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Sternby, Jan

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ADAPTIVE CONTROL OF EXTREMUM SYSTEMS

JAN STERNBY

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY
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ADAPTIVE CONTROL OF EXTREMUM SYSTEMS

Jan Sternby
Dept of Automatic Control
Lund Institute of Technology
Lund, Sweden

Abstract

Two model-based methods for extremum control are treated. The models consist of a dynamic linear part and a static nonlinearity which can be placed either at the input or at the output. It is shown that models with an input nonlinearity are easier to handle, but may give poor adaptive control laws. With a nonlinearity at the output, the optimal control law is dual, even if the system parameters are known. A way of rewriting this latter model is suggested to facilitate the use of parameter identification.

1. INTRODUCTION

Extremum control was a popular topic in automatic control some decades ago. A commonly considered problem was to find controller settings to minimize some nonlinear functional of the control error, such as the integral of the error squared. Nowadays that problem is rather solved using an adaptive scheme, and extremum control has not been discussed so much any longer.

There are, however, another class of problems, where the output should not be kept constant, but should rather be minimized or maximized. In these problems the nonlinearity is inherent in the problem and is not introduced by the designer. One example is the adjustment of the blade angles in a water turbine to achieve a maximum of produced power. This paper deals with the application of adaptive control laws to this type of extremum control problems.

In the past decades, computer technology has developed enormously. This is one reason why it might be rewarding to reconsider extremum control problems. It is now possible to implement rather complex control algorithms in low cost microcomputers, as has already been shown with adaptive control. It should then be possible to benefit from inserting more ideas from adaptive control and identification into the extremum control area. Moreover, with today's competition for market shares and increasing system complexity, even small gains may be very valuable.

A well-written survey of the older methods for extremum control is given in Blackman(1962). In recent years the idea of adaptive extremum control has been exploited by e.g. Keviczky and Haber(1974) and Bamberger and Isermann(1978).

In the present paper will be considered only methods based on the use of a system model. It will be shown that different models may differ drastically in their behaviour. For a simple example a comparison is also made between using a certainty equivalent control law and one which is dual in the sense of Feldbaum. It is noticable that this is a fairly realistic example where the dual control law is significantly better.

2. MODELS

As already mentioned, extremum control systems have one major characteristic in common. In the absense of disturbances, the steady-state relation between input and output should be a function with an extremum. The object of control is to stay as close to this extremum as possible despite the influence from dynamics, noise or drifts. In order to use optimal control theory, this desire must be translated into a formal loss function. There are several ways of doing this. One possibility is to use a system model to estimate the slope. The control law can then be designed to keep the slope as close to zero as possible, e.g. with its variance as a measure.

However, it is not at all clear what is the best and most natural way of modeling such a nonlinear dynamic system. To be able to use system identification it is of course desirable to have a model which is linear in its unknown parameters. Any a priori knowledge about the process should then be utilised in the choice of regressors. In this way it may be possible to handle quite complicated, but partially known nonlinear systems.

In general cases it is however difficult to find model structures that are general enough, and still allow calculations to be done. One attempt is to separate the linear and nonlinear parts into two blocks in series. There are then two possibilities: the nonlinear part can be placed either before or after the linear part. This choice will have a large influence on the behaviour of the model as can be seen from the following example.

Example

Consider a first order linear system with white equation noise, and a nonlinearity in the form of a squaring device. Then with the nonlinearity at the input of the linear part the overall system is

$$y(t+1) = ay(t) + bu(t)^2 + e(t)$$

where $e(t)$ is a white noise process. Suppose a stationary solution exists ($|a|< 1$). Expected values then are

$$Ey = \frac{b \cdot Eu^2}{1 - a}$$

If the goal is to minimize Ey (and $b>0$) the best performance is thus achieved by putting $u(t)=0$! Furthermore, if $|a|>1$ no stationary solution exists.

Now turn to the other case with an output nonlinearity described by

$$x(t+1) = ax(t) + bu(t) + e(t)$$

$$y(t) = x(t)^2$$

For $a=b=1$ this is the problem considered by Jacobs and Langdon(1970). They show that because of the nonlinear measurement this is a dual control problem in the sense of Feldbaum. The conditional distribution of the state x is discrete, the possible values being $x = \pm|x|$. The conditional mean of x can then be calculated. It is shown that it is <u>not</u> optimal in the long run to have $u(t) = -\hat{x}(t)$. These results would probably not change much if $a = 1-\varepsilon < 1$. Even if a is slightly greater than one, a stationary solution still seems possible. □

There are thus significant differences between the two cases in spite of their identical static response curves. In the first case with the nonlinearity at the input, the optimal control is constant, and thus contains no feedback. The solution to the second problem includes feedback and therefore seems more attractive. It is however more difficult to compute, because it has a dual nature even with known parameters.

May be an output nonlinearity is in general more important than an input nonlinearity for a good description of a nonlinear system. The only possible effect of a known nonlinearity at the input is to restrict the possible input values for the linear part. The nonlinear control problem can then be transformed to linear control with positive inputs. If the range of the nonlinearity is the whole of the real axis, then a change of control variable will reduce the problem into a linear one.

3. NONLINEARITY AT THE INPUT

With the nonlinearity at the input it is easy to set up a model which is linear in the parameters, and thus directly lends itself to parameter estimation and adaptive control. It was shown in the previous section that the optimal control in the case of known parameters is constant if the criterion is the mean output value. It will now be shown that, as expected, adaptive control may give a poor result in the corresponding case with unknown parameters.

