A Probing Control Strategy: Stability and Performance

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A probing control strategy: stability and performance

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***Abstract*— An extremum controller based on a pulse technique is examined. Plants consisting of the cascade of a piecewise linear static function and a LTI system are considered. Numerical methods for stability and performance analysis of the closed-loop systems are derived. Performance is measured by the ability to track a time-varying optimal point. An illustrative example where it is desirable to control the process to a saturation instead of an extremum is finally presented.**

I. Introduction

In most control problems the objective is to regulate the output around a desired reference value. In extremum control, the optimal setpoint is not known and is often given by the extremum of a static input-output map. The classical approach to this problem consists in adding a known time-varying signal to the process input and correlating the output with the perturbation signal to get information about the nonlinearity gradient. The controller adjusts continuously the control signal towards the optimum. A good overview of extremum control is given in [11].

In [9], the authors presented the stability analysis of an extremum seeking scheme for a general nonlinear dynamical system. Stability of the seeking scheme was proven under restrictive conditions: small adaptation gain and fast plant dynamic. In [8] they developped a tighter analysis where the process was modelled by a Wiener-Hammerstein system. No stability region was provided.

In [1] and [2] a probing controller based on a pulse technique is described. The main difference with the classical scheme is the separation in time of the correlation phase and the control phase. Pulses are periodically introduced at the process input and a control action is taken at the end of every pulse. This also allows the regulation of the process output by manipulation of a second control variable between two successive pulses. The control algorithm has been implemented and tested on real plants where good performance could be achieved, see for instance [3] or [5]. Although the control strategy is simple, it results in a complex closed-loop system that is nonlinear, time-varying with continuous as well as discrete states. Rigorous analysis of the closed-loop system would be valuable for a better understanding and tuning of the probing controller.

In a previous paper [12], we analysed the probing strategy for Hammerstein systems with a piecewise affine nonlinearity. Stability analysis could be performed by searching for piecewise quadratic Lyapunov functions and solving appropriate LMI, as described in [7] and [6]. The integrator from the controller was however approximated by a pole close to 1 and a time invariant nonlinearity was considered.

In the present paper we derive tools for piecewise affine systems in discrete time, which can be applied for rigorous analysis of the probing strategy. A reduced set of LMIs is derived to investigate closed-loop stability using standard optimization routines, such as the *LMI Control toolbox* in *matlab*. Performance of the probing controller is measured by the ability to track a time-varying optimum. Using the idea from [10] a quantitative result is given.

The paper is organized as follows. In section II, we describe the system to be optimized together with the probing controller. Section III presents a way to check global asymptotical stability of piecewise affine systems with integrator. Tools for performance assessement of the probing controller are also derived. We will finally illustrate in section IV the proposed methods with an example inspired by [1] where it is of interest to control the process to a saturation instead of an optimum. Tuning guidelines based on local analysis will also be provided.

II. A probing controller

The system to be optimized is of Hammerstein type: a static nonlinearity followed by a stable dynamical

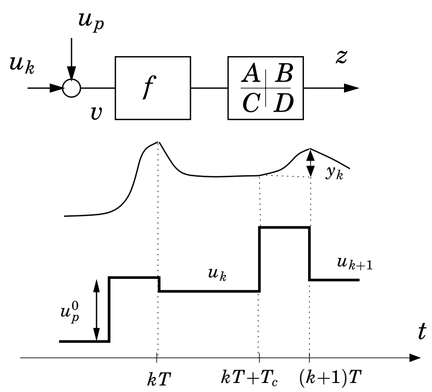

Fig. 1. Illustration of the probing controller. A pulse in the input signal v leads to a response in the output z. The size y_k of the pulse response is used to compute the change $u_{k+1} - u_k$.

linear plant. A state space representation of the process can be written as:

$$\begin{aligned} \dot{x} &= Ax + Bf(v) \quad x \in R^n \\ z &= Cx \end{aligned} \tag{1}$$

The control objective is to find and track the optimal point for which the gradient of f is small. The probing controller gets information about the nonlinearity from the response to pulses that are periodically superimposed to the control signal. The input signal v to the process is the sum of the control signal u_k and the perturbation signal $u_p(t)$:

$$v(t) = u_k + u_p(t) \qquad t \in [kT,\ kT + T] \tag{2}$$

$u_p(t)$ is taken to be a pulse train with period T and amplitude u_p^0:

$$u_p(t) = \begin{cases} 0 & t \in [kT,\ kT + T_c) \\ u_p^0 & t \in [kT + T_c, (k+1)T) \end{cases} \tag{3}$$

