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DEVELOPMENT OF A MODULAR SIMULATION MODEL  
FOR A WIND TURBINE SYSTEM

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DEVELOPMENT OF A MODULAR SIMULATION MODEL FOR A WIND TURBINE SYSTEM			
Abstract			
<p>A mathematical simulation model for a large horizontal axis wind turbine system is presented and discussed in detail. The model is intended for simulation of the synchronization of the wind turbine generator against the utility grid and the operation of the wind turbine system under different wind conditions and with different control algorithms.</p> <p>Particular attention has been given to the modularization. The model is divided into subsystems to make it easy to modify the model and adapt it to systems of similar type. The interactive simulation package SIMNON which allows good structuring and programming in a high level language has been used.</p>			
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**FOREWORD**

This report is an open version of a project report to Sydkraft AB. It is identical to that report, but the appendices containing the SIMNON program are excluded from this open version of the report according to agreements with the manufacturers of WTS-3 and Sydkraft AB.

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## 1 INTRODUCTION

This report presents a mathematical simulation model for a large horizontal axis wind turbine system.

A Wind Turbine System (WTS) differs conceptually from a conventional power production unit in the sense that the power source cannot be controlled in any way. If the available power is below the rated value, the system should be operated to extract maximum energy. Normally this is achieved by letting the blade angle be a static function of the wind and rotor speed. When the wind increases further, protection from over-power must be made by flapping and modulation of the pitch angle.

However, as the wind is of a stochastic nature and a WTS is a dynamically soft system, excitation of poorly damped modes are probable. In order to extract the maximum energy from the fluctuating wind and to minimize the dynamic loads on the WTS, a fast and well-tuned control system is required.

The WTS designer therefore includes the dynamic performance in the construction work and a great part of the design is to simulate system behaviour under different conditions. On the other hand, simulation is also good for design-verification and functional analysis work made e.g. by the customers. The need for good dynamical models and simulation systems is thus obvious.

The goal of this work was to develop a mainframe for a simulation package of a general Wind Turbine System, based on an existing and well-proven simulation package called SIMNON (Elmqvist(1975)). The simulation system should be good for design-verification and failure investigation in connection with faults in the electrical network. Furthermore, the goal was also to develop a simulator for educational purposes and future design of quite new controllers, like e.g. adaptive ones.

When developing a model for a complex system it is important to use a modular and well structured approach. In this report particular attention has been given to the modularization. The model is divided into subsystems in a logical way. This makes it easy to modify the model and use different complexity in the subsystems.

The plant to be considered in this study is a 3 MW wind turbine system, WTS-3, which now is built near the city of Trelleborg, in southern Sweden, by Karlskronavarvet AB, Sweden and Hamilton Standard, a division of United Technologies Inc, USA. A 4 MW plant (WTS-4) of the same design is built in Medicine Bow, Wyoming, USA. However, the modularization and use of a high level language make it easy to adapt the model to other systems of similar type.

A complex dynamical model must be validated in order to get a quality mark. As the physical system does not exist in the real world at present, the only validation so far has been made by simulation comparisons from runs made by the WTS designer (Östberg (1981)). During the full-scale testing period 1982-1985, system identification and model validation will be done.

A nice survey of wind power is given by Shepherd (1978). There is an extensive literature on models for the different components of the system. The proceedings from the workshop on wind turbine structural dynamics held at the NASA Lewis Research Center in Cleveland, Ohio, on November 15-17, 1977 (Miller (1978)) contain many interesting papers. Additional references are given in the text below. Four models for a complete system were found: Hwang and Gilbert (1978), Kos (1978), Edris (1979) and Krause and Man (1981) (also in Wasynczuk, Man and Sullivan (1981)). These models are discussed further in Section 11.

The report starts with considering the system. The system is then divided into smaller parts, which are discussed one by one. The report concludes with a comparison with other models. The simulation program is enclosed in the appendices.

## 2 THE WIND TURBINE SYSTEM - WTS-3

The wind turbine system WTS-3 is designed to supply power in parallel with other electrical generators to a large power utility grid and to operate in wind forces of 5-26 m/s. The rated power 3 MW is reached at 14 m/s.

WTS-3 has a horizontal axis downwind turbine with two blades mounted on a teetered hub. The rotor drives a 3-phase synchronous generator through a gearbox. The power is controlled by changing the blade pitch angle. The blade actuators are hydraulic positioning systems.

The tower is a 78 m high and 3.8 m wide cylindrical steel shell. The nacelle and rotor can turn freely around the axis of the tower. The rotor is aligned by an active yaw mechanism of the tower against the wind, downwind of the tower. The blades lean downwind at an angle of 6° and are designed as a monolithic base structure of epoxy plastic reinforced with glass fiber. They have a length of 39 m, a maximum corda of nearly 5 m and a weight of 14 tons each.

The generator is a 3-phase, 50 Hz, 1500 r.p.m synchronous machine. The rotor rotates at 25 r.p.m. and a multi-stage planetary gearbox steps up the rotation to 1500 r.p.m. A torsionally soft mounting (including hydraulic dampers) of the gearbox provides a torsionally soft connection. This soft connection compensates for rapid variations in rotor speed, since the generator must operate at constant r.p.m.

The control system consists of three processors, called supervisory controller, rotor controller and interface controller. The supervisory controller supervises the system and decides mode of operation. Depending on the mode of operation the rotor controller calculates a reference value to the hydraulic positioning system. The interface controller handles the operator communication. The three processors communicates with each other via serial lines.

The model is developed in a modularized, structured manner. It is decomposed into the subsystems:

- the aerodynamical part
  - the tower
  - the two teetered blades
  - the hydraulic pitch servo
- the electrical part
  - the synchronous generator
  - the connection to the utility grid



- the drive train
  - the shaft between the turbine rotor and the gear
  - the step-up gear
  - the shaft between the gear and the generator
- the control system
  - the supervisory system
  - the rotor controller
  - the voltage controller

A block diagram illustrating this decomposition can be found in Figure 2.1.

To make the model manageable and not unnecessary complicated some simplifying assumptions will be made.

#### Assumption 2.1:

The synchronous generator is connected via an impedance to an infinite bus.

