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NEW IMPLICIT ADAPTIVE POLE-ZERO-PLACEMENT ALGORITHMS FOR NONMINIMUM-PHASE SYSTEMS

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Table of contents

1. INTRODUCTION

There are several ways to design adaptive regulators based on pole placement. This paper is based on Åström, Westerberg and Wittenmark (1978), where the problem is formulated as a deterministic servo problem. It is assumed that the reader is familiar with that paper. The basic concept of the self-tuning regulator is used. The starting point is thus a pole-placement design method for a system with known parameters. When parameters are unknown they are estimated using a recursive estimator. The design method is then applied assuming that the model parameters are substituted by their estimates. A straightforward application of this idea leads to the algorithms based on estimation of an *explicit process model*. The algorithm can be simplified if the parameters of a transformed model are estimated. The parameters obtained are then an *implicit model* of the process. The explicit methods will often require that fewer parameters are estimated. The implicit methods are, however, often simpler. In many cases the structure of the implicit model can be chosen so that the regulator parameters are estimated directly. This means that the design calculations are avoided. In Åström, Westerberg and Wittenmark (1978) it was shown that efficient pole-placement algorithms could be obtained for minimum phase systems. In this case (Algorithm II) the structure of the implicit model was such that it was linear in the parameters. The identification could thus be done by simple least squares. In this paper it is shown that simple implicit algorithms also can be obtained for nonminimum-phase systems, provided that the implicit model is allowed to be nonlinear in the parameters.

The paper is organized as follows. A brief description of the design method for known parameters is given in Section 2. In Section 3 it is shown that an explicit model which contains the regulator parameters can be constructed. The particular model is such that the prediction error is *bilinear* in the parameters. A recursive estimator for the nonlinear model is then given. The complete algorithm is then easily obtained. The discussion in Sections 2 and 3 was purely deterministic. Disturbances were thus totally neglected. In Section 4 it is analysed how the algorithms behave when it is assumed that the controlled process is subject to stochastic disturbances. It is demonstrated that the algorithm given in Section 3 will give biased estimates. It is shown that the algorithm can be modified to give unbiased estimates if the correlation of the disturbances are known. Other algorithms which do not require this assumption are then presented in Section 4.

2. POLE-ZERO-PLACEMENT DESIGN

A brief description of the pole-placement design problem is given here. More details and references are given in Åström, Westerberg and Wittenmark (1978).

Problem Formulation

Consider a process characterized by the rational operator

$$G_p = \frac{B}{A}, \tag{2.1}$$

where A and B are polynomials in the forward shift operator. It is assumed that the polynomials A and B are relatively prime and that G_p is causal i.e. deg B < deg A. It is desired to find a regulator such that the transfer function from the command input u_r to the process output is given by the rational operator

$$G_M = \frac{Q}{P}, \tag{2.2}$$

where P and Q are polynomials in the forward shift operator. It is assumed that P and Q are relatively prime and that G_M is causal.

Design Procedure

The desired result can be obtained with the regulator

$$Ru = Tu_r - Sy. \tag{2.3}$$

The closed loop system is then characterized by the operator

$$G = \frac{TB}{AR + BS}.$$

Since G should be equal to the desired operator G_M the following equation is obtained:

$$\frac{TB}{AR + BS} = \frac{Q}{P}. \tag{2.4}$$

The design problem is thus equivalent to the algebraical problem of finding polynomials R, S, and T, such that (2.4) holds. Since deg P is normally less than deg (AR + BS) it is clear that there are factors in (2.4) which cancel. The canceled factors must of course be stable. In particular, if the polynomial B is factored as

$$B = B_1 B_2,$$

where the zeros of B_1 are well damped and the zeros of B_2 are unstable or stable but poorly damped, then the specifications must be such that

$$Q = Q_1 B_2 \ .$$

The design procedure can be described as follows:

Step 1: Determine the desired observer polynomial T_1 such that $\deg T_1 \geq \deg A - 1 - \deg B_1$. Introduce

$$T = T_1 Q_1.$$

Step 2: Solve the equation

$$AR_1 + B_2 S = PT_1 \tag{2.5}$$

with respect to R_1 and S.

Step 3: The regulator which gives a closed loop system with the desired response is then given by (2.3) with $R = R_1 B_1$, S and $T = T_1 Q_1$.