The pulse length is T_p such that

$$T = T_p + T_c \tag{4}$$

The piecewise constant control signal u_k is adjusted at the end of every pulse, depending on the size of the pulse response. Figure 1 illustrates the behavior of the probing controller. Since the input signal v defined by (2) and (3) is piecewise constant, equation (1) can be exactly integrated over one period T. The response to a pulse is the output $y_k = z((k+1)T) - z(kT + T_c)$ of a discrete-time system with sample interval T:

$$\begin{aligned} x_{k+1} &= A_o x_k + B_o \begin{bmatrix} f(u_k) \\ f(u_k + u_p^0) \end{bmatrix} \\ y_k &= C_o x_k + D_o \begin{bmatrix} f(u_k) \\ f(u_k + u_p^0) \end{bmatrix} \end{aligned} \tag{5}$$

where $x_k = x(kT)$ and

$$\begin{aligned} A_o &= e^{AT} \qquad C_o = C(e^{AT} - e^{AT_c}) \\ B_o &= \left[(e^{AT} - e^{AT_p}),\quad (e^{AT_p} - I)\right] A^{-1}B \\ D_o &= C(e^{AT_p} - I)\left[(e^{AT_c} - I),\quad I\right] A^{-1}B \end{aligned} \tag{6}$$

In [12], the output z was regulated between two successive pulses by an auxiliary control variable. Although the additional loop does not affect the structure of equation (5), it is not modelled in the present paper for the sake of simplicity.

For a better understanding of the probing controller, consider now the case of a large period T. When T_c goes to infinity, the influence of the state x_k vanishes and equation (5) reduces to a static input-output map:

$$y_k = C(e^{AT_p} - I)A^{-1}B \cdot (f(u_k + u_p^0) - f(u_k)) \tag{7}$$

Using the integrating feedback law

$$u_{k+1} = u_k + K(y_k - y_r) \tag{8}$$

with a desired pulse response $y_r = 0$, the equilibrium point u_∞ is such that $f(u_\infty + u_p^0) - f(u_\infty)$ vanishes, ie for a u_∞ that makes the gradient small.

This rough analysis indicates that a probing strategy using (8) can converge to the optimal point. What happens when the pulses are done more frequently? Do we have convergence to the optimal point when the dynamics in (5) is taken into account? Is the probing strategy able to track a time-varying nonlinearity? The paper aims at answering those questions.

We will only consider functions f that are piecewise affine. The closed-loop equations described by (8) and (5) have consequently a piecewise affine structure. Stability analysis can be performed by searching for piecewise quadratic Lyapunov functions. Modifications of the existing methods are however necessary to cope with the integrator that the control law (8) introduces. In the next section, tools for stability as well as performance analysis of piecewise affine systems will be derived. Evaluation of the probing strategy will be performed in section IV using those tools on an example.

III. Piecewise affine systems with integrators

Consider the piecewise affine system

$$\bar{X}^+ = \bar{A}_i \bar{X} \qquad \bar{X} \in \mathbf{X}_i \tag{9}$$

where

$$\bar{X} = \begin{bmatrix} X \\ 1 \end{bmatrix} \qquad \bar{A}_i = \begin{bmatrix} A_i & a_i \\ 0 & 1 \end{bmatrix}$$

$\mathbf{X}_i \subseteq R^n$ is a partition of the state space into convex polyhedral regions.We assume that there is only one equilibrium point, and that it is located in the region with index $i = 0$. The origin is shifted such that $a_0 = 0$.

