A Modular Simulation Model for a Wind Turbine System

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Abstract

A mathematical simulation model for a large horizontal axis wind turbine system is presented. The model is intended for simulation of the synchronization of the wind turbine generator against the utility grid and the operation of the wind turbine system under different wind conditions and with different control algorithms.

Particular attention has been given to the modularization. The model is divided into subsystems to make it easy to modify the model and adapt it to systems of similar type. The interactive simulation package SIMNON which allows good structuring and programming in a high level language has been used.

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Introduction

A Wind Turbine System (WTS) differs conceptually from a conventional power production unit in the sense that the power source cannot be controlled in any way. If the available power is below the rated value, the system should be operated to extract maximum energy. Normally this is achieved by letting the blade angle be a static function of the wind and rotor speed. When the wind increases further, protection from over-power is made by flapping and modulation of the blade angle.

However, as the wind is of a stochastic nature and a WTS is a dynamically soft system, excitation of poorly damped modes are probable. In order to extract the maximum energy from the fluctuating wind and to minimize the dynamic loads on the WTS, a fast and well-tuned control system is required.

The WTS designer therefore includes the dynamic performance in the construction work and a great part of the design is to simulate system behaviour under different conditions. On the other hand, simulation is also good for design-verification and functional analysis work made e.g. by the customers. The need for good dynamical models and simulation systems is thus obvious.

The Goal of the Study

The goal of this work was to develop a mainframe for a simulation package of a general Wind Turbine System, based on an existing and well-proven simulation package (SIMNON). The simulation system should be good for design-verification and failure investigation in connection with faults in the electrical network. Furthermore, the goal was also to develop a simulator for educational purposes and future design of quite new controllers, like e.g. adaptive ones.

The plant to be considered in this study is a 3 MW wind turbine system, WTS-3, which now is built near the city of Trelleborg, in southern Sweden, by Karlskronavarvet AB, Sweden and Hamilton Standard, a division of United Technologies Inc, USA. A 4 MW plant (WTS-4) of the same design is built in Medicine Bow, Wyoming, USA.

A complex dynamical model must be validated in order to get a quality mark. As the physical system does not exist in the real world at present, the only validation so far has been made by simulation comparisons from runs made by the WTS designer. During the full-scale testing period 1982-1985, system identification and model validation will be done.

The Wind Turbine System - WTS-3

The wind turbine system WTS-3 is designed to supply power in parallel with other electrical generators to a large power utility grid and to operate in wind forces of 5-26 m/s. The rated power 3 MW is reached at 14 m/s.

WTS-3 has a horizontal axis wind turbine with two teetered blades. The rotor drives a 3-phase synchronous generator through a gearbox. The power is controlled by changing the pitch angle. The blade actuators are hydraulic positioning systems.

The tower is a 78 m high and 3.8 m wide cylindrical steel shell. The nacelle and rotor can turn freely around the axis of the tower. The rotor is aligned by an active yaw mechanism of the tower against the wind, downwind of the tower. The blades lean downwind at an angle of 6° and are designed as a monolithic base structure of epoxy plastic reinforced with glass fiber. They have a length of 39 m, a maximum corda of nearly 5 m and a weight of 14 tons each.

The generator is a 3-phase, 50 Hz, 1500 r.p.m synchronous machine. The rotor rotates at 25 r.p.m. and a multi-stage planetary gearbox steps up the rotation to 1500 r.p.m. A torsionally soft mounting including hydraulic dampers of the gearbox provides a torsionally soft connection. This soft connection compensates for rapid variations in rotor speed, since the generator must operate at constant r.p.m.

The control system consists of three processors, called supervisory controller, rotor controller and interface controller. The supervisory controller supervises the system and decides mode of operation. Depending on the mode of operation, the rotor controller calculates a reference value to the hydraulic positioning system. The interface controller handles the operator communication. The three processors communicate with each other via serial lines.

SIMNON Overview

An interactive command driven simulation package has been developed at the Department of Automatic Control, Lund Institute of Technology. The package is called SIMNON (Simulation of Nonlinear Systems). A more complete description is found in Elmqvist (1975) [1]. The package has been used extensively since 1974 both for research, education and to solve industrial control and simulation problems.

In SIMNON the model can be described in a special high level model language or in Fortran. The model language is simple and easy to learn. The models are defined using state variable equations. The right hand sides of the differential

or difference equations are defined using simple assignment statements.

Often the system to be simulated can be divided into natural subsystems. A typical example is the process and a regulator. This should be used also in the simulation in order to simplify the programming. In SIMNON there are three different types of subsystems: continuous, discrete, and connecting. The first two types are used to define continuous time and discrete time systems respectively. The connecting subsystem defines, how different continuous and/or discrete subsystems are interconnected. The connecting subsystem can be said to define the block diagram, while the continuous and discrete subsystems define the blocks.

The models can easily be modified using a special editor. When the subsystems are defined, they are compiled and error checking is done. The compiler is included in the package and is working in parallel with the editor. This enables the user to correct erroneous lines immediately. For instance it is checked that all inputs and outputs are defined, and that all parameters are given values. When the total system has been compiled, it can be simulated and the results can be displayed, stored or printed.

The interactive package SIMNON is controlled by a small number of powerful commands. There are e.g. commands to change parameters of the model, perform simulation, plot the time response of selected variables on a display and modify the model. However, the structure of each command is flexible. The commands contain arguments which may be e.g. file names, variables, numbers and options. Arguments can in some cases be omitted. Default values are then used. One common situation when running the package is that the same sequence of commands is needed several times. The user can then define a MACRO containing the commands. The MACRO can then be used as a new command several times, possibly with different values of the arguments.

Loops and jumps can be used in a MACRO. It is even possible to make a MACRO which makes the dialog look like a question and answer program when executed. This possibility can e.g. be used to introduce SIMNON to new users.