Certain aspects of the design problem are seen more clearly if the calculations are carried out using the backward shift operator. For this purpose the reciprocal polynomials defined by

$$A^*(z) = z^{\deg A} A(z^{-1}) \tag{2.6}$$

are introduced. It is natural to normalize the polynomial A so that

$$A(z) = a_0 z^{\deg A} + a_1 z^{\deg A-1} + \ldots + a_{\deg A}$$

with $a_0 \neq 0$. The reciprocal polynomial A^* then becomes

$$A^*(\zeta) = a_0 + a_1 \zeta + \ldots + a_{\deg A} \zeta^{\deg A}$$

with $a_0 \neq 0$. The process model (2.1) can then be written as

$$y(t) = q^{-k} \frac{B^*(q^{-1})}{A^*(q^{-1})} u(t),$$

where

$$k = \deg A - \deg B \tag{2.7}$$

is the *pole excess*. The following abbreviated notation will also be used

$$y = q^{-k} \frac{B^*}{A^*} u. \tag{2.8}$$

Notice that the pole excess appears explicitly when the backward shift operator notation is used. Similarly the desired closed loop system is characterized by the operator

$$G_M^* = q^{-\ell} \frac{Q^*}{P^*} \tag{2.9}$$

where

$$\ell = \deg P - \deg Q. \tag{2.10}$$

If it is assumed that there are no computational delays, the control law (2.3) can be written as

$$R^*u = T^*u_r - S^*y, \tag{2.11}$$

where

$$R^*(0) \neq 0$$

$$T^*(0) \neq 0$$

$$S^*(0) \neq 0.$$

The design identity (2.4) then becomes

$$\frac{q^{-k} B^*T^*}{A^*R^* + B^*S^*} = \frac{q^{-\ell} Q^*}{P^*} \tag{2.12}$$

and the equation (2.5) can be written as

$$A^*R_1^* + q^{-k} B_2^*S^* = P^*T_1^* . \tag{2.13}$$

The closed loop system has the characteristic equation

$$B_1(z)\ T_1(z)\ P(z) = 0.$$

It thus follows that the polynomials B_1, T_1, and P must have all their zeros inside the unit disc for the closed loop system to be stable. It also follows from (2.12) that the design problem has a causal solution only if

$$\ell \geq k$$

i.e.

$$\deg A - \deg B \leq \deg P - \deg Q.$$

Since A and B are relatively prime, A and B_1 are also relatively prime. Equation (2.5) or equivalently (2.13) will then have solutions. There are in fact infinitely many solutions. If R_0 and S_0 are solutions, and if F is an arbitrary polynomial, then

$$R_1 = R_0 - B_2F$$
$$S_1 = S_0 + AF$$

is also a solution. The different solutions will all give the same transfer function from command signal to output. The transfer function from process disturbances to the process output will, however, be different for the different solutions. If we do not wish to specify the disturbances in detail, there are two natural choices:

(i) $\deg R_1^* < k + \deg B_2^*$

or

(ii) $\deg S^* < \deg A^*$.

The first choice corresponds to the situation when the number of delayed control signals appearing in the control law is made as small as possible. The other choice corresponds to choosing the number of delayed process outputs appearing in the control law as small as possible.

The polynomial T_1 can be interpreted as the characteristic polynomial of the observer. The choice deg T_1 = deg A-1 corresponds to a Luenberger observer and deg T_1 = deg A corresponds to a Kalman observer. In special cases there may be solutions to the design problem even if T_1 is chosen so that deg T_1 < deg A-1. It follows from the design procedure that the polynomial B_1 is cancelled. It must then be required that this polynomial corresponds to zeros that are sufficiently well damped.

3. THE ALGORITHM

To use the pole-placement design procedure described in the previous section the parameters describing the process operator G_p must be known. An algorithm, which will work even if the model parameters are not known, will now be derived.

Equation (2.13) gives

$$P^*T_1^* y = A^*R_1^* y + q^{-k} B_2^* S^*y. \tag{3.1}$$

Since the process was described by (2.1) it follows that

$$A^*y = q^{-k} B^*u.$$

Combining this with equation (3.1) gives

$$P^*T_1^* y = q^{-k} B^*R_1^* u + q^{-k} B_2^* S^*y = q^{-k} B_2^* (B_1^* R_1^* u + S^*y)$$

or

$$P^*T_1^* y = q^{-k} B_2^* (R^*u + S^*y). \tag{3.2}$$

This equation can be considered as a transformed process model. The basic process model (2.1) can be recovered from (3.2) if the regulator (2.3) is known. Notice that the model (3.2) contains the polynomials R and S which characterizes the regulator explicitly. Also notice that the model contains the polynomial B_2 i.e. the largest factor which is common to B and Q or equivalently the process zeros which are retained in the closed-loop transfer function.

