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ABSTRACT

The problem of design of simple self-tuning controllers is discussed. The basic idea is to estimate a low order model and to use pole-placement in order to obtain a desired closed loop performance. The controller has a three mode action and can be regarded as a generalized PID-controller. It is shown that it is possible to obtain a controller with only one tuning knob. This knob can be calibrated in the desired bandwidth of the closed loop system. Simulated examples as well as an experiment on a laboratory process illustrates the properties of the controller.

1. INTRODUCTION

One of the advantages of the well-known PID-controller is that it is a sufficiently flexible controller for many applications. The three parameters of the controller are generally tuned with the process in closed loop. The tuning is often easy. It may, however, be cases when tuning is difficult and time-consuming. Automatic tuning of the controllers is therefore of interest. The idea of self-tuning regulators was introduced in order to simplify the tuning of industrial controllers. The self-tuning regulators have, however, also tuning parameters. It can thus be said that one set of tuning parameters has been replaced by an other set. Hopefully the new parameters are easier to choose. In the early applications (Åström et al 1977), the self-tuners were applied to special problems. Good rules for choosing the parameters could then be found, Wittenmark (1973). Some parameters are, however, critical. For instance for self-tuners based on minimum variance control and least squares parameter estimation, it is crucial to have an upper bound on the time delay of the process.

The suitable parameterization of a self-tuner has been discussed widely. It has been suggested that there should be no adjustable parameters at all. A moment of reflexion shows that it is at least necessary to provide the controller with information about the desired specifications. The main idea is that the parameters selected by the operator should be related to the desired performance of the closed loop system. Such parameters are easier to choose than to choose parameters in the control law.

This paper describes a simple self-tuner intended for simple servo applications. It is assumed that the process can be described by a second order model. The regulator is based on recursive least squares estimation and pole-placement design, see Åström and Wittenmark (1979). The tuning parameters are the bandwidth of the closed loop system and possibly also the desired relative damping. In the paper it is only possible to give a brief description of the algorithm and its properties. Further details about simple self-tuners can be found in Wittenmark (1979) and Åström (1979c).

2. ALGORITHM DESIGN

The simple self-tuner is intended to solve simple servo problems for system which can be described by low order models. It is natural to characterize the performance of the servo by the bandwidth and the relative damping of the closed loop system. A servo problem is conveniently formulated as a pole-placement problem. It is then natural to use the formulation of self-tuning servos discussed in Åström and Wittenmark (1979). Since the low frequency properties of a system often can be approximated by a low order model it can be expected that a self-tuner based on a low order model will behave satisfactorily provided that the chosen bandwidth is sufficiently small, see Åström (1979a).

Problem formulation

Assume that the process can be described by the model

$$y(t) + a_1 y(t-h) + a_2 y(t-2h) = b_1 u(t-h) + b_2 u(t-2h) + b_3 \tag{2.1}$$

where h is the sampling time and b_3 is a bias. Introduce the polynomials $A(q^{-1}) = 1+a_1q^{-1} + a_2q^{-2}$ and $B(q^{-1}) = b_1 + b_2q^{-1}$ where q^{-1} is the backward shift operator.

The problem can be formulated as to find a feedback such that the closed loop system has poles that corresponds to the poles of a continuous time system with the characteristic polynomial $s^2+2\zeta\omega s+\omega^2$. For a sampled data system this means that the characteristic polynomial should be

$$P(q^{-1}) = 1 + p_1q^{-1} + p_2q^{-2} \tag{2.2}$$

where

$$p_1 = -2e^{-\zeta\omega h} \cos \omega h\sqrt{1-\zeta^2}$$

$$p_2 = e^{-2\zeta\omega h}$$

The process model (2.1) has a zero at $z = -b_2/b_1$. If this corresponds to a well damped mode the factor $b_1 + b_2q^{-1}$ can be cancelled by the regulator. This will be the case if

$$z_1 \leq -b_2/b_1 \leq z_2 \tag{2.3}$$

where the choice of z_1 and z_2 is discussed in Section 3. The desired closed loop response is then characterized by the pulse transfer function

$$G_d = \frac{q^{-1}(1 + p_1 + p_2)}{1 + p_1 q^{-1} + p_2 q^{-2}} \tag{2.4}$$

If the process zero corresponds to an unstable or poorly damped mode the zero cannot be cancelled and the desired pulse transfer function is instead

$$G_d = \frac{1 + p_1 + p_2}{b_1 + b_2} \cdot \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + p_1 q^{-1} + p_2 q^{-2}} \tag{2.5}$$

