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Mannerfelt, Carl Fredrik

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CARL FREDRIK MANNERFELT

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY

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Carl Fredrik Mannerfelt

Department of Automatic Control

Lund Institute of Technology

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LUND INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AUTOMATIC CONTROL
Box 725
S 220 07 Lund 7 Sweden

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Carl Fredrik Mannerfelt

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Abstract

The problem of control of systems with known dynamics and unknown disturbances is adressed. A solution to the problem is obtained by first assuming that the disturbance characteristics are known. The corresponding controller is a pole-placement regulator. Then the unknown parameters of the system that generates the disturbance are estimated by the least squares method. By using the parameter estimates as the true parameters in the regulator design, the controller is made self-tuning.

Simulations show that the proposed self-tuning regulator has attractive properties, also when the disturbance has time-variable parameters.

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1. INTRODUCTION

The theory for adaptive and self-tuning regulators has developed considerably during the recent years. There has appeared some new approaches to the problems of regulator design and parameter estimation. However, the basic design principles for self-tuners can still be divided into two cathegories:

I. The regulator problem formulated in a stochastic framework. This problem is often solved with optimal control techniques, e.g. self-tuning minimum variance control, see Åström and Wittenmark (1973).

II. The servo problem formulated in a completely deterministic framework. This is usually done by specifying a desired set of poles and zeros for the closed-loop system, e.g. self-tuning pole-placement design, see Åström and Wittenmark (1980).

It is assumed that the reader is familiar with these basic design principles.

A combination of the stochastic control problem and the deterministic servo and regulator problems, seems to be non-trivial to solve with self-tuners. This was indicated in Wittenmark (1973), where a self-tuning minimum variance regulator with forced integral action was simulated.

The present paper is concerned with self-tuning control of a special class of systems. The systems we consider have known dynamics, and are subjected to deterministic disturbances with unknown characteristics. The control situation implied by these assumptions is quite common. It is often disturbances of the type step, ramp or sinusoid, that is the main reason for control, while the dynamics of the process is well defined and is no source of trouble.

The design method is first formulated in the case of known process disturbances. The resulting controller is a pole-placement regulator, which has infinite gain at the frequencies of the disturbance. When the disturbance characteristics are unknown, they can be estimated with system identification methods. By estimating the parameters in a model of the disturbance and using estimates in the regulator design, the regulator is made adaptive. Simulations show that the self-tuning regulator has good ability to eliminate disturbances. They also show that the regulator can adapt itself to disturbances with slowly time-varying characteristics.

The paper is organized as follows: The proposed design method for systems with known disturbances, is presented in Section 2. In Section 3, the algorithm for the self-tuning regulator is given. The simulation experiments are presented in Section 4, and Section 5 contains the conclusions. Finally, the references are listed in Section 6.

2. DESIGN FOR SYSTEMS WITH KNOWN PARAMETERS

In this section we start with the formulation of the control problem. Then a design method for systems with both known dynamics and known disturbance characteristics is presented. The method is not new. It is rather the formulation of the control problem that makes the design method suitable for self-tuners.

Problem Formulation

Consider a process with the input-output relation

$$Ay = Bu + w \tag{2.1}$$

where u is the input, w is a disturbance and y is the process output. A and B are polynomials in the forward shift operator q. The process disturbance w is assumed to be the output of a linear autonomous dynamical system. The disturbance then satisfies

$$Dw = 0 \tag{2.2}$$

where the zeros of the polynomial D are the poles of the system that generates the disturbance. D is also referred to as the generating polynomial of the disturbance, see Åström (1979). It is assumed that the polynomials A and B are known, and that the polynomial D is unknown.

The design problem is to find a regulator such that the transfer function from command input u_c to output y is given by

$$B_M B/A_M \tag{2.3}$$

where A_M and B_M are given polynomials. Further, the disturbance should be totally eliminated in the system output.