A self-tuning algorithm can now be obtained as follows.

ALGORITHM I4 (General implicit algorithm for deterministic systems)

Data: Given the desired closed loop poles, the additional closed loop zeros, and the observer poles specified by the polynomials P, Q_1, and T_1.

Step 1: Estimate the parameters of the prediction model

$P^*T_1^*\,y(t) = B_2^*\,[R^*u(t-k) + S^*y(t-k)].$

Step 2: Normalize the polynomial Q_1^* as follows:

$\tilde{Q}_1^*(z) = Q_1^*(z)\,P^*(1)\,/\,[Q_1^*(1)\cdot B_2^*(1)].$

Step 3: Choose $T^* = T_1^*\tilde{Q}_1^*$ and compute the control signal from

$R^*u = T^*u_r - S^*y.$

The Steps 1, 2, and 3 are repeated at each sampling instant. □

This algorithm is similar to the algorithm I3 in Åström, Westerberg and Wittenmark (1978) but Steps 1 and 2 in that algorithm, which involve finding common factors of polynomials, are now replaced by the nonlinear estimation problem associated with the implicit model (3.2).

If the parameter estimates converge to their correct values the algorithm I4 will give a closed loop system with the rational transfer function Q_1B_2/P. This means that the zeros B_2 of the closed loop system may change if the process changes. This situation can, however, not be avoided when using the pole-placement design, because the poorly damped process zeros must by necessity also be zeros of the closed loop system.

<u>Details of the Parameter Estimation</u>

To obtain a complete description of the algorithm it is also necessary to give a recursive method for estimating the parameters in the model (3.1). Since no explicit assumptions have been made concerning the disturbances, it is simply assumed that the parameters are determined in such a way that the error defined by

$$\varepsilon = P^*T_1^*y - B_2^*q^{-k}(R^*u + S^*y) \tag{3.3}$$

is as small as possible in the least squares sense. A recursive estimation algorithm can then be constructed as follows.

Introduce the explicit polynomial notations

$$\begin{cases} R^*(\zeta) = r_0 + r_1 \zeta + \ldots + r_{n_R} \zeta^{n_R} \\ S^*(\zeta) = s_0 + s_1 \zeta + \ldots + s_{n_S} \zeta^{n_S} \\ B_2^*(\zeta) = b_0 + b_1 \zeta + \ldots + b_{n_{B2}} \zeta^{n_{B2}} \end{cases} \qquad (3.4)$$

Notice that it can always be assumed that $r_0 \neq 0$. Furthermore $s_0 \neq 0$, because it was assumed that the computational delay could be neglected. The coefficients n_R, n_S, and n_{B2} have the following interpretation:

n_R = number of delayed control variables in the control law

n_S = number of delayed outputs in the control law

n_{B2} = number of process zeros which are also zeros of the closed loop system.

Furthermore introduce

$$\begin{aligned} y_t &= [\ y(t) \quad y(t-1) \ \ldots \ y(t-n_S)\ ] \\ u_t &= [\ u(t) \quad u(t-1) \ \ldots \ u(t-n_R)\ ] \\ v_t &= [\ v(t) \quad v(t-1) \ \ldots \ v(t-n_{B2})\ ] \end{aligned} \qquad (3.5)$$

where

$$v(t) = R^*(q^{-1})\ u(t) + S^*(q^{-1})\ y(t) \qquad (3.6)$$

and

$$\psi(t) = \begin{bmatrix} B_2^*(q^{-1})\ y_t \\ B_2^*(q^{-1})\ u_t \\ v_t \end{bmatrix} . \qquad (3.7)$$

Let the vectors r, s, and b be defined by

$$s = [\ s_0 \ \ s_1 \ \dots \ s_{n_S}\ ]^T$$

$$r = [\ r_0 \ \ r_1 \ \dots \ r_{n_R}\ ]^T \qquad (3.8)$$

$$b = [\ b_0 \ \ b_1 \ \dots \ b_{n_{B2}}\ ]^T$$

and let θ be the vector

$$\theta = \begin{bmatrix} s \\ r \\ b \end{bmatrix}. \qquad (3.9)$$

The following algorithm is proposed for estimating the parameters of the process (3.2):

$$\theta(t+1) = \theta(t) + P(t+1)\ \psi(t-k+1)\ \varepsilon(t+1), \qquad (3.10)$$

where ε is given by (3.3) and

$$P^{-1}(t+1) = \lambda P^{-1}(t) + \psi(t-k+1)\ \psi^T(t-k+1). \qquad (3.11)$$

The following recursive equation can also be obtained for P:

$$P(t+1) = \frac{1}{\lambda}\Big(P(t) - P(t)\ \psi(t-k+1)\ [\lambda + \psi^T(t-k+1)\ P(t)\ \psi(t-k+1)]^{-1} \cdot \cdot\ \psi^T(t-k+1)\ P(t)\Big).$$