Control design for known parameters

The calculation of the control law when the process model is known is straight forward, see e.g. Åström (1979b). The control law is given by

$$Ru(t) = Ty_r(t) - Sy(t) \tag{2.6}$$

where y_r is the reference signal and R, S and T are polynomials in the backward shift operator, q^{-1}. In order to eliminate the bias term we assume that $R = R_1(1-q^{-1})$, i.e. there is an integrator in the controller. Other ways to eliminate the bias are discussed in Section 3. In order to treat the two cases above simultaneously we introduce

$$P'(q^{-1}) = \begin{cases} P(q^{-1})(1 + b_2/b_1 q^{-1}) & \text{if } z_1 \leq -b_2/b_1 \leq z_2 \\ P(q^{-1}) & \text{otherwise} \end{cases}$$

The control law is obtained by solving the polynomial equation

$$AR_1(1 - q^{-1}) + q^{-1}BS = P' \tag{2.7}$$

where R_1 and S are of order 1 and 2 respectively. The identity (2.7) has a unique solution provided $A(1-q^{-1})$ and B do not have a common factor. The correct steady state gain is obtained if we choose

$$T(q^{-1}) = S(1) = s_0 + s_1 + s_2 \tag{2.8}$$

The controller has four parameters, the coefficients of the polynomials $R_1=1+r_1q^{-1}$ and $S=s_0+s_1q^{-1}+s_2q^{-2}$. The closed loop system obtained when (2.6) is used will be

$$y(t) = \frac{q^{-1}TB}{AR+q^{-1}BS} y_r(t) + \frac{R\, b_3}{AR+q^{-1}BS} = \frac{q^{-1}TB}{P'} y_r(t) + \frac{R\, b_3}{P'} \tag{2.9}$$

The system will have the desired transfer function (2.4) or (2.5). Further if y_r is constant $y(t) \to y_r$ as $t \to \infty$.

Common factors in the process model

The polynomials A and B have a common facor if

$$T_{cf} = b_2^2 - a_1 b_1 b_2 + a_2 b_1^2 = 0$$

and B will contain the factor $1 - q^{-1}$ if $B(1) = 0$. If there is a almost common factor the solution of (2.7) will be poorly conditioned and that may result in very large control signals. To get dimension free test quantities the following test is used

$$T_{cf} \text{ or } (b_1 + b_2)^2 \leq \varepsilon \max(b_1^2, b_2^2) \tag{2.10}$$

to test for common or nearly common factor. The number ε is related to the maximum size of the feedback gain. When cancelling a common factor the transfer function of the process will be reduced to $b/(1+aq^{-1})$ where $b=b_1$ and $a=a_2 b_1/b_2 = a_1 - b_2/b_1$. The identity (2.7) can now be solved if R_1 and S both are of first order or if $R_1=1$ and S is of second order.

The sampling time

The desired performance of the closed loop system is determined by the damping, ζ, and the bandwidth or equivalently the natural frequency ω. It is then natural to have the sampling time inversely proportional to the bandwidth. A reasonable choice is

$$h = \frac{2\pi}{N\omega\sqrt{1-\zeta^2}} \tag{2.11}$$

where N is the number of samples per period. The choice of N is discussed in Åström (1979c). It is found that a reasonable choice is N=10-20. Further if we assume that the damping is $\zeta=1/\sqrt{2}$ then the sampling time should be choosen as

$$\omega h \approx 0.45 - 0.9.$$

If the parameter ω is changed during an experiment then the sampling time also changes. This will then influence the values of the parameters in the model (2.1).

