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Simple Drum-Boiler Models

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October 1988
Simple Drum-Boiler Models.

Abstract

This paper describes a simple nonlinear models for a drum-boiler. The models are derived from first principles. They can be characterized by a few physical parameters that are easily obtained from construction data. The models also require steam tables for a limited operating range, which can be approximated by polynomials. The models have been validated against experimental data. A complete simulation program is provided.

Simple Drum-Boiler Models

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Abstract
This paper describes a simple nonlinear model for a drum-boiler. The models are derived from first principles. They can be characterized by a few physical parameters that are easily obtained from construction data. The models also require steam tables for a limited operating range, which can be approximated by polynomials. The models have been validated against experimental data. A complete simulation program is provided.

1. Introduction

There are many models of drum-boilers in the literature. See the reference list. The models described in this paper are derived from first principles. They are characterized by a few parameters only which can be obtained from first principles. The models are validated by comparison with extensive plant data.

A key feature of a drum-boiler is that there is a very efficient energy and mass transfer between all parts that are in contact with the steam. The mechanism responsible for the heat transfer is boiling and condensation. A consequence of this is that it is a very good approximation to assume that all water steam and metal is in thermal equilibrium. This means that the total energy can be represented by a global energy balance. The validity of this approximation has been shown by many modeling exercises.

The paper is organized as follows. A first order model is presented in Section 2. This model is obtained from a global energy balance for the total plant. The model has one state variable which is chosen as the drum pressure. This model has the same structure as the model presented in Aström and Eklund (1972). The parameters are, however, obtained from first principles. To model the drum water level it is necessary to account for the shrink and swell phenomena. This is done in Section 3. A third order model is obtained. This model has drum pressure, water volume and steam quality in the risers as state variables. The model exhibits a complex behaviour in spite of being of low order. Simulation of step responses are presented in Section 4.

2. A First Order Model

Because of the efficient heat and mass transfer due to boiling and condensation all parts of the system which are in contact with the steam will be in thermal equilibrium. It is therefore natural to describe the plant with global mass and energy balances as was done in Aström and Eklund (1972). The global energy balance can be written as

\[ \frac{d}{dt}[\rho V_T V + \rho_w V_{WT} + m_0 T] = P + q_f h_{fw} - q_g h_s. \]  (1)

where \( \rho \) denotes specific density, \( h \) enthalpy, \( V \) volume and \( q \) mass flow. The indices \( s, w \) and \( fw \) refers to steam, water and feedwater respectively. The total mass of the metal tubes is \( m_0 \), the specific heat is \( q_s \) and the average metal temperature is \( T \).

The input power from the fuel is denoted by \( P \). The total steam volume is given by

\[ V_d = V_{dum} - V_w + \rho_0 V_r \]  (2)

where \( V_{dum} \) is the drum volume, \( V_w \) the volume of water in the drum, \( V_r \) the rise volume and \( \rho_0 \) the average steam-water volume ratio. The total water volume is

\[ V_w = V_w + V_{de} + (1 - \rho_0)V_r \]  (3)

The right hand side of equation (1) represents the energy flow to the system from fuel and feedwater and the energy flow from the system via the steam. Since all parts are in thermal equilibrium the state of the system can be represented by one variable which we choose as the steam pressure. Using steam tables the variables \( \rho, \rho_w, h_s, h_w \) can then be expressed as functions of steam pressure. Similarly \( T \) can be expressed as a function of pressure by assuming that \( T \) is equal to the saturation temperature of steam which corresponds to \( p \).

This model represents the dynamics due to input power well. When the feedwater flow or the steam flow is changed it is, however, necessary to also account into the amount that the sludge and steam masses are also changing. This can be accounted for in a global massbalance.

\[ \frac{d}{dt}[\rho TV + \rho_w V_{WT}] = q_f - \rho \]  (4)

The dynamics which describe how the drum pressure is influenced by input power, feedwater flow and steam flow is well captured by equations (1) and (4).