In order to simplify notations and analysis only a special low order case will be treated. The same type of problems that appear here will however show up also with dynamical models of any order. The system considered is of the Hammerstein type.

$$y(t) = k + b \cdot u(t-1) + c \cdot u(t-1)^2 + e(t) \tag{1}$$

The noise $\{e(t)\}$ is supposed to be a sequence of independent random variables with zero mean. Only the parameters k and b are supposed to be unknown and have to be estimated, but an unknown c-parameter can also be handled without any change of the results. This system should be controlled so that its output is kept as small as possible. To accomplish this, the criterion used is the steady state mean value of the output. Admissible control laws may use all information available, i.e. $u(t)$ may depend on $y(t)$, $u(t-1)$ and all previous inputs and outputs. If the parameters of the model (1) are all known the optimal control law is

$$u(t) = -b/2c \tag{2}$$

The optimal control law is thus no feedback controller. For the model (1) this controller minimizes the expected value of the output. But it also minimizes the output of the next step. With a more general system model where the output depends on the value of the input at several sampling points, the best steady state and the best one-step controllers will not coincide. For a further discussion on this point see Keviczky and Haber(1974).

Adaptive Control and Estimation

When the parameters of the model (1) are unknown, the control law (2) has to be modified. One approach is to replace the parameters by their estimates to form an adaptive control law, i.e.

$$u(t) = -\hat{b}(t)/2\hat{c}(t) \tag{3}$$

This control law is also one step ahead optimal for the chosen criterion. Since the process noise e in (1) is assumed to be white the process parameters can be estimated with an ordinary least squares estimator. It is also possible to use a stochastic approximation type algorithm. This variant is easier to analyze, and will be discussed in detail in the sequel. Let $\hat{x}(t)$ be the column vector of parameter estimates obtained at time t and

$$\theta(t) = [\ 1 \quad u(t-1)\ ] \tag{4}$$

Then

$$\hat{x}(t) = \hat{x}(t-1) + \frac{C}{t}\cdot\theta(t)^{T}[y(t) - cu(t-1)^{2} - \theta(t)\hat{x}(t-1)] \tag{5}$$

Analysis

For the case of least squares estimation, the general Bayesian convergence results of Sternby(1977) show that the estimates will converge with probability one. But the limits will in general differ from the true values if the conditional variance does not tend to zero. This is what happens here.

The behaviour of the algorithm can also be analyzed using the technique derived by Ljung(1977). In order to apply his results a number of technical conditions must be fulfilled. Some of these conditions are difficult to check mathematically, but are relatively easy to accept intuitively. In this paper no strict proofs of non-consistency will be given. Instead the differential equations of Ljung will be used to show the expected paths of the parameter estimates. The results are confirmed by simulations of the original algorithms. Many identification procedures may be described in terms of the general recursive algorithm of Ljung(1977), which is in the time-invariant case with $\gamma(t)=1/t$

$$\hat{x}(t) = \hat{x}(t-1) + \frac{1}{t}\cdot Q(\hat{x}(t-1),y(t)) \tag{6}$$

where the measurements y are generated as one component of the vector φ in

$$\varphi(t) = A(\hat{x}(t-1))\cdot\varphi(t-1) + B(\hat{x}(t-1))e(t) \tag{7}$$

The noise vectors $e(\cdot)$ are supposed to be independent. According to Ljung(1977) (6) will asymptotically behave like the solution to

$$\dot{\underline{x}} = f(\underline{x}) = EQ(\underline{x},y) \tag{8}$$

In calculating the expectation of (8) $\underline{x}$ is a fixed vector and the measurement y is generated from (7) with this fixed $\underline{x}$-value. In our case we have from (5)

$$f(\underline{x}) = E\,\theta(t)^T[(k-\underline{k}) + (b-\underline{b})u(t-1) + e(t)] \tag{9}$$

where $\underline{x}$ is a column vector containing $\underline{k}$ and $\underline{b}$. The expectation shall be calculated for every fixed value of the vector $\underline{x}$. With $u = -\underline{b}/2c$ let

$$\varepsilon = (k-\underline{k}) + (b-\underline{b})\cdot u = (k-\underline{k}) - (b-\underline{b})\cdot\underline{b}/2c \tag{10}$$

Then (8) is

$$\dot{\underline{k}} = \varepsilon \quad ; \quad \dot{\underline{b}} = \varepsilon\cdot u \tag{11}$$

The parameters may converge to any point on the curve $\varepsilon=0$ for which (11) is stable. Convergence to the correct point may happen since it satisfies $\varepsilon=0$. There are, however, infinitely many other possible convergence points which form a parabola in the $\underline{b}$-$\underline{k}$-plane. The trajectories of (11) are easily found by dividing the two equations. They satisfy

$$\underline{b}(\tau) = \underline{b}(0)\cdot\exp\{[\underline{k}(0) - \underline{k}(\tau)]/2c\} \tag{12}$$

Figs. 1 and 2 show parameter phase planes for the algorithm and its associated differential equations respectively with the true parameter values $k=b=0.4$, $c=0.2$ and the standard deviation of the noise $\sigma=0.03$. Note the parabola of stationary points (dashed in both figs.).

For small c-values part of the stationary points are unstable. This can be studied through a linearization around the stationary points, i.e. by looking at the derivative matrix of the right member of (11). The unstable points must satisfy

$$b - \sqrt{b^2 - 32c^2} < \underline{b} < b + \sqrt{b^2 - 32c^2} \tag{13}$$

This will only happen if the square root exists. A phase plane for (11) in this case is shown in fig. 3.

