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Ivar Gustavsson and Haldo Östlund

ABSTRACT

Results from time series analysis of data characterizing standing are presented. Parametric models are estimated by the maximum likelihood method. Data from healthy people as well as from people suffering from different diseases are used. Characteristics are found for some of the diseases but as a whole the results are quite heterogeneous even within the healthy group. It is questionable if anything can be gained by parametric estimation compared with e.g. spectral analysis as long as a model structure cannot be derived from physiological reasons.

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1. INTRODUCTION

Methods for analysis of complex motor behaviour, e.g. the examination of standing, are subjective. The result is then dependent upon the examiner's skill and practice. It would be valuable to characterize standing more objectively.

It is well-known that nobody is able to stand perfectly still [13]. People trying to stand still sway both in the sagittal and the frontal plane. The movements are usually small in both directions, but somewhat larger in the sagittal direction [17].

The human body may be considered as an unstable system which is retained in upright position by some kind of control system. This control system is very complex and its detailed function largely unknown. Individual differences in strategy, probably determined by physiological factors, are supposed to be small. There seem, however, to be different needs of stability for different persons. Large deviations from normal strategy are probably interpretable as disturbances, errors or malfunctions in the control system.

The theme of this paper is to attempt to characterize the system by analyzing the control error as a time series. This signal has been obtained by indirect measuring of the location of the mass centre of the body. Data from healthy people as well as from people suffering from different diseases have been used. In earlier studies such time series have been characterized by an estimate of their total variance and by an estimate of the power spectrum, e.g. [19]. In this paper it is attempted to characterize the time series by linear dynamic systems with independent normal variables as input.

The parameters of the linear discrete models have been determined using the maximum likelihood method [2]. The method was used because programs for this method were already available and because it has been efficient for many other applications, cf [12]. The same method has been used successfully also for identification of models from input/output data, e.g. [11].

Parameters like time constants may perhaps be used to characterize the balancing system. Time constants of first order models have been compared. A description by a first order model is, however, often not sufficient, which is clearly indicated for some of the series. But low order models can be used. The model order may actually be another characteristic of the system. Qualitatively and to some extent quantitatively these characteristics can of course also be observed in autocorrelation functions and in power spectra for the data. In fact it seems that only small further information about the system can be gained from parametric models for these data, at least as long as no more detailed information about the structure of the system is available.

In Section 2 some physiological considerations are presented. An attempt to interpret the balancing system as a control system is made in Section 3. The experiments and the material for the investigation are described in Section 4. The maximum likelihood method is shortly described in Section 5 and in Section 6 some of the results are given and discussed.

2. PHYSIOLOGICAL AND CLINICAL CONSIDERATIONS

Attitudes are repeatedly changing during spontaneous standing in various situations such as waiting for a bus or queueing, revealing an inherent need of motion. Also during sleep changes of position are observed, and their elimination might cause damage, for example during intoxicated sleep.

The statement of Denny-Brown that stability in progression and standing is secured by the righting reflexes [18] is supposed to be valid not only for spontaneous standing but also in static standing, although very restricted in postural movements in accordance with a test instruction.

The postural reflexes can be primarily divided into tonic (static) or attitudinal fixating the body configuration and position and the phasic (kinetic) or adjustmental controlling postural stability during active and passive movements. Further, they can be classified as local or general, depending upon the level of the central nervous system (CNS) from which they originate [14] . Configuration of the body is to a great extent dependent on an intact proprioceptive input concerning position and change of position of body parts, supplemented by the sensory input from muscle spindles and tendon organs. The proprioceptive system has access both to a short-circuit spinal and a long-circuit cerebral system.

The position and movements of the head are then important for the adequacy of general postural reflexes such as neck, labyrinthic and optical reflexes organized at the brain-stem, mid-brain and cortical levels [14].

Input/output relationships having their primary mechanisms in various structures of the CNS seem to be integrated in the cerebellum [7]. The linked Alpha-Gamma action, one characteristic feature of motor activity according to Granit [9], is to a great extent due to intact cerebellar function. The coordinated Alpha-Gamma action enables the segmental myotatic reflex system to perform its stabilizing function as a continuous feedback also during movements.

In ablation experiments postural reflexes are characteristically altered in association with removal of various structures and disconnections of the CNS. Sections of the dorsal spinal nerve-roots or of the dorsal columns of the spinal medulla, interfering with the proprioceptive sensory input convert the rather slow, well sustained postural reactions into fast erratic vestibular and optical reflexes that are badly sustained [7, 18]. The analogous state of sensory ataxia of the clinic is characterized by bad positioning, loose, flailing movements and a substantial worsening on blind-folding, the latter being the pragmatic basis of Romberg's test [16]. The clinical phenomena of ataxia have been interpreted as a consequence of abnormal irregular interruptions in the maintenance of a constant degree of muscular contraction during isotonic, isometric and shifting conditions [18]. Besides sensory ataxia we recognize cerebellar ataxia, in which the abnormal movements, however, are of a more stiff oscillating nature [18]. In cerebellectomized animals postural reflexes have a high threshold but are hyperactive, erratic and stereotyped when finally in action.

The subcortical mechanisms provide not only the tonic background for movement but also the associated movements themselves [18]. In lesions of basal ganglia due to diseases such as Parkinson's disease static as well as kinetic postural reflexes are more or less deteriorated.

We recognize states of rigidity and hypokinesia meaning increased resistance to passive movements, approximately constant over the whole range, and poverty, slowness and delayed initiation of movements [6, 15]. The basic mechanism of postural reflexes seems to be the fundamental deficit while the problem concerning a possible unbalance between the Alpha and Gamma system, causing for instance the phenomenon of rigidity, still is disputed.