This means that the bus voltage and bus frequency are not affected by the wind turbine system. The system is designed to supply power in parallel with other electrical generators to a large utility grid. Consequently, the power supplied by the wind turbine system will only constitute a small portion. By making the bus frequency and bus voltage time varying it is possible to model a large utility grid in both normal operation and during faults.

#### Assumption 2.2:

The shaft between the turbine rotor and the gearbox, and the shaft between the gearbox and the synchronous generator are rigid compared to the mounting of the gearbox.

This assumption means that, disregarding the gearing, the drive train can be modelled as one spring with a damper. Simulations performed by ASEA (1980) show that this is a very good approximation. There is no information available on how the moments of inertia of the gearbox are distributed within the system. Only the sum is given. If reduced to the generator side, it is less than 10 % of the moment of inertia of the generator and, if reduced to the turbine side, it is less than 2 % of the moment of inertia of the wind turbine rotor, so it cannot be crucial. For simplicity, the moment of inertia of the gearbox is placed on the turbine side.

The WTS-3 has an active yaw mechanism, but the yawing will not be modelled here. It will be assumed that

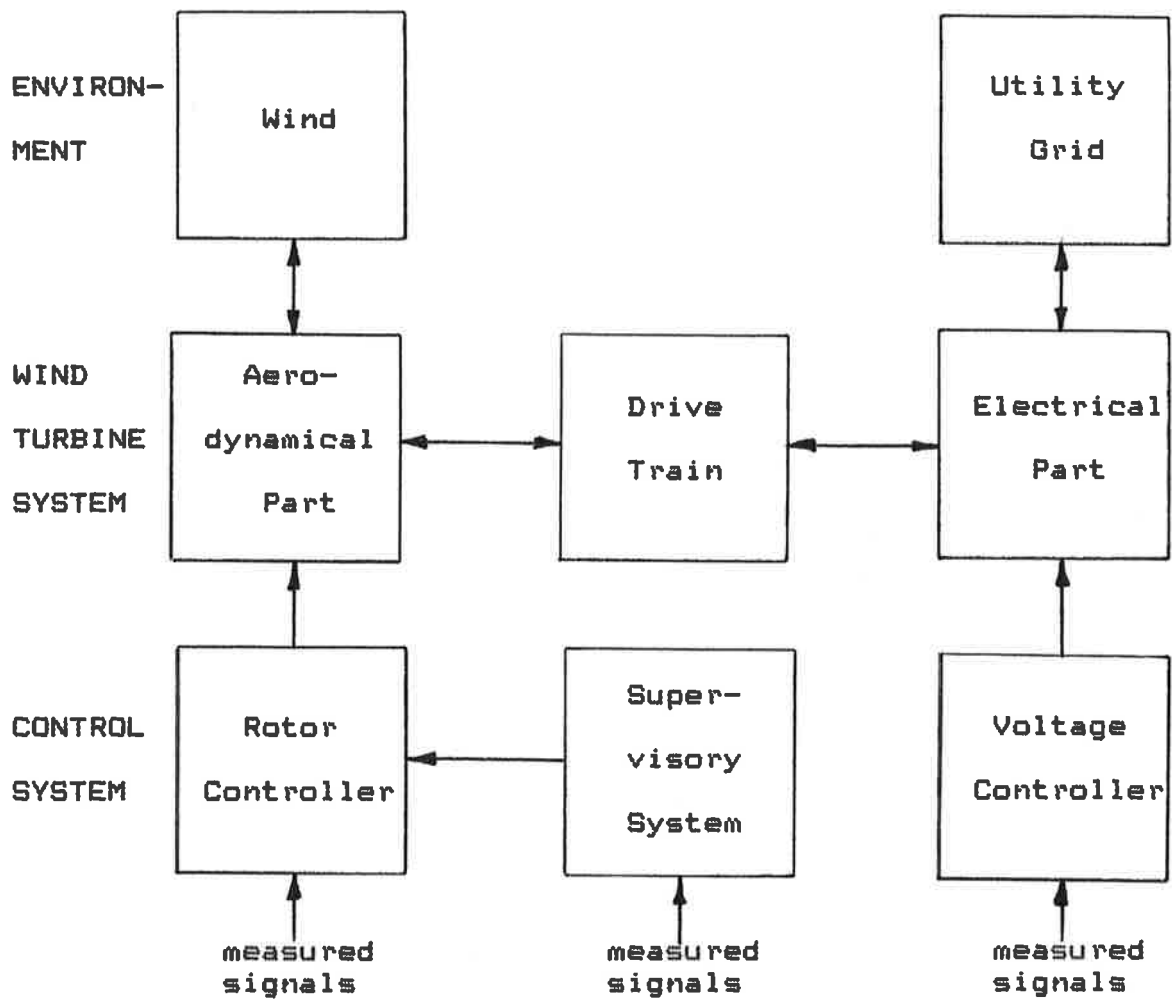


Figure 2.1: The structure of the wind turbine system.

Assumption\_2.3:

The nacelle is aligned in the direction of the wind.

Assumption\_2.4:

The blades are torsionally rigid and the pitch servo is not affected by the wind.

Friedmann (1976) states that the blades in a typical wind turbine system are torsionally rigid and this is confirmed by Gustavsson and Montgomerie (1980). The first frequency of torsional mode is high (about 37 rad/s).

Assumptions 2.1-2.4 make it possible to draw a simple block diagram (Figure 2.2) that directly can be translated to a SIMNON program. This block diagram will in the following be discussed subsystem by subsystem. The SIMNON subsystems can be found in the appendices. They are hopefully self-documenting. To increase the flexibility, the inputs and outputs are chosen as unscaled, physical quantities. SI-units are used. However, to improve the model numerically scaled quantities are used inside the subsystems.

Two examples of connecting systems are given in Appendix A1 and A2.