A. Stability analysis

The search for piecewise quadratic Lyapunov functions as in [7] and [6] is a powerful tool for stability analysis. Let denote by V the Lyapunov function candidate:

$$V(X) = \begin{cases} X^T P_i X & X \in \mathbf{X}_i \quad i = 0 \\ \bar{X}^T \bar{P}_i \bar{X} & X \in \mathbf{X}_i \quad i \neq 0 \end{cases} \tag{10}$$

For V to be a Lyapunov function, one should have for $i \neq 0$:

$$\begin{aligned} V(X) &= \bar{X}^T \bar{P}_i \bar{X} > 0 && X \in \mathbf{X}_i \\ \Delta V(X) &= \bar{X}^T (\bar{A}_i{}^T \bar{P}_j \bar{A}_i - \bar{P}_i) \bar{X} < 0 && X \in \mathbf{X}_i \\ & && X^+ \in \mathbf{X}_j \end{aligned} \tag{11}$$

and similarly for $i = 0$. The search for the matrices $\bar{P}_i$ can be formulated as an optimization problem in terms of LMIs. The stability conditions (11) take the form:

$$\begin{aligned} \bar{P}_i - \bar{R}_i &> 0 \\ \bar{A}_i{}^T \bar{P}_j \bar{A}_i - \bar{P}_i + \bar{S}_{ij} &< 0 \end{aligned} \tag{12}$$

where $\bar{R}_i$ and $\bar{S}_{ij}$ are matrices used in the S-procedure. They express the fact that the inequalities are only required to hold for particular X, e.g.. X in $\mathbf{X}_i$. More details on how to find such matrices can be found in [7].

A solution to (12) implies the existence of $\gamma > 0$ such that $\Delta V(X) < -\gamma \|X\|^2$ for all X. When the state partition contains an unbounded region with an integrator, it may not be possible to bound ΔV quadratically in all directions although it is strictly negative. Modifications of equations (12) for the regions with an integrator are therefore necessary. Our approach is similar to that in [4] for linear systems and consists in deriving a reduced LMI set after removal of the nullspace of $A_i - I$.

Consider a region $\mathbf{X}$ of the state partition, where the dynamics contains an integrator. By a change of coordinates $Z = T^{-1}X$, the dynamics equation in $\mathbf{X}$ can be put in the form:

$$\bar{Z}^+ = \underbrace{\left[\begin{array}{cc|c} A_s & 0_{n-1\times 1} & T^{-1}a \\ 0_{1\times n-1} & 1 & \\ \hline 0_{1\times n-1} & 0 & 1 \end{array}\right]}_{\bar{D}} \bar{Z}, \qquad \bar{Z} = \left[\begin{array}{c} z_s \\ z_u \\ \hline 1 \end{array}\right]$$

where A_s has all its eigenvalues in the open unit ball. Defining $\bar{T} = \text{diag}\{T, 1\}$ and $\bar{Q} = \bar{T}^{-T} \bar{P} \bar{T}$, one can express $\Delta V(X)$ using the new coordinates $\bar{Z}$:

$$\begin{aligned} \Delta V(\bar{T}\bar{Z}) &= \bar{Z}^T (\bar{D}^T \bar{Q} \bar{D} - \bar{Q}) \bar{Z} \\ &= \left[\begin{array}{c} z_s \\ z_u \\ \hline 1 \end{array}\right]^T \left[\begin{array}{cc|c} M_{ss} & M_{su} & M_s \\ M_{su}^T & 0 & m_u \\ \hline M_s^T & m_u & m \end{array}\right] \left[\begin{array}{c} z_s \\ z_u \\ \hline 1 \end{array}\right] \end{aligned} \tag{13}$$

The absence of quadratic term in z_u is a consequence of the eigenvalue 1. Application of the S-procedure can introduce a negative quadratic term in z_u only if the region $\mathbf{X}$ is bounded in z_u direction. When the state can pass to infinity along the eigendirection defined by z_u, the cell description can be rewritten as:

$$\mathbf{X} = \{Z \mid z_u < \begin{bmatrix} G_j & g_j \end{bmatrix} \begin{bmatrix} z_s \\ 1 \end{bmatrix} \text{ for } j = 1 \cdots p\} \tag{14}$$

Define matrices $\bar{N}_j, j = 1 \cdots p$ such that for all $i \neq j$

$$\begin{bmatrix} z_s \\ 1 \end{bmatrix}^T \bar{N}_j \begin{bmatrix} z_s \\ 1 \end{bmatrix} > 0 \text{ when } \begin{bmatrix} G_i - G_j & g_i - g_j \end{bmatrix} \begin{bmatrix} z_s \\ 1 \end{bmatrix} \prec 0$$