The estimator in the self-tuner will of course adjust to these changes. It is, in principle, easy to compute how the model is changed. This can be done by transforming the model to a continuous time system and then sample this system with the new sampling time. It is, however, possible that the estimated model does not have a continuous time counterpart. A simplified method is to approximate $z=\exp(sh)$ as $z=1+sh$. Simple calculations will lead to a transformation which relates the parameters of the model for different sampling times, see Åström (1979c).

Estimation procedure

A self-tuning controller contains a parameter estimator. In this case a recursive least squares estimation with exponential forgetting of old data is used. The controller discussed here contains an integrator. This implies that the bias term b_3 in (2.1) does not need to be estimated. The other parameters are estimated from the differences of the inputs and outputs, i.e. using $u(t)-u(t-h)$ and $y(t)-y(t-h)$ respectively.

A simplified self-tuning controller

The discussion above can now be summarized into the following algorithm, where Steps 1-4 are repeated at each sampling time.

Data: The operator selects ω and ζ which determines the closed loop characteristic polynomial. The sampling time is choosen according to (2.11).

Step 1: Estimation. The parameters a_1, a_2, b_1 and b_2 in the process model are estimated. The previous estimates are transformed if the sampling time has been changed.

Step 2: Test of the model. Common or nearly common pole and zero are removed using the test (2.10). The desired characteristic polynomial P' is determined based on the test (2.3).

Step 3: Controller parameter determination. The parameters of the controller are determined by solving the polynomial equation (2.7) and using (2.8).

Step 4: Control. The control signal is determined from

$$u(t)=\mathrm{sat}[t_0y_r(t)-s_0y(t)-s_1y(t-h)-s_2y(t-2h)+(1-r_1)u(t-h)+r_1u(t-2h)]$$

to avoid saturation and reset windup.

3. DISCUSSION OF THE ALGORITHM

The algorithm presented in the previous section contains some parameters that have to be determined. This together with a discussion of the properties of the algorithm are given in this section.

Choice of parameters

The choice of the parameters in the algorithm is discussed and examplified in Åström (1979c). Some nominal values that can be used are given below. The initial values in the estimator can be $\hat{a}_1(0)=-1.5$, $\hat{a}_2(0)=0.7$, $\hat{b}_1(0)=0.1$ and $\hat{b}_2(0)=0$. The inital covariance matrix in the estimator can be 100 times a unit matrix and the exponential forgetting factor approximately 0.95-0.99. If it is desirable to have only one tuning parameter the damping could be fixed to $\zeta=0.7$. A reasonable value of ε in (2.10) is 0.01. The zero of the process may be removed if $(z_1,z_2)=(-0.1, 0.99)$. If a smaller value of z_1 is used the control signal usually starts to oscillate. The values given above are reasonable rules of thumb values. It has been found in simulations that none of the values are very critical. The inital estimates of $\hat{b}_1(0)$ and $\hat{b}_2(0)$ will, however, have crucial influence on the initial transient.

Reset action

There are several ways to eliminate steady state errors due to bias or load disturbances. The way used here is to postulate that the controller has an integrator. In Åström (1979c) it is shown that the parameter estimator can take care of the bias automatically. This will in general give unsymmetrical responses for positive and negative steps. A third way is to estimate the bias b_3 in the model (2.1) and compensate for it. It has been found advantageous to have a smaller forgetting factor for the bias than for the dynamic parameters. Finally, the bias can be eliminated by having a self-tuning controller in an inner loop and a fixed integral controller in an outer loop. All methods have been investigated through simulations and there are no drastic differences in the performances.

