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Moment-Method Calculations on Apertures Using Basis Singular Functions

Tommy Andersson

Abstract—**The transmission properties of perforated perfectly conducting screens are of practical interest. The treatment of nonperiodical structures by numerical techniques, such as the method of moments, is very computer intensive. In this paper it is shown that using basis functions that incorporate the edge as well as the corner singularities, the number of unknowns can be drastically reduced. Advantages and limitations of the method are discussed. Numerical results are presented illustrating transmission properties of arrays of square and rectangular apertures.**

I. Introduction

Perforated metallic screens and wire mesh screens are often used for electromagnetic shielding and filtering purposes. The reflection and transmission properties of infinite wire grids and periodically apertured screens have been analyzed by many authors in the past, e.g., [1]–[4]. However, the treatment of finite structures is also of practical interest. During the last decades different numerical methods, such as the method of moments (MoM) [5], have been successfully applied to aperture problems [6], [7]. An attempt to apply the MoM to an array of apertures, constituting one wire mesh covered aperture, was made in [8]. Truncated period structures have been considered by, e.g., [9] and [10]. Recursive schemes have recently been suggested to handle finite, nonperiodic structures [11], [12].

However, the performance of numerical methods seems to depend heavily on the ability to handle the singular behavior of the fields near the rims of the apertures. The MoM involves the expansion of the unknown function in terms of a set of basis functions. Basis functions with correct edge behavior have successfully been used to calculate the polarizability of electrically small apertures [13], [14]. In the treatment of the complementary problem, scattering by conducting strips and plates, the incorporation of the edge behavior in the basis functions has become an established technique used even for more applied problems [15], [16]. Recently, the scattering by a thin conducting square plate was treated by the author [17]. Basis functions with not only correct edge singularities but also with correct corner singularities were used, giving greatly enhanced convergence. Here, this approach is applied to calculate the transmission properties of an array of rectangular apertures in an infinite, thin, perfectly conducting screen.

In Section II the MoM approach and the basis functions are briefly described. In Section III numerical results are presented. The enhanced convergence, due to the singular basis functions, is illustrated and the limitations of the method are discussed. The transmission coefficients and transmission cross sections are calculated for different geometries varying the distance between the apertures. Finally, some conclusions are given in Section IV.

II. Method of Moments Approach

The diffracted electromagnetic field through apertures in a perfectly conducting, thin, plane screen can, according to Babinet's principle, be found by solving the complementary problem, i.e., the electromagnetic scattering by perfectly conducting plates in free space replacing the apertures. However, here we adopt the well known formulation using the equivalence principle and magnetic sources, i.e., the problem is formulated as an integral equation with the equivalent magnetic sources in the apertures as the functions to be solved for [6], [7], [18].

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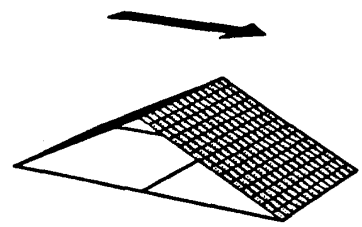

Fig. 1. The interior basis functions. The magnetic current is approximated by rooftop functions.

The Matrix Equation

To obtain a matrix equation the MoM is applied to the integral equation. The magnetic surface current is expanded in terms of a set of vector basis functions f_p with unknown coefficients a_p. We use Galerkin's method, i.e., we use testing functions that are identical to the basis functions. The integral equation formulation and the MoM approach are described in detail in [19].

The matrix equation can be expressed as

$$\sum_p a_p \int_S \int_S \left(\nabla_S \cdot f_p(\mathbf{r}') \nabla_S \cdot f_q(\mathbf{r}) - k^2 f_p(\mathbf{r}') \cdot f_q(\mathbf{r}) \right) G(\mathbf{r}, \mathbf{r}') \, dS' \, dS$$

$$= \frac{i\omega\mu}{2} \int_S \mathbf{H}^{in}(\mathbf{r}) \cdot f_q(\mathbf{r}) \, dS, \qquad q = 1, 2, 3 \cdots. \quad (1)$$

Here S is the surface of the apertures in the screen, the operator $\nabla_S \cdot$ represents surface divergence, k is the wave number and $G(\mathbf{r}, \mathbf{r}')$ is the free space Green function. $\mathbf{H}^{in}$ is the magnetic field of the incident wave, ω the angular frequency, and μ the permeability of the surrounding homogeneous media.

Basis Functions

In [17] the scattering by a perfectly conducting square plate is calculated. At the edges and corners of a thin, perfectly conducting plate the fields and source distributions have known singular behavior, see, e.g., [20]. It is shown in [17] that the use of basis functions with correct edge and corner behavior greatly enhance the convergence of the scattering problem compared to the use of ordinary "rooftop" functions.

The aperture problem, as it is formulated in this paper, leads to an equation and to singular behavior of the fields that are similar to the scattering problem [17]. As a consequence of that, the singular basis functions used in [17] can be used also in this aperture problem. We give here just a short description of these basis functions, for more details cf. [17].