The derivative of the total water volume \( dV_{WT}/dt \) can be eliminated between equations (1) and (4). Multiplication of (4) by \( h_w \) and subtracting from (1) gives

\[ \frac{d}{dt}(\rho TV) + \left[ \rho TV \frac{dh_s}{dt} + \rho_w V_{WT} \frac{dh_w}{dt} + m_0 \frac{dT}{dt} \right] = P - q_f (h_w - h_{fw}) - q_g h_s. \]  (5)

The condensation enthalpy \( h_c = h_w - h_{fw} \) has also been introduced. If the boiler is provided with a good level control system the total water volume \( V_{WT} \) and the total steam volume \( V_d \) do not change much. Equation (5) can then be simplified to

\[ \frac{d}{dt} P = P - q_f (h_w - h_{fw}) - q_g h_s. \]  (6)

where

\[ \epsilon_{11} = h_N \frac{d\rho}{dt} + \epsilon_{1f} \frac{dh_s}{dt} + \epsilon_{1w} \frac{dh_w}{dt} + m_0 \frac{dT}{dt} \]

Apart from steam table data it is thus sufficient to know total steam and water volumes and total metal mass. The model (6) is identical to the model in Aström and Eklund (1972). Notice however that in this case the parameters are obtained from construction data. Also notice that the term

\[ \epsilon_s = \frac{\rho TV}{m_0} \frac{dh_s}{dt} + \rho_w V_{WT} \frac{dh_w}{dt} + m_0 \frac{dT}{dt} \]
can be interpreted as the total condensation flow. It is observed that the terms \( dh_u/dp \) and \( dh_a/dp \) are key quantities in predicting the energy and mass transfer between steam and water. These terms also appeared in the drum-boiler model of Morton and Price (1977).

### 3. Shrink and Swell

For some control tasks e.g. drum level control it is necessary to model the dynamics of the drum level. This is more difficult because of the shrink and swell effect. To describe this it is necessary to account for the distribution of steam and water and the transfer of mass and energy between steam and water.

The steam-water distribution varies along the risers. Partial differential equations are needed to describe this properly. To keep a finite dimensional model we will assume that the shape of the distribution is known. The assumed shape is based on solving the partial differential equations in the steady state. This gives a linear distribution of the steam-water mass ratio along the risers. We will therefore assume that the ratio varies

\[
\varepsilon(\xi) = \varepsilon \xi \quad 0 \leq \xi \leq 1
\]

where \( \xi \) is a normalised length coordinate along the risers and \( \varepsilon \) is the steam-water mass ratio at the riser outlet. The transfer of mass and energy between steam and water by condensation and evaporation is a key element in the modelling. When modelling steam and water separately the transfer must be accounted for explicitly. This can be avoided by writing joint balance equations for water and steam. The global mass balance for the riser section is

\[
\frac{d}{dt} (\varepsilon \rho_a V_r) + \frac{d}{dt} (\rho_w (1 - \varepsilon) V_r) = \dot{q}_{wa} - \dot{q}_w
\]

where \( \dot{q}_w \) is the total mass flow out of the risers. The global energy balance for the riser section is

\[
P + \dot{q}_{wa} - \dot{q}_w = \frac{d}{dt} (\varepsilon \rho_a V_r) + \frac{d}{dt} (\rho_w (1 - \varepsilon) V_r)
\]

The flow out of the risers \( \dot{q}_w \) can be eliminated by multiplying equation (8) by \( -/(h_m + \varepsilon_h \dot{h}_w) \) and adding to equation (9). Hence

\[
\frac{d}{dt} (\varepsilon \rho_a V_r) - (h_m + \varepsilon_h \dot{h}_w) \frac{d}{dt} (\varepsilon \rho_a V_r) + \frac{d}{dt} (\rho_w (1 - \varepsilon) V_r) - (h_m + \varepsilon_h \dot{h}_w) \frac{d}{dt} (\rho_w (1 - \varepsilon) V_r) = P - \dot{q}_w - \dot{q}_{wa}
\]

This can be simplified to

\[
h_m (1 - \varepsilon) \frac{d}{dt} (\varepsilon \rho_a V_r) + \rho_w (1 - \varepsilon) \frac{d}{dt} (\rho_w V_r) =
\]

Drum Level

To calculate the drum level it is necessary to know the average steam-water volume ratio in the risers \( (\varepsilon_m) \). We have

\[
\varepsilon = \frac{\dot{q}_w}{\dot{q}_w + \dot{q}_a (1 - \varepsilon)}
\]