3. THE BALANCE SYSTEM AS A FEEDBACK CONTROL SYSTEM

It is clear from the discussion in Section 2 that the human balance system is very complex if all details are taken into account. Many of the mechanisms involved are not yet quite fully understood. However, from the point of view of control systems a few facts are obvious. The system without control is unstable. In control system terminology the balancing system is thus an unstable system with a stabilizing feedback, Fig. 1, which can be considered as an attempt to use an aggregated model of a very complex process.

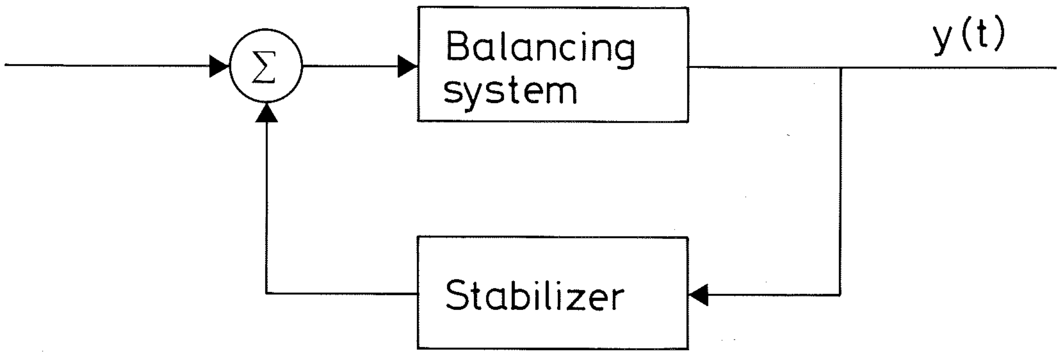

Fig. 1. The balancing system as a feedback control system. y(t) denotes the output.

From Section 2 we conclude that the system actually is multivariable and has several feedback loops. The information of the system for the stabilizer is received e.g. from receptors in the muscles, in the joints and in the soles of the feet, from the vestibular apparatus and as visual information [4, 15]. The control actions are results of decisions on different levels of the neurological system which somehow are coordinated. It is difficult to distinguish between the effects of the different feedback loops. Specially designed experiments could here be of some help as well as further studies of patients with certain malfunctions in their balancing system.

Studies of patients suffering from sensory ataxia seem to show that when the brain is deprived of information from some of the proprioceptors the output of the system has larger variance and contains more power in high frequencies compared to the normal case. This may be taken as an indication that the stabilizing system can be divided into two parts, one part with fast response because of a short information and control path via the spinal cord, and one slower part where the control path is passing the brain, Fig. 2. This model with a division of the stabilizer into two parts is of course not in any way claimed to be a complete and not even a sufficient description of the system. However, it explains some of the observed properties. Both parts are aggregations of different control loops and are of course actually not strictly separated either.

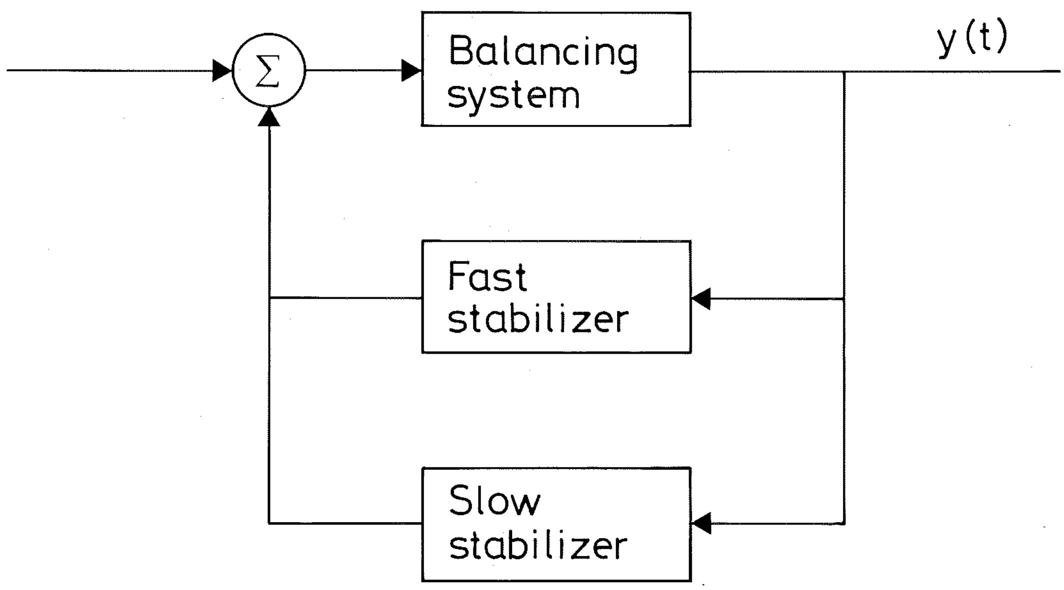

Fig. 2. Division of the stabilizer into one part with fast response and another part with slow response.

The conclusions from the observations of the sensory ataxia patients is that the system cannot be controlled satisfactorily when it is deprived of some of the normal information and control paths. In this case the slow stabilizer does not get full information. An additional effect could be caused by an adaption of the remaining control system to the new situation.