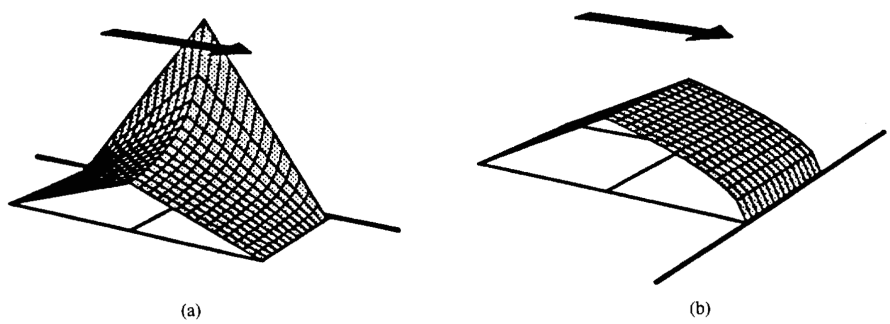

Fig. 2. The basis functions used to approximate the magnetic current tangential (a) and perpendicular (b) to an edge.

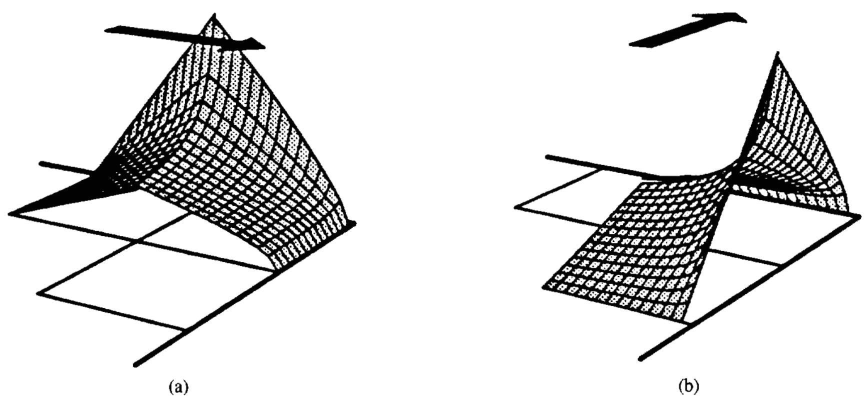

Fig. 3. The corner basis functions used to obtain the correct magnetic singularity. The Cartesian components of the basis functions for the radial magnetic current are illustrated.

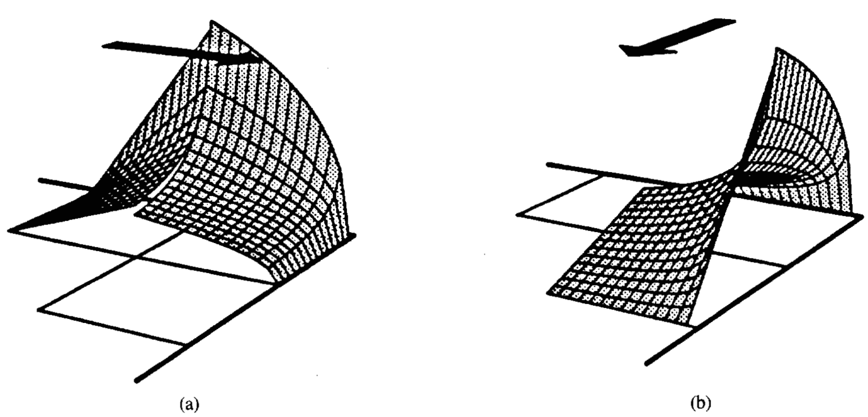

Fig. 4. The corner basis functions used to obtain the correct electric singularity. The Cartesian components of the basis functions for the tangential magnetic current are illustrated.