The Design Procedure

If the generating polynomial D of the disturbance is known, the posed problem is easy to solve with the pole-placement design method, see Åström and Wittenmark (1980). A general linear regulator can be represented as

$$Ru = - Sy + Tu_c \tag{2.4}$$

where R, S and T are polynomials in the forward shift operator. By combining (2.1) and (2.4), we can represent the closed-loop system as

$$y = \frac{BT}{AR + BS} u_c + \frac{R}{AR + BS} w \tag{2.5}$$

The condition that the effects of the disturbance should be totally eliminated in the output, leads to

$$Rw = 0 \tag{2.6}$$

This, and equation (2.2), give that R can be factorized as

$$R = R_1 D \tag{2.7}$$

Since the poles of the regulator are the zeros of the polynomial R, relation (2.7) shows that the regulator has infinite gain at the frequencies (modes) of the disturbance.

The other condition of the design, is that

$$\frac{T}{AR + BS} = B_M/A_M \tag{2.8}$$

which implies that T can be factorized as

$$T = A_o B_M \tag{2.9}$$

The polynomial A_o can be interpreted as an observer polynomial, see Åström (1976), and it should be chosen to have zeros inside the unit circle |z| = 1 (the stability region). The regulator polynomials R_1 and S can now be derived from the equation

$$A_o A_M = ADR_1 + BS \tag{2.10}$$

3. A SELF-TUNING REGULATOR

We will now proceed to derive a self-tuning regulator for the control problem described in the previous section.

The idea behind self-tuning regulators is the following: Start with some suitable design method for systems with known parameters. Then use some identification procedure, such as the Recursive Least Squares (RLS) method, to estimate the unknown parameters of the system. The regulator is then redesigned at each sampling step, using the estimated parameters.

We start with a discussion of the parameter estimation.

Parameter Estimation with RLS

In the posed control problem, the characterization of the disturbance, i.e. the coefficients in the D-polynomial, are assumed to be unknown. The disturbance w is itself also unknown, but it can be computed from known data:

$$w = Ay - Bu \tag{3.1}$$

Since w is known to satisfy

$$Dw = 0 \tag{3.2}$$

for some polynomial D, it is straightforward to estimate the unknown coefficients in D. Practically any estimation method can be used. Here we proceed with the recursive least squares method, RLS.

First specify n, the number of parameters in D that is to be estimated. the structure of D is thus assumed to be

$$D(q) = q^n + d_1 q^{n-1} + \ . \ . \ . + d_n \tag{3.3}$$

Introduce a vector of parameter estimates

$$\theta = [\ \hat{d}_1 \ . \ . \ . \ . \ \hat{d}_n\ ]^T \tag{3.4}$$

and a vector of regression variables

By examining equation (2.10) it is possible to get conditions on the degrees of the polynomials R_1 and S. This is fully elaborated in Åström and Wittenmark (1982).

The resulting design procedure can be summarized as:

Data: Given polynomials A,B and D that represent the process, and polynomials A_M and B_M that describe the desired closed- loop system.

Step 1: Determine a desired observer polynomial A_o, of degree

$$\deg A_o \geq 2 \deg A + \deg D - \deg A_M - 1 \qquad (2.11)$$

Step 2: Solve the equation

$$A_o A_M = ADR_1 + BS$$

with respect to R_1 and S. The degrees of R_1 and S are

$$\deg R_1 = \deg A_o + \deg A_M - \deg A - \deg D$$

$$\deg S = \deg A + \deg D - 1$$

Step 3: Form the control law

$$R_1 Du = - Sy + A_o B_M u_c \qquad (2.12)$$

In the next section we will present an algorithm for a self-tuning regulator, based on the proposed design method.