The Model

The model is developed in a modularized, structured manner.
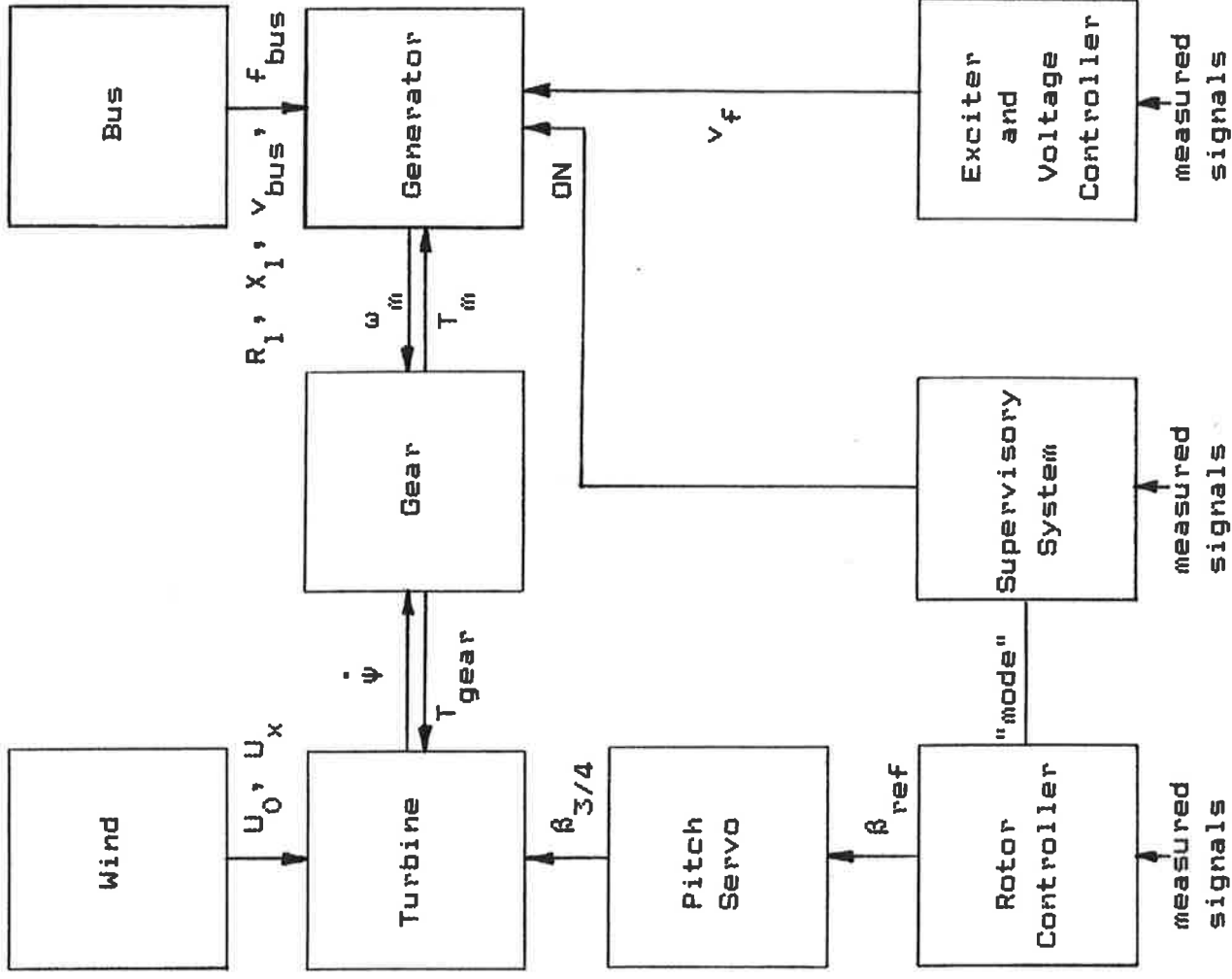The decomposition into subsystems is showed in Figure 1.

Figure 1: Model structure.

To increase the flexibility, the inputs and outputs are chosen as unscaled, physical quantities. SI-units are used. However, to improve the model numerically scaled quantities are used inside the subsystems. An overview of the model is given below; more detailed information can be found in Mattsson (1981) [2].

The Wind

It is assumed that the nacelle is aligned in the direction of the wind and that the wind profile is linear. Consequently, the yawing is not modelled. The wind can be modelled in different ways. A model for discrete longitudinal wind gusts is given by Hwang and Gilbert (1978) [3]. The framework of stochastic processes can be used to model turbulence. SIMNON provides noise generators. It is in SIMNON also possible to use measured wind data.

The Pitch Servo

A hydraulic servo system is used to vary the pitch angle. The servo is modelled as a first order system with limits on the rate. It is assumed that the blades are torsionally rigid and that the pitch servo is not affected by the wind. Friedmann (1976) [4] states that the blades in a typical wind turbine system are torsionally rigid. The first frequency of torsional mode is high (about 37 rad/s).

The Wind Turbine

Mechanical instabilities and vibrations may result from interaction of the flexible rotating blades with the base motions of the supporting tower. The motion of the nacelle in the thrusting direction is modelled. However, since this simulation model is not intended for studying the mechanical properties, it is assumed that the system is aerodynamically and mechanically well-designed, so higher modes of the vibrations can be neglected.

Equations of Motion. Consider Figure 2. Since the blades are teetered, the flapping angles satisfy

$$\varphi_1 + \varphi_2 = 2\varphi_0 = \text{constant} \qquad (1)$$

and it is possible to describe the flapping motion by one variable:

$$\varphi = (\varphi_1 - \varphi_2)/2 \qquad (2)$$

The motion in the ψ and φ directions assuming a rigid tower is modelled in Hultgren (1979) [5]. This model, extended with the motion in the z_T direction, will be used.
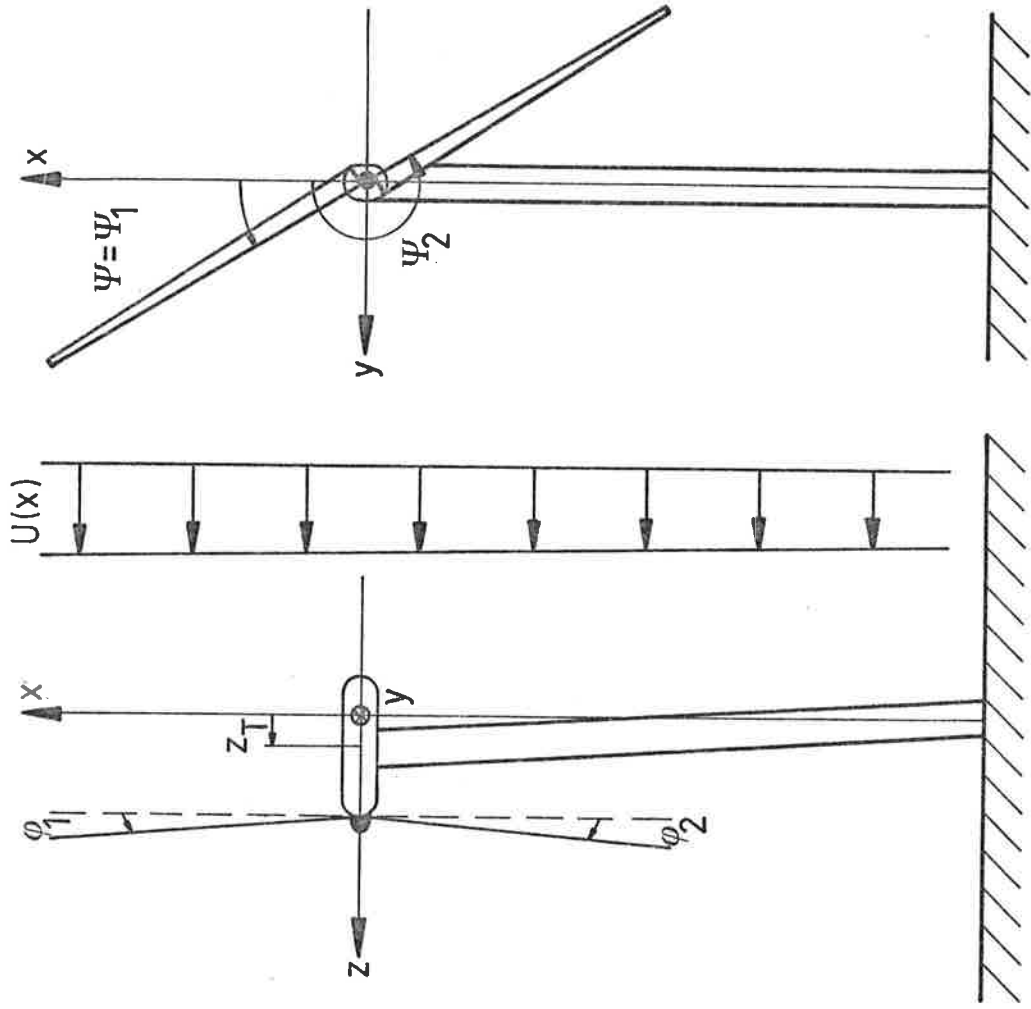
Figure 2: The wind turbine.

Applications of Lagrange's equations yield the following equations of motion

$$\{J_B(1 + \cos 2\varphi_0 \cos 2\varphi) + J_{hub} + J_{gear}\}\, \ddot{\psi}$$

$$- 2J_B \dot{\varphi}\, \dot{\psi} \cos 2\varphi_0 \sin 2\varphi + 2gS_B \sin \varphi_0 \sin \varphi \sin \psi$$

$$= T_{\psi 1} + T_{\psi 2} - T_{gear} \qquad (3)$$

$$2J_B \ddot{\varphi} - \{2S_B \sin \varphi_0 \sin \varphi\}\, \ddot{z}_T + J_B \dot{\psi}^2 \cos 2\varphi_0 \sin 2\varphi$$

$$- 2gS_B \sin \varphi_0 \cos \varphi \cos \psi = T_{\varphi 1} - T_{\varphi 2} \qquad (4)$$

$$\{-2S_B \sin\varphi_0 \sin\varphi\}\varphi + M_T \ddot{z}_T - 2S_B \dot{\varphi}^2 \sin\varphi_0 \cos\varphi = F_{zT1} + F_{zT2} + F_T \quad (5)$$

where

g   Gravitational acceleration

J_B Moment of inertia of one blade

J_{hub} Moment of inertia of the hub

J_{gear} Moment of inertia of the gear

S_B Static moment of one blade

M_T Mass of the nacelle

$T_{\psi i}$, $T_{\varphi i}$ Aerodynamical torques

F_{zTi} Aerodynamical thrust

T_{gear} Driving torque to the gearbox

F_T Reaction thrust from the tower

Aerodynamical Thrust and Torques. The aerodynamical thrust and torques can be obtained by applying static and two-dimensional airfoil theory to each cross section of the blades.

The profiles of the incomming wind at far distance before $U_\infty(t,x)$ and at the rotor disc $U_d(t,x)$ are assumed to be linear

$$U(t,x) = (U_0(t) + U_x(t) x)\hat{z} \qquad -R \le x \le R \quad (6)$$

and to be related as

$$U_d(t,x) = (1-a(t)) U_\infty(t,x) \quad (7)$$

U_0 is the wind at the hub, U_x is a measure of the wind shear, R is the length of a blade and a(t) is the

interference factor.

The tower shadow has a significant impact for wind turbines downwind of the tower. Seidel (1977) [6] reports that the Mod-0 wind turbine (The ERDA-NASA 100 kW wind turbine near Sandusky, Ohio) momentarily loses more than 60 % of the rotor torque as a blade swings behind the tower. The wake depends critically on the aerodynamical properties of the tower and is difficult to model. However, the form of the wake is probably not critical for the intended use of this model. A common modification for the wind at the i:th blade is given by the factor

$$q_i(\psi_i) = \begin{cases} 1 - C\cos(\dfrac{\pi}{2}\dfrac{\tilde{\psi}_i - \pi}{\alpha}), & |\tilde{\psi}_i - \pi| < \alpha \\ 1, & \text{otherwise} \end{cases} \qquad (8)$$

where

$$\tilde{\psi}_i = \psi_i \bmod 2\pi \qquad (9)$$

The pitch distribution of the blades is of the form

$$\beta(s) = \beta_1 + \frac{R}{s}\beta_2 \qquad (10)$$

The profile lift increment is assumed to depend linearly on the angle of attack. The profile drag increment is assumed to be independent of the angle of attack. Consequently, the stalling is not modelled. This is justified by the fact that the controller must prevent the blades from going into stalling. Introduce

$$\lambda = U_{dO}/(R\dot{\psi}) \quad \text{(the inflow ratio)} \qquad (11)$$

$$u_s = RU_{dx}/U_{dO} \qquad (12)$$

Lengthy calculations, ignoring terms of the orders:
$O(\lambda^4)$, $O(\lambda^3\cdot\dot{\varphi}_i)$, $O(\lambda\varphi_i)$, $O(\dot{\varphi}_i^2)$, $O(z_T^2)$, $O(u_s^2)$, $O(u_s\dot{\varphi}_i)$, $O(z_T\dot{\varphi}_i)$ and