From other studies [21] it is obvious that the individual differences between healthy people are large. They may perhaps be explained as adaption of the control system to different intentions of stability. The possibility to compensate missing or defective control loops is exemplified in the following way. When a man closes his eyes the variance of the sway usually increases noticeably. A blind man, however, manages perfectly well on the remaining control system [4]. When a sensory ataxia patient closes his eyes the standing stability is impaired much more than for healthy people [21]. One possible explanation is that in this case the slow stabilizer is almost not working at all because the brain is deprived most of the information that it normally receives about the state of standing. Considering Fig. 2 the effect of disconnecting the slow loop would be an increased variance, just as observed.

Disturbances enter the system in many ways in the primary sensors, in the feedback paths and in the actuators. Other disturbances are external and there might also be internally generated intentional motions present in order to fulfil the physiological needs. Due to all these disturbances the system will in normal operation show fluctuations in the output. With no disturbances at all acting on a stabilizing system it would settle. The characteristics of the fluctuations will reflect both the disturbances acting on the system and the features of the feedback system. It would therefore at least in principle be possible to find some of the internal properties of the system simply by analysing the output, $y(t)$. To draw such conclusions it is necessary to have detailed understanding of how the properties of the different subsystems will influence the closed loop system. From the foregoing discussion it follows that such knowledge is not available. Some mechanisms are known qualitatively but not quantitatively. It is thus not possible to try to give a mathematical model which describes the phenomenon in detail.

To get some insight into the problem we will attempt to describe the system with a very simple model which contains the gross features. Since only small motions are considered it is reasonable to use a linear model. If the disturbances are appropriate and stationary conditions are assumed the output would be a stationary stochastic process. The spectral density of the output would thus reflect the properties of both the system and the disturbances. Spectral densities have been computed for a large material [20, 21] and they indicate that the assumption of stationarity is not unreasonable.

The simplest possible model that can be used is a first order system which can be characterized simply by a time constant, T and a gain, K. A simple model of the disturbances would be to assume that they can be modelled as a white noise source. In such a case the spectral density $\phi(\omega)$, of the output would be of the form

$$\phi(\omega) = \frac{K^2}{1 + (\omega T)^2}$$

A check of the computed spectral densities shows, however, that a system of higher order is often indicated. Fig. 3 shows two typical power spectra.

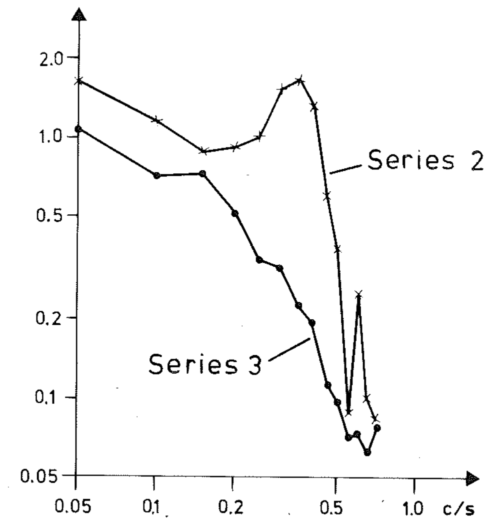

Fig. 3. Power spectra for time series 2 and 3 (cf Table 2) respectively.

One of them might reflect the properties of a first order system, the other one certainly a higher order system. The difference in appropriate model order for different time series is of course another characteristic that can be used to distinguish between them. This is discussed a little more in Section 6.

4. EXPERIMENTAL CONDITIONS AND MATERIAL

The test subjects were standing on two scales with one foot on each and with the feet 7 cm apart, Fig. 4. The difference between

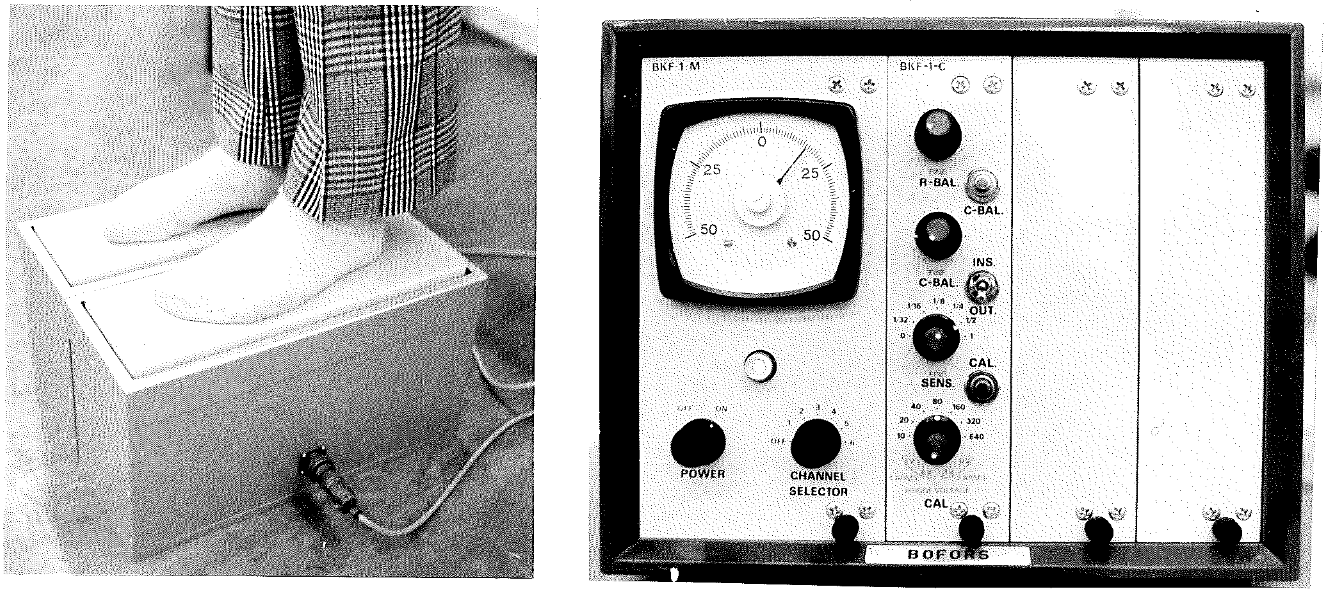

Fig. 4. Photo showing test equipment and conditions.