$$\varphi(t) = [\, -\hat{w}(t-1) \;.\;.\;.\;.\; -\hat{w}(t-n) \,]^T \tag{3.5}$$

where $\hat{w}$ is computed according to (3.1), but with delayed inputs and outputs

$$\hat{w} = q^{-\deg A}[\, Ay - Bu \,] \tag{3.6}$$

The RLS estimate is then given by

$$\theta(t) = \theta(t-1) + P(t)\varphi(t)\varepsilon(t) \tag{3.7}$$

where ε is the equation error

$$\varepsilon(t) = \hat{w}(t) - \varphi(t)^T\theta(t-1) \tag{3.8}$$

and

$$P(t) = \left[P(t-1) - \frac{P(t-1)\varphi(t)\varphi(t)^T P(t-1)}{\lambda(t) + \varphi(t)^T P(t-1)\varphi(t)} \right] / \lambda(t) \tag{3.9}$$

The forgetting factor $\lambda(t)$ can be chosen as a constant, usually in the range of 0.95 to 1.0, or as a time-varying variable, see e.g. Åström (1982). Use of a forgetting factor that is less than unity has the positive effect that the estimation algorithm (3.4) to (3.9) is able to track the parameters of slowly time-varying disturbances. This will be illustrated in the next section.

If it is known that the D-polynomial can be factorized as

$$D = D_1 D_2 \tag{3.10}$$

where D_1 is known, only D_2 needs to be estimated. In this case we redefine the variable $\hat{w}$ as

$$\hat{w} = q^{-(\deg A + \deg D_1)} D_1 [\, Ay - Bu \,] \tag{3.11}$$

and the θ-vector contains only the estimates of the coefficients in the unknown D_2-polynomial. An example of this is when it is known that the disturbance w has a constant level. The D-polynomial can then be factorized as

$$D = (q - 1)D_2 \tag{3.12}$$

In some situations it may be known that the system is subjected to high-frequency disturbances. Such disturbances are difficult to eliminate since this often requires large control signals. It is then no longer interesting to estimate the corresponding factors in the D-polynomial. A way to avoid this is to filter the $\hat{w}$-variables before using them in the estimation algorithm, according to

$$\hat{w} = q^{-degA} H [Ay - Bu] \tag{3.13}$$

Here H is a (causal) low-pass filter with appropriate bandwidth. This measure also reduces the disturbing effect that high-frequency modelling errors in the process model, i.e. the A and B polynomials, may have on the parameter estimation.

In the original formulation of the problem, it was assumed that the disturbance satisfied

$$Dw = 0 \tag{3.14}$$

for some D. This assumption may in fact be relaxed to

$$Dw = \delta \tag{3.15}$$

where δ is zero for most times. At the time instants when $\delta \neq 0$, the system that generates the disturbance is excitated by some mechanism. The RLS estimation of the D-polynomial works well also in this case. If we use the assumption that the sequence $\{\delta(t)\}$ is a stochastic process which can be represented as independent white noise, the parameter estimation still works well.

A Self-Tuning Regulator

The algorithm for an adaptive regulator for the control problem can now be formulated. The following steps are repeated at each sampling instant:

Step 1: Estimate the parameters of the D-polynomial with the recursive least squares method.

Step 2: Design the regulator according to the algorithm in Section 2, using the estimated D-polynomial.

The assumption (2.2) of the disturbance means that future values of the disturbance can be exactly predicted, if the D-polynomial is known. When the D-polynomial now instead is estimated, the self-tuning regulator can be interpreted as a combination of feedback from a pole-placement controller and feedforward from an adaptive predictor.

In the next section we illustrate the properties of the self-tuning regulator with some simulation examples.

4. SIMULATIONS

In this section, two simulation examples are presented to illustrate some properties of the self-tuning regulator. The simulations are made using the simulation package Simnon, see Elmquist (1977).

Example 1

Assume that the system to be controlled can be represented as

$$y(t+1) = y(t) + u(t) + w(t) \tag{4.1}$$

where the disturbance w is the sum of a constant level and a sinusoidal component

$$w(t) = w_0 + w_1 \sin(2\pi t/T_p) \tag{4.2}$$

This process model could e.g. be used as a description of a tank with variable inflow and outflow, or as a model of a product inventory when the demand for the stored article is varying periodically.