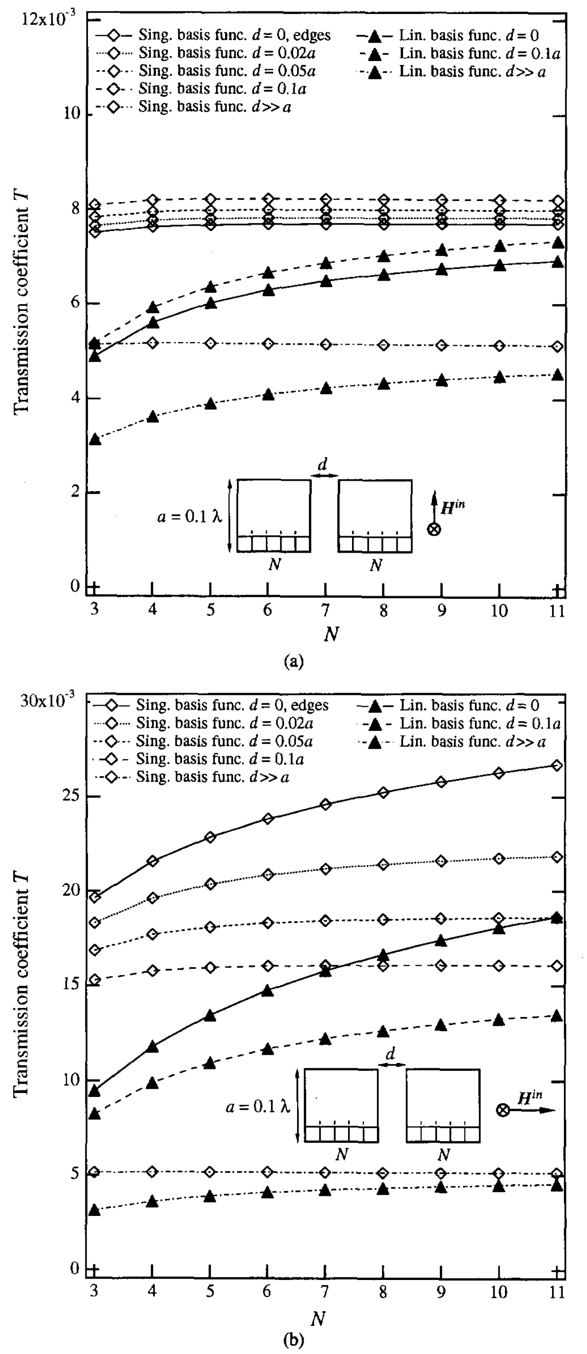

Fig. 5. Convergence of the transmission coefficient T for two square apertures, each with length of side $a = 0.1\lambda$. Normal incidence. Each aperture is divided into $N \times N$ "subsquares." The convergence is shown for different distances d between the apertures. The polarization of the incident magnetic field is (a) transverse; (b) parallel to the row of apertures.

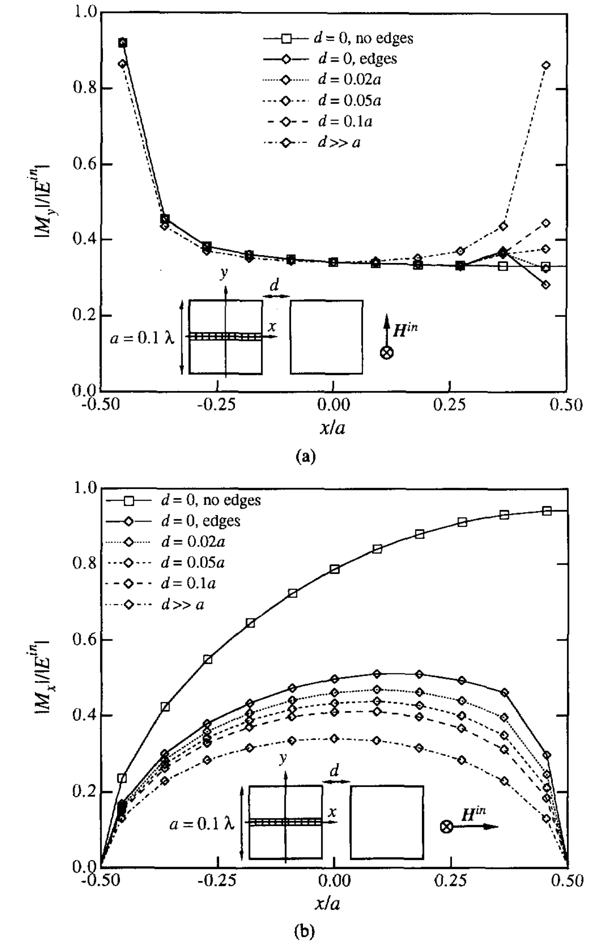

Fig. 6. The dominant component of the magnetic current density in one of two adjacent square apertures. Normal incidence. The mean values of the current density on "subsquares" in the middle row parallel to the row of apertures are shown. The length of the side of each aperture is 0.1λ. The apertures are divided into 11×11 "subsquares." The polarization of the incident magnetic field is (a) transverse; (b) parallel to the row of apertures.

Subdomain basis functions are used, i.e., basis functions with support only in subsections of the domain. Due to simplicity we have chosen rectangular subsections. This limits the adaptation to apertures of rectangular shape. However, the implementation of the main ideas to more general triangular subsections is possible. To assure continuity of the magnetic current density in the direction of flow an overlapping technique is used. This prevents fictitious magnetic line charges at the boundaries of the subsections.

In the interior of the domain the current is approximated by ordinary rooftop basis functions, cf. Fig. 1. At the edges two kinds of basis functions are used, cf. Fig. 2. The magnetic surface current tangential to an edge is expanded in basis functions with the singularity $1/\sqrt{d}$, where d represents the perpendicular distance to the edge, but have rooftop character in the direction of flow. The current flowing perpendicular to the edge is approximated by basis functions that go to zero as $\sqrt{d}$ near the edge, which agrees with the singularity $1/\sqrt{d}$ for the magnetic charge.