the signals from the scales was used as the output signal. This is approximatively proportional to the angle between the location of the mass centre and the vertical if the signal is compensated for the length and weight of the test subject. The commercially available scales are built on strain-gauge principles. The resonance frequency and coefficient of damping for the scales were estimated to 80 Hz and 0.2 respectively [21]. The resonance frequency was the same for the apparatus unloaded and loaded with a test subject of ordinary weight. A more detailed description of the measuring devices is given in [21]. Outputs from two experiments are given in Fig. 5.

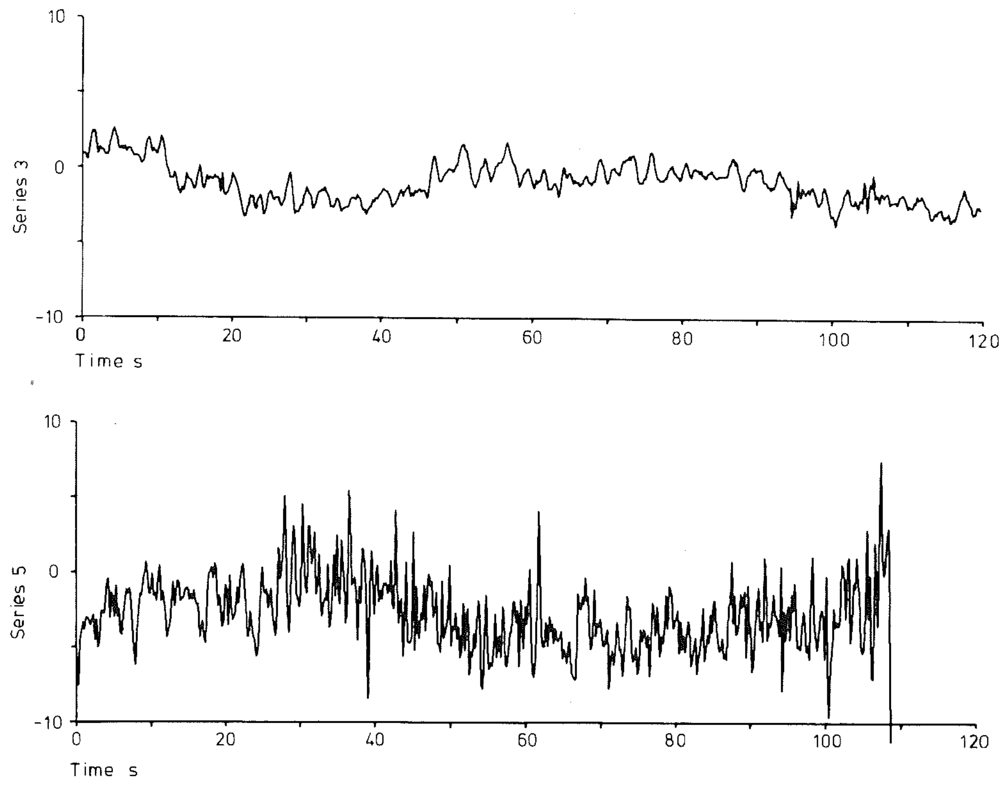

Fig. 5. Series 3 and 5 (cf Table 2). The sampling interval was 0.2 s.

The lengths of the experiments were about two minutes each. The continuous signal was recorded on a magnetic tape. The frequencies of interest were considered to be lower than 5 Hz [21] and the signal was filtered by a low pass filter with a cutting-off frequency of about 10 Hz. The signal was then sampled with a frequency of 25 Hz. The results of this study are, however, based on a sampling frequency of 5 Hz, i.e. every fifth value out of the original sampled series was used. The reason was to reduce the computing time. To test if this reduction of data was crucial for the analysis some computations using the original series were also performed. However, the results did not differ significantly from those obtained with the reduced sampling frequency. The lower sampling frequency can also be justified by the fact that the spectra of the series had only small contributions from frequencies above 2.5 Hz.

In order to keep the conditions as equal as possible for the different test subjects all were standing with their feet about parallel and told to keep them still, to stand erect, looking straight forward and to stand as still as possible through the whole experiment.

The study is based on measurements on 19 individuals, 4 healthy persons, 4 patients suffering from sensory ataxia, 3 from cerebellar ataxia, and 8 from Parkinson's disease. Out of those suffering from Parkinson's disease 4 patients had no treatment and 4 selected at random had been given L-Dopa. Two patients in the group of cerebellar ataxia were suffering from cortical cerebellar degeneration and one from a cholesteatoma in the posterior fossa of the skull. Two of the patients suffering from sensory ataxia had week spastic symptoms at the same time. The ages were from 50 to 60 years but for one in the Parkinson group who was 38 and one in the group of sensory ataxia who was 80 years old.