The desired input-output relation of the system is

$$y(t+1) = u_c(t) \tag{4.3}$$

The relevant polynomials thus are

$$A(q) = q - 1 \qquad \text{and} \qquad B(q) = 1$$

$$A_M(q) = q \qquad \text{and} \qquad B_M(q) = 1$$

$$D(q) = q^3 + d_1 q^2 + d_2 q + d_3$$

From relation (2.11) we see that the observer polynomial A_o is at least of 3:rd order, and we choose it arbitrarily as

$$A_o(q) = q^3$$

This leads to the following structure of the regulator polynomials

$$R_1(q) = 1$$

$$S(q) = s_0 q^3 + s_1 q^2 + s_2 q + s_3$$

$$T(q) = A_o(q) B_M(q) = q^3$$

By solving the polynomial equation we get

$$s_0 = 1 - d_1$$

$$s_1 = d_1 - d_2$$

$$s_2 = d_2 - d_3$$

$$s_3 = d_3$$

In Figs. 4.1 and 4.2 are shown the simulations of the process when the system is controlled by the self-tuning regulator. The command input u_c is a square wave signal. The characteristics of the disturbance is $w_0 = 2$, $w_1 = 0.5$ and $T_p = 10$. The forgetting factor was $\lambda = 0.95$. It is seen that the control rapidly is becoming very good. The estimates of the D-coefficients quickly converge to constant values. At time t = 100 the period of the disturbance is changed to the correct
$T_p = 20$. The self-tuner is seen to rapidly adapt itself to the new disturbance.