$\dot{O}(z_T u_{T_s})$ give (Hultgren (1979) [5] extended with the motion

in the z_T direction)

$$T_{\psi i} = \frac{1}{2}\dot{\psi}^2\cos^3\varphi_i\left\{\Lambda_i\{A_1(\Lambda_i-r)-A_2\beta_1\} - \frac{1}{2}A_i^3(A_0\beta_1+A_0 r)_{-1} - B_3\right\}$$

$$-\frac{1}{2}\dot{\psi}\cos^2\varphi_i\left\{\{A_2(2\Lambda_i-r) - A_3\beta_1 - \frac{3}{2}A_i^2(A_0\beta_1+A_0 r)_{-1}\}\dot{\varphi}_i\right.$$

$$\left.+ \{A_1(2\Lambda_i-r) - A_2\beta_1 - \frac{3}{2}A_i^2(A_0\beta_1+A_0 r)_{-1}\}\dot{z}_T\cos\varphi_i\right\},$$

$$i = 1, 2 \qquad (13)$$

$$T_{\varphi i} = \frac{1}{2}\dot{\psi}^2\cos^2\varphi_i\left\{-A_2\{A_0(\frac{1}{2}\Lambda_i-r)-A_3\beta_1\} + A_2(\Lambda_i-r)-A_3\beta_1+B_2\Lambda_i\right\}$$

$$-\frac{1}{2}\dot{\psi}\cos^2\varphi_i\left\{\{A_3 + \Lambda_i[A_1(\frac{1}{2}\Lambda_i-r)-A_2\beta_1] + B_3\}\dot{\varphi}_i\right.$$

$$\left.+\{A_2 + \Lambda_i[A_0(\frac{1}{2}\Lambda_i-r)-A_1\beta_1] + B_2\}\dot{z}_T\cos\varphi_i\right\},$$

$$i = 1, 2 \qquad (14)$$

$$F_{zTi} = \frac{1}{2}\dot{\psi}^2\cos^3\varphi_i\left\{-A_2\{A_1(\frac{1}{2}\Lambda_i-r)-A_0\beta_1\} + A_1(\Lambda_i-r)-A_2\beta_1+B_1\Lambda_i\right\}$$

$$-\frac{1}{2}\dot{\psi}\cos^2\varphi_i\left\{\{A_2 + \Lambda_i[A_0(\frac{1}{2}\Lambda_i-r)-A_1\beta_1] + B_2\}\dot{\varphi}_i\right.$$

$$\left.+\{A_1 + \Lambda_i[A_{-1}(\frac{1}{2}\Lambda_i-r)-A_0\beta_1] + B_1\}\dot{z}_T\cos\varphi_i\right\},$$

$$i = 1, 2 \qquad (15)$$

where

$$\Lambda_i = q_i \lambda R \tag{16}$$

$$r = R\beta_2 \tag{17}$$

$$\dot{\Phi}_i = \dot{\phi}_i - q_i U_{idx}^2 \cos\phi_i \cos\psi_i \tag{18}$$

A_{-1}, A_0, A_1, A_2, A_3, B_1, B_2 and B_3 are blade constants.

$$A_\alpha = \varrho_a \int_0^R cas^\alpha\, ds \tag{19}$$

$$B_\alpha = \varrho_a \int_0^R cC_D s^\alpha\, ds \tag{20}$$

where ϱ_a is the density of air, c the local cord length, a is the lift curve slope of the blade section and C_D the profile drag coefficient of the section.

The Reaction Torque from the Tower. The influence of the tower on the nacelle is modelled as a spring with damper

$$F_T = -(K_T z_T + D_T \dot{z}_T) \tag{21}$$

The Interference Factor. The interference factor a(t) can be calculated by using momentum theory (e.g. Shepherd (1978) [7]).

The generated power is

$$P = n_p\, 2\varrho_a \pi R^2\, a(1-a)^2\, U_\infty^2 \tag{22}$$

where n_p is the degree of power efficiency of the blades. (13), (17), (22) and the relation

$$P = \dot{\psi}\,(T_{\psi 1} + T_{\psi 2}) \tag{23}$$

give (neglecting the effects of wind shear, the tower motion and the tower shadow)

$$b^3 - (1-c_1 A_1 \frac{\dot\psi}{U_e})b^2 - c_1(A_2\beta_1 + A_1 r)(\frac{\dot\psi}{U_e})^2 b - c_1 B_3 (\frac{\dot\psi}{U_e})^3 = 0 \qquad (24)$$

where

$$b = 1 - a \qquad (25)$$

$$c_1 = 1/(n_p\, 2\rho_a\, \pi R^2) \qquad (26)$$

The Gearbox

The shaft between the turbine rotor and the gearbox, and the shaft between the gearbox and the synchronous generator are assumed to be rigid compared to the mounting of the gearbox. This means that, disregarding the gearing, the drive train can be modelled as one nonlinear spring with a linear damper. The SIMNON code for this subsystem is given in Figure 3. The angular position of the turbine rotor and the generator rotor are increasing and of the same size. The calculation of γ by taking the difference causes numerical difficulties. However, to improve the numerical properties γ is calculated by integrating the difference between the angular velocities on the input and output side. Torque and energy balance give the driving torque from the turbine (Tgear) and the input torque to the generator (Tm) expressed in the reaction torque from the gearbox mounting (Tgamma). Note, that double quote (") in the SIMNON code means a comment and is supplied for a proper documentation.

```
CONTINUOUS SYSTEM GEARBOX

"Description:
" The gearbox is modelled as a nonlinear spring
" with a linear hydraulic damper

INPUT psidot wm

OUTPUT Tgear Tm

STATE gamma

DER Dgamma

"Inputs:
"   psidot    angular velocity of wind turbine [rad/s]
"   wm        mechanical angular velocity of generator
"             [rad/s]
"
"Outputs:
"   Tgear     driving torque from the turbine [Nm]
"   Tm        input torque to generator [Nm]
"
"States:
"   gamma     torsion angle [rad]

Dgamma = (psidot - wm/Ng)/(1-1/Ng)

Tgear = -(Tgamma/(1-1/Ng)
Tm = -Tgamma/(Ng-1)

Tgamma = Tsp + Td

Tsph = -(Kg0*gmax + Kg1*(gamma-gmax))
Tsp = if gamma<gmax then -Kg0*gamma else Tsph

Tdul = -Dg*Dgamma
Td = max(-Tdmax, (min(Tdmax, Tdul) )

"Parameters: (fictive values)

Ng:      60.0      "gear ratio between generator side
                   "and turbine side
Kg0:     5.3E3     "lower spring coefficient (ref. to
                   "the turbine side) [Nm/rad]
Kg1:     2.3E20    "higher spring coefficint (ref. to
                   "the turbine side) [Nm/rad]
Dg:      6.5E2     "hydraulic damping torque coefficient
                   "(ref. to the turbine side) [Nms/rad]
Tdmax:   3.4E5     "limit of damper torque [Nm]
gmax:    1.0       "limit of torsion angle [rad]

END
```

Figure 3: SIMNON code of Gearbox submodel.