4. STOCHASTIC CONSIDERATIONS

Additional insight into the properties of the algorithm can be obtained by considering the stochastic aspects. This will also make it possible to obtain algorithms which work for the combined servo and regulator problem.

Assume that the process to be controlled is governed by the model

$$A^*y = B^*q^{-k} u + C^*e. \tag{4.1}$$

Equation (2.13) gives

$$P^*T_1^* y = A^*R_1^* y + q^{-k} B_2^* S^*y.$$

Using (4.1) to eliminate y in the first term of the right hand side we get

$$P^*T_1^* y = B^*R_1^* q^{-k} u + B_2^* S^*q^{-k} y + R_1^* C^*e =$$

$$= q^{-k} B_2^* (B_1^* R_1^* u + S^*y) + R_1^* C^*e$$

or

$$P^*T_1^* y = q^{-k} B_2^* (R^*u + S^*y) + R_1^* C^*e. \tag{4.2}$$

Analysis

Equation (4.2) makes it possible to understand what happens if the algorithm of Section 3 is applied to a process with random disturbances. If $C^* \neq 1$ the residual $R_1^* C^*e(t)$ will in general be correlated with the regressors $u(t-k), u(t-k-1), \ldots, y(t-k), y(t-k-1), \ldots$ and the estimates will consequently be biased. This is also true if $T_1^* = C^*$. It can thus be expected that the algorithm will *in general* give biased estimates of B_2, R, and S. Consequently it can never converge to the correct control law in the presence of coloured disturbances (i.e. $C \neq 1$). It is, however, possible to modify the algorithm so that there is a possibility to obtain unbiased estimates. The modification discussed below is based on the same arguments which led to the simple self-tuning algorithm STURE, see Åström and Wittenmark (1973), Åström et al. (1977, p. 470), Clarke and Gawthrop (1975), and Egardt (1978). Consider the process model (4.2) and the desired response model

$$P^* y_r = q^{-k} B_2^* Q_1^* u_r .$$

Hence

$$P^*(y - y_r) = q^{-k} \frac{B_2^*}{T_1^*} [R^* u + S^* y - T^* u_r] + \frac{R_1^* C^*}{T_1^*} e. \qquad (4.3)$$

Since the control law is chosen so that the term in the square brackets is zero, this term can be operated upon by a stable operator without changing the model. The following implicit algorithm can thus be constructed.

ALGORITHM I5

Data: Given the desired closed loop poles and the additional desired closed loop zeros specified by the polynomials P and Q_1.

Step 1: Estimate the parameters of the prediction model

$$P^*[y(t) - y_r(t)] = B_2^*[R^* u(t-k) + S^* y(t-k) - T_1^* Q_1^* u_r(t-k)], \qquad (4.4)$$

where u_r is the command input and y_r the desired output, by nonlinear recursive least squares as was described in Section 3. One of the coefficients of the polynomials R^*, S^*, or T_1^* can be fixed in the estimation.

Step 2: Compute the control signal from

$$R^* u = T^* u_r - S^* y, \qquad (4.5)$$

where $T^* = T_1^* Q_1^*$.

Repeat Steps 1 and 2 at each sampling period. □

Remark

Notice that there is a fundamental problem in estimating the parameters of a model of type (4.4) which is bilinear in the parameters. The main difficulty is that the representation (4.4) is unique only if it is required that all zeros of B_2 are outside the unit disc. There is for example always a representation of type (4.4) with $B_2 = 1$ which corresponds to the regulator obtained when all process zeros are cancelled. When parameters are estimated it may easily happen that the degree of $\hat{B}_2$ may change. Special precautions will therefore have to be taken in the details of the algorithm. □