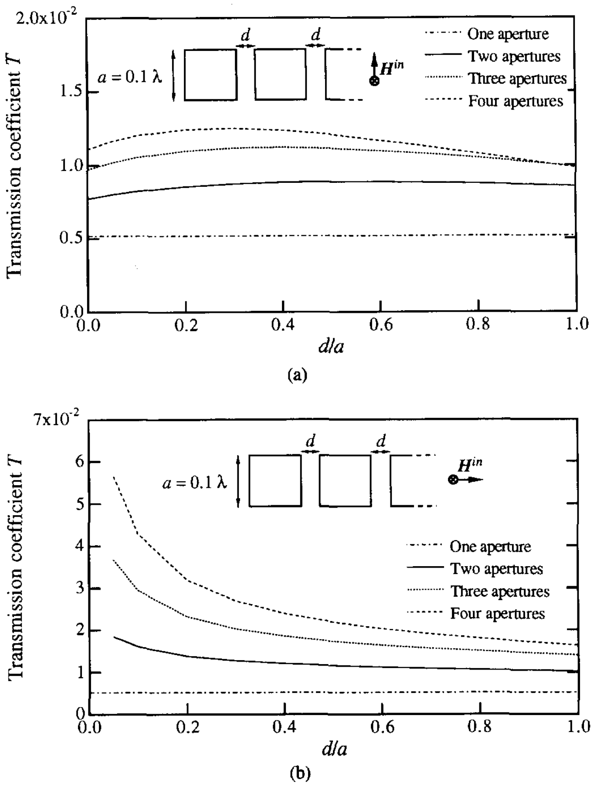

Fig. 7. The transmission coefficient for a row of square apertures versus the distance between the apertures. Normal incidence with the polarization of the magnetic field transverse (a) and parallel (b) to the row of apertures. The length of the side of each aperture is 0.1λ.

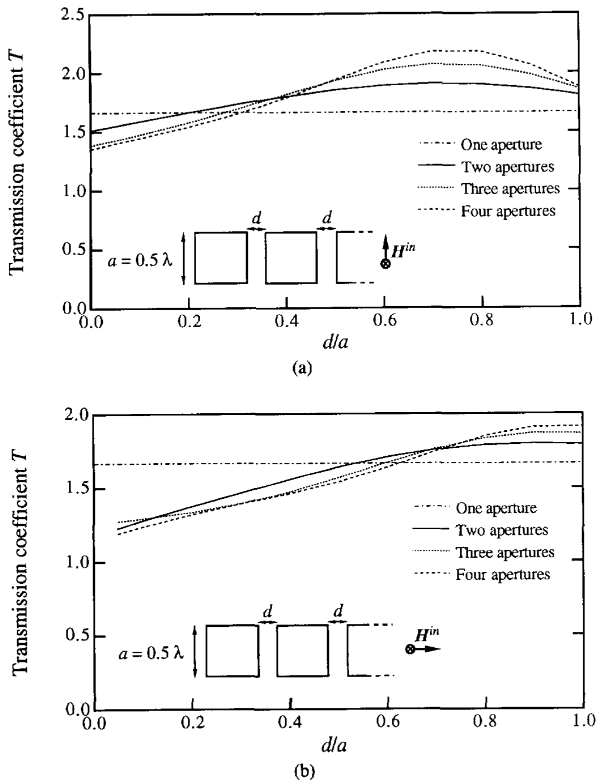

Fig. 8. The transmission coefficient for a row of square apertures versus the distance between the apertures. Normal incidence with the polarization of the magnetic field transverse (a) and parallel (b) to the row of apertures. The length of the side of each aperture is 0.5λ.

At the corners the electric and magnetic fields have different singular behavior. We use two kinds of basis functions corresponding to these two singularities.

The first kind is used to expand the magnetic current flowing *in the radial direction towards the corner*. The singularity of the related accumulated magnetic charge density $r^{\nu-1}$ corresponds to the singular behavior of the magnetic field. Here r denotes the distance to the corner and the approximate value $\nu = 0.30$ according to [20]. The x and y components of these magnetic current basis functions are shown in Fig. 3. Note the correct edge behavior and the linear parts used to connect the corner subsection with the adjacent edge subsections.

The second kind of corner basis functions expands the tangential magnetic current near the corner. This current has the same singularity as the electric field, $r^{\tau-1}$. The approximate value of the exponent is given by [20] as $\tau = 0.82$. Besides the correct corner and edge behavior these basis functions are also solenoidal. Hence, no magnetic charge is accumulated which otherwise would influence the expansion of the magnetic field. The tangential current basis functions are illustrated in Fig. 4. The discontinuity of the current density that can be seen in Fig. 4 is due to computational considerations, cf. [17].

Numerical Treatment of the Matrix Equation

The use of complicated basis and testing functions like those described above, makes it essential to find numerically efficient methods to calculate the matrix elements given by (1). A multipole expansion technique to calculate the non self-patch terms of the matrix elements is described by the author in [17] and [21]. The translation properties of the spherical scalar wave functions [22] imply that the matrix elements can be expressed as a series of multipole moments. Moreover, this technique can be used to calculate the right-hand side of (1) and the diffracted field. This subject is, however, not pursued in the present paper. The reader is referred to the above mentioned references.