The following result should be combined with (12) to investigate stability of piecewise affine systems

Theorem 1: If there exist $\bar{Q}$ with $\bar{Q}_{su} = 0_{n-1\times 1}$ and $\bar{R}$ such that for $j = 1 \cdots p$

$$\begin{aligned} &\bar{T}^T \bar{Q} \bar{T}^{-1} - \bar{R} > 0 \\ &\begin{bmatrix} M_{ss} & M_s + m_u G_j \\ (M_s + m_u G_j)^T & m + m_u g_j \end{bmatrix} + \bar{N}_j < 0 \\ &m_u > 0 \end{aligned}$$

then $V(X) > 0$ and $\Delta V(X) < 0$ in $\mathbf{X}$.

Proof:

The first inequality in the statement guarantees positivity of V in $\mathbf{X}$.

From equation (13) we have

$$\Delta V(Z) = \begin{bmatrix} z_s \\ 1 \end{bmatrix}^T \begin{bmatrix} M_{ss} & M_s^T \\ M_s & m \end{bmatrix} \begin{bmatrix} z_s \\ 1 \end{bmatrix} + 2 \begin{bmatrix} z_s \\ 1 \end{bmatrix}^T \begin{bmatrix} M_{su} \\ m_u \end{bmatrix} z_u$$

M_{su} can be easily computed to be of the form:

$$M_{su} = (A_s - I)\bar{Q}_{su}$$

Since Q is such that $\bar{Q}_{su} = 0$, the cross-term $z_u z_s$ in ΔV vanishes:

$$\Delta V(Z) = \begin{bmatrix} z_s \\ 1 \end{bmatrix}^T \begin{bmatrix} M_{ss} & M_s \\ M_s^T & m \end{bmatrix} \begin{bmatrix} z_s \\ 1 \end{bmatrix} + 2 m_u z_u$$

By assumption, we have

$$m_u > 0$$

it follows from (14) that

$$m_u z_u < m_u \begin{bmatrix} G_j & g_j \end{bmatrix} \begin{bmatrix} z_s \\ 1 \end{bmatrix}, \quad j = 1 \cdots p$$

For less conservative bounds we add the relaxation term involving $\bar{N}_j$

$$m_u z_u < m_u \begin{bmatrix} G_j & g_j \end{bmatrix} \begin{bmatrix} z_s \\ 1 \end{bmatrix} + \begin{bmatrix} z_s \\ 1 \end{bmatrix}^T \bar{N}_j \begin{bmatrix} z_s \\ 1 \end{bmatrix}, \quad j = 1 \cdots p$$

which leads to p upper-bounds for ΔV

$$\begin{aligned} \Delta V(Z) < &\begin{bmatrix} z_s \\ 1 \end{bmatrix}^T \begin{bmatrix} M_{ss} & M_s + m_u G_j \\ (M_s + m_u G_j)^T & m + m_u g_j \end{bmatrix} \begin{bmatrix} z_s \\ 1 \end{bmatrix} \\ &+ \begin{bmatrix} z_s \\ 1 \end{bmatrix}^T \bar{N}_j \begin{bmatrix} z_s \\ 1 \end{bmatrix}, \quad j = 1 \cdots p \end{aligned}$$

and each of them is negative by hypothesis.

Remark 1: The condition $\bar{Q}_{su} = 0$ is not restrictive but actually necessary for ΔV to be negative in the unbounded region.

Remark 2: Along the z_u direction, V is decreasing linearly and the inequality $m_u > 0$ imposes the correct sign of the slope to ΔV.

Remark 3: Since the search for Lyapunov functions is done in terms of $\bar{Q}$, i.e. in the new coordinate systems, it is easy to impose $\bar{Q}_{su} = 0$. $\bar{Q}$ is used in equations (12) as $\bar{P}_i = \bar{T}^T \bar{Q} \bar{T}^{-1}$ for some i.

B. Servo-problem

The previous result provides a way to check global stabilty of piecewise affine systems with integrator, using standard LMI solvers. However, it does not guarantee a good behavior in presence of disturbances.

In [10], the authors propose a way to analyse the servo-problem for piecewise affine systems in continuous-time. Performance was evaluated by computing the L_2 gain between the derivative of the exogeneous input $\dot{r}$ and the error $x - x_r$ between the system trajectory x and a predetermined trajectory x_r.