Solving this equation for \( a \) we get

\[
a = a(\varepsilon) = \frac{\dot{q}_w}{\dot{q}_w + \dot{q}_a (1 - \varepsilon)}
\]

Assume that the steam-water mass ratio is linear along the riser as expressed by equation (7). The average steam-water volume ratio in the risers is

\[
a_m = \int_0^1 a(\varepsilon, \xi) d\xi = \frac{1}{\varepsilon} \int_0^1 a(\varepsilon, \xi) d(\varepsilon, \xi)
\]

\[
= \frac{1}{\varepsilon} \int_0^1 a(\varepsilon) d\varepsilon
\]

\[
= \frac{\dot{q}_w}{\dot{q}_w - \dot{q}_a} \left[ 1 - \frac{\dot{q}_w}{\dot{q}_w - \dot{q}_a} \ln \left( 1 + \frac{\dot{q}_w - \dot{q}_a}{\dot{q}_w} \right) \right]
\]

We can now obtain the following equation for the drum level

\[
\frac{d}{dt} (\varepsilon_m V_r) + \frac{d}{dt} (\rho_a V_r) = \frac{\dot{q}_w}{\dot{q}_w - \dot{q}_a} \left( \frac{\dot{q}_w - \dot{q}_a}{\dot{q}_w} \right) \frac{d}{dt} (\rho_a V_r)
\]

### 4. Simulations

The equations derived in Section 3 will now be summarized. The state equations are given by (11) and (12). The state variables are chosen as drum pressure \( p \), water volume in drum \( V_w \) and average steam quality at riser outlet \( \varepsilon_r \). Equation (11) and (12) can then be written as

\[
\begin{align*}
\frac{dp}{dt} &= \frac{\dot{q}_w}{\dot{q}_w - \dot{q}_a} \frac{dV_r}{dt} + \frac{dV_r}{dt} \frac{d\varepsilon_m}{d\varepsilon_r} = P + \dot{q}_w h_w \rho_a, \\
\frac{dV_r}{dt} &= \frac{\dot{q}_w}{\dot{q}_w - \dot{q}_a} \frac{dV_r}{dt} + \frac{dV_r}{dt} \frac{d\varepsilon_m}{d\varepsilon_r} = P - \dot{q}_w h_a
\end{align*}
\]

where

\[
\begin{align*}
\varepsilon_{11} &= \left( \frac{\dot{q}_w}{\dot{q}_w - \dot{q}_a} \frac{dV_r}{dt} + \frac{dV_r}{dt} \frac{d\varepsilon_m}{d\varepsilon_r} \right) V_r + \left( \frac{\dot{q}_w}{\dot{q}_w - \dot{q}_a} \frac{dV_r}{dt} + \frac{dV_r}{dt} \frac{d\varepsilon_m}{d\varepsilon_r} \right) V_r + \\
\varepsilon_{12} &= \frac{\dot{q}_w}{\dot{q}_w - \dot{q}_a} \frac{dV_r}{dt} + \frac{dV_r}{dt} \frac{d\varepsilon_m}{d\varepsilon_r} = \frac{\dot{q}_w}{\dot{q}_w - \dot{q}_a} \frac{dV_r}{dt} + \frac{dV_r}{dt} \frac{d\varepsilon_m}{d\varepsilon_r}
\end{align*}
\]

Assume that the steam-water mass ratio is linear along the riser as expressed by equation (7). The average steam-water volume ratio in the risers is

\[
\dot{a} = \frac{\dot{q}_w}{\dot{q}_w + \dot{q}_a (1 - \varepsilon)}
\]

Assume that the steam-water mass ratio is linear along the riser as expressed by equation (7). The average steam-water volume ratio in the risers is
To execute the simulation equation (15) has to be solved for the derivatives of the state variables. The right hand side of (15) contain input variables \( P, q_{fu} \) and \( q_s \), and functions of the state variables. Notice that downcomer flow \( q_d \) is given by equation (13). A detailed description of the simulation is given in the code in the Appendix.

Parameters

The model is characterized by the variables

- \( V_{drum} \): drum volume
- \( V_r \): riser volume
- \( V_{dc} \): downcomer volume
- \( m \): total metal mass
- \( c_p \): specific heat of metal
- \( k \): friction coefficient

and the functions \( g_f(p), g_m(p), h_s(p), h_w(p), T(p), h_fu(p) \) which are obtained from steam tables. Quadratic approximations to the steam tables are given in the program listing in the Appendix.