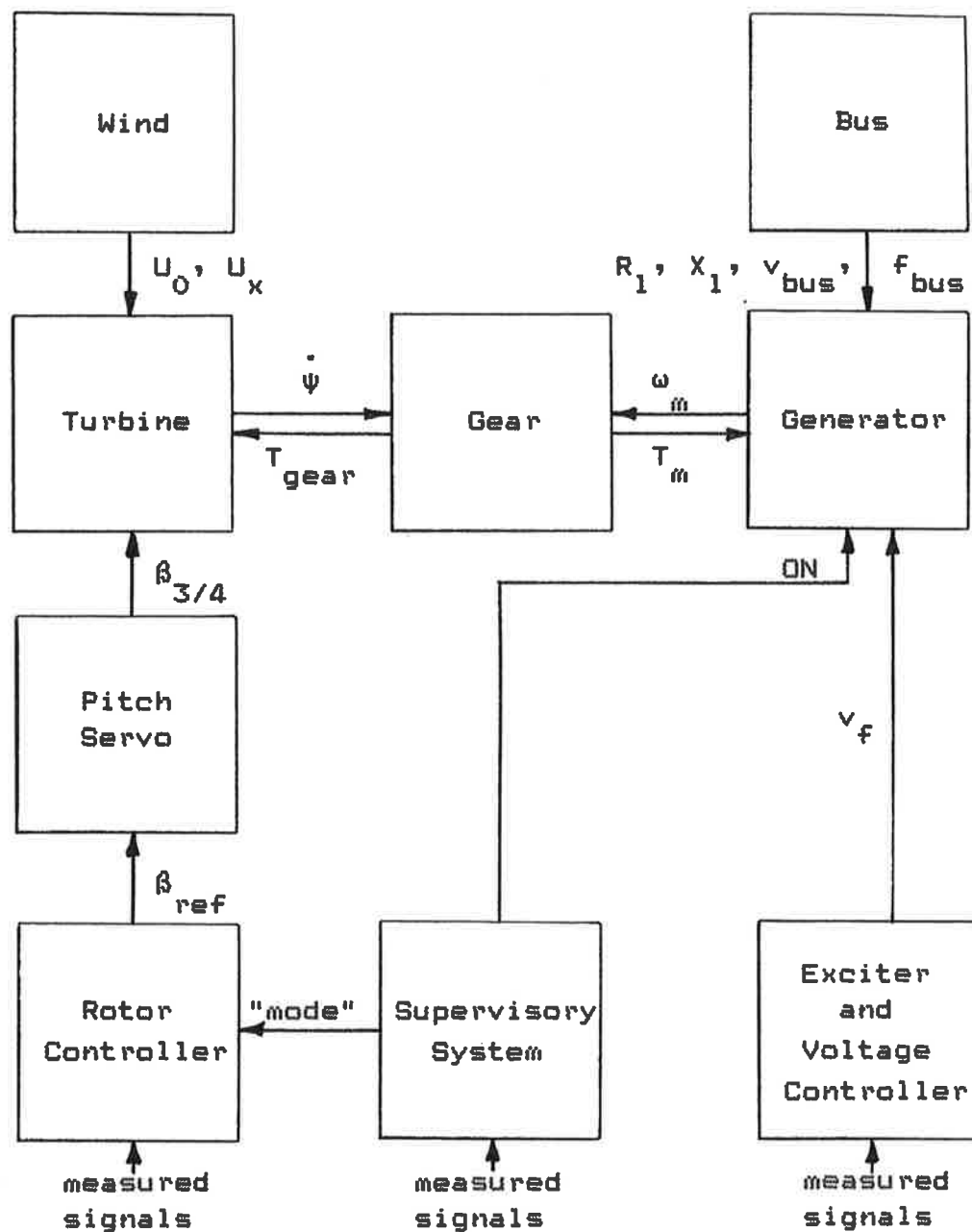


Figure 2.2: Model structure.



Consider Figure 3.1. Since the blades are teetered, the flapping angles satisfy

$$\varphi_1 + \varphi_2 = 2\varphi_0 = \text{constant} \quad (3.1)$$

and it is possible to describe the flapping motion by one variable:

$$\varphi = (\varphi_1 - \varphi_2)/2 \quad (3.2)$$

The motion in the  $\psi$  and  $\varphi$  directions assuming a rigid tower is modelled in Hultgren (1979). This model, extended with the motion in the  $z_T$  direction, will be used. The notations

used here differ somewhat from Hultgren's in order to conform with those used by Karlskronavarvet AB and Hamilton Standard.

Applications of Lagrange's equations yield the following equations of motion

$$\begin{aligned} \{J_B (1 + \cos 2\varphi_0 \cos 2\varphi) + J_{\text{hub}} + J_{\text{gear}}\} \ddot{\psi} \\ - 2J_B \dot{\varphi} \dot{\psi} \cos 2\varphi_0 \sin 2\varphi + 2gS_B \sin \varphi_0 \sin \varphi \sin \psi \\ = T_{\psi 1} + T_{\psi 2} - T_{\text{gear}} \end{aligned} \quad (3.3)$$

$$\begin{aligned} 2J_B \ddot{\varphi} - \{2S_B \sin \varphi_0 \sin \varphi\} \ddot{z}_T + J_B \dot{\psi}^2 \cos 2\varphi_0 \sin 2\varphi \\ - 2gS_B \sin \varphi_0 \cos \varphi \cos \psi = T_{\varphi 1} - T_{\varphi 2} \end{aligned} \quad (3.4)$$

$$\begin{aligned} \{-2S_B \sin \varphi_0 \sin \varphi\} \ddot{\varphi} + M_T \ddot{z}_T - 2S_B \dot{\varphi}^2 \sin \varphi_0 \cos \varphi \\ = F_{zT1} + F_{zT2} + F_T \end{aligned} \quad (3.5)$$

where

$g$	Gravitational acceleration
$J_B$	Moment of inertia of one blade
$J_{hub}$	Moment of inertia of the hub
$J_{gear}$	Moment of inertia of the gear
$S_B$	Static moment of one blade
$M_T$	Mass of the nacelle
$T_{\psi_i}, T_{\phi_i}$	Aerodynamical torques
$F_{zTi}$	Aerodynamical thrust
$T_{gear}$	Driving torque to the gearbox
$F_T$	Reaction thrust from the tower

#### Aerodynamical Thrust and Torques

The aerodynamical thrust and torques can be obtained by applying static and two-dimensional airfoil theory to each cross section of the blades.