An extension of the method to piecewise affine systems in discrete time will be performed. Consider the following piecewise affine system with input r:

$$x(k+1) = A_i x(k) + B_i r(k) + a_i \tag{15}$$

The reference trajectory is determined by the sequence of equilibrium points $x_r(k)$:

$$x_r(k) = (I - A_0)^{-1} B_0 r(k) \tag{16}$$

and the performance is measured with the following cost function:

$$J(x,r) = \sum_{k=0}^{\infty} (x(k) - x_r(k))^T \bar{Q} (x(k) - x_r(k))$$

Suppose that for any constant $r \in \mathcal{R}$ the piecewise linear system has a unique equilibrium point located in $\mathbf{X}_0$.

Define

$$\bar{A}_i = \begin{bmatrix} A_i & a_i & B_i + (A_i - I)(I - A_0)^{-1} B_0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{B}_i = \begin{bmatrix} -(I - A_0)^{-1} B_0 \\ 0 \\ 1 \end{bmatrix}, \quad \bar{I} = \mathrm{diag}(I, 0, 0)$$

and the matrices $\bar{S}_i$ such that

$$\begin{bmatrix} x - x_r \\ 1 \\ r \end{bmatrix}^T \bar{S}_i \begin{bmatrix} x - x_r \\ 1 \\ r \end{bmatrix} > 0 \text{ for } x \in \mathbf{X}_i, r \in \mathcal{R}$$

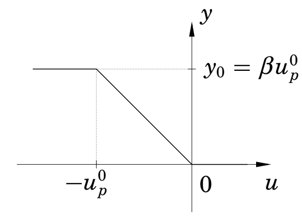

Fig. 2. Stationary amplitude of the pulse response y as a function of u. Small responses indicate that u is close to the saturating point $r = 0$.

We have then the following statement:

Theorem 2: If there exist $\gamma > 0$ and $P_i > 0$ such that $\bar{P}_i = \mathrm{diag}\{P_i, 0\}$ satisfies

$$\begin{bmatrix} \bar{A}_i^T \bar{P}_j \bar{A}_i - \bar{P}_i + \bar{Q} + \bar{S}_i & (\bar{B}_i^T \bar{P}_j \bar{A}_i)^T \\ \bar{B}_i^T \bar{P}_j \bar{A}_i & \bar{B}_i^T \bar{P}_j \bar{B}_i - \gamma^2 \end{bmatrix} < 0, \quad i, j \neq 0$$

and similarly for $i = j = 0$, then every trajectory defined by (15) with $x(0) = 0$ satisfies

$$J(x,r) < \gamma^2 \sum_{k=0}^{\infty} (r(k+1) - r(k))^2$$

The proof is similar to that in continuous case [10] and it will not be repeated here.

IV. Case study

In this section, we will illustrate the performance of the probing controller on an example inspired by [1]. It is desirable to control the process to a saturation instead of an extremum. The process is modeled by a first order system:

$$\begin{aligned} \dot{x} &= -ax + f(v) \qquad a > 0 \\ v &= u_k + u_p(t) \end{aligned}$$

The static nonlinearity f that models the saturation in the cell respiration system is taken to be a min function:

$$f(v) = \min(v, r_k) = \begin{cases} v, & v \le r_k \\ r_k, & v > r_k \end{cases} \tag{17}$$

To start with we assume that the saturating point r_k is constant $r_k = 0$. Time-varying r_k will later be considered for performance evaluation of the controller. We will also assume that r_k does not vary under a period T.

The stationary amplitude of the pulse response as a function of u_k is plotted in Figure 2 where the constant β is a function of the plant dynamics:

$$\beta = (1 - e^{-aT_p})(1 - e^{-aT_c})(1 - e^{-aT})^{-1} a^{-1}$$

The static pulse response indicates that the integrating feedback law (8) with $y_r > 0$ can drive the input u_k close to the saturation $r = 0$. The reference value y_r for the pulse response affects the stationary distance to the saturating point.

