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Johansson, Rolf

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ROLF JOHANSSON

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY

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ROLF JOHANSSON

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY

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LUND INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AUTOMATIC CONTROL
Box 725
S 220 07 Lund 7 Sweden

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Author(s)
Rolf Johansson

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Abstract

The problem of adaptive prediction and direct adaptive control for systems described by delay-differential systems is considered. A model reference approach to direct adaptive control is shown to be feasible for a class of 2-D systems. A reparametrization of the 2-D system is given and the identification problem is converted into a linear estimation problem. Some necessary conditions for existence of solutions to the adaptive prediction and direct adaptive control problem are also given.

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INTRODUCTION

The problem of direct adaptive control and adaptive filtering for systems described by delay-differential equations is considered. This kind of system descriptions is natural to obtain for processes with flows, transport delays, and refluxes, see [Mor] and [EÖK]. A quite different application described by similar system equations is image processing systems. This latter application will however not be considered here.

A model reference approach to direct adaptive control is shown to be feasible for a class of 2-D systems. A reparametrization of the 2-D system description is given and the identification problem is converted into a linear estimation problem. Some necessary conditions for existence of solutions to the adaptive prediction and direct adaptive control problem are also given.

PRELIMINARIES

Let $R[z]$ denote the ring of polynomials in z with real coefficients. Here we will also consider systems defined over $R[z_1,z_2]$ i.e. the ring of polynomials with real coefficients in two indeterminates - z_1 and z_2. Such systems are for example continuous-time systems with delays and two-dimensional image processing filters. The systems may as well be described by polynomials in $R(z_1)[z_2]$ or $R[z_1][z_2]$ i.e. polynomials in z_2 whose coefficients are rational functions and polynomials in z_1, respectively. For the theory of polynomials of several variables, see [K&R] or [Van]. The algebraic properties of $R[z_1,z_2]$ are different from those of $R[z]$. The ring of polynomials $R[z]$ is e.g. a Euclidean domain while $R[z_1,z_2]$ is not, see [Sha], chapter 6.

Traditional methods to solve model matching problems in linears systems theory, see [A&W] or [Kai], are based on the Diophantine equation

$$AR + BS = P \tag{1}$$

which for given $A, B, P \in R[z]$ has solutions $R, S \in R[z]$ if A and B are coprime. In the two-dimensional case there are solutions $R, S \in R(z_1)[z_2]$ for $A, B, P \in R[z_1][z_2]$ when A and B are coprime i.e. when the Bezout identity exists, see [MLK]. There are also solutions $R, S \in R[z_1][z_2]$ for coprime A, $B \in R(z_1)[z_2]$ and $P \in R[z_1][z_2]$, see [K&S] and [EÖK], theorem 2.11. For further investigations on the coprimeness problem of 2-D polynomials, see [MLK].

PROBLEM FORMULATION

The problem of direct adaptive control of 2-D systems will now be considered. The treatment is based on a model reference approach to direct adaptive control. A closer account is given in e.g [A&W] or [Lan]. The model reference approach is closely connected with the problem of exact model matching. Treatments of the 2-D case of model matching are found in e.g. [E&E] and [Seb].

The problem is to find a continuous-time regulator described by

$$u = - \frac{S}{R} y + \frac{T}{R} u_c \tag{2}$$

to control a continuous-time object described by the factorized input-output model

$$y = \frac{B}{A} u \tag{3}$$

with input u, output y, and command signal u_c. The problem of existence of polynomial factorizations is treated in [Kha]. Here A, B, R, S, T are polynomials with unknown real coefficients in some operator(s). The operators may be e.g. the differential operator p and some time delay operator. In this paper the following operators will be used

$$z_1 = \frac{a}{p+a} \qquad \text{and} \qquad z_2 = e^{-\tau p} \tag{4}$$

for some given, positive constant a. The operator z_1 is a low pass filter operator and z_2 represents a time delay τ. These operators commute in the context of this presentation i.e. the input-output point of view of signal processing.