5. AN OUTLINE OF THE TIME SERIES ANALYSIS, PARTICULARLY THE MAXIMUM LIKELIHOOD TECHNIQUE

The analysed data are assumed to be stationary time series or such series superposed on a linear trend. At least for short measurements such an assumption can be justified. The reason for introducing the trend is that it is quite clear from almost all the experiments that the time series contain rather slow modes together with the faster ones. The slow modes are of the order half a minute or more and may perhaps be interpreted as motions caused of the tiring procedure to stand in upright position. The test subject thus to some extent sway over from one foot to the other. Rather short parts of the series were used in the analysis in order to reduce the effect of these slow modes. The linear trend was estimated by the method of least squares and removed from data before further analysis.

Time series can be represented in different ways. Stationary time series can be represented e.g. by covariance functions or by spectral density functions. If we furthermore restrict ourselves to stationary time series with rational spectra, all such processes can be generated by sending discrete white noise through asymptotically stable dynamical systems [1]. A further restriction would be to allow only autoregressive processes. Even if a stationary time series does not have a rational spectrum it may be well approximated by such a spectrum and even as an autoregressive process. However, the order of an autoregressive model may become high if a good approximation is desired. For this problem therefore the so-called mixed autoregressive moving average process, i.e. the dynamical system mentioned above, was chosen.

The estimation of the parameters of the model is performed by the maximum likelihood method introduced in [2]. This estimation procedure also provides an estimate of the accuracy of the parameters and the possibility to test what order is appropriate for the model. The obtained estimates are consistent, asymptotically normal and efficient under mild conditions [3]. It must be emphasized that it is crucial for the statistical interpretations that the assumption of the structure of the model is correct. However, in any case the procedure gives the estimate that minimizes the sum of squares of the one step ahead prediction error. The model can be directly used for prediction. It is also easy to calculate spectrum and covariance function from the model. Other ways of estimating the parameters are given in e.g. [5, 8]. For the computations a Fortran program was available [10]. The computer used was a UNIVAC 1108.

The estimation problem can be formulated in the following way: Given the observations $\{y(t),\ t = 1,2,\ldots,N\}$ of a stationary time series, find an estimate of the parameters of the model

$$A^*(q^{-1})\ y(t) = \lambda\, C^*(q^{-1})\ e(t) \tag{1}$$

where $\{e(t)\}$ is a sequence of independent gaussian variables with zero mean and variance one. Furthermore q denotes the shift operator

$$q\ x(t) = x(t + h)$$

where h is the sampling interval,
and $A^*(q^{-1})$ and $C^*(q^{-1})$ are polynomials

$$A^*(q^{-1}) = 1 + a_1\, q^{-1} + \ldots + a_n\, q^{-n}$$
$$C^*(q^{-1}) = 1 + c_1\, q^{-1} + \ldots + c_n\, q^{-n}$$

Assume that the functions $A(q) = q^n \cdot A^*(q^{-1})$ and $C(q) = q^n \cdot C^*(q^{-1})$

have all their zeroes inside the unit circle and that there are no common factors to the polynomials $A^*(q^{-1})$ and $C^*(q^{-1})$.

This problem of parametric estimation of rational spectra is a special case of identification of linear discrete time systems with gaussian disturbances with rational spectra, which was described in [2]. A short summary of the method is given here. The proofs of the statistical properties of the estimates and convergence etc. can be found in [3].

The problem stated above is a statistical estimation problem and will be solved by the method of maximum likelihood. It follows from (1) that the residuals $\varepsilon(t)$ defined by

$$C^*(q^{-1})\,\varepsilon(t) = A^*(q^{-1})\,y(t) \tag{2}$$

are independent and normal $(0, \lambda)$. The logarithm of the likelihood function L, for the problem becomes

$$-2 \log L = \lambda^{-2} \sum_{k=1}^{N} \varepsilon^2(k) + 2N \log \lambda + N \log 2\pi$$

Maximizing L with respect to λ, $a_1, \ldots, a_n$, $c_1, \ldots, c_n$ is equivalent to minimizing the loss function

$$V(\theta) = \frac{1}{2} \sum_{k=1}^{N} \varepsilon^2(k) \tag{3}$$

which is readily shown. θ is defined by

$$\theta = (a_1, \ldots, a_n, c_1, \ldots, c_n)^T$$

When $\hat{\theta}$, such that $V(\theta)$ is minimal, has been found, the maximum likelihood estimate of λ is obtained from

$$\hat{\lambda}^2 = \frac{2}{N} V(\hat{\theta})$$

Notice that λ^2 can be interpreted as the variance of the one step ahead prediction error.

The problem of minimizing $V(\Theta)$ can be solved in many different ways by using different optimization procedures. The difference between the procedures is essentially the amount of information of the function $V(\Theta)$ that is required to find the minimum. In the used program a modified Newton-Raphson algorithm is used for the iteration towards the minimum, i.e.

$$\Theta^{k+1} = \Theta^k - \alpha\{V_{\Theta\Theta}^*(\Theta^k)\}^{-1} V_\Theta(\Theta^k) \tag{4}$$

where k denotes the iteration number,

V_Θ = the gradient vector of $V(\Theta)$,

$V_{\Theta\Theta}^*$ = the matrix of second partial derivatives of $V(\Theta)$ or an approximation of this matrix.

α is used to modify the step length in each iteration.

The reason for using a Newton-Raphson algorithm is that the convergence rate will be fast near the minimum and that the calculation of the second derivative matrix can be performed relatively economically. Furthermore this matrix provides the accuracy estimates.