Interpretation of the controller

The controller (2.6) with the number of parameters used here can be interpreted as a PID-controller with a special structure. Consider the PID-controller given by

$$u(t) = \frac{\alpha_1}{(1-q^{-1})R_1}(y_r(t) - y(t)) - \alpha_0 y(t) - \frac{\beta(1-q^{-1})}{R_1}y(t)$$

The three terms on the right hand side are the integral, proportional and derivative parts. The factor R_1 can be interpreted as the filter that should be used to obtain the derivative. Notice that the proportional and derivative parts only works on the output and not on the error. The controller can be written as

$$(1-q^{-1})R_1\, u(t) = \alpha_1\, y_r(t) - [\alpha_1 + (1-q^{-1})(\alpha_0 R_1 + \beta(1-q^{-1}))]y(t)$$

This controller has exactly the same structure as (2.6) and the parameters α_0, α_1 and β can be obtained from the parameters s_0, s_1 and s_2 if $r_1 \neq -1$. This and other structures for self-tuning PID-controllers are discussed in Wittenmark (1979).

Higher order processes

The discussed self-tuning controller will work well if the process can be well approximated by the second order model (2.1) and if the desired bandwidth is not too large. In Åström (1979a) results are given which show that the closed loop system designed on the basis of an approximative model will be stable if the desired bandwidth of the closed loop system is sufficiently small.

The tuning rule for the regulator is thus very simple. Start with a small bandwidth. Establish the possible range of bandwidths for which the regulator will work by increasing the specified bandwidth until the performance deterioates. If the desired bandwidth is outside the range found it is necessary to use a more complex regulator or to change the specifications. Tuning is simple because it involves only one parameter.

4. EXAMPLES

Three examples will be given which will illustrate some of the properties of the simple self-tuner. The first two examples are simulations while the the third is level control of a laboratory process

Example 4.1 Second order system

The system

$$G(s) = \frac{1}{(s+1)^2}$$

is controlled with the self-tuner described in the previous sections. The specifications are $\omega = 1.5$ and $\zeta = 1/\sqrt{2}$. Fig.1 shows the output, the reference value, and the control signal. Already at the second step there is a good agreement between the desired output and the process output. The first transient will of course depend on the chosen initial values in the estimator. For $t \geq 15$ a load disturbance $v = 1$ is added to the input of the process. The controller eliminates the effect of the disturbance.

Example 4.2 Fourth order system

The system has the transfer function

$$G(s) = \frac{1}{(s+1)^4}$$

In this case it is more difficult to find a good second order approximation of the process. The desired bandwidth has to be chosen quite small. Fig. 2 shows the output and the control signal at a step in the reference signal when the estimator has converged. For $\omega = 0.3$ the control is good. The behaviour starts to deteriorate when ω is increased to 0.4 and further to 0.45. In all three cases the desired damping has been 0.7.

Example 4.3 Level control

One variant of the simple self-tuner has been implemented on a LSI-11 computer. The communication with the operator is done through commands. The different parameters in the controller can be easily changed on-line. The controller and operator communication is written in Pascal. Further details about the implementation is given in Wittenmark, Hagander and Gustavsson (1980).

As an example the controller has been used to control a laboratory process consisting of a pneumatic valve and a small water tank. The position of the valve is the control signal and the output is the level in the tank. Fig. 3 shows the level and the control signal when the reference level is changed in steps about each 45 second. Each step is about 10 % of the maximum level which is 0.5 m. The specifications where $\omega = 0.45$ and $\zeta = 1$ and the sampling time was 1 s. From the figure it can be seen that the controller gives the same response over the whole range of levels. This is not possible with a fixed controller. The parameters in the controller changed about 20 - 40 % going from the minimum to the maximum level.

5. CONCLUSIONS

The report presents a simple self-tuner for typical servo problems. The self-tuner has one major adjustable parameter which is proportional to the desired bandwidth of the closed loop system. All other parameters are fixed or related to the bandwidth. It is shown by simulations that the algorithm works well in many circumstances. However, the simple self-tuning controller which can be interpreted as a PID-controller cannot control all processes. It can only behave as a well tuned PID-controller. It is thus possible to use the self-tuner on the same type of processes as the conventional PID-controller can be used on. Many common processes in practice belong to this class and manual tuning can thus often be eliminated.