The Synchronous Generator

When the behaviour of synchronous machines in transient conditions are studied by simulation, it is essential that the system is modelled in sufficient detail. A model on a form suitable for our purpose is given by Olive (1968) [8]. The damper windings are modelled as two circuits, one in the direct axis and the other in the quadrature axis. In addition the rotor has one field circuit. Symmetrical load is assumed.

The Voltage Control System

The excitation system of the generator uses an AC alternator and a rotating rectifier to produce the current needed for the generator field. The voltage control system is supplied to achieve a constant machine terminal voltage, independent of the load conditions and to distribute reactive power among the synchronous machines working in parallel at the grid. The model of the excitation system and voltage controller uses a Type 1 standard IEEE representation [9] including non-linear saturation effects.

The Electrical Network

The generator is assumed to be connected to an infinite bus via an impedance $R_1 + jX_1$. This means that the bus voltage and bus frequency are not affected by the wind turbine system. However, by making the bus voltage and the bus frequency time varying it is possible to model a large utiliy grid in both normal operation and during faults.

The Rotor Control System

The WTS-3 Control System is a complex, microprocessor based and distributed system which has been partitioned into a number of subsystems. The rotor controller is one of these subsystems which is used to establish a blade angle reference signal to the electrohydraulical positioning system. Four modes of operation are defined: In acceleration control mode a safe start up is ensured by controlling the maximum acceleration of the rotor and by preventing excessive blade stall. In rotor speed control mode the generator speed is matched to the line frequency for a safe synchronization. In power control mode the generator is connected to the power grid and the blade angle is modulated to extract maximum power from the wind. The deceleration control mode is used for normal shutdown and controls the deceleration torque to an acceptable level.

Simulation Results

Due to the modularization each submodel could easily be tested independently of the others and when accepted, the complete WTS-3 model was formed. A number of studies with the package have been performed.

Model Validation Studies

For this purpose, a well-defined excitation of the system from the wind was selected. In Figure 4 an input step-change of the mean wind from 14 to 16 m/s is shown and a good agreement with the results from Hamilton Standard is found. However, a later system redesign was also investigated by increasing the model complexity to include the mentioned tower motions and shadow effect as well as the redesigned control system. The model validation, so far, only indicates minor discrepancies in the aerodynamical part while the other subsystems still show good agreement.
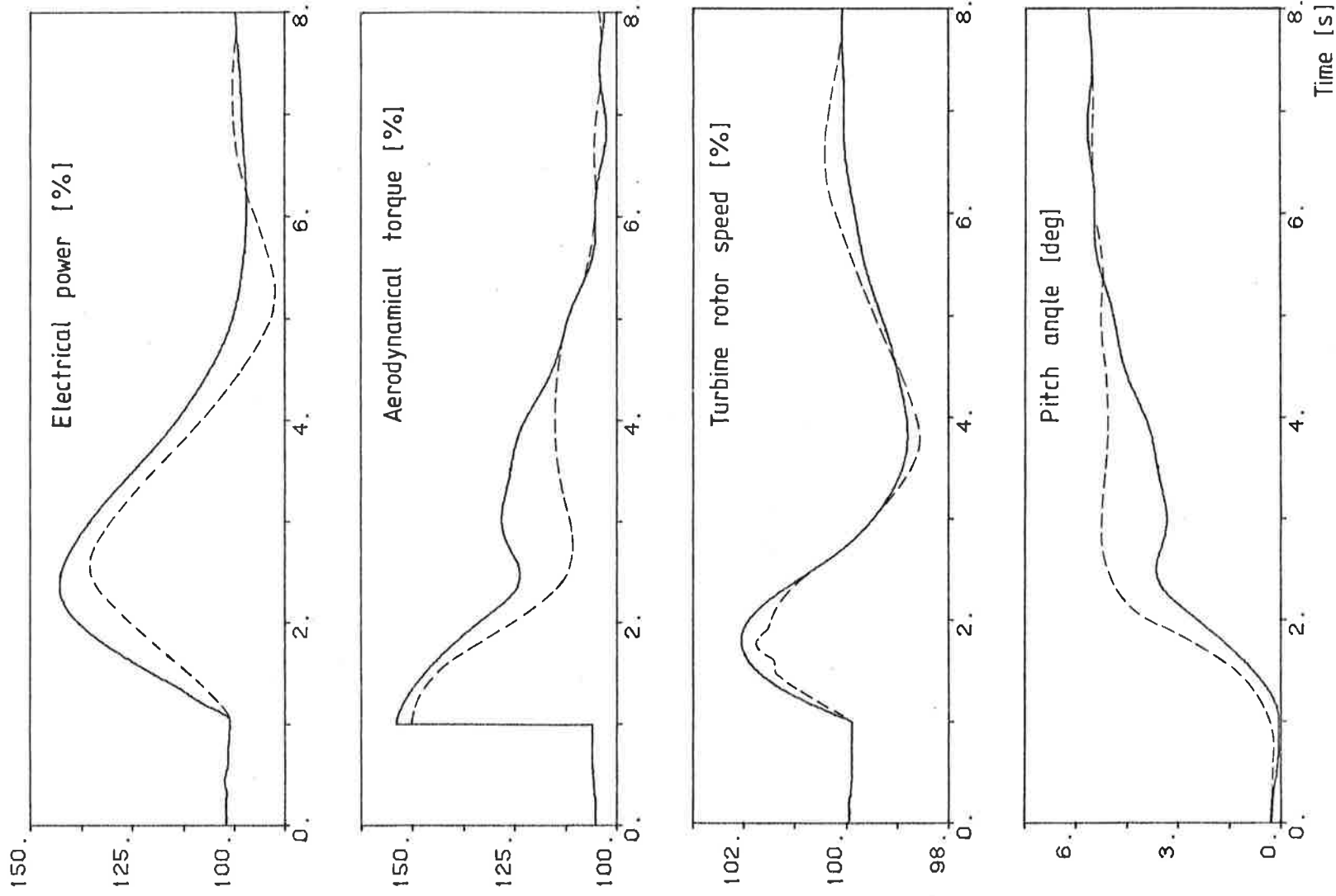
Figure 4: Model validation simulations from wind step-change 14-16 m/s (dashed line Hamilton Standard).