Differentiating (3) gives

$$\frac{\partial V}{\partial \Theta_i} = \sum_{k=1}^{N} \varepsilon(k) \frac{\partial \varepsilon(k)}{\partial \Theta_i} \tag{5}$$

$$\frac{\partial^2 V}{\partial \Theta_i \partial \Theta_j} = \sum_{k=1}^{N} \frac{\partial \varepsilon(k)}{\partial \Theta_i} \frac{\partial \varepsilon(k)}{\partial \Theta_j} + \sum_{k=1}^{N} \varepsilon(k) \frac{\partial^2 \varepsilon(k)}{\partial \Theta_i \partial \Theta_j} \tag{6}$$

If only the first term of (6) is used in the iterative minimizing algorithm, it is secured that the matrix is at least positive semidefinite and in most cases positive definite. This means that the iteration will yield a smaller function value. Far from the minimum only the first term is used. The second term of (6) is included in $V^{*}_{\Theta\Theta}(\Theta)$ when it is probable that the algorithm is "sufficiently" near a local minimum.

Differentiating (2) twice with respect to Θ gives the difference equations which are used to calculate the first and second derivatives of the residuals with respect to Θ. These derivatives are needed for the calculation of V_{Θ} and $V^{*}_{\Theta\Theta}$, (5) and (6) respectively. The difference equations can be solved faster by introducing appropriate state variable representations, [3, 10]. Another problem is the choice of initial values for the difference equations. They are here chosen to zero because of the assumption that $\{\varepsilon(t)\}$ has zero mean and thus $\{y(t)\}$ also has zero mean.

The algorithm for the minimization thus becomes principally

i) Put $\Theta^{k} = \Theta^{0}$ (starting value of Θ)

ii) Evaluate $V_{\Theta}(\Theta^{k})$ and $V^{*}_{\Theta\Theta}(\Theta^{k})$ using (5) and (6)

iii) Calculate Θ^{k+1} from (4) and repeat from ii) unless some specified convergence criteria are fulfilled.

The recursive formula (4) requires an initial value, Θ^{0}. Letting $c_i = 0$, $i = 1, 2, \ldots, n$ the least squares estimate a^{0} of a is obtained in one step. Then the initial value for the iteration is taken as $\Theta^{0} = (a_1^{0}, \ldots, a_n^{0}, 0, \ldots, 0)^{T}$.

An estimate of the covariance matrix for the parameter estimates is given by $\lambda^2\{V_{\theta\theta}(\hat{\theta})\}^{-1}$. As the minimization algorithm uses this matrix the accuracy is directly available. For simulated data these accuracy estimates most often are very good estimates of the Cramér-Rao lower bound for the estimation problem.

The computing time for a time series of 1000 values and for models of order 1-4 is roughly 25 seconds on a UNIVAC 1108. The computing time is almost linear in both n and N, i.e. proportional to the product nN.

Most often the correct order of the system is not known in advance. Thus some criteria for deciding appropriate model order have to be developed. Many tests can be proposed, e.g. from a statistical point of view. The assumptions made for the method have to be checked, so that they are not violated. Among the different tests we can mention

a) statistical test of the significance of the decrease of the loss function,

b) test of the independence of the residuals,

c) examination of the factors of the polynomials $A^*(q^{-1})$ and $C^*(q^{-1})$,

d) test of the significance of the parameters a_n and c_n.

If the order of the system is unknown the identification has to be performed for increasing order of the model. The tests given above are applied to the models of different orders. These tests work very well for artificially generated data. For real data, however, the tests are often not so clear-cut and subjective decisions have to be made. A reason for this may be that the structure of the model is not quite correct. We are only trying to find a linear model of a certain order approximating the real process. It is very important to check the relevance of the model, obtained from one set of data, by simulating the model on another set of data from the same process. The reason is that even if e.g. a high order model is a very good approximation of one specific realization of the process, it may not be a good model of the process itself. The model may include specific properties of this realization which are not typical for the process. In such a case the model may give bad predictions for another realization. This is one reason for choosing a model of lowest possible order. This discussion thus indicates the need of several realizations for the analysis.

The model (1) can be considered as a pulse transfer function or a filter with discrete time white noise as input, Fig. 6.

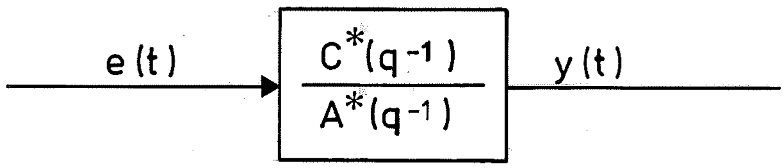

Fig. 6. Model for stationary time series with rational spectrum.

As such it can be represented by concepts like gain and time constants. In the following we therefore often prefer to talk about time constant rather than about the parameter a_1 etc.

6. RESULTS AND DISCUSSION

In this section some of the results from the maximum likelihood identification of the time series are presented. A short discussion of possible conclusions and relevance of the results are also given. In Table 1 the total variance, σ^2, and the time constant, T, of the first order models for the series are listed. The results are based on parts of the original measurements. The parts are chosen so that slow modes (cf Section 5) and irregularities are eliminated. The series are then compensated for linear drift, estimated by ordinary least squares method.

Clinic state	σ^2	T
Healthy	0.13	0.45
	0.62	0.82
	0.43	1.78
	1.19	1.93
Parkinson's disease untreated	1.08	0.55
	0.38	1.26
	0.86	1.60
	1.93	2.51
Parkinson's disease treated with L-Dopa	0.33	0.79
	0.50	0.96
	0.55	1.21
	0.45	2.26
Cerebellar ataxia	15.0	1.10
	11.5	1.23
	4.57	1.65
Sensory ataxia	4.72	0.19
	5.18	0.20
	53.3	0.40
	15.9	0.74

Table 1. Total variance, σ^2, and time constant, T, of first order models for the time series used.