The polynomials A and B are then

$$A(z_1, z_2) = \sum_{i=0}^{n_{iA}} \sum_{j=0}^{n_{jA}} a_{ij} z_1^i z_2^j \qquad ; \qquad a_{00} = 1$$

$$B(z_1, z_2) = \sum_{i=0}^{n_{iB}} \sum_{j=0}^{n_{jB}} b_{ij} z_1^i z_2^j \tag{5}$$

for some finite orders n_{iA}, n_{jA}, n_{iB}, n_{jB}. The condition on a_{00} is imposed for the purpose of normation. The transfer operator of (3) may also be factorized as

$$\begin{cases} A(z_1, z_2)\xi(t) = u(t) \\ y(t) = B(z_1, z_2)\xi(t) \end{cases} \tag{6}$$

for some internal state variable ξ.

The polynomial B may be further decomposed into a number of factors. The main concern is the locations of zeros. Introduce for this reason the following factorizations

$$B = b_0 B_1 B_2 \quad ; \quad B_1(0,0)=1, \;\; B_2(1,1)=1 \tag{7}$$

The polynomial B_1 is supposed to be such that all solutions $s=s_i$ to the equation

$$B_1\left(\frac{a}{s+a}, \; e^{-s\tau}\right) = 0 \tag{8}$$

have $Re(s_i)<0$ while B_2 contains the remaining factors. The factor b_0 is a scalar i.e. a pure constant gain. The problem to find a B_2 of least possible complexity will not be considered here but is of some practical importance. The reason to make this factorization is to assure internal stability in the controlled system. The factor B_2 will play the same role of system invariant as the relative degree, time delay etc. in 1-D model matching problems, see [A&W].

ASSUMPTIONS

A number of restrictions on the class of control objects will now be given in order to assure solutions with good properties. Assume that

A1: $b_{00}=0$ i.e. there is no direct term from input to output

A2: The polynomial degrees n_{iA}, n_{jA}, n_{iB}, n_{jB} of A and B are all known

A3: B_2 is known

A4: A and B are coprime i.e. $\exists M,N \in R(z_1,z_2)$: $AM+BN=1$

A5: The time delay τ is known

It should be said already here that the requirement to know B_2 is impractical except when B_2 may be reduced to pure powers of z_1 and z_2 i.e. low pass filters and time delays in series.

SOLUTION

The closed-loop system of (2) and (3) with time-invariant A, B, R, S, and T gives the transfer operator as

$$y = \frac{BT}{AR + BS} u_c \tag{9}$$

where R, S, T may be tuned to fulfil specifications in terms of a reference model. This model should reflect the design trade-offs and is of the type

$$y_m = \frac{B_2 B_m}{A_m} u_c \quad ; \quad A_m(0,0) = 1 \tag{10}$$

where A_m, B_m are desired pole- and zero-polynomials and where B_2 is the part of the B-polynomial of the control object which should not be cancelled in the closed-loop system (9).

$$P = A_m B_1$$

$$T = t_0 B_m \quad ; \quad t_0 = \frac{1}{b_0} \tag{11a-b}$$

where b_0 is the constant gain introduced in (7). The model matching problem is then solved via the Diophantine equation

$$AR + BS = P \tag{12}$$

It is necessary to obtain polynomials R and S from (12) in order to have solutions that are suitable for direct adaptive control. Such solutions do not always exist, see example 2 below.

When there are polynomial solutions R, S in two indeterminates, it is possible to express them as

$$R(z_1, z_2) = \sum_{i=0}^{n_{iR}} \sum_{j=0}^{n_{jR}} r_{ij} z_1^i z_2^j \quad ; \quad r_{00} = 1$$

$$S(z_1, z_2) = \sum_{i=0}^{n_{iS}} \sum_{j=0}^{n_{iS}} s_{ij} z_1^i z_2^j \tag{13}$$

for some numbers n_{iR}, n_{jR}, n_{iS}, n_{jS} which are determined in the process of solving (12).

The model matching control law (2) is then expressed explicitly as

$$u = - \sum_{i=1}^{n_{iR}} \sum_{j=1}^{n_{jR}} r_{ij}(z_1^i z_2^j u) - \sum_{i=1}^{n_{iR}} r_{i0}(z_1^i u) - \sum_{j=1}^{n_{jR}} r_{0j}(z_2^j u) \tag{14}$$

$$- \sum_{i=0}^{n_{iS}} \sum_{j=0}^{n_{jS}} s_{ij}(z_1^i z_2^j y) + t_0(B_m u_c)$$

The data filters

$$z_1^i z_2^j \quad \forall i, j \geq 0 \tag{15}$$

require for their implementation the causal operators of low pass filters and time delays and are therefore realizable.