The closed-loop system described by (8) and (5) is a discrete piecewise linear system with a state space partioned into 3 regions:

$$\begin{aligned} \mathbf{X}_1 &= \{X \in R^2, u < -u_p^0\} \\ \mathbf{X}_2 &= \{X \in R^2, -u_p^0 < u < 0\}\ , \quad X = \begin{bmatrix} x \\ u \end{bmatrix} \\ \mathbf{X}_3 &= \{X \in R^2, u > 0\} \end{aligned}$$

The system equations can be written as

$$X_{k+1} = A_i X_k + a_i, \quad \text{for} X_k \in \mathbf{X}_i, \quad i \in I = \{1, 2, 3\}$$

where A_i and a_i are matrices given in the appendix. The equilibrium point, if it exists, is located in the middle region. Integrators are always present in the extreme regions.

A. Tuning guidelines

The probing control strategy has a few parameters to be chosen. Some tuning guidelines are necessary for the strategy to work well. Local analysis of the closed-loop system can be performed to derive necessary conditions for global stability.

It is easy to derive a necessary condition for A_2 to be Hurwirtz:

$$0 < K < \frac{2}{\beta} \frac{1 + e^{-aT}}{1 - e^{-aT}} \tag{18}$$

The presence of the integrator in the extreme regions may give rise to situations where the state vector tends to infinity along the critically stable eigendirections. The vector field should therefore be oriented towards the middle region on these directions. Inspection of the vector field leads to the following inequality:

$$0 < \frac{y_r}{u_p^0} < \beta \tag{19}$$

and

$$0 < K < \frac{e^{aT_c} - 1}{\beta} \tag{20}$$

Equation (19) relates the size of the desired pulse response to the dynamics of the open-loop: y_r should not exceed the largest pulse response βu_p^0 that one gets at stationarity, see Figure 2. Equations (18) and (20) give bounds on the controller gain K and therefore limit the convergence speed to the middle region.

B. Numerical computations

For numerical computations we take

$$a = 1, \quad u_p^0 = 1, \quad T_p = 1$$

Conditions (18) and (20) impose constraints on K and T_c. It can be shown that large adaptation gains K are not allowed when the pulses are performed too frequently, i.e. when T goes to $T_p = 1$.

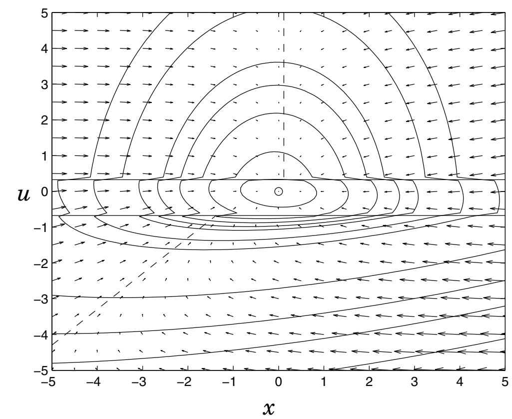

Fig. 3. Phase plane with some level curves of the Lyapunov function. The solid lines represent the cell borders and the dashed line are the eigendirections corresponding to the eigenvalue 1.

We choose $T = 4$ and $K = 2$ to get a fast convergence and some robustness margin.

Equation (19) provides an interval in which y_r should be: $0 < y_r < 0.612$. We choose $y_r = 0.3$ to get a symmetric behavior above and below the saturation.

The parameters of the probing controllers have now been chosen such that all necessary conditions are fulfilled. Local stability alone is however not a satisfactory result. The equilibrium point of the closed-loop system is indeed located close to a cell border.

1) Global stability: Global stability can be investigated using Theorem 1. The LMIs are implemented and solved using *matlab*. Stability of the closed-loop system can be proved for gains $K < 3.1$, which is very close to the upper bound from equation (18) when $T = 4$. Level curves of the piecewise quadratic Lyapunov function as well as the phase plane are shown in Figure 3. Convergence of u to a neighborhood of the saturation can be guaranteed for all initial values of u and x.

2) Performance: For a better understanding of the probing controller, a simulation with a particular trajectory r_k has been performed. The result is shown in Figure 4. The probing controller succeeds to track the time-varying saturation by using the pulse responses for feedback. Theorem 2 from last section can be used to quantify the performance of the closed-loop system for all variations of $r_k \in [-5, 5]$. It can be checked that for those r_k the equilibrium point is always in the middle region defined by $r_k < u_k < r_k - u_p^0$. The integrator in the control law is replaced by a pole close to 1.