Stochastic Wind Studies

Normal operation in on-line and off-line mode have been studied using a simple stochastic wind model. A pseudo random wind disturbance was generated and the spectral property of the longitudinal wind velocity was approximated by a first order model. However, to account for the spatial filtering effect due to averaging over a large area in space, typical for large scale wind turbines, a first order spatial filter was also introduced. In Figure 5 the system response from a wind-series with a turbulence factor close to 0.1 and on-line mode of operation is shown.
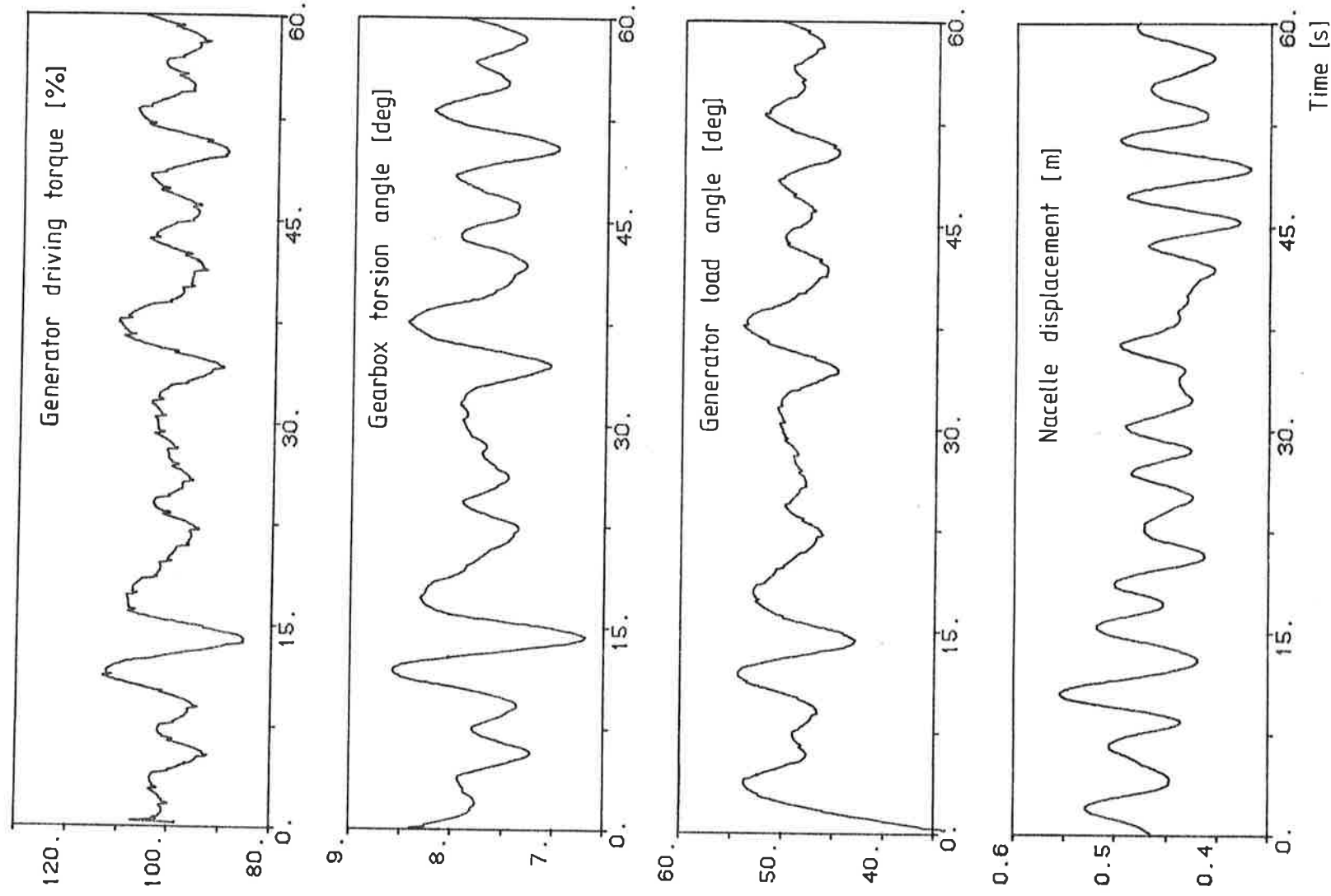

Wind speed [m/s]

Electrical power [%]

Aerodynamical torque [%]
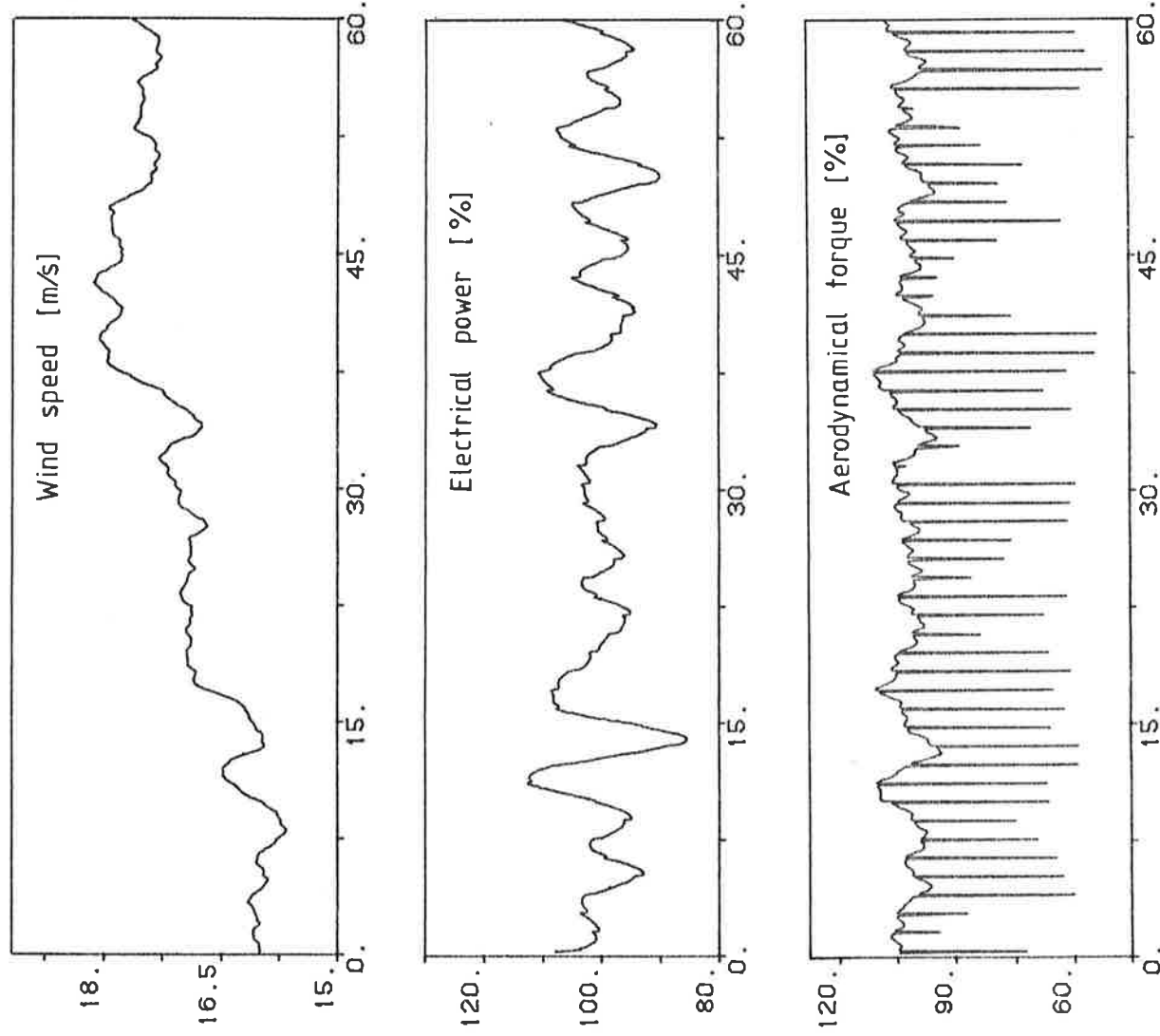
Figure 5: Stochastic wind simulation WTS-3.