Equilibrium Conditions

Equilibrium conditions are obtained from (15). Hence

\[
q_{fu} = g_u \quad (17)
\]

\[
P = g_s(h_s - h_{fu}) \quad (18)
\]

\[
P = g_s + h_s \quad (19)
\]

The equilibrium value of the drum pressure can be determined from equation (18) since \( h_s \) and \( h_{fu} \) depend on the pressure.

Dynamic Response

Responses to steps in fuel flow and steam flow are given in Figures 1 and Figure 2. The simulations illustrate the dynamic features that are captured by the model. Figure 1 shows the response to a step change in fuel flow. The pressure responds like a pure integrator. The total amount of water in the drum increases because steam is generated in the risers. The total amount of steam in the risers increases because of the increased steam generation. The steam quality in volume ratios increases initially but it will later decrease because of the compression effect.

The drum level increases rapidly at first but the rate of increase decreases. The downcomer flow matches the steam fraction volume ratio. There is an instantaneous increase of the riser flow at the beginning of the step. The riser flow will then decrease at the same rate as the downcomer flow. Figure 2 shows the response to a step change in steam flow. The global effect is that the pressure and the volume will respond like integrators. There will, however, be a swell effect because of the initial evaporation of steam.

5. Conclusions

This paper has presented simple models for a drum boiler system. The models capture the major dynamical behaviour. They are derived from first principles and require only a few physical parameters that are easily obtained from construction data and steam tables. The behaviour has been shown by simulating step responses to fuel and steam flow changes. Reasonable results are obtained even for the difficult problems of predicting circulation flow and drum water level shrink and swell. The model can easily be augmented by equations for turbine and electrical output given in Åström and Eklund (1972, 1975) or Bell and Åström (1979) to produce a simple model for a complete boiler-turbine alternator system. A strong feature is that the model capture the essence of the steam generation in a heated pipe. It has also been used successfully to model steam generation in a nuclear plant. It can also be adapted to model once-through boilers.
6. References


Appendix

CONTINUOUS SYSTEM DRUM

"Nonlinear third order model for drum-downcomer-riser"

"Author K J Åström 670805"

INPUT pow qf3 treq qe
OUTPUT dV dV dV dxr
STATE p V x x
DER dp dV dV dxr
"Properties of steam and water in saturated state

\[
\begin{align*}
\text{hs} & = a01 + (a11 + a21) \cdot (p-10) \cdot (p-10) \\
\text{dhsdp} & = a12 + a22 \cdot (p-10) \\
\text{a01} & = 2.72666 \\
\text{a11} & = -1.79224 \\
\text{a21} & = -924.0 \\
\text{rw} & = a02 + (a12 + a22) \cdot (p-10) \cdot (p-10) \\
\text{drgdp} & = a12 + a22 \cdot (p-10) \\
\text{a02} & = 55.43 \\
\text{a12} & = 7.136 \\
\text{a22} & = 0.224 \\
\text{hv} & = a03 + (a13 + a23) \cdot (p-10) \cdot (p-10) \\
\text{dhwdp} & = a13 + a23 \cdot (p-10) \\
\text{a03} & = 1.400866 \\
\text{a13} & = 4.56524 \\
\text{a23} & = -110.0 \\
\text{rw} & = a04 + (a14 + a24) \cdot (p-10) \cdot (p-10) \\
\text{drgdp} & = a14 + a24 \cdot (p-10) \\
\text{a04} & = 691.36 \\
\text{a14} & = -1.867 \\
\text{a24} & = 0.081 \\
\text{ts} & = a05 + (a15 + a25) \cdot (p-10) \cdot (p-10) \\
\text{dtasdp} & = a15 + a25 \cdot (p-10) \\
\text{a05} & = 311.0 \\
\text{a15} & = 7.022 \\
\text{a25} & = -0.32 \\
\end{align*}
\]