The parameters are thus likely to converge to some point which gives a nonoptimal input value. The same thing will happen also in more general cases as shown in Sternby(1978a). Such cases may include least squares identification, dynamics in the model or the input modification of stochastic approximation type discussed by Keviczky and Haber(1974).

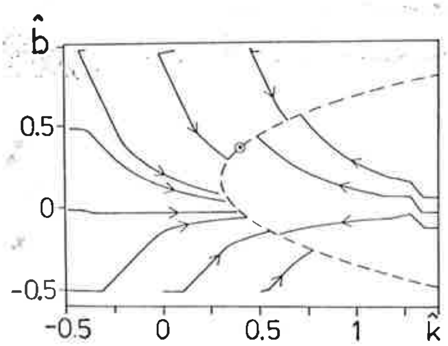

Figure 1-Phase plane for algortithm

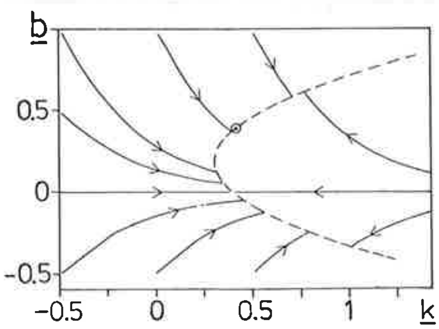

Figure 2-Phase plane for diff. equation

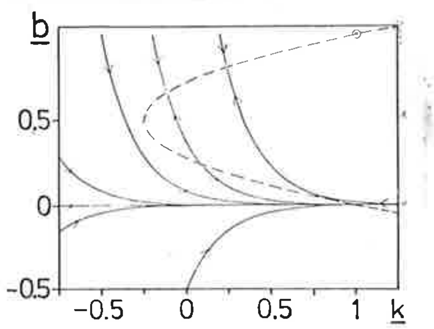

Figure 3-Unstable stationary points.

Improved Control

The control law must be changed to get convergence to the correct values. It is then important to find the reason for nonconsistency. Here the problem is that the same factor ε appears in both equations of (11). This happens because the input depends only on the estimates, and is therefore constant in the calculation of $f(\underline{x})$ of (9). Both components of the vector $\theta(t)$ are then constant. All attempts of improvement must therefore aim at increasing the variation of the input.

A straightforward way is to add a perturbation signal to the input. This works, but convergence was very slow in a few test simulations performed. Also, one more parameter, the perturbation signal amplitude, has to be chosen in advance.

Another possibility could be to introduce a forgetting factor in the identification procedure to prevent the estimates from converging too fast. If the forgetting factor is allowed to tend to one, then the estimates will finally converge. Simulations have indicated that this method may give a nonsatisfactory behaviour. The estimates tend to stay constant for a while and then suddenly jump to another constant value.

The control law (3) is one-step ahead optimal for the chosen criterion. But the dynamic programming could be pursued one more step to give a two-step ahead optimal control. This would introduce a tendency in the control law to actively reduce the parameter uncertainty. In our case, however, this optimal control would still depend only on the estimates, and the problem remains unsolved. But if dynamics are included in the model (1) this might be a possible method, since the input will then also be a function of the measured output, and $\theta(t)$ is no longer a constant in the calculation of $f(\underline{x})$ of (9).

4. NONLINEARITY AT THE OUTPUT

The example in the models section shows that a nonlinearity at the output of a linear system is much more difficult to handle than one at the input. Even if the system parameters are known, the optimal control is e.g. of a dual nature. It tries actively to improve the estimate of the input x to the nonlinearity at the price of worse short term control.

In one of the oldest and most used methods of extremum control the slope of the nonlinearity is estimated by applying a perturbation signal at the input and observe its effect at the output. This method was e.g. discussed already by Leblanc(1922). Taking the so estimated slope as an output, a self-tuning regulator of suitable order could be used to determine the input. This possibility will not be pursued any further here.

Things are much simplified if it is possible to measure the intermediate signal between the linear part and the nonlinearity. It is then, in principle, possible to do system identification separately for the two parts with the nonlinearity modelled as e.g. a second order polynomial. A self-tuning regulator could again be used to keep the output of the linear part with minimal variance around the estimated position of the extremum of the nonlinearity. The results of the previous section will then apply to the nonlinear part, the only difference being that its input is not determined directly, but through linear dynamics. Noise on this intermediate signal will then act as a perturbation signal and improve identifiability, but as stated in the previous section convergence may be slow.

When the intermediate signal is not measurable the problem is more difficult. Even with known parameters the optimal control law is dual. A simple example of this type will now be discussed to show the difficulties and suggest a solution method that can possibly be extended to the case with unknown parameters.

Example

Consider the system

$$y(t) = [\, x(t) - c \,]^2 + v(t) \tag{14}$$

$$x(t+1) = x(t) + u(t) + w(t+1) \tag{15}$$

where c is an unknown constant and $v(\cdot)$ and $w(\cdot)$ are zero mean disturbances. Only y(t) is measurable, and the object of control is to minimize the mean value of the output. First let c=0. This is no restriction here, since c can be subtracted from both members of (15). A reason for choosing this example is that the optimal control has been calculated by Jacobs and Langdon(1970) and is available for comparison.

One possibility is to apply certainty equivalence and let $u(t) = -\hat{x}(t)$. The problem is then to calculate a good estimate $\hat{x}(t)$. In the special case v(t)=0 the conditional distribution for x(t) can be tracked exactly as shown by Jacobs and Langdon(1970). Florentin(1964) treated the case $v(t) \neq 0$ by approximating the conditional distribution by the sum of two Gaussian distributions. Another possibility is to rewrite the system as follows to be able to use some identification method directly. If R is the known variance of w(t) and c=0, then inserting (15) into (14) gives

$$\begin{aligned} y(t) = {} & y(t-1) + u(t-1)^2 + R + 2u(t-1)x(t-1) + \\ & + e(t) + (w(t)^2 - R) + 2w(t)[x(t-1) + u(t-1)] \end{aligned} \tag{16}$$

where e(t) = v(t)-v(t-1).