III. Numerical Results

In the following section we consider plane wave incidence. This is due to simplicity, the described method allows any incident field.

Transmission Quantities

The far diffracted field, the transmission cross section and the transmission coefficient are conveniently computed using the multipole technique, cf. [17], [19].

We define the transmission cross section τ, cf., e.g., [6], as

$$\tau(\theta, \phi) = \lim_{r\to\infty} 2\pi r^2 \frac{|\mathbf{H}^d|^2}{|\mathbf{H}^{in}|^2}, \tag{2}$$

where $\mathbf{H}^d$ is the diffracted field at the observation point (r, θ, ϕ) $(z > 0)$.

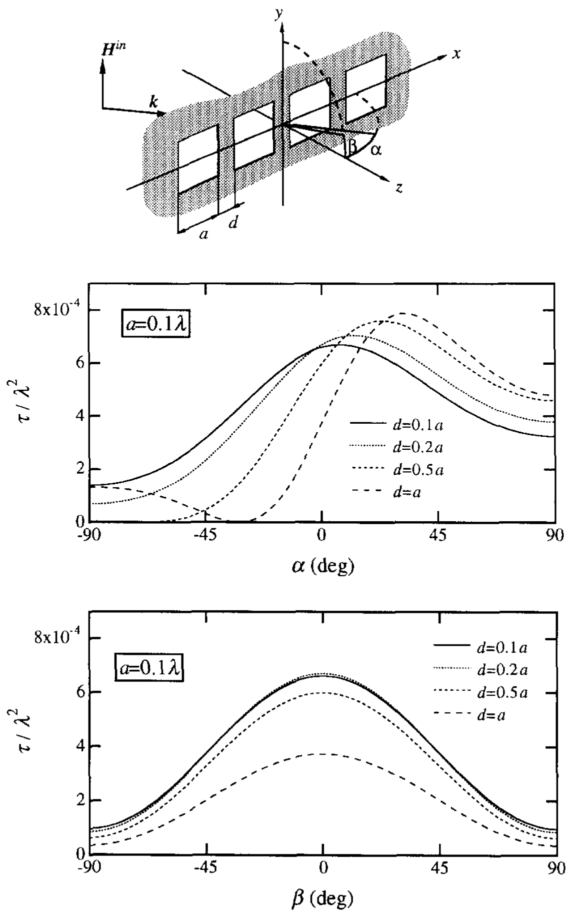

Fig. 9. Transmission cross section of an array of four apertures oriented along the x axis. The plane of incidence is equal to the x-z plane and the incidence angle is 45°. The polarization of the magnetic field is parallel to the y axis. The length of the side of each aperture is 0.1λ.

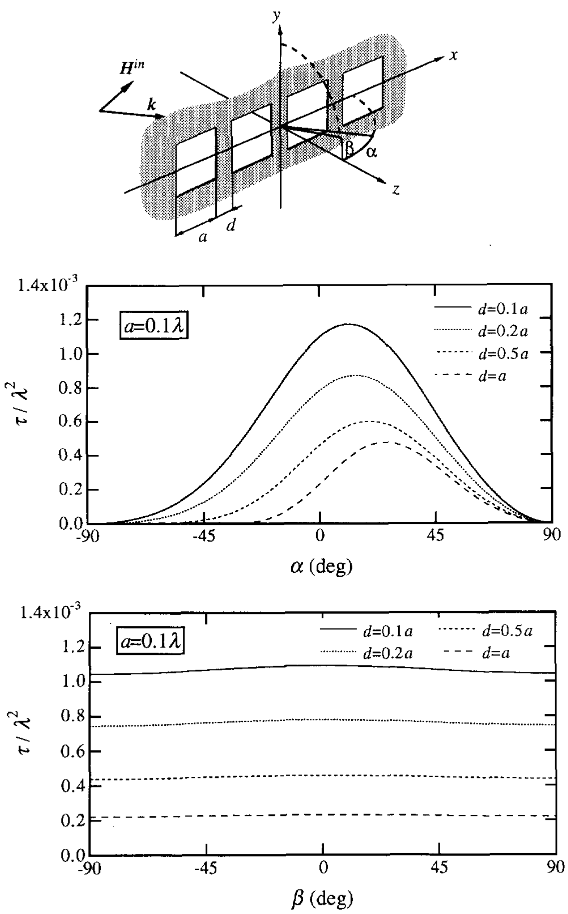

Fig. 10. Transmission cross section of an array of four apertures oriented along the x axis. The plane of incidence is equal to the x-z plane and the incidence angle is 45°. The polarization of the magnetic field is perpendicular to the y axis. The length of the side of each aperture is 0.1λ.

We define the transmission coefficient T of an array of apertures as the ratio of the power transmitted through the apertures to the power incident on the apertures, cf. [23].

