS}$  and  $N_S$  on the collisional parameter seems to suggest that direct collisions of single or binary stars are not the main formation mechanism of BSSs. At first glance, the evolution of primordial binaries

also does not seem to be the dominant formation mechanism for BSSs in all GCs. In the simple hypothesis that the binary fraction is the same in all clusters, we would expect the BSS density to show a behavior similar to that of the HB and RGB stars, in Figure 2. On the other hand, the evolution of primordial binaries is affected by the cluster environment, and, in particular, it is accelerated in clusters where the encounter probability is higher. Indeed, in a paper parallel to this one, using the mechanism proposed by Davies & Hansen (1998) to explain the production of millisecond pulsars in GCs, Davies, Piotto, & De Angeli (2004) demonstrate that in clusters with high encounter probability the formation of BSSs from primordial binaries has been favored in the past. Now these binaries cannot form BSSs anymore (they have already evolved), and this explains the observed relative absence of BSSs in many highmass, high collision rate clusters. It also explains the relatively larger fraction of BSSs among the field stars, where the even lower density environment makes the evolution of binaries via encounters slower than in any GC, allowing them to produce BSSs for a more extended time interval (till the present).

Davies et al. (2004) show also that only in the most luminous GCs (specifically, clusters with  $M_v < -8.8$ ) do the BSSs start to be produced predominantly by stellar collisions. A better way to characterize the physical properties of the BSSs is to look at their luminosity function (LF). In order to overcome possible dependencies of the LF on the cluster metallicity, distance, and reddening, we have divided the luminosity of each BSS by the turnoff luminosity of the parent cluster. Figure 3 shows the LFs for GCs with different total luminosity. The cut in  $M_v$  has been set at  $M_v = -8.8$ , where the theory (Davies et al. 2004) predicts that the BSSs should become predominantly collisional.

Interestingly enough, clusters with  $M_V < -8$ , 8 have a BSS LF that is significantly different from the BSS LF of less luminous clusters (Fig. 3), in that the LFs for the most luminous clusters have a brighter peak and are significantly shifted toward brighter magnitudes. If the relative importance of the BSS production mechanisms depends on the cluster mass, we would then expect to see a dependence of the BSS LF on  $M_V$ , as is

Fig. 3.—BSS LFs for clusters with different integrated magnitude

observed in Figure 3. In general, a BSS produced by collision is expected to have a different luminosity with respect to a BSS from mass transfer or merger of binaries, due to the resulting interior chemical profile. How much different is still controversial. Indeed, recent detailed smoothed particle hydrodynamic simulations performed by Sills et al. (2002) have shown that collision products are not chemically homogeneous. This has the effect of producing a BSS structure less blue and less bright than expected on the basis of the "fully mixed" models (e.g., Bailyn & Pinsonneault 1995). Nevertheless, Sills et al. (2002) have also shown that collision products emerge as rapidly rotating blue stragglers, and so far we lack a full understanding of the changes in the evolutionary properties due to rotationally induced mixing.

It is also worth noting that PCC clusters seem to have a normal BSS population. This might be due to the fact that the core-collapse phase is very short and confined to the very central part of the clusters and therefore does not affect the BSS production over the last few gigayears.

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