"Properties of water in subcritical state

\[
\begin{align*}
\text{bd} & = \text{hv} \cdot \text{a06} + \text{a16} \cdot (p-10) \cdot (p-10) \\
\text{dhdwp} & = \text{dhwdp} + \text{a16} \cdot (p-10) \cdot (p-10) \cdot \text{dtasdp} \\
\text{cp} & = \text{a06} \cdot \text{a16} \cdot (p-10) \\
\text{a06} & = 5900 \\
\text{a16} & = 260 \\
\text{rd} & = \text{rw} \cdot \text{a07} + \text{a17} \cdot (p-10) \cdot (p-10) \\
\text{drdwp} & = \text{drwp} + \text{a17} \cdot (p-10) \cdot (p-10) \cdot \text{dtasdp} \\
\text{ddrwt} & = \text{a07} \cdot \text{a17} \cdot (p-10) \\
\text{a07} & = 2.4 \\
\text{a17} & = 0.2 \\
\text{kfw} & = \text{hv} \cdot \text{a06} + \text{a16} \cdot (p-10) \cdot (p-10) \cdot \text{tfw} \cdot \text{ts} \\
\text{hc} & = \text{hs} \cdot \text{hw} \\
\text{hr} & = \text{xh} \cdot \text{hs} \cdot (1-xr) \cdot \text{hw} \\
\text{Drum level} \\
\text{lv} & = \text{Vw} \cdot \text{adrain} \\
\text{lr} & = \text{an} \cdot \text{Vr} \cdot \text{adrain} \\
\text{dl} & = \text{lr} \cdot \text{lv} \\
\text{Average steam quality volume ratio} \\
\text{s2} & = \text{rs} / (\text{xs} \cdot \text{rv} \cdot \text{rs}) \\
\text{s3} & = 1 - \text{xs} / (\text{rs} \cdot \text{rv} \cdot \text{rs}) \\
\text{am} & = \text{rs} / (\text{rv} \cdot \text{rs}) \cdot (1-a-n) \cdot \text{ln} (a3) \\
\text{dram} & = \text{rs} \cdot \text{e2} \cdot \text{ln} (a3) / (\text{rs} \cdot \text{rv} \cdot \text{rs}) - 1 / s3 \cdot \text{rs} \\
\end{align*}
\]
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Simple Drum-Boiler Models

1. Introduction
2. First Order Model
3. Shrink and Swell
4. Simulations
5. Experiments
6. Conclusions
INTRODUCTION

Motivation
Simple physics based models for system studies

Experimental verification
Industrial collaboration with Sydkraft AB Malmö Sweden

Progress
Slow painstaking
Eklund 1968
Åström Eklund 1972, 1975
Bell and Åström 1979
Bell and Åström 1987
Simple Drum-Boiler Models

1. Introduction

2. First Order Model

3. Shrink and Swell

4. Simulations

5. Experiments

6. Conclusions
Global Energy Balance

\[
\frac{d}{dt} \left[ \rho_s h_s V_{st} + \rho_w h_w V_{wt} + mc_p T \right] = P + q_f h_f - q_s h_s
\]  
(1)

Total Steam Volume

\[
V_{st} = V_{drum} - V_w + a_m V_r
\]  
(2)

Total Water Volume

\[
V_{wt} = V_w + V_{dc} + (1 - a_m) V_r
\]  
(3)

Global Mass Balance

\[
\frac{d}{dt} \left[ \rho_s V_{st} + \rho_w V_{wt} \right] = q_f - q_s
\]  
(4)

Eliminate \(dV_{wt}/dt\) between (1) and (4)

\[
h_c \frac{d}{dt} (\rho_s V_{st}) + \left[ \rho_s V_{st} \frac{dh_s}{dt} + \rho_w V_{wt} \frac{dh_w}{dt} + mc_p \frac{dT}{dt} \right] = P - q_f (h_w - h_{fw}) - q_s h_c
\]  
(5)
\[ h_c \frac{d}{dt} (\rho_s V_{st}) + \left[ \rho_s V_{st} \frac{dh_s}{dt} + \rho_w V_{wt} \frac{dh_w}{dt} + mc_p \frac{dT}{dt} \right] = P - q_{fw} (h_w - h_{fw}) - q_s h_c \] 