Convergence

To check the validity of the method we first study the convergence of the transmission coefficient T for a configuration consisting of two square apertures and normal incidence. The length of the side of each aperture is 0.1λ. The results are highly dependent on the polarization of the incident wave. We assume in the following that the apertures are placed in the x-y plane along the x axis.

When the incident magnetic field is polarized in the $\hat{y}$ direction, the convergence is fairly independent of the distance between the plates, cf. Fig. 5(a). At zero distance the transmission coefficient agrees completely with the transmission coefficient of a corresponding, single, rectangular aperture. The magnetic current distribution, which corresponds to the tangential electric field in the aperture, is illustrated in Fig. 6(a). As the distance between the apertures vanishes, the singular behavior of the magnetic current flowing along the adjacent rims of the two apertures should disappear. However, as seen in Fig. 6(a), the singular source distributions prescribed by the basis functions cause nonphysical oscillations. Still, using singular basis functions the convergence of the transmission coefficient is, as seen from Fig. 5, greatly enhanced compare to the use of ordinary, linear basis functions.

A different situation arises when the incident magnetic field is polarized in the $\hat{x}$ direction. When the apertures get close to each other the convergence becomes very slow, cf. Fig. 5(b). This is due to the increasing edge singularities of the sources as the distance between the apertures decreases. The corresponding phenomenon has recently been reported in the literature when calculating scattering by strips [12]. This type of phenomena is, however, not mentioned in [8] for an aperture with a wire grid. The magnetic current distribution is shown in Fig. 6(b). Notice that the curves representing the distances $d = 0$ and $d = 0.02a$ are not valid, since the convergence at these distances is too slow, while the other curves correspond to convergent solutions, cf. Fig. 5(b). The current distribution of the limiting case, a single rectangular aperture, is also shown. Obviously, the slow convergence at small distances may cause misleading results.

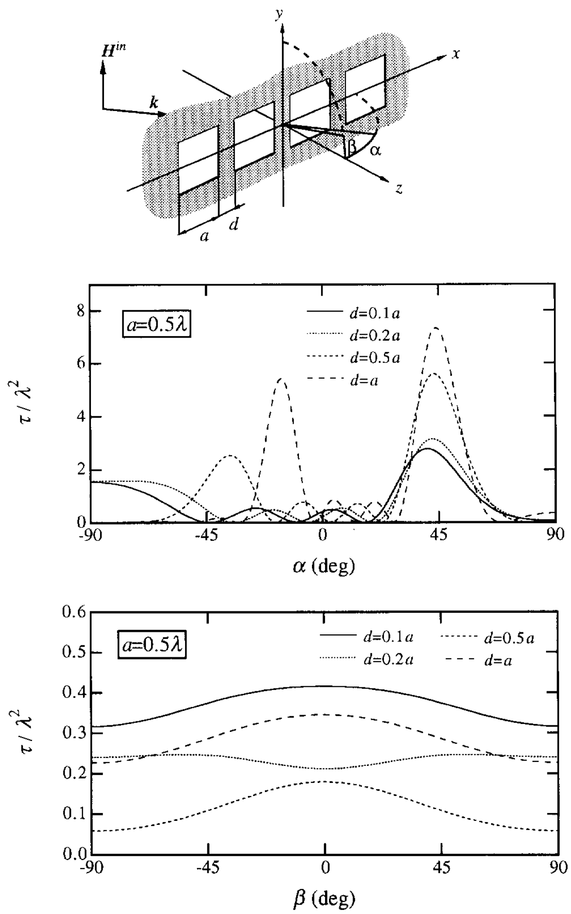

Fig. 11. Transmission cross section of an array of four apertures oriented along the x axis. The plane of incidence is equal to the x-z plane and the incidence angle is 45°. The polarization of the magnetic field is parallel to the y axis. The length of the side of each aperture is 0.5λ.

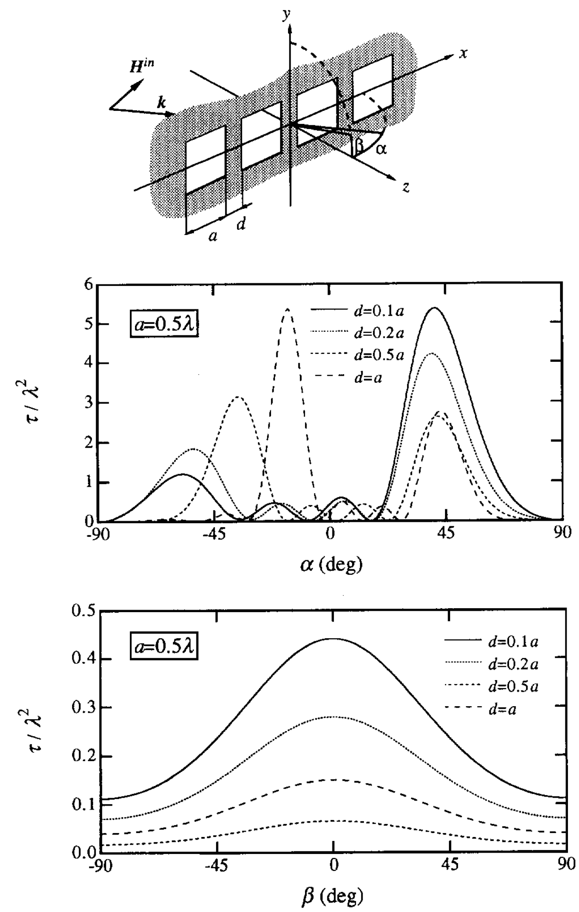

Fig. 12. Transmission cross section of an array of four apertures oriented along the x axis. The plane of incidence is equal to the x-z plane and the incidence angle is 45°. The polarization of the magnetic field is perpendicular to the y axis. The length of the side of each aperture is 0.5λ.