The matrice $\bar{Q}$ that penalizes the state deviation $x - x_r$ from its equilibrium point is taken to be $\bar{Q} = \text{diag}\{1, 1, 0, 0\}$. The LMIs from theorem 2 turn out to be feasible. Minimizing γ subject to the constraints, one obtains $\gamma = 30$.

Minimization of γ has been performed for different values of the gain K. The result is plotted in Figure 5

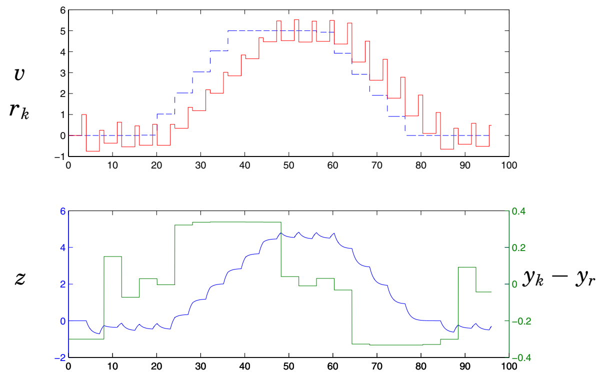

Fig. 4. Simulation of the probing controller with a time-varying optimal point. Top: The input signal v with superimposed pulses is tracking the reference trajectory defined by r_k. Bottom: Output signal z and deviation $y_k - y_r$ of the pulse responses from the reference value.

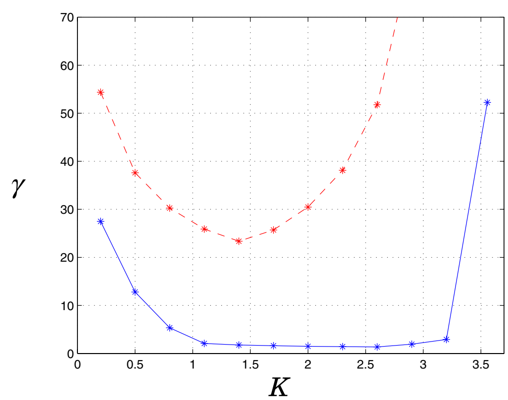

Fig. 5. Performance measurement for different values of the probing controller gain K. The dashed line represents the γ values obtained by numerical computations whereas the solid line is the result of simulations.

together with the gain obtained by simulation with a particular r. A better agreement between the gain obtained by simulations and γ can be achieved by looking for a worst case disturbance r. The graph is however helpful for design purposes. The slow convergence speed for small K values is indicated by the large γ values. The plot suggests a K value of about 1.5. Larger K do not improve much the performance and may give poor robustness properties, as it is seen in simulations.

V. Conclusion

A probing strategy has been analysed for plants of Hammerstein type. When the nonlinearity is piecewise affine, the problem can be formulated in a piecewise affine framework. Numerical algorithms have been proposed to study stability and performance of the closed-loop system. The efficiency of methods have been illustrated on an example where it is desirable to track a saturating point.

Acknowledgements

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Appendix

The closed-loop dynamics in the 3 regions of the state partition is given by the matrices

$$A_1 = \begin{bmatrix} e^{AT} & (e^{AT} - I)A^{-1}B \\ KC(e^{AT} - e^{AT_c}) & 1 + KC(e^{AT} - e^{AT_c})A^{-1}B \end{bmatrix}$$

$$a_1 = \begin{bmatrix} 0 \\ -Ky_r \end{bmatrix} + \begin{bmatrix} I \\ KC \end{bmatrix} (e^{AT_p} - I)A^{-1}Bu_p^0$$

$$A_2 = \begin{bmatrix} e^{AT} & (e^{AT} - e^{AT_p})A^{-1}B \\ KC(e^{AT} - e^{AT_c}) & 1 + KC(e^{AT_p} - I)(e^{AT_c} - I)A^{-1}B \end{bmatrix}$$

$$a_2 = \begin{bmatrix} 0 \\ -Ky_r \end{bmatrix}$$

$$A_3 = \begin{bmatrix} e^{AT} & 0 \\ KC(e^{AT} - e^{AT_c}) & 1 \end{bmatrix} \qquad a_3 = \begin{bmatrix} 0 \\ -Ky_r \end{bmatrix}$$

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