6. ACKNOWLEDGEMENTS

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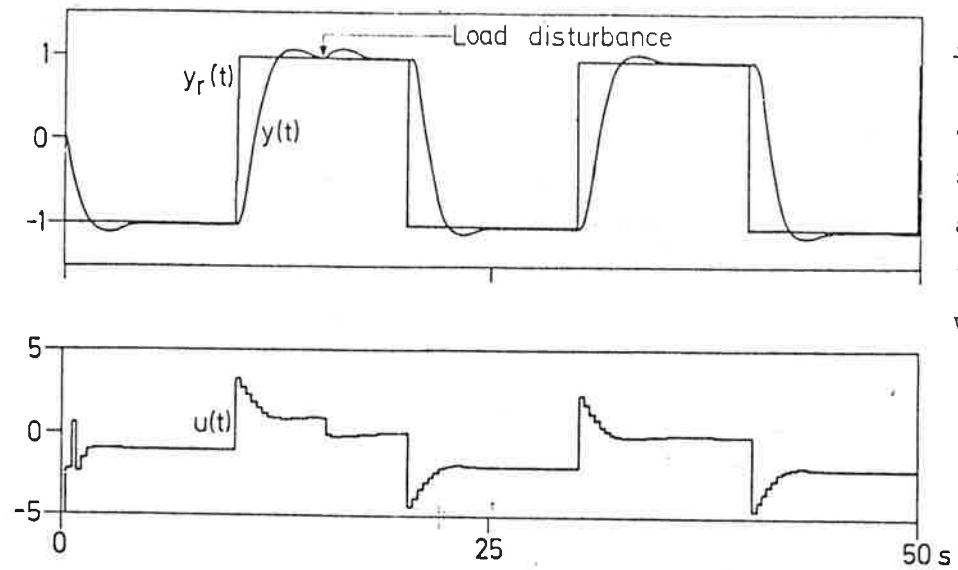


Fig 1. Output and control signals when controlling a second order system. A step load of size 1 is applied at the input at t=15. The specifications were ω=1.5 and ζ=0.7.

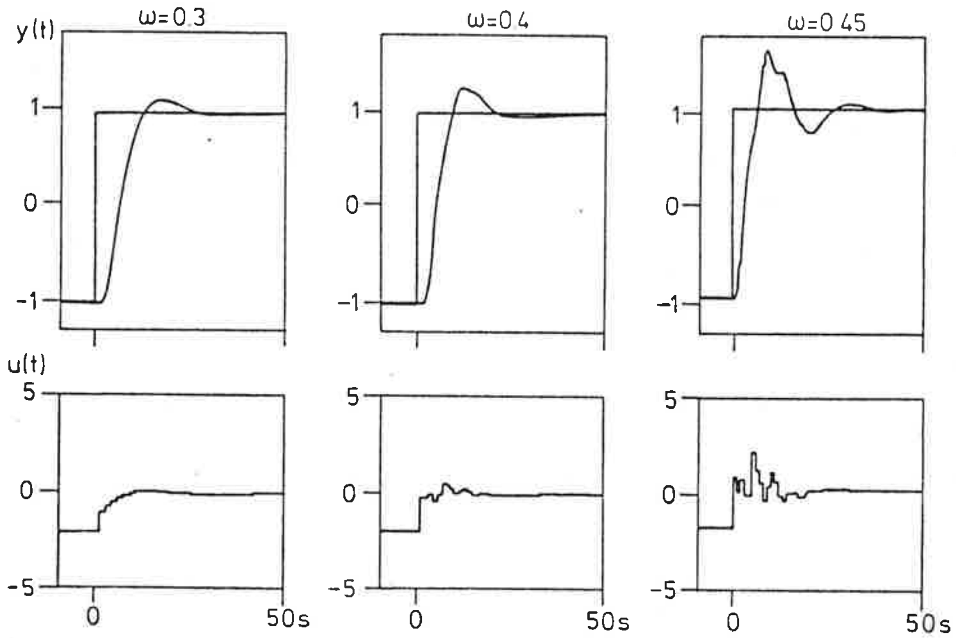


Fig 2. The output and the control signals for a fourth order system when ω=0.3, 0.4 and 0.45. For ω=0.3 the control is good but the performance starts to deteriorate when ω is increased.