We have also made some studies of the convergence of the transmission coefficient at higher frequency $a = 0.5\lambda$. The above described characteristics seem to remain, although less accentuated. It is conjectured that this is due to the decreased influence from the singular magnetic charge distribution at higher frequencies, cf. the term $\nabla_S \cdot f_p \, \nabla_S \cdot f_q$ in (1). We therefore refrain from depicting this higher frequency case since the main features of the problem are illustrated at low frequency.

Numerical Examples

The fast convergence achieved with the singular basis functions makes the described method well suited for calculating the transmission properties of arrays of apertures. As just a few unknowns are needed for each aperture to obtain reasonable accuracy, configurations with multiple apertures can be treated with limited computational effort. (All computations presented have been carried out on a Sun 3/80 work station.)

Figs. 7 and 8 illustrate the variation of the transmission coefficient T as the distances between apertures, arranged in a row, are varied. In the computation each aperture was divided into 7×7 "subsquares." Note that the transmission coefficient shows a small maximum for the transverse polarization, while it is a monotonic function for the parallel polarization at the lower frequency $a = 0.1\lambda$.

In Figs. 9–12 a series of numerical computations of the transmission cross section is given for four apertures oriented along a line. Oblique incidence is considered and both polarizations are depicted for two different frequencies ($a = 0.1\lambda$ and $a = 0.5\lambda$).

Figs. 13 and 14 show transmission cross sections for a more complicated configuration consisting of six rectangular apertures. The size of each aperture is $0.5\lambda \times 1\lambda$ and the distances between the apertures are 0.25λ. Both polarizations at oblique incidence are illustrated. Notice the excellent convergence, actually even the most coarse discretization gives good results.

IV. Summary and Conclusions

We have presented a moment method approach to calculate the transmission properties of arrays of rectangular apertures in

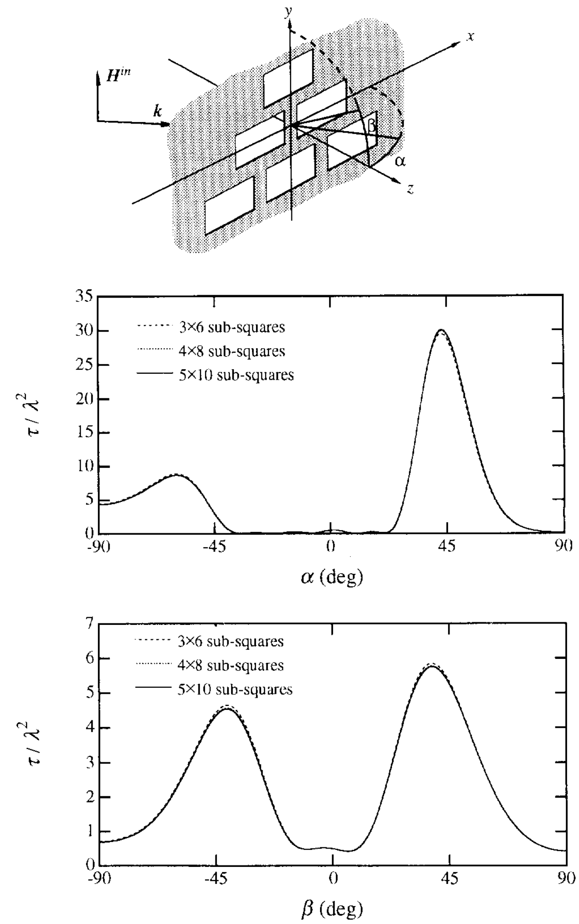

Fig. 13. Transmission cross section of a configuration of six apertures. The size of each aperture is $0.5\lambda \times 1\lambda$ and the distances between the apertures are 0.25λ. The results of three different discretizations of the apertures are illustrated. The plane of incidence is equal to the x-z plane and the incident angle is 45°. The polarization of the magnetic field is parallel to the y axis.