Rewritten as

\[ e_{11} \frac{dp}{dt} = P - q_{fw} (h_w - h_{fw}) - q_s h_c \] (6)

\[ e_{11} = h_c V_{st} \frac{d\rho_s}{dp} + \rho_s V_{st} \frac{dh_s}{dp} + \rho_w V_{wt} \frac{dh_w}{dp} + mc_p \frac{dT_s}{dp} \]

Total condensation flow

\[ q_c = \frac{1}{h_c} \left[ \rho_s V_{st} \frac{dh_s}{dt} + \rho_w V_{wt} \frac{dh_w}{dt} + mc_p \frac{dT_s}{dt} \right] \]
Simple Drum-Boiler Models

1. Introduction
2. First Order Model
3. Shrink and Swell
4. Simulations
5. Experiments
6. Conclusions
THE VOID MODEL

A distributed parameter system

Assuming a void distribution gives a lumped parameter model

The PDEs gives a steady state solution with a linear steam water mass ratio

Use static relation also for dynamics

Model explored for nuclear reactor models where elaborate simulation models are available
Mass Balance for Riser Section

\[
\frac{d}{dt}(\rho_s a_m V_r) + \frac{d}{dt}(\rho_w (1 - a_m)V_r) = q_{dc} - q_r
\]  \hspace{1cm} (8)

Energy Balance

\[
\frac{d}{dt}(\rho_s h_s a_m V_r) + \frac{d}{dt}(\rho_w h_w (1 - a_m)V_r) \\
= P + q_{dc} h_w - x_r q_r h_s - (1 - x_r) q_r h_w
\]  \hspace{1cm} (9)

Eliminate \(q_r\) between (8) and (9)

\[
\frac{d}{dt}(\rho_s h_s a_m V_r) - (h_w + x_r h_c) \frac{d}{dt}(\rho_s a_m V_r) \\
+ \frac{d}{dt}(\rho_w h_w (1 - a_m)V_r) - (h_w + x_r h_c) \\
\frac{d}{dt}(\rho_w (1 - a_m)V_r) = P - x_r h_c q_{dc}
\]

Simplify to

\[
h_c (1 - x_r) \frac{d}{dt}(\rho_s a_m V_r) + \rho_w (1 - a_m) V_r \frac{d h_w}{dt} \\
- x h_c \frac{d}{dt}(\rho_w (1 - a_m)V_r) + \rho_s a_m V_r \frac{d h_s}{dt}
\]  \hspace{1cm} (10)

\[
= P - x_r h_c q_{dc}
\]
Drum Level

Average steam-water volume ratio

\[ x = \frac{\rho_s a}{\rho_s a + \rho_w (1 - x)} \]

Solving with respect to \( a \)

\[ a = a(x) = \frac{\rho_w x}{\rho_s + (\rho_w - \rho_s) x} \]

Assume

\[ x(\xi) = x_r \xi \quad 0 \leq \xi \leq 1 \] (11)

Hence

\[ a_m = \int_0^1 a(x_r \xi) d\xi = \frac{1}{x_r} \int_0^1 a(x_r \xi) d(x_r \xi) \]

\[ = \frac{1}{x_r} \int_0^{x_r} a(x) dx \]

\[ = \frac{\rho_w}{\rho_w - \rho_s} \left[ 1 - \frac{\rho_s}{(\rho_w - \rho_s) x_r} \ln \left( 1 + \frac{\rho_w - \rho_s}{\rho_s} x_r \right) \right] \] (11)

\[ \ell = \frac{V_w + a_m V_r}{A} \] (12)
Downcomer Flow

Momentum balance

\[ a_m V_r (\rho_w - \rho_s) = \frac{1}{2} k q_{dc}^2 \]  \hspace{1cm} (13)

Riser flow from (8)

\[ q_r = q_{dc} - \frac{d}{dt} (\rho_s a_m V_r) - \frac{d}{dt} (\rho_w (1 - a_m) V_r) \]  \hspace{1cm} (14)
Summary

\[ \frac{d}{dt} \left[ \rho_s h_s V_{st} + \rho_w h_w V_{wt} + m c_p T \right] = P + q_{fw} h_{fw} - q_s h_s \] (1)

\[ \frac{d}{dt} [\rho_s V_{st} + \rho_w V_{wt}] = q_{fw} - q_s \] (4)

\[ h_c(1 - x_r) \frac{d}{dt} (\rho_s a_m V_r) + \rho_w (1 - a_m) V_r \frac{dh_w}{dt} \]
\[ - x h_c \frac{d}{dt} (\rho_w (1 - a_m) V_r) + \rho_s a_m V_r \frac{dh_s}{dt} \]
\[ = P - x_r h_c q_{dc} \] (10)

Choose \( p, V_w, \) and \( x_r \) as state variables.