Estimation

Assuming that {e(t)} and {w(t)} are independent zero mean sequences, the least squares method can be used to estimate x(t-1) from (16). The last row is then regarded as zero mean noise with zero autocorrelation. The first three terms are known at time t-1. The measurement equation (16) will thus give $\hat{x}(t-1|t)$ from $\hat{x}(t-1|t-1)$. Then (15) is used to get $\hat{x}(t|t)$ from $\hat{x}(t-1|t)$ as

$$\hat{x}(t|t) = \hat{x}(t-1|t) + u(t-1)$$

This is an approximation since w(t) is actually partly known at time t through the measurement $y(t)$. Now denote $\hat{x}(t|t)$ by $\hat{x}(t)$. The approximate least squares estimation equations then are

$$\hat{x}(t) = \hat{x}(t-1) + u(t-1) + K(t)\varepsilon(t) \tag{17}$$

$$\varepsilon(t) = y(t) - y(t-1) - u(t-1)^2 - R - 2u(t-1)\hat{x}(t-1) \tag{18}$$

$$K(t) = 2P(t-1)u(t-1)/[\sigma(t-1)^2 + 4P(t-1)u(t-1)^2] \tag{19}$$

$$P(t) = \sigma(t-1)^2 P(t-1)/[\sigma(t-1)^2 + 4P(t-1)u(t-1)^2] + R \tag{20}$$

From (16) with $u(t) = -\hat{x}(t)$ a suitable value for the standard deviation $\sigma(t-1)$ of the measurement noise is

$$\sigma(t-1)^2 = \sigma^2 + 2R^2 + 4RP(t-1) \tag{21}$$

where σ^2 is the variance of $e(t)$.

Control

As shown by simulations, the certainty equivalence controller $u(t) = -\hat{x}(t)$ does not work very well for this example. For the case $R=0$ this can be explained by the consistency results of Sternby(1977), which applied here say that for consistency, the sum of the inputs squared must diverge. With the certainty equivalence controller, (17) shows that $u(t)=-K(t)\varepsilon(t)$. But Corollary 3 of Sternby(1977) tells that $K(t)$ is square summable, and the same thing is then true for $u(t)$ since $\varepsilon(t)$ is mean square bounded. It is also obvious that the controller may be trapped at the value $u(t)=0$ as $K(t+1)$ will then also be zero.

Some feature is needed in the controller to improve the estimation of x. This can be achieved by adding a perturbation signal to the input. The simulations show two disadvantages with that method. The perturbation amplitude must be chosen accurately by the user, and it is nevertheless not possible to get a performance close to the optimal.

Dual Control

Another method is to minimize the criterion two steps ahead as was suggested in Sternby(1978b) for linear systems with unknown parameters. Thus $u(t)$ should be chosen to minimize $E[y(t+1) + y(t+2)|t]$. In the following derivation will be used the approximations

$$\hat{x}(t) = E[x(t)|t] \quad \text{and} \quad P(t) = Var[x(t)|t]$$

Then with $u(t+1) = -\hat{x}(t+1)$

$$\text{Min } E[y(t+2)|t+1] = y(t+1) + R - \hat{x}(t+1)^2 \tag{22}$$

The best two-step $u(t)$ should therefore minimize

$$V[u(t)] = E[2y(t+1) + R - \hat{x}(t+1)^2|t] \tag{23}$$

With the use of (16)-(21) V can be written in the form

$$V(u) = (u - K)^2 + f(u) + \text{constant} \tag{24}$$

with $K = -\hat{x}(t)$ and

$$f(u(t)) = \sigma(t)^2 P(t)/[\sigma(t)^2 + 4P(t)u(t)^2] \tag{25}$$

Utilising the structure of f(u), V(u) can be approximately minimized. In the neighbourhood of u=0 a linear approximation of V"(u) is made from u=0 to the point where f"(u)=0. Outside this area V"(u) is approximated by a constant. In both areas the minimizing u is then found by equating the corresponding approximation of V'(u) to zero.

Simulation

The certainty equivalence control (CE), with and without an added perturbation signal, and the two-step control were tested by simulation. The system used was described by (14)-(15) with c=0, R=1 and v(t)=0 in order to allow a comparison with the optimal control derived by Jacobs and Langdon(1970). The amplitude of the perturbation signal was adjusted to best possible performance.

It was found that with no perturbation signal, the certainty equivalent control behaved much better with a constant $\sigma(t)$ than that given by (21). The reason is probably that with that control the uncertainty P is often large, so that $\sigma(t)$ of (21) would also be large and decrease the influence of the measurements on the estimation by keeping K(t) small. This control law was therefore tuned manually with respect to the constant $\sigma(t)$. The results are shown in fig. 4, where the mean values of the output together with estimated standard deviations over 20 runs of 500 steps are displayed.

The accuracy (standard deviation) in the mean value estimation is better than 0.1 and there is thus a significant difference in the behaviour of the three algorithms. It can be concluded that dual control is needed in this case. Fig. 5 shows the state x controlled by the twostep controller in a run of 400 steps with R=0.16 and σ=0.2. At t=200 c changes from c=0 to c=3. Because of the integrator, x will track the change in c.