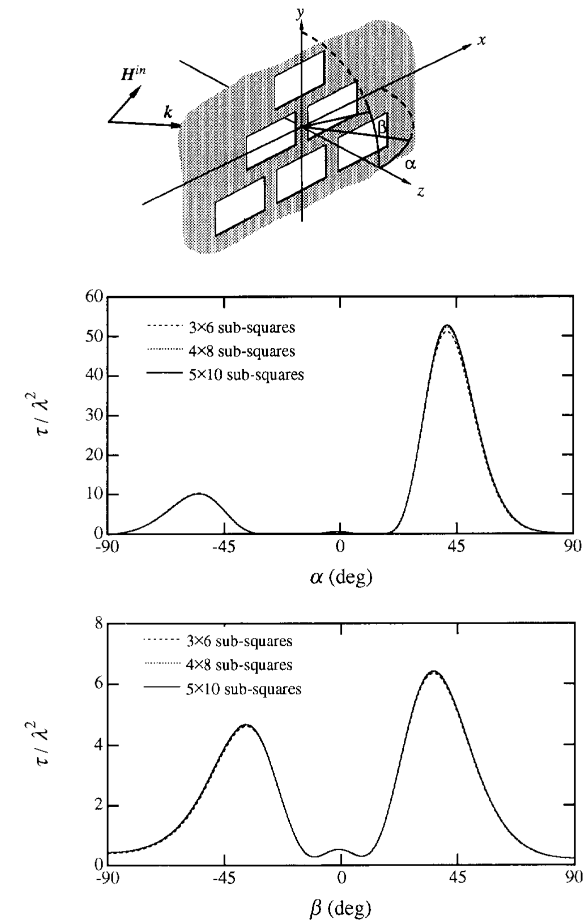

Fig. 14. Transmission cross section of a configuration of six apertures. The size of each aperture is $0.5\lambda \times 1\lambda$ and the distances between the apertures are 0.25λ. The results of three different discretizations of the apertures are illustrated. The plane of incidence is equal to the x-z plane and the incident angle is 45°. The polarization of the magnetic field is perpendicular to the y axis.

a perfectly conducting screen. Basis functions with correct singular behavior were used to enhance the convergence and thus reducing the necessary truncation size of the moment method matrix. Numerical results were presented illustrating the performance of the method.

The method is less suited when treating apertures very close to each other, due to the special singular behavior of the sources in those cases. Thus, an attractive extension of the technique would be to incorporate this behavior into the basis functions. Another approach, would be to combine the presented method with some recursive technique, and thus, possibly, further reduce the computational costs.

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An Exact Solution of the Generalized Exponential Integral and Its Application to Moment Method Formulations

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***Abstract*—The generalized exponential integral is one of the most fundamental integrals in antenna theory and for many years exact solutions to this integral have been sought. This paper considers an exact solution to this integral which is completely general and independent of the usual restrictions involving the wavelength, field point distance, and dipole length. The generalized exponential integral has traditionally been evaluated numerically or by making certain convenient but restrictive assumptions. The exact series representation presented in this paper converges rapidly in the induction and near-field regions of the antenna and therefore provides an alternative to numerical integration. Two method of moments formulations are considered which use the exact expression for the generalized exponential integral in the computation of the impedance matrix elements. It is demonstrated that, for very thin straight-wire antennas, an asymptotic expansion can be used to obtain a numerically convenient form of the generalized exponential integral.**

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I. Introduction

Numerical methods for the analysis of cylindrical wire antennas of arbitrary shape have become increasingly important since the advent of high speed computers. These numerical techniques often require the evaluation of certain types of integrals which are associated with mutual impedance or interaction effects. One of the most common of these integrals is a generalized exponential integral [1], [2].

An exact solution to a related integral was found by Weinbaum [3]. The approach which was used to evaluate this integral involved finding an associated differential equation and solving it by the method of power series expansion. This procedure resulted in a double infinite series representation of the integral. Wait [4] and King [1] expressed the generalized exponential integral in terms of three integrals. The first of these integrals has a closed form solution while the second and third are equal to tabulated generalized sine and cosine integrals, respectively. Various forms of generalized sine and cosine integrals have been tabulated in [5] and [6]. Harrington [7] discusses several useful approximations to the generalized exponential integral which are valid for different conditions on the field point location and dipole length. These approximations are based on Maclaurin series expansions of either the exponential contained in the integrand or the integrand itself, followed by a term-by-term integration of the resulting series. Similar expressions for a related integral have been studied by Preis [8].

An exact method of integration for the vector potential of a uniform current infinitesimally thin dipole antenna was recently found by Overfelt [9]. Werner [10] extended this method to include cylindrical dipole antennas of arbitrary nonzero radii. This was accomplished through the use of an exact expression for the cylindrical wire kernel which was first derived by Wang [11] and later modified by Werner [10]. Also introduced in [10] are exact expressions for the generalized exponential integral and higher-order associated integrals. Of particular significance is the exact solution obtained for the generalized exponential integral because of its rapid convergence in the induction and near-field regions of the antenna. Specific forms of this exact solution which are numerically convenient to evaluate will be emphasized in this paper. In order to demonstrate their usefulness, an application of these expressions to two different method of moments formulations for very thin straight-wire antennas is considered.

II. Theory

In this section an exact solution to the generalized exponential integral associated with a uniform current thin-wire vector potential will be introduced which is useful for computational

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