The profiles of the incoming wind at far distance before  $U_{\infty}(t,x)$  and at the rotor disc  $U_d(t,x)$  are assumed to be linear

$$U(t,x) = (U_0(t) + U_x(t) x) z \quad -R \leq x \leq R \quad (3.6)$$

and to be related as

$$U_d(t,x) = (1-a(t)) U_{\infty}(t,x) \quad (3.7)$$

$U_0$  is the wind at the hub,  $U_x$  is a measure of the wind shear,  $R$  is the length of a blade and  $a(t)$  is the interference factor. Models for the incoming wind are discussed further in Section 8.

The tower shadow has a significant impact for wind turbines downwind of the tower. Seidel (1977) reports that the Mod-0 wind turbine (The ERDA-NASA 100 kW wind turbine at the NASA Plume Brook Station near Sandusky, Ohio) momentarily loses more than 60 % of the rotor torque as a blade swings behind the tower. The wake depends critically on the aerodynamical

properties of the tower and is difficult to model. However, the form of the wake is probably not critical for the intended use of this model. A common modification for the wind at the  $i$ :th blade is given by the factor

$$q_i(\psi_i) = \begin{cases} 1 - C \cos\left(\frac{\pi}{2} \frac{\tilde{\psi}_i - \pi}{\alpha}\right), & |\tilde{\psi}_i - \pi| < \alpha \\ 1 & \text{otherwise} \end{cases} \quad (3.8)$$

where

$$\tilde{\psi}_i = \psi_i \bmod 2\pi \quad (3.9)$$

The pitch distribution of the blades is of the form

$$\beta(s) = \beta_1 + \frac{R}{s} \beta_2 \quad (3.10)$$

The profile lift increment is assumed to depend linearly on the angle of attack. The profile drag increment is assumed to be independent of the angle of attack. Consequently, the stalling is not modelled. This is justified by the fact that the controller must prevent the blades from going into stalling. Introduce

$$\lambda = U_{d0} / (R\dot{\psi}) \quad (\text{the inflow ratio}) \quad (3.11)$$

$$u_s = RU_{dx} / U_{d0} \quad (3.12)$$

Lengthy calculations, ignoring terms of the orders:

$$O(\lambda^4), O(\lambda^3 \dot{\varphi}_i), O(\lambda^2 \dot{\varphi}_i^2), O(\lambda^2 \dot{u}_s), O(\lambda^2 \dot{z}_T), O(\lambda^2 \dot{u}_s \dot{\varphi}_i), O(\lambda^2 \dot{z}_T \dot{\varphi}_i) \text{ and}$$

$O(\lambda^2 \dot{z}_T \dot{u}_s)$  give (Hultgren (1979) extended with the motion in the  $z_T$  direction)



$$\begin{aligned}
T_{\psi i} = & \frac{1}{2} \dot{\psi}^2 \cos^3 \varphi_i \left\{ \Lambda_i \{A_{i1} (\Lambda_i - r) - A_{21} \beta_i\} - \frac{1}{2} \Lambda_i^3 (A_{01} \beta_i + A_{-1} r) - B_{3i} \right\} \\
& - \frac{1}{2} \dot{\psi} \cos^2 \varphi_i \left\{ \{A_{21} (2\Lambda_i - r) - A_{31} \beta_i - \frac{3}{2} \Lambda_i^2 (A_{01} \beta_i + A_{-1} r)\} \dot{\varphi}_i \right. \\
& \quad \left. + \{A_{11} (2\Lambda_i - r) - A_{21} \beta_i - \frac{3}{2} \Lambda_i^2 (A_{01} \beta_i + A_{-1} r)\} \dot{z}_T \cos \varphi_i \right\}, \\
& \quad i = 1, 2 \quad (3.13)
\end{aligned}$$

$$\begin{aligned}
T_{\varphi i} = & \frac{1}{2} \dot{\psi}^2 \cos^2 \varphi_i \left\{ \frac{1}{2} \Lambda_i^2 \{A_{i0} (\frac{1}{3} \Lambda_i - r) - A_{11} \beta_i\} + A_{21} (\Lambda_i - r) - A_{31} \beta_i + B_{2i} \Lambda_i \right\} \\
& - \frac{1}{2} \dot{\psi} \cos \varphi_i \left\{ \{A_{31} + \Lambda_i [A_{i1} (\frac{1}{2} \Lambda_i - r) - A_{21} \beta_i] + B_{3i}\} \dot{\varphi}_i \right. \\
& \quad \left. + \{A_{21} + \Lambda_i [A_{i0} (\frac{1}{2} \Lambda_i - r) - A_{11} \beta_i] + B_{2i}\} \dot{z}_T \cos \varphi_i \right\}, \\
& \quad i = 1, 2 \quad (3.14)
\end{aligned}$$

$$\begin{aligned}
F_{zTi} = & \frac{1}{2} \dot{\psi}^2 \cos^3 \varphi_i \left\{ \frac{1}{2} \Lambda_i^2 \{A_{i-1} (\frac{1}{3} \Lambda_i - r) - A_{01} \beta_i\} + A_{11} (\Lambda_i - r) - A_{21} \beta_i + B_{1i} \Lambda_i \right\} \\
& - \frac{1}{2} \dot{\psi} \cos^2 \varphi_i \left\{ \{A_{21} + \Lambda_i [A_{i0} (\frac{1}{2} \Lambda_i - r) - A_{11} \beta_i] + B_{2i}\} \dot{\varphi}_i \right. \\
& \quad \left. + \{A_{11} + \Lambda_i [A_{i-1} (\frac{1}{2} \Lambda_i - r) - A_{01} \beta_i] + B_{1i}\} \dot{z}_T \cos \varphi_i \right\}, \\
& \quad i = 1, 2 \quad (3.15)
\end{aligned}$$

where

$$\Lambda_i = q_i \lambda R \quad i = 1, 2 \quad (3.16)$$

$$r = R \beta_2 \quad (3.17)$$

$$\dot{\phi}_i = \dot{\varphi}_i - q U_i \cos^2 \varphi_i \cos \psi_i \quad i = 1, 2 \quad (3.18)$$

$A_{-1}$ ,  $A_0$ ,  $A_1$ ,  $A_2$ ,  $A_3$ ,  $B_1$ ,  $B_2$  and  $B_3$  are blade constants.

$$A_\alpha = \rho_a \int_0^R c a s^\alpha ds \quad (3.19)$$

$$B_\alpha = \rho_a \int_0^R c C_D s^\alpha ds \quad (3.20)$$

where  $\rho_a$  is the density of air,  $c$  the local cord length,  $a$  is the lift curve slope of the blade section and  $C_D$  the profile drag coefficient of the section.