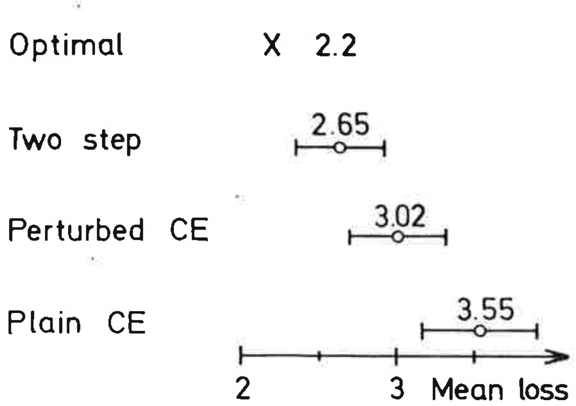

Figure 4-Control laws compared

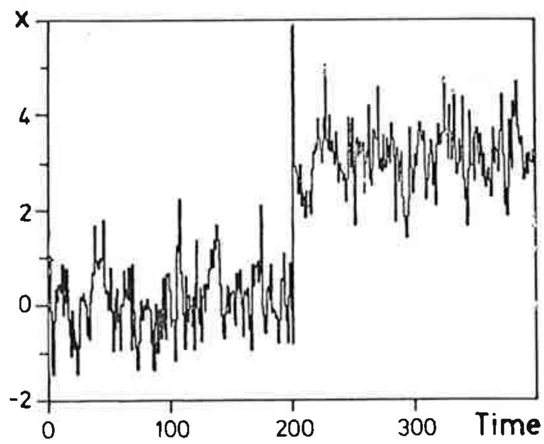

Figure 5-Tracking a moving optimum

5. CONCLUSION

Two possible models for extremum systems have been discussed, both consisting of a dynamic linear part and a static nonlinear part. It was shown that with the nonlinearity at the input, performance may be unsatisfactory when using a straightforward adaptive control law based on certainty equivalence.

With an output nonlinearity the calculations are more difficult even with known system parameters. For a special example the system equations were rewritten to allow the application of the least squares method for identification, and a dual control law could be computed. To make the method interesting, it should be extended to cases with unknown parameters and higher order dynamics. This has not yet been done, but it seems to be possible, maybe with slight extensions of existing identification procedures.

6. ACKNOWLEDGEMENT

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A P P E N D I X 1

An approximate function minimization

APPENDIX 1.

Approximate minimization of

$$V(u) = (u - K)^2 + f(u)$$

where

$$f(u) = \sigma^2 P/(\sigma^2 + 4Pu^2)$$

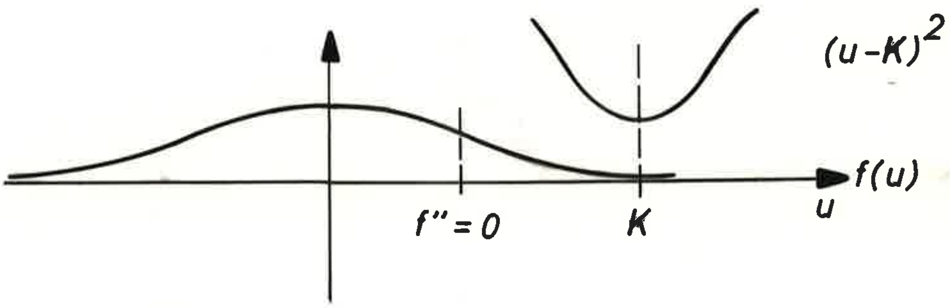
We have

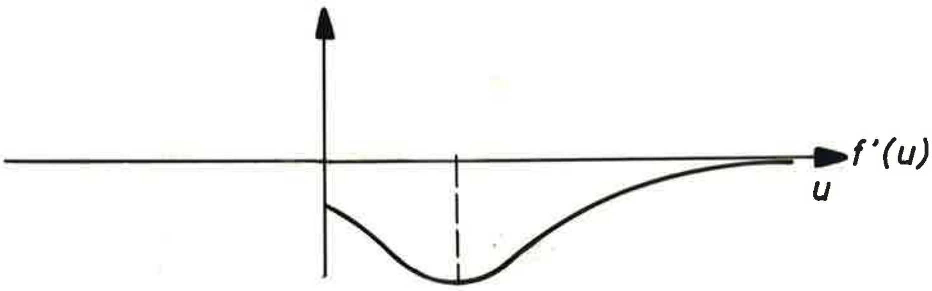
$$f'(u) = -\frac{8uP^2\sigma^2}{(\sigma^2 + 4Pu^2)^2} < 0$$

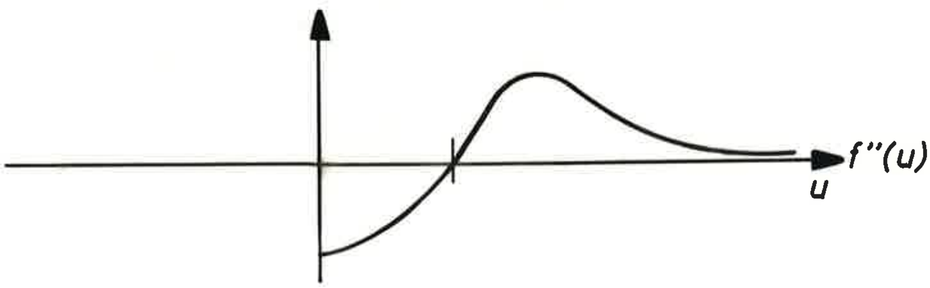
$$f''(u) = 8P^2\sigma^2 \cdot \frac{12Pu^2 - \sigma^2}{(\sigma^2 + 4Pu^2)^3}$$