Algorithm I5 can be expected to have interesting properties which is seen from the following *heuristic* argument. Assume that the algorithm is applied to a process described by (4.1) with $C \neq 1$. Then the correct regulator for the process is

$$R^*u = T^*u_r - S^*y,$$

where

$$T^* = C^*Q_1^* .$$

Let the parameter estimates, obtained from the adaptive algorithm, be denoted by the superscript " ^ ". The residual of the least squares estimation is then given by

$$\varepsilon = P^*[y-y_r] - \hat{B}_2^*q^{-k}[\hat{R}^*u + \hat{S}^*y - \hat{T}^*u_r] = P^*[y - y_r] \tag{4.6}$$

where the last equality follows from (4.5). But it follows from (4.2) that

$$P^*T_1^*[y - y_r] = q^{-k} B_2^*[R^*u + S^*y - T^*u_r] + R_1^*C^*e.$$

Hence if $R = \hat{R}$, $S = \hat{S}$, and $T = \hat{T}$ it follows from (4.5) that

$$P^*T_1^*[y - y_r] = R_1^*C^*e,$$

and furthermore, if $T_1 = C$,

$$P^*[y - y_r] = R_1^*e .$$

Equation (4.6) thus gives

$$\varepsilon = P^*[y - y_r] = R_1{}^* e.$$

This residual is clearly not correlated with the regressors in the model (4.4). The correct control law is consequently a possible convergence point if the algorithm I5 is applied to the process (4.1). It has thus been indicated that the algorithm I5 is an algorithm which corresponds to STURE for the combined servo and regulator problem.

Another Algorithm

It is also possible to devise other algorithms which can be expected to have interesting properties. Introduce the filtered signals $\bar{u}$ and $\bar{y}$ defined by

$$T_1{}^* \bar{u} = B_2{}^* u$$

$$T_1{}^* \bar{y} = B_2{}^* y.$$

Equation (4.2) gives

$$P^* y = q^{-k}(R^* \bar{u} + S^* \bar{y}) + \frac{C^*}{T_1{}^*} R_1 e. \tag{4.7}$$

Since deg $R_1 < k$ it is clear that the parameters of the model (4.7) can be estimated consistently with nonlinear least squares if $T_1 = C$. If the polynomial C is known it is thus possible to obtain a good estimator. When C is not known the following algorithm can also be used.

ALGORITHM I6

Data: Given the desired closed loop poles and the desired additional zeros specified by the polynomials P and Q_1.

Step 1: Estimate the parameters of the model

$$A^* y = q^{-k} B^* u + C^* e \tag{4.8}$$

by extended least squares or recursive maximum likelihood.

Step 2: Filter the signals u and y by $1/C^*$ where C^* is the polynomial whose coefficients were estimated in the first step. Let the filtered signals be

$$\begin{cases} C^*\overline{u} = B_2^*u \\ C^*\overline{y} = B_2^*y. \end{cases} \tag{4.9}$$

Step 3: Estimate the parameters of the model

$$P^*y = q^{-k}(R^*\overline{u} + S^*\overline{y}) \tag{4.10}$$

by nonlinear least squares as was described in Section 3.

Step 4: Use the control law

$$R^*u = T^*u_r - S^*y \tag{4.11}$$

where

$$T^* = Q_1^*C^* \tag{4.12}$$

and C^* is the estimate obtained in the first step. □

In this algorithm the design calculations are thus eliminated by using a multi-step estimation procedure. There are clearly many possible variations on this theme.

5. CONCLUSIONS

Adaptive control algorithms based on the pole-placement design were discussed in Åström et al. (1978). A distinction was made between explicit and implicit algorithms. In the *explicit algorithms* an explicit process model is first estimated, then the design procedure is applied to the estimated model. In the *implicit algorithms* a modified process model where the regulator parameters appear directly is instead estimated. This means that the design calculations are avoided in the implicit algorithms. For the general pole-placement problem the design involves factorization of a polynomial into parts which contains well damped and poorly damped or unstable parts, and solution of a linear equation.

In this paper three new implicit algorithms have been proposed. The first algorithm is for the pure servo problem when the disturbances can be neglected. The estimated model is bilinear in the parameters. The parameter estimation is done by nonlinear, recursive least squares. The other two algorithms deal with the combined servo and regulator problem. One algorithm is a direct generalization of the simple algorithm STURE. The other algorithms is based on a multi-step estimation procedure.

Heuristic arguments which indicate that the algorithms have interesting properties are presented. Much work, both analysis and simulation, remains before the properties of the algorithms are fully explored. This report does, however, support the conjecture that for each explicit adaptive algorithm it is possible to find corresponding implicit algorithms. The report also shows that there are algorithms for the mixed servo-regulator problems with different structures.

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