Transient Stability Studies

The integration of the WTS-3 prototype into the local electrical grid in southern Sweden is an area of a great interest. A simplified bus model was developed for the grid. With the model shown in Figure 6 the transient behaviour of the wind turbine generator during electrical disturbances was investigated. With the system initially operating at rated condition, 3 MW, a 3-phase fault was applied at the bus at different locations (a-g). After a given time the fault was cleared by breaker action in the line where the fault was applied. A severe case, a fault at c) is shown in Figure 7. However, for simulations the connection between the generator and the infinite bus is assumed to be modelled as a serial impedance. The model in Figure 6, must be transformed to serial form. The transient stability studies were performed under the assumption of stochastic disturbances in the wind and in the bus frequency. However, the standard deviation and spectral properties were matched to conform with normal operating records.
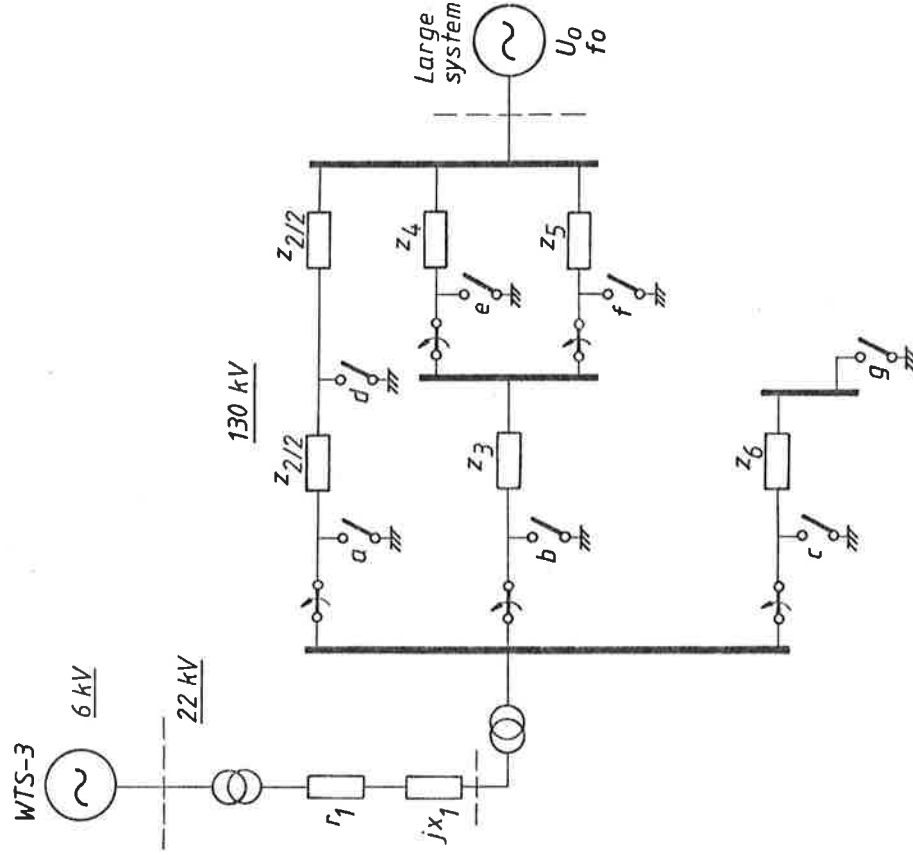