Let the control law (14) be formulated as the scalar product of the parameter vector θ and the data vector ϕ. The vector ϕ thus contains filtered and delayed inputs u and outputs y. Then we have

$$u(t) = - \theta^T \phi(t) \tag{16}$$

with

$$\theta^T = \left[r_{10} \ldots r_{ij} \ldots s_{00} \ldots s_{kl} \ldots t_0 \right] \tag{17}$$

and

$$\phi = \left[(z_1 u) \ldots (z_1^i z_2^j u) \ldots y \ldots (z_1^k z_2^l y) \ldots -(B_m u_c) \right] \tag{18}$$

The estimation problem of direct adaptive control is now to calculate θ from

PARAMETER ESTIMATION

The parameters of the regulator may now be estimated from a linear model obtained by manipulations of the factorization (6) using (12) and (6), respectively.

$$y = B\,\xi = B\,\frac{RA + SB}{P}\,\xi = \frac{B}{P}\left[\, Ru + Sy \,\right] \qquad (19)$$

Assume that B_2 is known, cf. (A3), and define

$$\bar{u} = B_2 u \;,\quad \bar{y} = B_2 y \quad \text{etc.} \qquad (20)$$

Introduce also the data vector φ

$$\varphi = \left[(z_1\bar{u}) \ldots (z_1 z_2 \bar{u}) \ldots \bar{y} \ldots (z_1^k z_2^l \bar{y}) \ldots -(A_m y)\right] \qquad (21)$$

in accordance with standard notation of recursive estimation, see [L&S]. Then it is possible to rearrange (19) to the scalar product

$$\bar{u}(t) = -\,\theta^T \varphi(t) \qquad (22)$$

The constant parameters θ may now be estimated by any suitable method for identification of parameters of a linear model with known components of φ. Recall that the estimates $\hat{\theta}$ of the parameters θ in (22) are intended for use in the continuous-time regulator (14). The parameters may however be estimated by discrete-time methods. One natural choice is to sample corresponding $\bar{u}$ and φ and to fit parameter estimates to the sampled data by recursive identification. For further details on recursive estimation, see [L&S].

ADAPTIVE FILTERING

It is possible to carry through all of the arguments above with the purpose to derive adaptive filters for $P\xi$ with any P such that there is a solution to (12). There is in this case no need to factorize B into B_1 and B_2.

EXAMPLE 1

Consider the following delay-differential control object

$$\begin{cases} \dot{x}(t) = -a_1 x(t) + a_2 x(t-\tau) + bu(t) \\ y(t) = x(t) \end{cases} \tag{25}$$

with input u and output y. The time delay τ is supposed to be known. The problem is to design an adaptive controller

$$u(t) = -\frac{S(z_1, z_2)}{R(z_1, z_2)} y(t) + \frac{T(z_1, z_2)}{R(z_1, z_2)} u_c(t) \tag{26}$$

such that the closed-loop system matches the reference model

$$y_m(t) = \frac{1}{2p+1} u_c(t) \tag{27}$$

A polynomial description of the control object in terms of the operators

$$z_1 = \frac{1}{p+1} \quad \text{and} \quad z_2 = e^{-\tau p} \tag{28}$$

is obtained via

This yields the transfer function

$$y(t) = \frac{bz_1}{1 + (a_1-1)z_1 - a_2z_1z_2}u(t) \triangleq \frac{B(z_1,z_2)}{A(z_1,z_2)}u(t) \tag{30}$$

It is here easy to see that A and B are coprime. The zero of B appears at

$$z_1 = 0 \tag{31}$$

where the polynomial A certainly is non-zero. The zero of (31) is - in terms of the differential operator - an infinite zero of degree 1 of the transfer function. The reference model

$$\frac{1}{2p+1} = \frac{0.5z_1}{1-0.5z_1} \triangleq \frac{B_m}{A_m}z_1 \tag{32}$$

has the same type of zero and it is therefore possible to find a solution to the pole placement problem with a regulator (26) which is causal and without derivatives. A solution with $A_m=1-0.5z_1$ and $B_m=0.5$ is obtained for

$$\begin{cases} R = 1 \\ S = \left[-\frac{3}{2} + a_1 + a_2z_2\right]/b = s_{00} + s_{01}z_2 \\ T = \frac{1}{b}\cdot 0.5 = t_0B_m \end{cases} \tag{33}$$