#### The Reaction Torque from the Tower

The influence of the tower on the nacelle is modelled as a spring with damper

$$F_T = -(K_T z_T + D_T \dot{z}_T) \quad (3.21)$$

#### The Interference Factor

The interference factor  $a(t)$  can be calculated by using momentum theory (e.g. Shepherd (1978)).

The generated power is

$$P = n_p 2 \rho_a \pi R^2 a(1-a)^2 U_\infty^2 \quad (3.22)$$

where  $n_p$  is the degree of power efficiency of the blades.

3.13, 3.22 and the relation

$$P = \dot{\psi} (T_{\psi 1} + T_{\psi 2}) \quad (3.23)$$

give (neglecting the effects of wind shear, the tower motion and the tower shadow)

$$b^3 - (1 - c_{11} A_{11} \frac{\dot{\psi}}{U}) b^2 - c_{12} (A_{21} \beta + A_{11} r) (\frac{\dot{\psi}}{U})^2 b - c_{13} B_{13} (\frac{\dot{\psi}}{U})^3 = 0 \quad (3.24)$$

where

$$b = 1 - a \quad (3.25)$$

$$c_{11} = 1 / (n_p \frac{2\theta}{a} \pi R^2) \quad (3.26)$$

### Representation of the Azimuth Angle $\psi$

From the equations 3.1-3.26 it can be seen that the azimuth angle of blade ( $\psi$ ) enter the model only as  $\sin \psi$ ,  $\cos \psi$  and  $\psi \bmod 2\pi$  (c.f. the discussion in Section 5). Since  $\psi$  is increasing with time, it is better numerically to have  $\sin \psi$  and  $\cos \psi$  as states. The state equations are

$$\frac{d}{dt}(\cos \psi) = -\dot{\psi} \sin \psi + \cos \psi (1 - \sin^2 \psi - \cos^2 \psi) \quad (3.27)$$

$$\frac{d}{dt}(\sin \psi) = \dot{\psi} \cos \psi + \sin \psi (1 - \sin^2 \psi - \cos^2 \psi) \quad (3.28)$$

The second term in the right hand of (3.27) and the second term in the right hand of (3.28) guarantee amplitude stability ( $\sin^2 \psi + \cos^2 \psi = 1$ ).

### Conclusion

The model given in Appendix B models a horizontal axis, variable pitch wind turbine with two teetered blades downwind of the tower. The wind is assumed to be aligned in the direction of the nacelle (the yawing is not modelled) and the wind profile is assumed to be linear over the disc. The aerodynamical thrust and torques are calculated under the assumption that the inflow ratio  $\lambda$  is small ( $0.05 \leq \lambda \leq 0.2$  according to Gustavsson and Montgomerie (1980)). The tower shadow is taken into consideration. The stalling is not modelled. The blades, the nacelle and the tower are modelled as rigid bodies except for the bending motion of the tower in the wind direction which is modelled as well as the flapping motion of the two teetered blades.

#### 4 THE SYNCHRONOUS GENERATOR

Since extensive documentation of the modelling of a synchronous generator exists, only the equations will be given here. See for example Elgerd (1971) and Nordanlycke, Paulsson and Wredenberg (1974). The models of interest are Park models of different complexity (Park (1929)). Three models of different complexity will be given. The discussion starts with the most complex model and then are simplifications discussed.

A model on a form suitable for our purposes is given by Olive (1968) and this model is also used by Hwang and Gilbert (1978). Olive's paper contains a detailed and interesting discussion of the model and its features.

The standard notation will be used and the SIMNON subsystem in Appendix C1 contains a list of used symbols. To simplify the model and to increase its numerical properties, the voltages, currents and impedances are in per unit based upon the machine rating.

##### Synchronous Machine Equations

The damper windings are modelled as two circuits, one in the direct axis and the other in the quadrature axis. In addition the rotor has one field circuit. Symmetrical load is assumed.

$$\dot{e}'_q = \left( v_f - \frac{X'_d - X''_d}{X'_d - X''_d} e'_q + \frac{X'_d - X'_d}{X'_d - X''_d} e''_q \right) / T'_{d0} \quad (4.1)$$

$$\dot{e}''_d = -(e''_d - (X_q - X''_q) i_q) / T''_{q0} \quad (4.2)$$

$$\dot{e}''_q = (e'_q - e''_q - (X'_d - X''_d) i_d) / T''_{d0} \quad (4.3)$$

$$v_d = -R_a i_d + (e''_d + X''_q i_q) \frac{\omega}{\omega_0} + \dot{\lambda}_d \quad (4.4)$$

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$$\dot{e}''_d = -(e''_d - (X_q - X''_q) i_q) / T''_{q0} \quad (4.2)$$

$$\dot{e}''_q = (e'_q - e''_q - (X'_d - X''_d) i_d) / T''_{d0} \quad (4.3)$$

$$v_d = -R_a i_d + (e''_d + X''_q i_q) \frac{\omega}{\omega_0} + \dot{\lambda}_d \quad (4.4)$$

$$v_q = -R_a i_q + (e''_q - X''_{d'} i_d) \frac{\omega}{\omega_0} + \dot{\lambda}_q \quad (4.5)$$

$$\lambda_d = (e''_q - X''_{d'} i_d) / \omega_0 \quad (4.6)$$

$$\lambda_q = -(e''_d + X''_{q'} i_q) / \omega_0 \quad (4.7)$$

The armature voltage and current are given by

$$v_a = v_d \cos \theta - v_q \sin \theta \quad (4.8)$$

$$i_a = i_d \cos \theta - i_q \sin \theta \quad (4.9)$$

where  $\theta$  is the electrical position of the rotor:

$$\dot{\theta} = \omega \quad (4.10)$$

The generator is assumed to be connected to an infinite bus through a line and transformer with the impedance  $R_1 + jX_1$