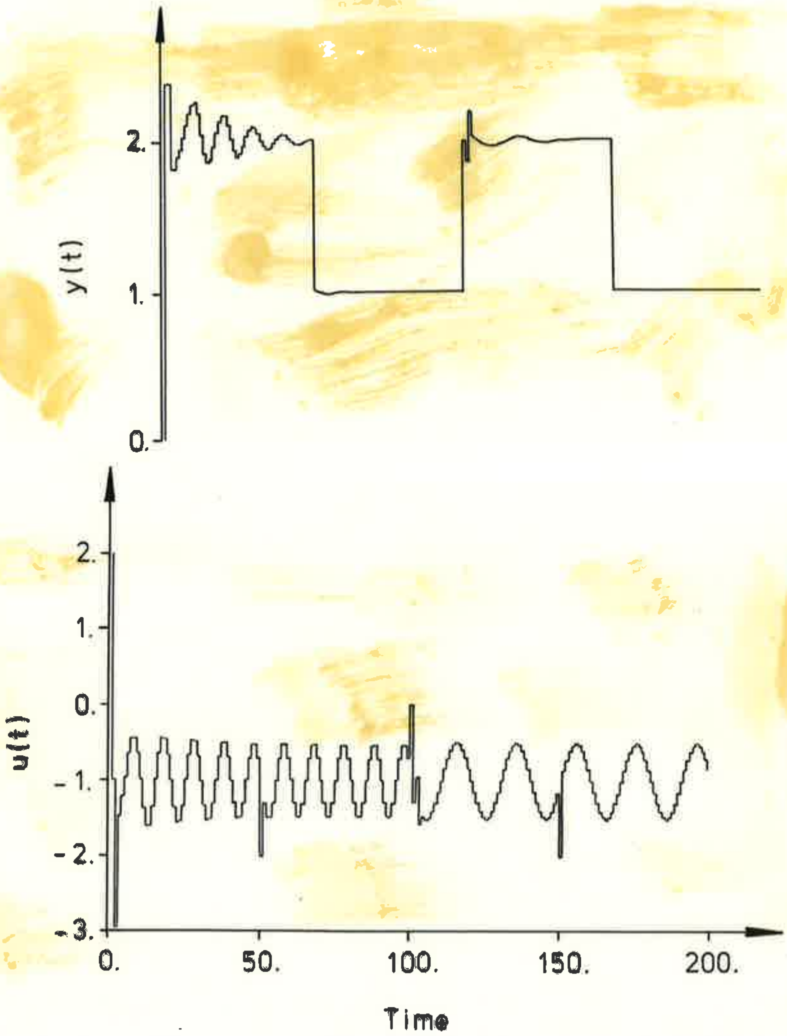

Fig. 4.1 - The variables of the simulated process. At time $t = 100$ the period of the disturbance is changed to $T_p = 20$.

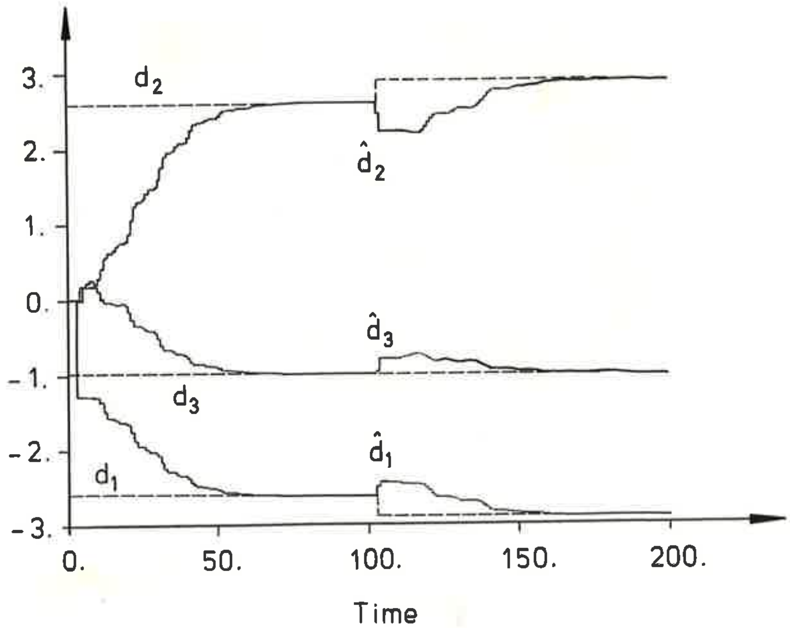

Fig. 4.2 - The D-coefficients and their estimates.

Example 2

Assume that we want to control the speed of a DC servo motor, described by the set of differential equations

$$\dot{x}_1 = - x_1 + 25u - 20\sin(x_2/100)$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_3 = - 30x_3 - 2500x_4 + 2500x_1$$

$$\dot{x}_4 = x_3$$

$$y = x_4 \tag{4.4}$$

Here x_1 is the angular velocity and x_2 is the angle of the motor axis. The states x_3 and x_4 represent a resonance mode that is due to a flexible coupling between the motor and a tachometer generator. This mode is poorly damped, $\zeta = 0.3$ and $\omega_0 = 50$. The motor is subjected to a periodic load disturbance that is dependent of the angle of the axis, via a gear with ratio 100:1. The system thus is nonlinear. This type of disturbance is common for instance when motors are used to drive rotating pumps.

To avoid aliasing effects the process is sampled with a short sampling interval, h = 0.01. We use a simplified linear process model for the sampled system

$$Ay = Bu + w \tag{4.5}$$

where y is the measured angular velocity, u is the control signal and w is a periodic disturbance. The polynomials are

$$A(q) = q - 0.99 \quad \text{and} \quad B(q) = 0.25$$

We require that the regulator should have integral action, and the relevant D-polynomial therefore has the structure

$$D(q) = D_1(q)[\, q^2 + d_1q + d_2 \,]$$

where

$$D_1(q) = q - 1$$

The closed-loop system transfer function is desired to be represented by the polynomials (according to (2.3))