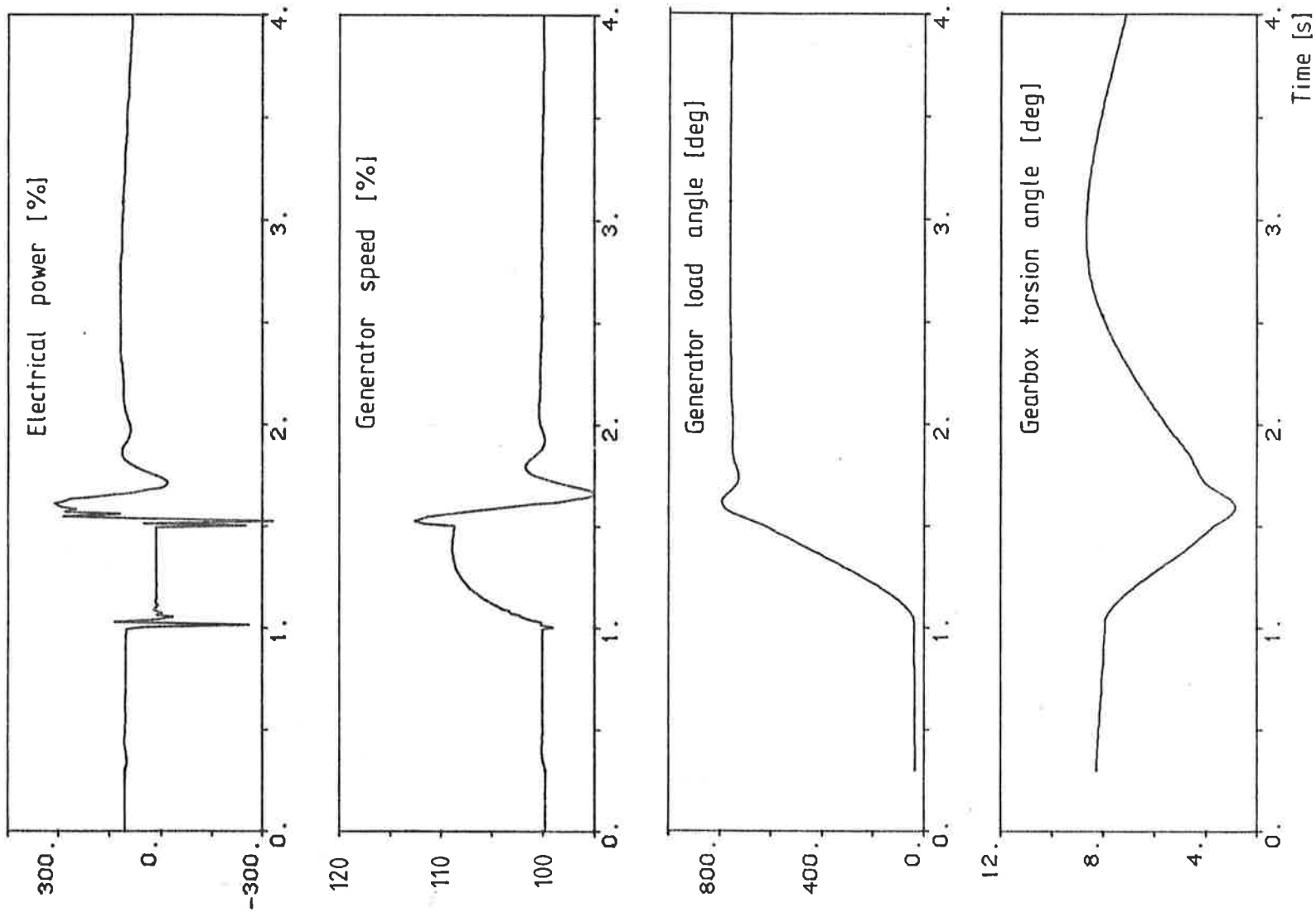Figure 6: Simplified bus model for transient stability simulations.

Figure 7: Transient stability simulation WTS-3. 3-phase fault applied at c) at t=1 s, cleared at t=1.5 s.

Conclusions

By the modularization and use of a high level simulation language a Wind Turbine Simulation package was developed.

The flexible model structure has pointed out the advantage of "step-by-step" model-building and implementation procedures. Increasing submodel complexity and changing complete parts are easy to handle and affect normally the connecting system only. The use of the MACRO facility for system setup, state-initialization, data storage plotting etc. has shown the advantage of using a well-designed simulation package (SIMNON) and reduces preparation time to a minimum.

The package described here was adapted to the design of the WTS-3. However, incorporation of submodels for asynchronous generators, up-wind turbines etc. will expand the simulation package further and is planned as a forthcoming phase.

The submodel validation procedure for the WTS-3 package was made by comparisons with runs from the designer. Only minor discrepancies were found, mainly in the aerodynamical part. The results from stochastic wind simulation show significant dynamical properties of the different components and the response in the quality of control was investigated. The results from the transient stability studies show the system behaviour during faults in the network and will be used for further investigations and a selective design of the generator protection system.

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$$T_{\psi i} = \frac{1}{2}\dot{\psi}^2 \cos^3\varphi_i \left\{\Lambda_i\{A_1(\Lambda_i - r) - A_2\beta_1\} - \frac{1}{2}\Lambda_i^3(A_0\beta_1 + A_{-1}r) - B_3\right\}$$

$$- \frac{1}{2}\dot{\psi}\cos^2\varphi_i \left\{A_2(2\Lambda_i - r) - A_3\beta_1 - \frac{3}{2}\Lambda_i^2(A_1\beta_1 + A_0 r)\right\}\dot{\Phi}_i$$

$$+ \{A_1(2\Lambda_i - r) - A_2\beta_1 - \frac{3}{2}\Lambda_i^2(A_0\beta_1 + A_{-1}r)\}\dot{z}_T\cos\varphi_i ,$$

$$i = 1, 2 \qquad (3.13)$$

$$T_{\varphi i} = \frac{1}{2}\dot{\psi}^2 \cos^2\varphi_i \left\{-\Lambda_i^2\{A_0(\frac{1}{3}\Lambda_i - r) - A_1\beta_1\} + A_2(\Lambda_i - r) - A_3\beta_1 + B_3\right\}$$

$$- \frac{1}{2}\dot{\psi}\cos^2\varphi_i \left\{A_3 + \Lambda_i\{A_1(\frac{1}{2}\Lambda_i - r) - A_2\beta_1\} + B_3\right\}\dot{\Phi}_i$$

$$+ \{A_2 + \Lambda_i\{A_0(\frac{1}{2}\Lambda_i - r) - A_1\beta_1\} + B_2\}\dot{z}_T\cos\varphi_i ,$$

$$i = 1, 2 \qquad (3.14)$$

$$F_{zTi} = \frac{1}{2}\dot{\psi}^2 \cos^3\varphi_i \left\{-\Lambda_i^2\{A_{-1}(\frac{1}{3}\Lambda_i - r) - A_0\beta_1\} + A_1(\Lambda_i - r) - A_2\beta_1 + B_1\right\}$$

$$- \frac{1}{2}\dot{\psi}\cos^2\varphi_i \left\{A_2 + \Lambda_i\{A_0(\frac{1}{2}\Lambda_i - r) - A_1\beta_1\} + B_2\right\}\dot{\Phi}_i$$

$$+ \{A_1 + \Lambda_i\{A_{-1}(\frac{1}{2}\Lambda_i - r) - A_0\beta_1\} + B_1\}\dot{z}_T\cos\varphi_i ,$$

$$i = 1, 2 \qquad (3.15)$$

where

$$\Lambda_i = q_i\lambda R \qquad i = 1, 2 \qquad (3.16)$$

$$r = R\beta_2 \qquad (3.17)$$