Simulation Model

\[
\begin{cases}
    e_{11} \frac{dp}{dt} + e_{12} \frac{dV_w}{dt} + e_{13} \frac{dx_r}{dt} = P + q_{fw} h_{fw} - q_s h_s \\
    e_{21} \frac{dp}{dt} + e_{22} \frac{dV_w}{dt} + e_{23} \frac{dx_r}{dt} = q_{fw} - q_s \\
    e_{31} \frac{dp}{dt} + e_{33} \frac{dx_r}{dt} = P - q_{dc} x_r h_c
\end{cases}
\] (15)
\[
e_{11} = \left( \frac{d\rho_s}{dp} h_s + \rho_s \frac{d\sigma_s}{dp} \right) V_{st} \\
+ \left( \frac{d\rho_w}{dp} h_w + \rho_w \frac{d\sigma_w}{dp} \right) V_{wt} + mc_p \frac{dT_s}{dp}
\]

\[
e_{12} = \rho_w h_w - \rho_s h_s
\]

\[
e_{13} = (\rho_s h_s - \rho_w h_w) V_r
\]

\[
e_{21} = \frac{d\rho_s}{dp} V_{st} + \frac{d\rho_w}{dp} V_{wt}
\]

\[
e_{22} = \rho_w - \rho_s
\]

\[
e_{23} = (\rho_s - \rho_w) V_r \frac{da_m}{dx_r}
\]

\[
e_{31} = \left[ (1 - x_r) h_c \frac{h\rho_s}{dp} + \rho_s \frac{d\sigma_s}{dp} \right] a_m V_r
\]

\[
+ \left[ \rho_w \frac{d\sigma_w}{dp} - x_r h_c \frac{d\rho_w}{dp} \right] (1 - a_m)V_r
\]

\[
e_{33} = [(1 - x_r) \rho_s + x_r \rho_w] h_c V_r \frac{da_m}{dx_r}
\]
Parameters

$V_{drum}$  drum volume
$V_r$  riser volume
$V_{dc}$  downcomer volume
$m$  total metal mass
$c_p$  specific heat of metal
$k$  friction coefficient
Simple Drum-Boiler Models

1. Introduction

2. First Order Model

3. Shrink and Swell

4. Simulations

5. Experiments

6. Conclusions
Step in Fuel Flow

Figure 1. Responses to a step in fuel flow.
Step in Steam Flow

Figure 2. Response to a step
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EXPERIMENTS

P16-G16 at Öresundsverket

Steinmuller boiler Stal-Laval turbine. Active power 160MW.

Controllers disconnected. PRBS like perturbations introduced in fuel flow, feedwater flow and steam valve at high and low load.
Fuel Flow Changes at Low Load

Drum pressure. 1=model, 2=plant

Electrical output. 1=model, 2=plant

Drum water level. 1=model, 2=plant
Turbine Valve Changes at Low Load

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Drum pressure. 1=model, 2=plant

Electrical output. 1=model, 2=plant

Drum water level. 1=model, 2=plant
Feedwater Flow Changes at Low Load

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Drum pressure. 1=model, 2=plant

Electrical output. 1=model, 2=plant

Drum water level. 1=model, 2=plant
Fuel Flow Changes at High Load

Drum pressure. 1=model, 2=plant

Electrical output. 1=model, 2=plant

Drum water level. 1=model, 2=plant
Turbine Valve Changes at High Load

Drum pressure. 1=model, 2=plant

Electrical output. 1=model, 2=plant

Drum water level. 1=model, 2=plant
Feedwater Flow Changes at High Load

88.02.05 - 19:45:37 nr: 1

Drum pressure. 1=model, 2=plant

Electrical output. 1=model, 2=plant

Drum water level. 1=model, 2=plant
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CONCLUSIONS

Promising results

Some details remain

Further experiments

Simplifications

Control design