The results seem rather heterogenous, also within the healthy group. A few observations can, however, be made. The ataxia groups have obviously a very high total variance compared to the other groups. The sensory ataxia group shows in general a rather short time constant. For the Parkinson's disease groups it seems that the total variance is smaller for the treated patients. However, they are not matched pairs in the sense that the same patient is compared before and after treatment. Therefore the difference can perhaps be explained by the individual differences. The results for the Parkinson's groups can also be influenced by tremor. Because of the limited material in statistical sense it is dangerous to draw definite conclusions. It must also be emphasized that the first order model is used here, even if higher order models were indicated by the maximum likelihood identification (see below). Thus the comparison of T for different series is not quite fair, because several of the series need more parameters to be appropriately characterized.

In the following some of the series are treated in more detail. The series chosen for the discussion are plotted in Fig. 7 and characteristics of them are given in Table 2.

Series	Clinic state	σ^2	T
1	Healthy	0.62	0.82
2	Parkinson's disease untreated	0.86	1.60
3	Parkinson's disease treated with L-Dopa	0.50	0.96
4	Cerebellar ataxia	11.5	1.23
5	Sensory ataxia	4.72	0.19

Table 2. Total variance, σ^2 and time constant, T, for the first order models of five chosen series.

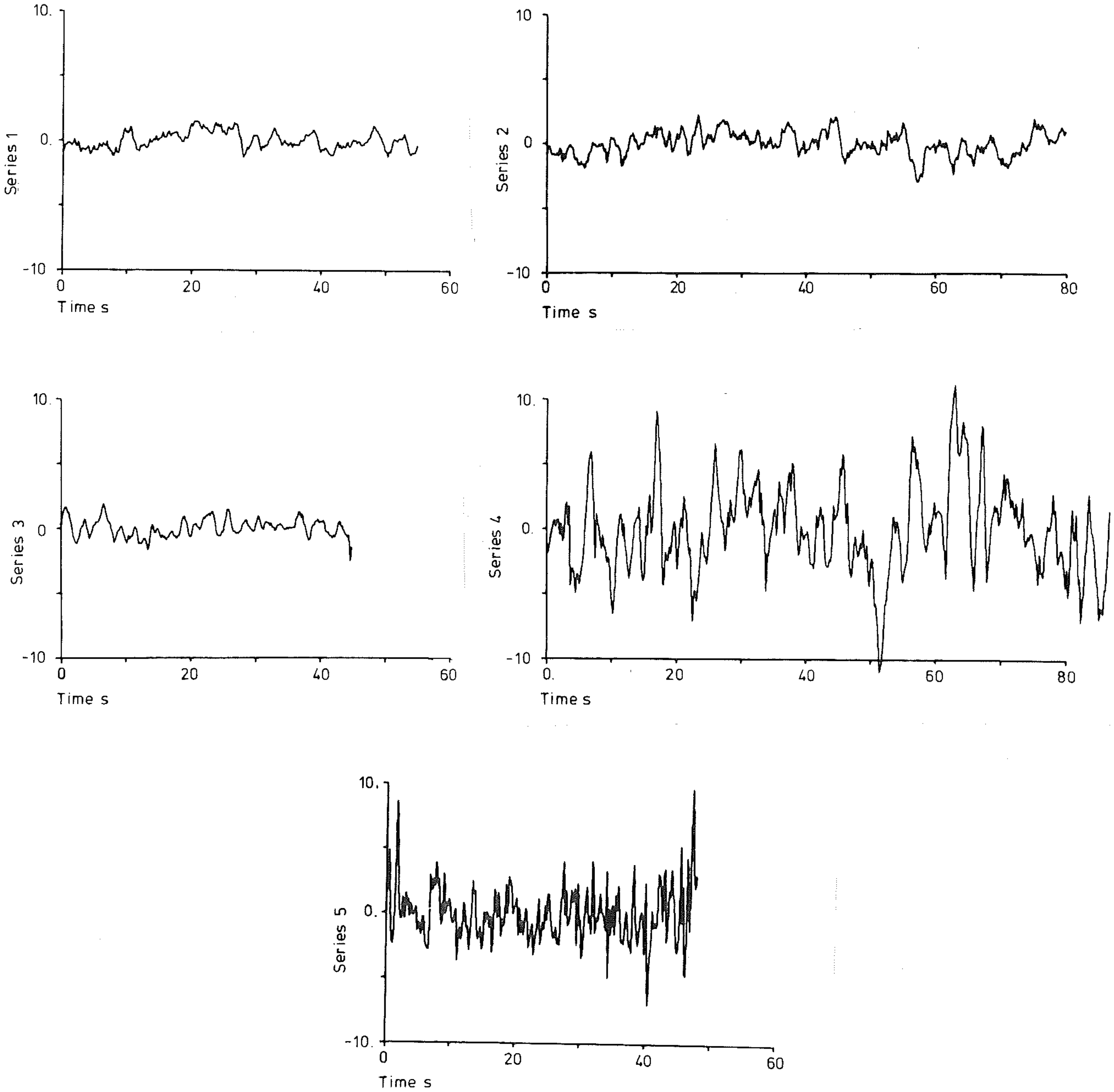

Fig. 7. Parts of series 1-5 used for the estimation.