(Assumption 2.1). Kirchhoff's law equations describing this connection are

$$X_{1d} \dot{i}_d = \omega_0 (v_d - v_{bus} \sin \delta) - \omega R_1 i_d + \omega X_{1q} i_q \quad (4.11)$$

$$X_{1q} \dot{i}_q = \omega_0 (v_q - v_{bus} \cos \delta) - \omega X_{1d} i_d - \omega R_1 i_q \quad (4.12)$$

$v_{bus}$  is the bus voltage and  $\delta$  is the power angle.

$$\delta = \theta - \theta_{bus} \quad (4.13)$$

The equations must be rearranged, because 4.4, 4.6 and 4.11 and 4.5, 4.7 and 4.12 contain algebraic loops. Eliminating  $\lambda_d$  and  $\lambda_q$  and solving for  $v_d$ ,  $v_q$ ,  $\dot{i}_d$  and  $\dot{i}_q$  give

$$v_d = \{ (e_d'' + X_{dq}'' i_q) \omega + \dot{e}_q'' - X_{dd}'' \dot{i}_d \} / \omega_0 - R_a i_d \quad (4.14)$$

$$v_q = \{ (e_q'' - X_{dq}'' i_d) \omega - \dot{e}_d'' + X_{qq}'' \dot{i}_q \} / \omega_0 - R_a i_q \quad (4.15)$$

$$\dot{i}_d = \{ \omega [e_d'' + (X_{dq}'' + X_{dl}'' ) i_q] - \omega_0 [(R_a + R_l) i_d + v_{bus} \sin \delta] + \dot{e}_q'' \} / (X_{dd}'' + X_{dl}'') \quad (4.16)$$

$$\dot{i}_q = \{ \omega [e_q'' - (X_{dq}'' + X_{ql}'' ) i_d] - \omega_0 [(R_a + R_l) i_q + v_{bus} \cos \delta] - \dot{e}_d'' \} / (X_{qq}'' + X_{ql}'') \quad (4.17)$$

### Equation of Motion

The mechanical angular speed of the rotor is

$$\omega_m = \omega / (p/2) \quad (4.18)$$

where  $p$  is the number of poles. Differentiation of 4.13 gives

$$\dot{\delta} = \frac{p}{2} \omega_m - \omega_{bus} \quad (4.19)$$

The equation of motion, or swing equation is

$$\dot{\omega}_m = (T_m - D_m \omega_m - T_e) / J_{gen} \quad (4.20)$$

$$T_e = \frac{p}{2} \frac{S_{base}}{\omega_0} (e_q'' i_q + e_d'' i_d - (X_{dd}'' - X_{qq}'') i_d i_q) \quad (4.21)$$

$T_m$  is the input torque,  $D_m$  represents constant friction torque,  $T_e$  is the torque developed by the generator (in SI-units) and  $J_{gen}$  is the moment of inertia of the generator.

### Disconnection

When the generator is disconnected, the currents  $i_d$  and  $i_q$  should be reduced to zero. However, they are states and SIMNON does not permit direct assignments to states. They can only be reduced to zero through manipulations of their derivatives. The derivatives are given constant values so that the currents are reduced to zero within a few milliseconds. When the generator is disconnected from the grid,  $\delta$  may increase. Since it is only  $\delta \bmod 2\pi$  that are of interest  $\cos \delta$  and  $\sin \delta$  can be used as states instead of  $\delta$  itself as done in (3.27) and (3.28). A system where  $\cos \delta$  and  $\sin \delta$  are used as states is given in Appendix C2.

### Model Simplification

The generator model given above and in Appendix C1 contains both fast modes (in the ms range) and slow modes (in the 1 s range) which makes the model computationally slow. The fast modes are only of interest if the behavior of the electrical variables are studied. If only the mechanical oscillations are investigated, the fast modes can be neglected. The improvement of computational speed is considerable. Janischewsky and Prabhaskar reports in the discussion of Olive's paper (1968) that in a particular study it was possible to increase the step size 50 times (from 1 ms to 50 ms), if the  $\dot{\lambda}_d$  and  $\dot{\lambda}_q$  terms in 4.4 and 4.5 were neglected.

This means that 4.4 and 4.5 do not depend on  $\dot{i}_d$  and  $\dot{i}_q$ . The algebraic loops have disappeared and only 4.11 and 4.12 have terms dependent on  $\dot{i}_d$  and  $\dot{i}_q$ . If  $X_1$  is small, these derivatives can be eliminated from the model. Further assuming  $\omega = \omega_0$  in 4.4, 4.5, 4.11 and 4.12 gives

$$v_d = e_d'' + X_q'' i_q - R_a i_d \quad (4.22)$$



$$v_q = e_q'' - X_d'' i_d - R_a i_q \quad (4.23)$$

$$i_d = \{(R_a + R_l)(e_q'' - v_{bus} \sin \delta) + (X_d'' + X_l)(e_q'' - v_{bus} \cos \delta)\} / \Sigma \quad (4.24)$$

$$i_q = \{(R_a + R_l)(e_q'' - v_{bus} \cos \delta) - (X_d'' + X_l)(e_q'' - v_{bus} \sin \delta)\} / \Sigma \quad (4.25)$$

$$\Sigma = (R_a + R_l)^2 + (X_d'' + X_l)(X_q'' + X_l) \quad (4.26)$$

The equations 4.1-4.3, 4.8-4.10, 4.13 and 4.18-4.26 constitute a simplified model which can be found in Appendix C3. Krause, Nozari, Skvarenia and Olive (1979) discuss the implications of these approximations. Hallingstad (1980) uses this model. From his analysis it can be seen that there still is a large difference in the time constants of the modes (a typical set of eigenvalues of the linearized system about the steady state are  $-3 \pm 20i$ , 15, 100 and 200). If the dynamics of  $e_d''$  and  $e_q''$  are neglected 4.1-4.3 and 4.21-4.26 can be further reduced to

$$\dot{e}_q' = (v_f - e_q' - (X_d' - X_d'') i_d) / T_{d0}' \quad (4.27)$$

$$v_d = X_q' i_q - R_a i_d \quad (4.28)$$

$$v_q = e_q' - X_d' i_d - R_a i_q \quad (4.29)$$

$$i_d = \{-(R_a + R_l)v_{bus} \sin \delta + (X_q' + X_l)(e_q' - v_{bus} \cos \delta)\} / \Sigma_1 \quad (4.30)$$

$$i_q = \{(R_a + R_l)(e_q' - v_{bus} \cos \delta) + (X_d' + X_l)v_{bus} \sin \delta\} / \Sigma_1 \quad (4.31)$$

$$\Sigma_1 = (R_a + R_l)^2 + (X'_d + X_l)(X_q + X_l) \quad (4.32)$$

$$T_e = \frac{p}{2} \frac{S_{base}}{\omega_0} (e'_q + (X_q - X'_d) i_d) i_q \quad (4.33)$$

The equations 4.8-4.10, 4.13, 4.18-4.20 and 4.27-4.33 constitute a further simplified model, which can be found in Appendix C4. This model is the same as was used by Demello and Concordia (1969).