$$A_M(q) = (q - 0.95)^2 \quad \text{and} \quad B_M(q) = 0.01q$$

This choice represents a moderate speeding-up of the motor, but is otherwise arbitrary. With the above polynomials, it follows from relation (2.11) that the observer polynomial A_o is at least of 2:nd order, and we choose it arbitrarily as

$$A_o(q) = (q - 0.9)^2$$

The regulator polynomials then have the structures

$$R_1(q) = 1$$

$$S(q) = s_0q^3 + s_1q^2 + s_2q + s_3$$

$$T(q) = (q - 0.9)^2 0.01q$$

By solving the polynomial identity we obtain

$$s_0 = - 6.84 - 4d_1$$

$$s_1 = 16.57 + 7.96d_1 - 4d_2$$

$$s_2 = -12.654 - 3.96d_1 + 7.96d_2$$

$$s_3 = 2.924 - 3.96d_2$$

We tried to simulate the self-tuning regulator based on the data above, but the control of the motor was rather poor.

The reason for this was that the resonant mode in the system destroyed the estimation of the parameters of the D_2-polynomial. We then used a low-pass filter in the parameter estimator, according to (3.13), to reduce the effects of the high frequency modelling errors. The filter was

$$H(q) = \frac{0.05\, q^2}{(q - 0.95)(q - 0.995)}$$

This choice was not critical. The remedy proved to be satisfactory. A simulation of the closed-loop system is shown in Figs. 4.3 and 4.4. It is seen that the control is quite satisfactory. The command input is a square wave signal, switching between the levels 100 and 50 radians/second. The forgetting factor in the estimator was chosen to $\lambda = 0.95$.

It is obvious in this example that the sampling rate is unnecessarily high, a sampling interval in the range 0.1 to 0.5 would probably have been more appropriate. In this case, however, a presampling filter would have been necessary in order to avoid aliasing. Since the inclusion of such a filter partly would have destroyed the illustrating effect of the example, we prefered to use the high sampling rate instead.

Here it is appropriate to stress that the proposed self-tuning regulator eliminates the disturbances in the sampled system. In the corresponding continuous time system, the disturbance is in general only eliminated at the sampling instants and not during the sampling intervals. The faster the process is sampled, the better the control becomes. A common choice of sampling period for a system with desired bandwidth ω_b, is given by $\omega_b h < 0.5$. Since the disturbances we want to eliminate are within the closed-loop system bandwidth, this choice of h usually gives sufficient disturbance attenuation.