Then

$$\begin{cases} f''(u) = 0 \quad \text{for} \quad u^2 = \sigma^2/12P \\ f''(0) = -8P^2/\sigma^2 < 0 \end{cases}$$







The basic idea is now the following. In the area where $f''(u) > 0$ the derivative $f'(u)$ is approximated by a straight line. Close to $u = 0$, where $f''(u) < 0$, this second derivative is approximated by a straight line. Since the problem is symmetric with respect to the sign of K, we will assume that $K > 0$.

a) The case $K^2 > \sigma^2/12P$

First assume that $f'(u) = f'(K)$ for all u, i.e. find the point where

$$2(u - K) = - f'(K)$$

The solution is $u_1 = K - \frac{1}{2} f'(K)$. Then calculate $V'(u_1)$, and make a linear interpolation between K and u_1 to find approximately the point where $V'(u) = 0$. This gives

$$u_{optimal} = K + \frac{1}{2} \frac{(f'(K))^2}{f'(u_1) - 2f'(K)}$$

b) The case $K^2 < \sigma^2/12P$

Make a linear interpolation of f'' between $u = 0$ and $u = \sigma/\sqrt{12P}$ to give

$$f''(u) \simeq \frac{\sqrt{12P}}{\sigma} \cdot 8 \frac{P^2}{\sigma^2} \cdot \left(u - \frac{\sigma}{\sqrt{12P}} \right)$$

Integrate this with the initial condition $f'(0) = 0$ to form a second order polynomial approximation to $V'(u)$. Solving for $V'(u) = 0$ with the positive square root chosen will then give

$$u_{optimal} = \frac{\sigma}{\sqrt{12P}} \cdot \left\{ 1 - \frac{\sigma^2}{4P^2} + \sqrt{\left(1 - \frac{\sigma^2}{4P^2} \right)^2 + \frac{\sigma \sqrt{12P}}{2P^2} \cdot K} \right\}$$

A P P E N D I X 2

Program listings

```
DISCRETE SYSTEM SYS
TIME T
TSAMP TS
INPUT E1 E2 C U
OUTPUT Y
STATE X
NEW NX
TS=T+DT
DT:1
Y=(X-C)*(X-C)+S1*E1
DYNAMICS
NX=X+U+S2*E2
S1:1
S2:0
END
```

- -

```
DISCRETE SYSTEM REG
TIME T
TSAMP TS
INPUT YI
OUTPUT UT
STATE YO UO XH P
NEW NYO NUO NXH NP
TS=T+DT
DT:1
EPS=YI-YO-UO*UO-R-2*UO*XH
SIG2=IF SIGV<0.5 THEN SIG*SIG ELSE SIG*SIG+2*R*R+4*R*P
HP=P/(SIG2+4*P*UO*UO)
K=2*UO*HP
UL=XH+UO+K*EPS
PT=SIG2*HP+R
SG2=SIG*SIG+2*R*R+4*R*PT
KA=ABS(UL)
KMX=SQRT(SG2/12/PT)
HL1=SG2+4*KA*KA*PT
DU=4*KA*PT*PT*SG2/HL1/HL1
HL2=SG2+4*PT*(KA+DU)*(KA+DU)
DF=8*PT*PT*SG2*(KA+DU)/HL2/HL2
U2=IF KA>KMX THEN KA+2*DU*DU/(4*DU-DF) ELSE 0
BETA=SG2/4/PT/PT
U3=KMX*(1-BETA+SQRT((1-BETA)*(1-BETA)+2*BETA*KA/KMX))
U4=IF KA>KMX THEN -SIGN(UL)*U2 ELSE -SIGN(UL)*U3
UT=DELT*SIN(2*T)+ (IF REG<1.5 THEN -UL ELSE U4)
DYNAMICS
NXH=UL
NYO=YI
NUO=UT
NP=PT
P:10
R:0
SIG:1
DELT:0
REG:0
SIGV:1
END
```

```
DISCRETE SYSTEM LOSS
TIME T
TSAMP TS
INPUT VI
STATE VOL
NEW NVOL
TS=T+DT
DT:1
V=VOL+VI
VSTEP=IF T>0.5 THEN V/T ELSE 0
VS=VSU+VSTEP
VSS=VSSU+VSTEP*VSTEP
VSU:0
VSSU:0
DYNAMICS
NVOL=V
END
```

```
CONNECTING SYSTEM CON1
TIME T
E1[SYS]=E1[NOIS1]
E2[SYS]=E2[NOIS1]
C[SYS]=IF T<T0 THEN NIV1 ELSE NIV2
T0:200
NIV1:0
NIV2:0
YI[REG]=Y[SYS]
U[SYS]=UT[REG]
VI[LOSS]=Y[SYS]
END
```

```
MACRO START
LET N.NOIS1=2
LET NODD.NOIS1=19
SYST SYS REG LOSS NOIS1 CON1
PLOT
AXES
INIT U0:-1
PAR S1:0
PAR S2:1
PAR R:1
PAR SIG:0
END
```