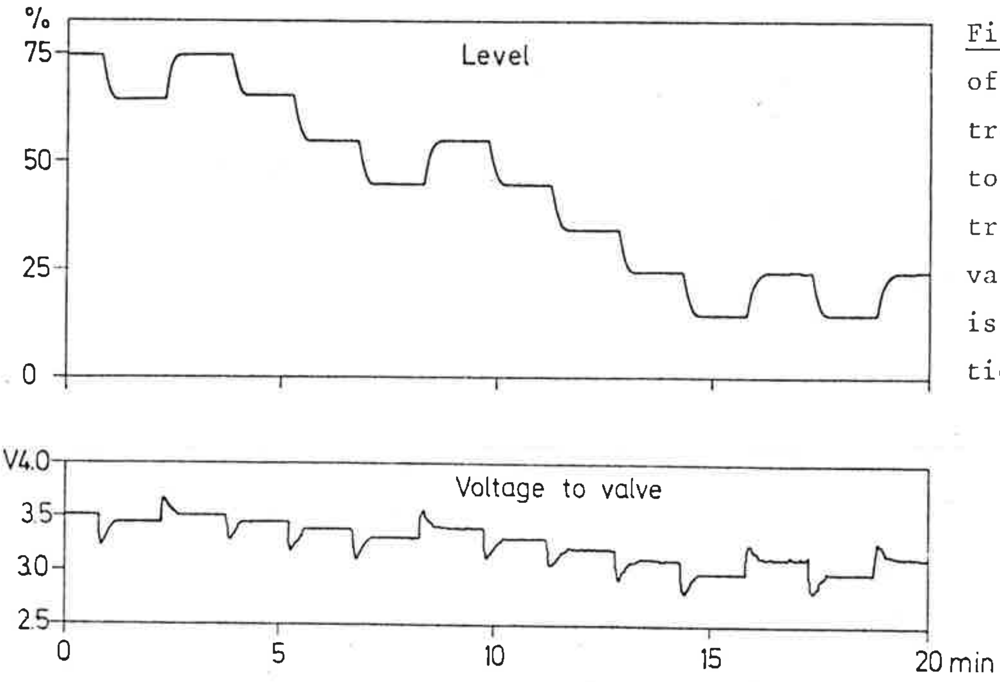


Fig 3. Water level control of a small tank. The control signal is the voltage to a voltage/pressure transducer of a pneumatic valve. The reference value is changed over the operational range of the tank.