Input-matching estimation based on the linear model

$$(z_1u) = -\left[s_{00} + s_{01}z_2\right](z_1y) + t_0(A_my) \tag{34}$$

EXAMPLE 2

Consider the following delay-differential equation for a control object

$$\begin{cases} \dot{x}(t) = -a_1 x(t) + a_2 x(t-\tau) + bu(t-\tau) \\ y(t) = x(t) \end{cases} \tag{36}$$

This equation differs from that of the previous example in the time delay of u. The polynomial representation now becomes

$$y(t) = \frac{bz_1 z_2}{1 + (a_1-1)z_1 - a_2 z_1 z_2} u(t) \tag{37}$$

The A- and B-polynomials now both vanish at

$$(z_1, z_2) = \left(\frac{1}{1-a_1}, 0\right) \tag{38}$$

For this reason it is not possible to solve the polynomial Diophantine equation

$$AR + BS = P \tag{39}$$

for an arbitrary P. It is necessary to include in P a factor with a zero at (38). The factor

$$\left(1 + (a_1-1)z_1\right) \tag{40}$$

CONCLUSIONS

It has been possible to formally carry over many of the algebraic results from 1-D direct adaptive control. There is however a strong practical difference between the required <u>prior information</u> in the two cases.

The question of coprimeness is also more complicated in the 2-D case. Any solution $(z_1,z_2)=(a,b)$ to the equation

$$A(z_1, z_2) = 0$$

$$B(z_1, z_2) = 0 \tag{41}$$

puts the constraint on P that P(a,b) must be zero in order to assure the existence of a solution to (12). In contrast to the 1-D case it is not in general possible to get rid of the common zero by pole-zero cancellation between A and B, see example 2 and [MLK].

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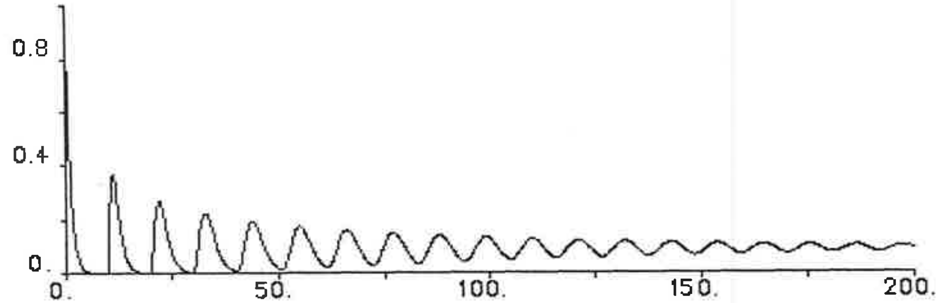
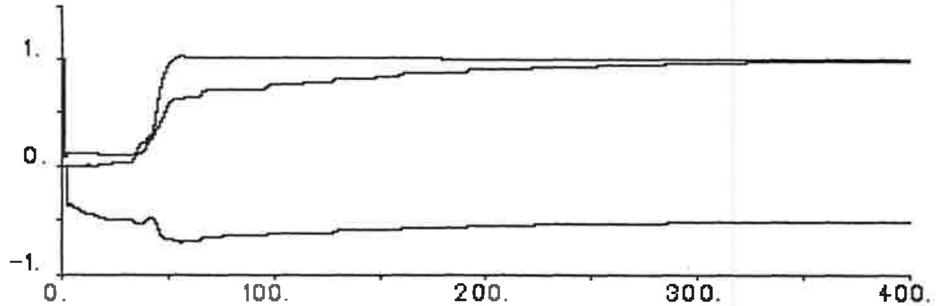
The following simulations show the results when (34) has been sampled with sampling period h=1. Recursive least-squares-identification has been applied to estimate the parameters.

<u>Simulations</u>



A: Impulse response of open-loop system with $a_1=1$, $a_2=1$, b=1 and $\tau=10$.



B: Parameters estimates $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$ from recursive LS-identification with sampling period h=1 s and $\tau=10$.