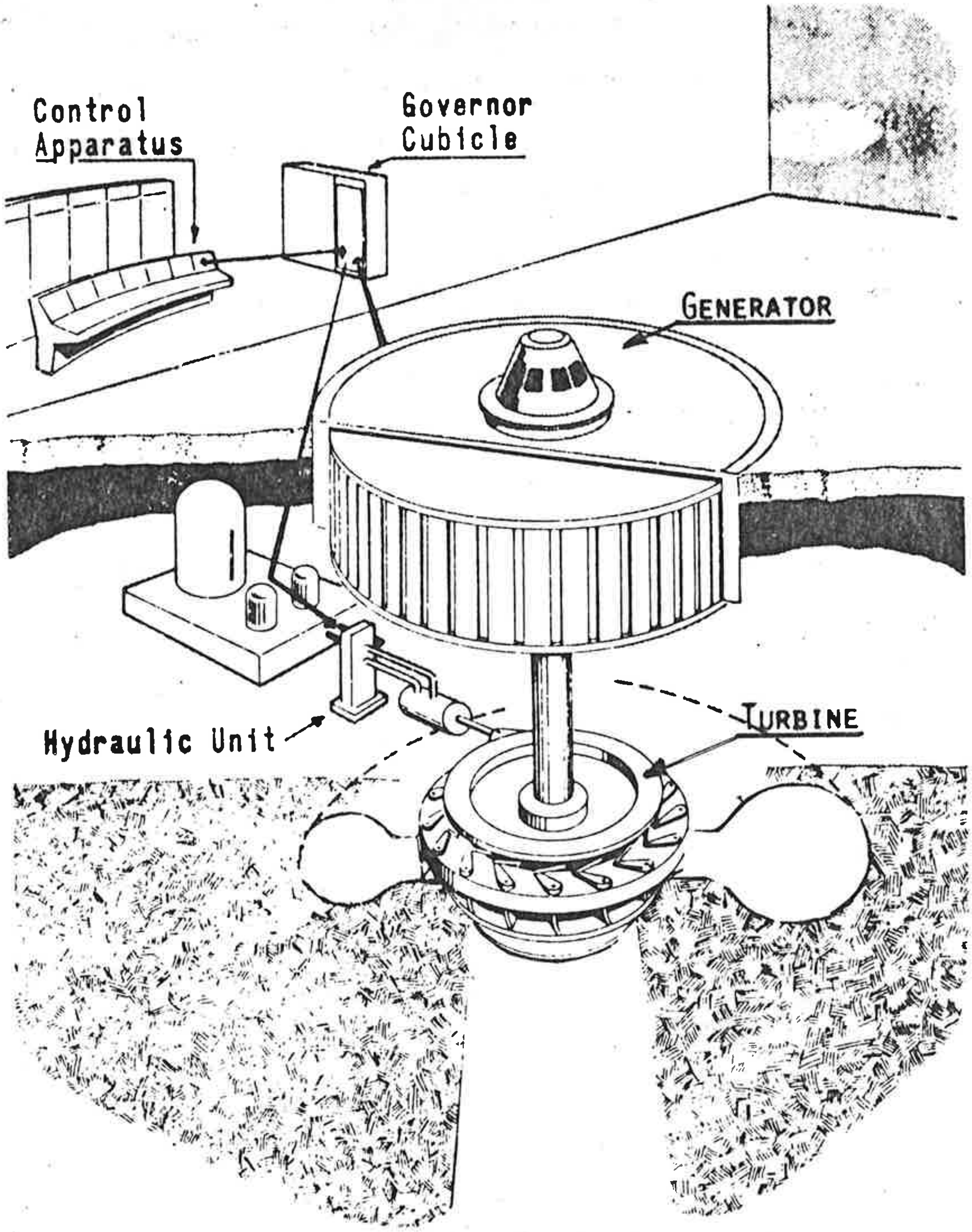
APPENDIX 3

Overhead pictures

Adaptive Control of Extremum Systems

Jan Sternby
Dept. of Automatic Control
Lund Institute of Technology
LUND, Sweden

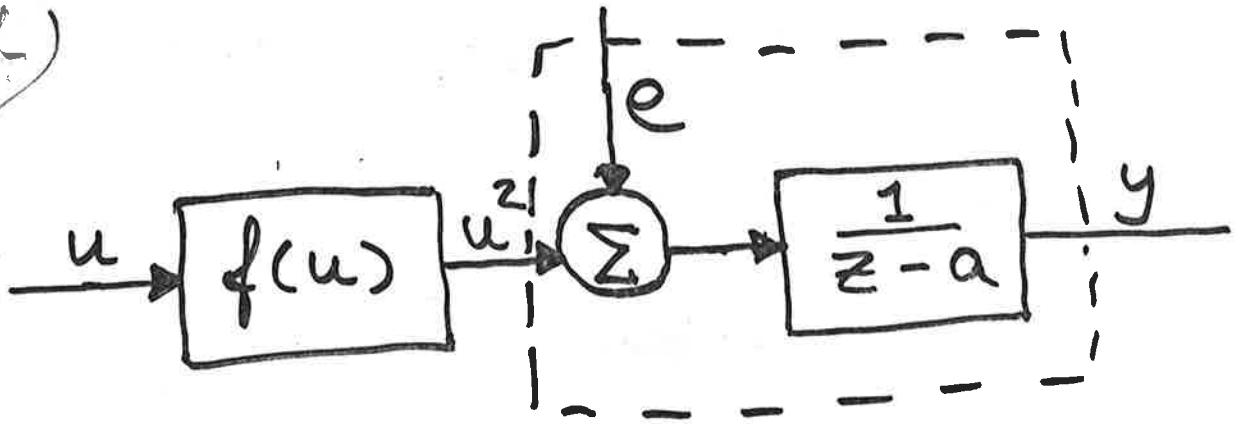
WATER TURBINE



* Introduction
* Two Models
* Input Nonlinearity
* Output Nonlinearity
* Conclusions

Example:

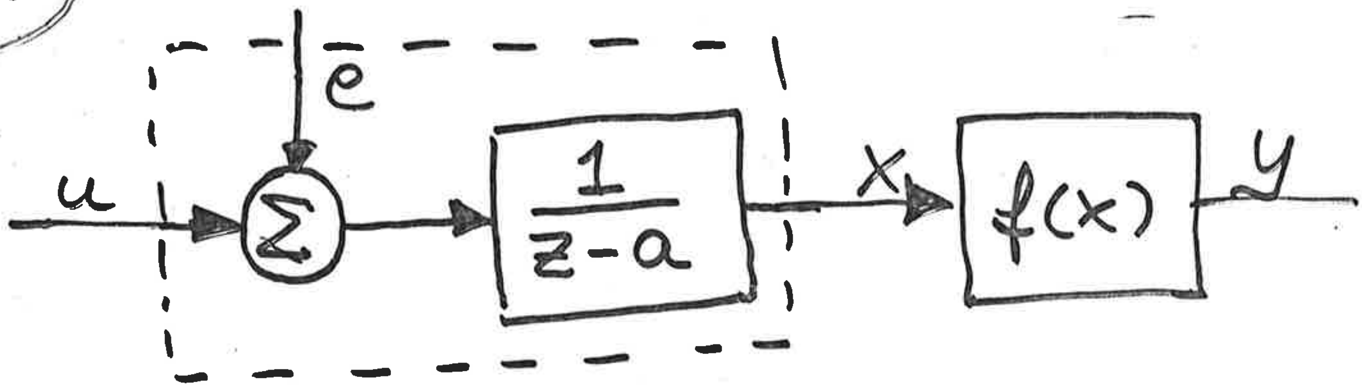
i)



$$y_{t+1} = a y_t + u_t^2 + e_t$$

$$Ey = \frac{Eu^2}{1-a} \Rightarrow \boxed{u_{opt} \equiv 0}$$

ii)



$$x_{t+1} = a x_t + u_t + e_t$$

$$y_t = x_t^2$$

$\boxed{u_{opt} \quad \text{complicated!}}$

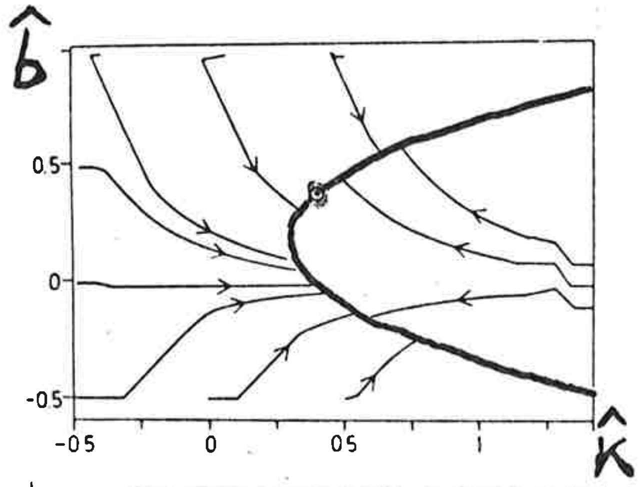
INPUT NONLINEARITY

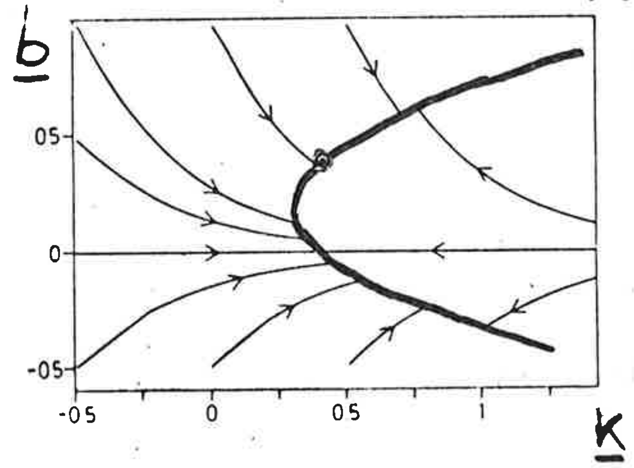
$$y(t) = k + b\,u(t-1) + c\,u(t-1)^2 + e(t)$$

$$u_{CE} = -\hat{b}/2\hat{c} = -\hat{b}/2c$$

Let

$$\hat{x} = [\hat{k} \quad \hat{b}]^T \qquad \theta = [1 \quad u]$$





ALGORITHM (SA):

$$\hat{x}(t) = \hat{x}(t-1) + \frac{1}{t}\,\theta(t)^T \eta(t)$$

ANALYSIS:

$$\dot{\underline{x}} = E(\underline{\theta}^T \cdot \underline{\eta})$$

OUTPUT NONLINEARITY

$$x(t+1) = x(t) + u(t) + w(t+1)$$

$$y(t) = x(t)^2 + v(t)$$

$$= y(t-1) + u(t-1)^2 + R$$

$$+ \underline{2u(t-1)\,x(t-1)}$$

$$+\Delta v(t) + (w(t)^2 - R) + 2w(t)\left[x(t-1) + u(t-1)\right]$$

LS - estimation of x!

Control:

Minimize $E \sum y(k)$

$$u_{one-step}(t) = \boxed{-\hat{x}(t)} = u_{CE}(t)$$

$$u_{two\text{-}step}(t) = \arg\min \left\{ [u(t) + \hat{x}(t)]^2 + \sigma(t)^2 P(t) / [\sigma(t)^2 + 4P(t)u(t)^2] \right\}$$

Approximation!

Simulations:

$R = 1 \qquad c = 0 \qquad v(t) \equiv 0$