Damping Torques. The expression 4.21 for the electrical torque only contains the fundamental torque if the rate of stator flux is neglected as in the second and third model (Olive (1968)). To take the damping torque into account, equation 4.20 is usually modified with an extra term

$$\dot{\omega}_m = (T_m - D_m \dot{\omega}_m - T_e - D_e \dot{\delta}) / J_{gen} \quad (4.34)$$

$$D_e = \frac{p}{2} \frac{S_{base}}{\omega_0} D_{pu} \quad (4.35)$$

where  $T_e$  is calculated as before. The constant coefficient  $D_{pu}$  can be between 5-50 depending on the design of the generator.

Discussion. A comparison between these models can be found in Östberg (1981). There were no problems in using the most complex one (Appendix C1) and no reason for not using it were found.

## 5 THE GEARBOX

The wind turbine is connected to the synchronous generator through a multi-stage planetary step-up gearbox. The gearbox is softly mounted and the mounting includes hydraulic dampers. According to Assumption 2.2, the drive train can be modelled as a spring with dampers. The torsion of the spring ( $\gamma$ ) viewed from the wind turbine side is

$$\gamma = (\psi - \theta / N_g) / (1 - 1/N_g) \quad (5.1)$$

where  $\psi$  is the azimuth angle of the wind turbine,  $\theta$  is the mechanical position of the rotor of the generator and  $N_g$  is the step-up gear ratio. However, 5.1 causes numerical difficulties, since  $\psi$  and  $\theta / N_g$  are increasing and of the same size. If the relation is expressed as

$$\dot{\gamma} = (\dot{\psi} - \omega / N_g) / (1 - 1/N_g) \quad (5.2)$$

the numerical difficulties are avoided.

Torque and energy balance give

$$T_{\text{gear}} = -T_\gamma / (1 - 1/N_g) \quad (5.3)$$

$$T_m = -T_\gamma / (N_g - 1) \quad (5.4)$$

where  $T_{\text{gear}}$  is the driving torque from the wind turbine,  $T_m$  is the torque driving the generator and

$$T_\gamma = T_{\text{sp}} + T_d \quad (5.5)$$

$T_{\text{sp}}$  is the reaction torque from the mounting of the gearbox.

$$T_{sp} = \begin{cases} -K_{g1} y - (K_{g0} - K_{g1}) y_{min}, & y < y_{min} \\ -K_{g0} y, & y_{min} < y < y_{max} \\ -K_{g1} y - (K_{g0} - K_{g1}) y_{max}, & y_{max} < y \end{cases} \quad (5.6)$$

The mounting is soft in the interval  $y_{min} < y < y_{max}$  and outside this it is much stiffer.

$T_d$  is the damping torque from the hydraulic dampers. It can be modelled to have either a linear or a quadratic characteristic. If the switch LIN is set to true

$$T_d = - \max(-T_{dmax}, \min(T_{dmax}, D_{g1} |\dot{y}| \dot{y})) \quad (5.7)$$

otherwise

$$T_d = - \max(-T_{dmax}, \min(T_{dmax}, D_{g2} \dot{y}^2)) \quad (5.8)$$

This model can be found in Appendix D.

## 6 THE PITCH SERVO

The pitch angles of the blades can be controlled by changing the pitch angle reference signal to the hydraulic pitch change mechanism. This servo system is modelled as a first order system with limits on the rate.

$$\dot{\beta}_{75} = \min(\dot{\beta}_{\max}, \max(\dot{\beta}_{\min}, (\beta_{\text{ref}} - \beta_{75})/\tau)) \quad (6.1)$$

The variable  $\beta_{75}$  is the pitch angle of blade at 3/4 radius (convention of Hamilton Standard).  $\dot{\beta}_{\max}$  and  $\dot{\beta}_{\min}$  are the maximum and minimum rate limits.  $\beta_{\text{ref}}$  is the reference to the pitch servo and  $\tau$  is the time constant of the pitch servo. This model can be found in Appendix E.

## 7 THE BUS

The generator is assumed to be connected to an infinite bus via an impedance  $R_1 + jX_1$  (Assumption 2.1). This means that

the bus voltage and bus frequency are not affected by the wind turbine system. A model of the bus with constant bus voltage and bus frequency and with constant line impedance is given Appendix F1. Furthermore, by making the bus frequency and bus voltage time varying it is possible to model a large utility grid in both normal operation and during faults. Consider the following example.

The integration of the WTS-3 prototype into the local electrical grid in southern Sweden is of great interest. A simple model was developed for the grid (Figure 7.1). A 3-phase fault in different locations (a-g) can be modelled by reducing the faults to step-changes in the bus voltage. Impedance variations are very small due to the transformers and can be neglected. The system in Appendix F2 simulates 3-phase faults by changing the bus voltage to a given level at a given point of time. After a given time the bus voltage is reset to normal level, simulating breaker action in the line where the fault was applied. The system also models stochastic variations in the bus frequency.

## 8 THE WIND MODEL

It is possible to use measured wind data sequences when simulating the system in SIMNON. Below some analytical models for the wind are discussed.