SIMPLE SELF-TUNING CONTROLLERS

B. WITTENMARK
K.J. ÅSTRÖM

BACKGROUND

THE IDEA

CONTROLLER DESIGN

ALGORITHM

EXAMPLES

DISCUSSION

THE IDEA

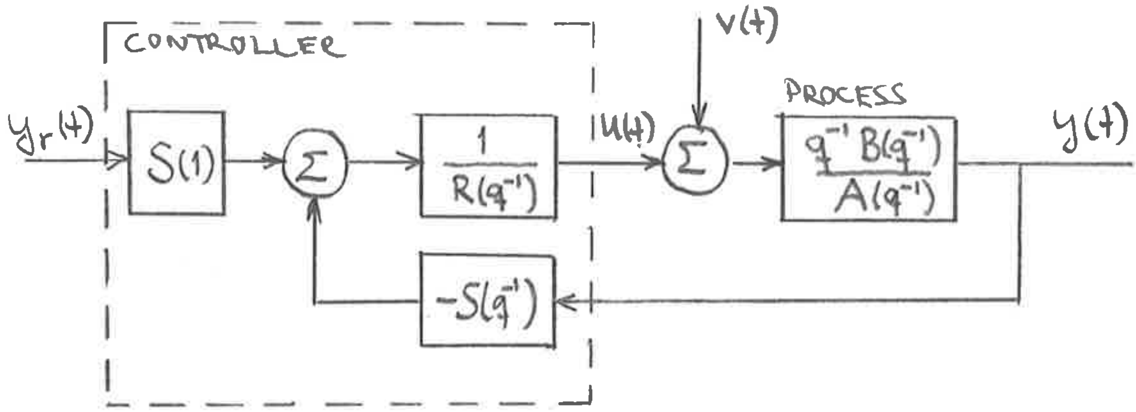
RESET ACTION

CLOSED LOOP BANDWIDTH AND DAMPING

LOW ORDER MODEL

POLEPLACEMENT

CONTROLLER DESIGN



$$y(t) = \frac{S(1)B}{AR + q^{-1}BS}\, y_r(t-1) + \frac{BR}{AR+q^{-1}BS}\, v(t-1)$$

→ POLE PLACEMENT

$$AR + q^{-1}BS = P$$

→ $$\frac{1}{s^2 + 2\zeta\omega s + \omega^2} \xrightarrow{h} \frac{\sim}{1 + p_1 q^{-1} + p_2 q^{-2}}$$

→ $\omega h \approx 0.45 - 0.9$

$$\overbrace{(1-q^{-1})(1+r_1 q^{-1})}^{R(q^{-1})} u(t) = \overbrace{(s_0 + s_1 + s_2)}^{S(1)} y_r(t) - \overbrace{(s_0 + s_1 q^{-1} + s_2 q^{-2})}^{S(q^{-1})} y(t)$$

THE ALGORITHM

DATA: $\omega, \xi \Rightarrow h$

ESTIMATION LS

$$y(t) + a_1 y(t-h) + a_2 y(t-2h) =$$
$$= b_1 u(t-h) + b_2 u(t-2h) + b_3$$

OR DIFFERENCES

TEST

- COMMON POLE - ZERO
- WELL DAMPED ZERO

PARAMETER DETERMINATION

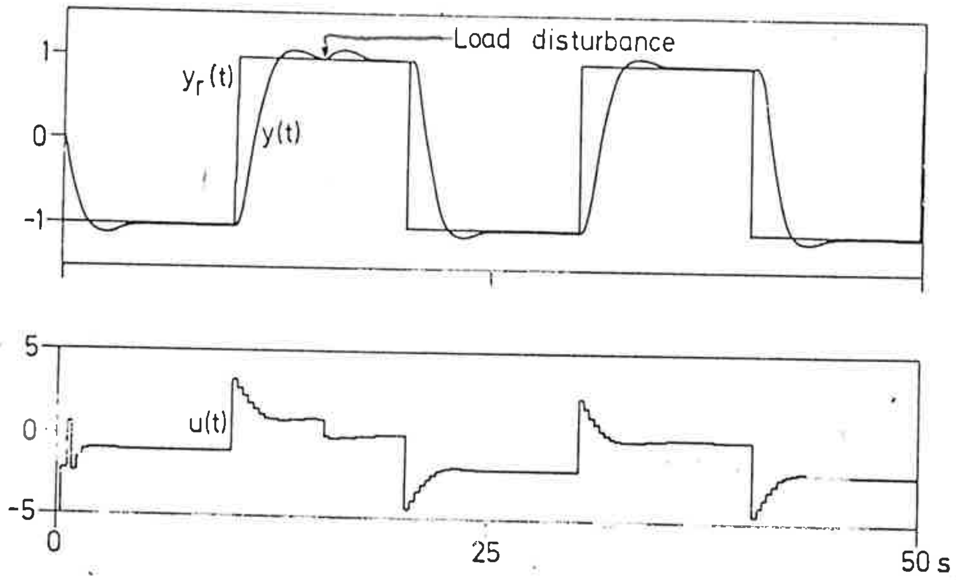
$$AR + q^{-1} BS = P'$$

CONTROL

$$u(t) = \mathrm{SAT}\Big[t_0 y_r(t) - s_0 y(t) - s_1 y(t-h) - s_2 y(t-2h)$$
$$+ (1-r_1) u(t-h) + r_1 u(t-2h)\Big]$$