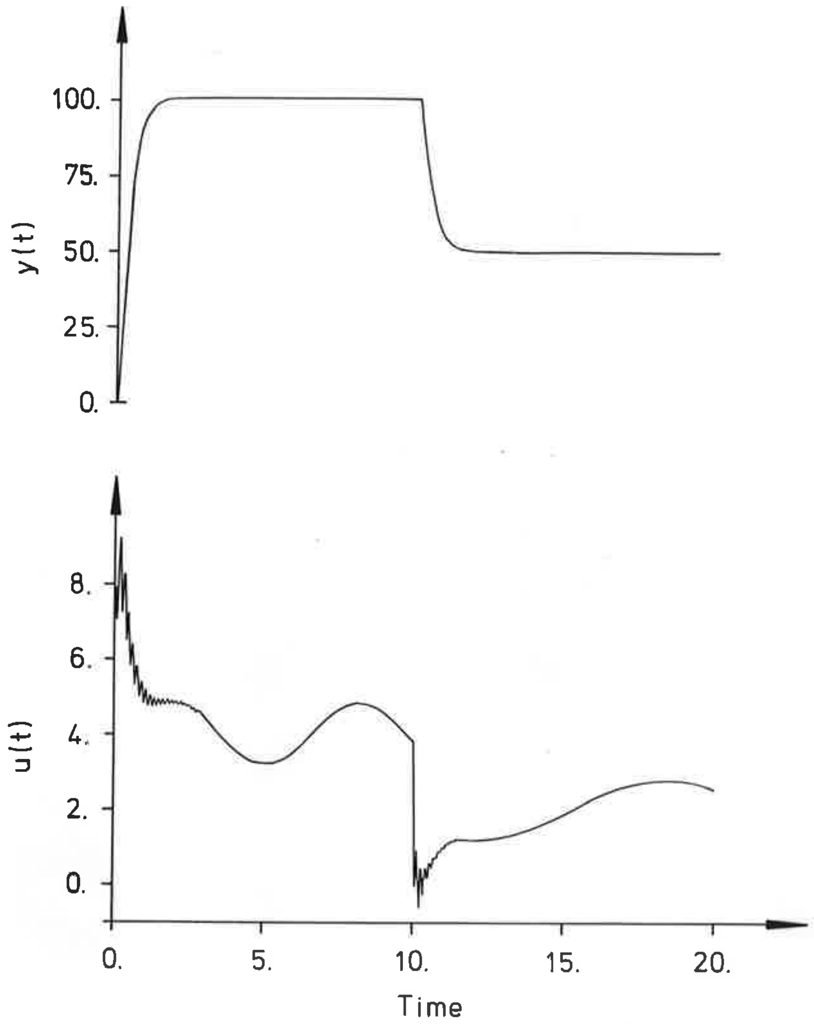

Fig. 4.3 - The variables of the simulated process.

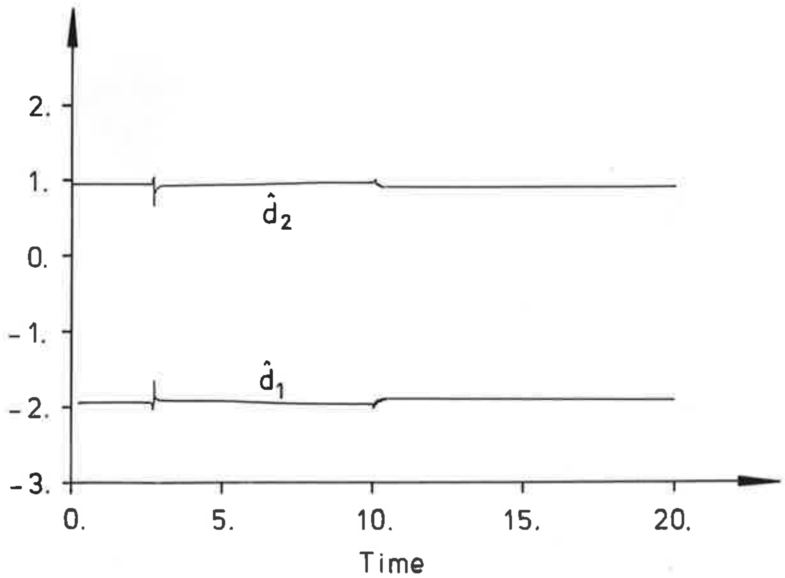

Fig. 4.4 - The estimated parameters of the D_2-polynomial.

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5. CONCLUSIONS

We have presented a self-tuning regulator for systems with known dynamics and unknown disturbance characteristics. The regulator is based on a design method for systems with known parameters, which is a variant of pole-placement design. It is assumed that the disturbance acting on the process is a completely deterministic signal, which thus can be described by its generating polynomial. It is this polynomial that is estimated in the proposed adaptive regulator. Some properties of the regulator are illustrated by two simulation examples. The controlled systems behave satisfactory, and the estimation of the unknown (and sometimes timevarying) coefficients of the generating polynomial of the disturbance is quite acceptable. A means to reduce effects of badly modelled system dynamics in the estimation, is also presented.

An interesting goal for future research is to extend the present self-tuning regulator to additionally estimate the dynamics of the system.