### Mean\_Wind

Shepherd (1978) gives a survey over wind characteristics and the discussion of the effect of height shows that the mean wind velocity  $\bar{U}(z)$  at the height  $z$  over the ground level can be modelled as

$$\bar{U}(z_2)/\bar{U}(z_1) = \ln(z_2/z_0)/\ln(z_1/z_0) \quad (8.1)$$

$z_0$  is a measure of terrain roughness. The roughness length is not an actual dimension, but an equivalent parameter which has been deduced from actual wind profiles. Level ground is approximated by  $z_0 = 0.03$  m and rough terrain by  $z_0 = 1.0$  m. Above the reference level of 10 m the values for level ground are expressed with negligible error by the simple power relationship

$$\bar{U}_2/\bar{U}_1 = (z_2/z_1)^{1/7} \quad (8.2)$$

For rough terrain an exponent of 0.4 gives a useful simple expression although it is in error by about 5 % at 30 m.

In Section 3 it is assumed that the wind profile at the rotor disc is linear

$$U(t,x) = U_0(t) + U_x(t) x \quad -R \leq x \leq R \quad (8.3)$$

where  $x$  is the height with reference to the hub. 8.1 can be well approximated by a linear expression over the rotor disc.

### Gusts

Hwang and Gilbert (1978) model discrete longitudinal wind gusts as

$$U(z) = \begin{cases} \bar{U}(z) + A(\tau, z) \left(1 - \cos \frac{2\pi t}{\tau}\right), & 0 \leq t \leq \tau \\ \bar{U}(z), & \text{otherwise} \end{cases} \quad (8.4)$$

$$A(\tau, z) = 1.5 \frac{\bar{U}(z_1)}{\ln(z_1/z_0)} \{1 - \exp[-\bar{U}(z) \tau / (1.48z)]\}^{1/2} \quad (8.5)$$

8.4 can also be approximated by a linear expression of the form 8.3. A wind model of this kind can be found in Appendix G1.

### Turbulence

The framework of stochastic processes can be used to model turbulence. In Gregefors (1980) AR- and ARMA-models are identified from real wind data.

The system in Appendix G2 generates a pseudo random wind sequence. The spectral property of the longitudinal wind velocity is approximated by a first-order model. However, accounting for the spatial filtering effect over a large area in the space, typical for a large scale wind turbines, a first order spatial filter is also introduced.

## 9 THE SUPERVISORY SYSTEM AND THE ROTOR CONTROLLER

The WTS-3 control system is a complex, microprocessor based, distributed system which has been partitioned into a number of subsystems. The supervisory controller is one of these subsystems. It supervises the system, decides mode of operation and governs the rotor controller which is another subsystem. The rotor controller is used to establish a blade angle reference signal to the electrohydraulic positioning system.

### The Supervisory System

In Figure 2.2 the supervisory system is a special subsystem and it is recommendable to have it like this when simulation complex sequences like synchronization of the generator against the grid. However, in simple cases where the supervisory system is uninteresting like those given in Appendix A1 and A2 it is not necessary to have a special subsystem, but it can be handled in the connecting system.

### The Rotor Controller

It is possible to use different rotor controllers when simulating the system. The controller given in Appendix H1 follows the specifications given in "System Requirements Specification of Rotor Controller for WTS-3", No. 246x93, Revised 1/28/80. Four modes of operation are defined: Acceleration control mode, rotor speed control mode, power control mode and deceleration control mode. In acceleration control mode a safe startup is ensured by controlling the maximum acceleration of the rotor and by preventing excessive blade stall. In rotor speed control mode the generator speed is matched to line frequency for a safe synchronization. In power control mode the generator is connected to the power grid and the blade angle is modulated to extract maximum power from the wind. The deceleration control mode is used for normal shutdowns and controls the maximum deceleration torque to an acceptable level. Because of the assumptions made on the inflow ratio  $\lambda$  ( $0.05 \leq \lambda \leq 0.2$ ) in Section 3, the maximum limit  $\beta_{\max}$  is not

scheduled, but kept constant. The parameters of the controller are set to the nominal values given in the specifications.

In Appendix H2 a modified version of that one in Appendix H1 is given. Lag-lead compensation for control/tower interaction in on-line mode and a notch filter to attenuate 2p-oscillations are added. The scheduling of  $K_Q$  and  $K_{S1}$  is changed. The value of  $T_2$  is changed.

## 10 THE EXCITATION SYSTEM AND THE VOLTAGE CONTROLLER

The excitation system of the generator use an AC alternator and a rotating rectifier to produce the direct current needed for the generator field.

The voltage control system is supplied to achieve a constant terminal voltage, independent of the load conditions and to distribute reactive power among the synchronous machines working in parallel at the grid.

The model of the excitation system and voltage controller given in Appendix I uses a Type 1 standard IEEE (IEEE Committee Report (1968)) representation including non-linear saturation effects.



## 11 DISCUSSION

Four models for the complete systems were found in literature: Hwang and Gilbert (1978), Kos (1978), Edris (1979) and Krause and Man (1981) (also in Wasynczuk, Man and Sullivan (1981)). These four models are similar to that one given in this report. The differences are mainly in the aerodynamical parts. All the four models have a Park model for the synchronous generator. Hwang and Gilbert's is identical to that of Olive (1968) and which also is given in Appendix C1. Edris and Krause and Man have a model of the same complexity. Kos states that he has tried complex Park models, but that the swing equation is satisfactory for his purposes. Hwang and Gilbert have a torsionally rigid drive train, while the others have a torsionally soft drive train as that in this report. The pitch servo is a first order system with limits on the rate in all the four models. The tower, the nacelle and the blades are modelled as rigid bodies in Hwang and Gilbert (1978) and Edris (1979). Kos (1978) and Krause and Man (1981) model the first edgewise oscillations by a second order system. The yawing, the tower shadow or the flapping is not considered in any model. However, Kos and Krause and Man consider the shear effects when calculating the driving aerodynamical torque. They represent the blade by a point at  $3/4$  of the distance from the hub to the blade tip, when calculating the aerodynamic torques.

None describes how the models are programmed. The advantages with the model presented here compared to their models, since the equations are almost identical, is that this model is modularized and programmed in a high-level language, which make the model flexible, readable and easy to modify and use.

Each submodel is carefully tested and the model was preliminary validated by simulation comparisons with runs from the designer (See Östberg (1981)). Only minor discrepancies were found. Furthermore (Östberg (1981)), the model has been used to study the dynamical properties of WTS-3 in on-line and off-line mode in turbulent wind and to study the transient behaviour during electrical disturbances.

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