EXAMPLE 1

$$G(s) = \frac{1}{(s+1)^2} \qquad \omega = 1.5 \quad \zeta = 0.7$$



IMPLEMENTATION

- LSI-11
- PASCAL

  FORGROUND

  BACKGROUND

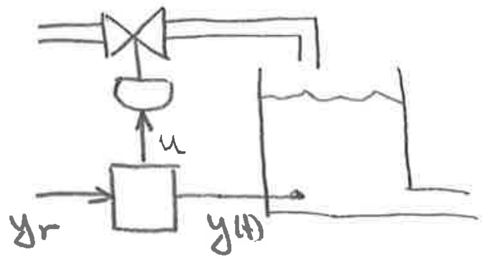
  2-3 MANWEEKS

- MEMORY REQUIREMENT

REGULATOR	4.9 kB
COMMUNICATION	8.5 kB
LIBRARY	5.1 kB

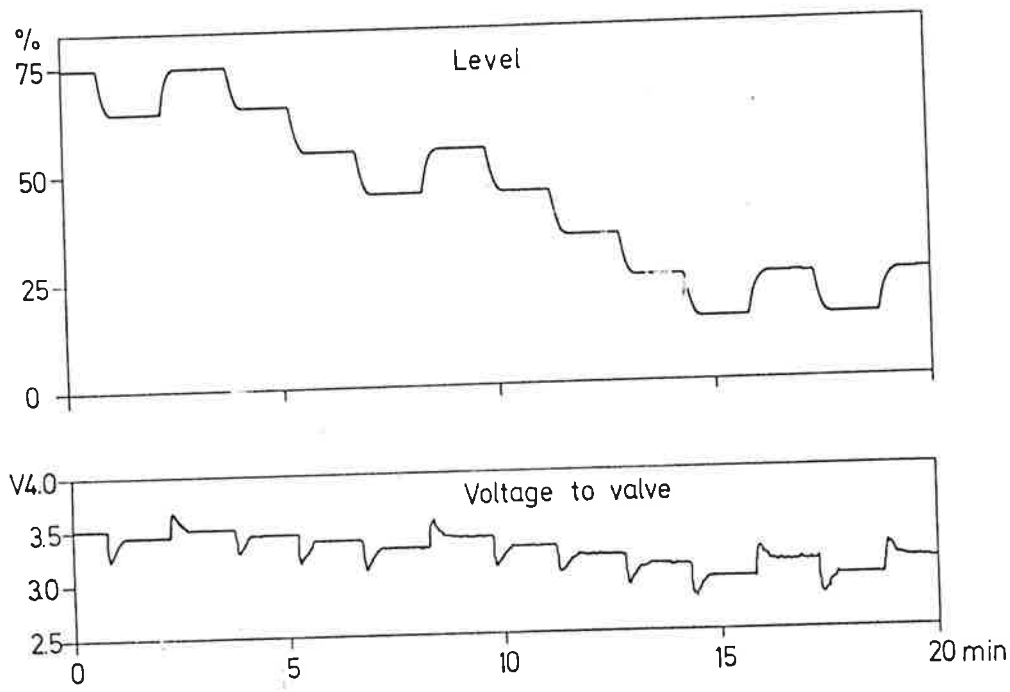
- COMMAND DRIVEN

EXAMPLE 2 LEVEL CONTROL



$\omega = 1$

$\xi = 1.$



20 - 40% CHANGE IN CONTROLLER PARAMETERS

DISCUSSION

PARAMETERIZATION $\quad \zeta \quad \omega$

RESET ACTION $\quad R = (1 - q^{-1}) R_1$

PID - INTERPRETATION