Just to compare with other techniques the autocorrelation functions are shown in Fig. 8 for the five time series. The fast mode in the time series 5 from the patient suffering from sensory ataxia can easily be observed. For some of the series there also seem to be an oscillating component present which is in accordance with the indication of higher order models discussed in Section 3. The oscillation seems to have a frequency of about 0.3 - 0.5 Hz, and might be caused by the breathing.

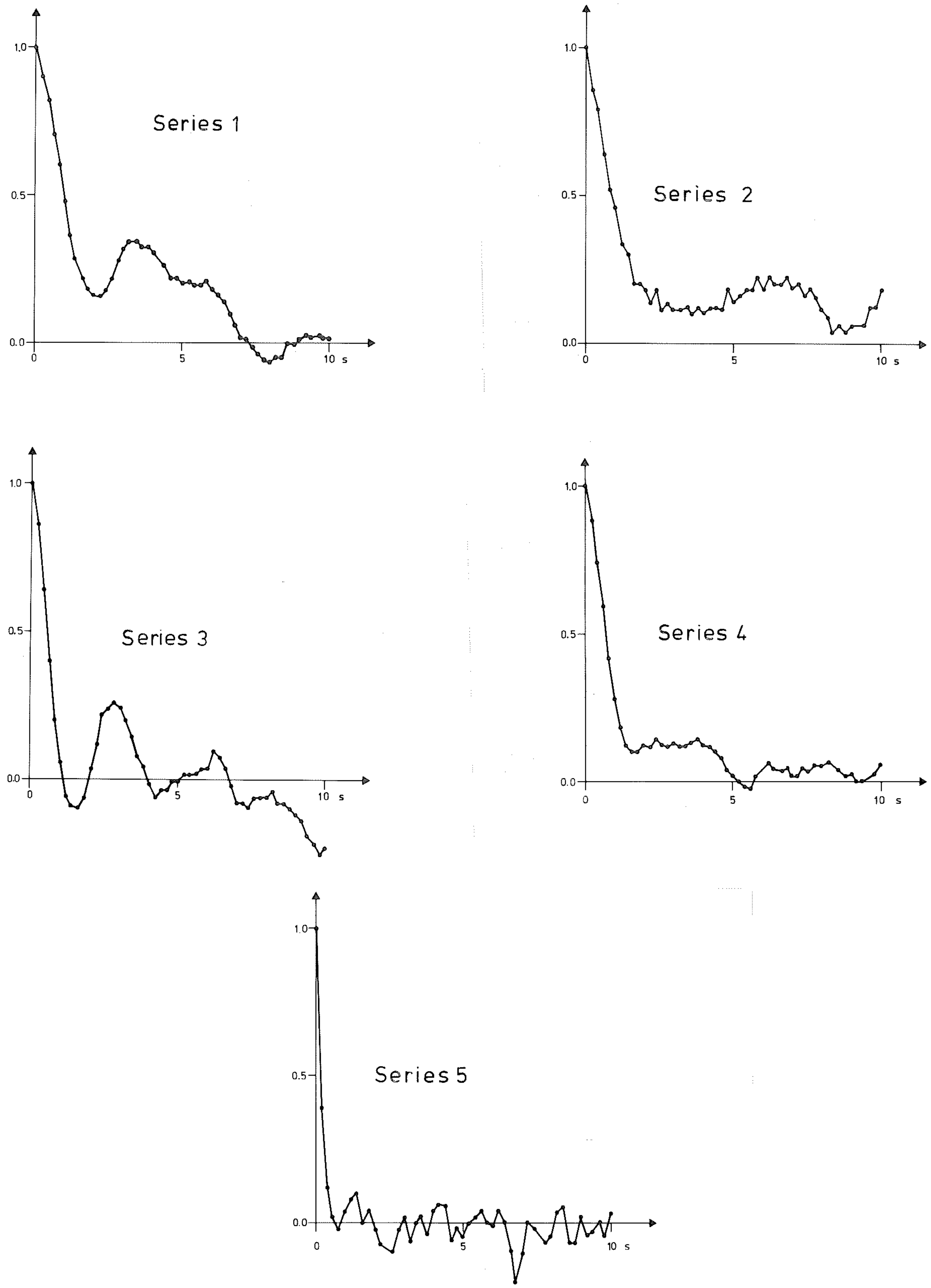

Fig. 8. Autocorrelation functions for series 1 - 5.

In order to give a more complete picture of how the maximum likelihood method works, we also show results from identification with higher order models for three of the series, Table 3, indicating that for different series different orders of the model should be chosen according to statistical tests.

Series	2			3			5		
Order	V	λ	$F_{n,n-1}$	V	λ	$F_{n,n-1}$	V	λ	$F_{n,n-1}$
0	172.33	0.928	-	409.72	1.930	-	49.37	0.686	-
1	42.31	0.460	633	353.29	1.792	17.6	10.84	0.321	370
2	41.28	0.454	4.9	347.03	1.776	1.9	8.74	0.288	24.8
3	41.27	0.454	0.1	337.46	1.752	3.0	8.67	0.287	0.8
4	33.33	0.408	46.7	332.32	1.738	1.6	8.45	0.285	2.6
5	32.90	0.406	2.5						
6	32.79	0.405	0.6						

Table 3. Loss function values, V, prediction errors, λ, and the values of the F-test quantities for the series 2, 3 and 5.

The statistical test of the significance of the decrease of the loss function is based on the fact that the quantity

$$F_{n,\,n-1} = \frac{V_n - V_{n-1}}{V_n} \cdot \frac{N - 2n}{2}$$

has an F (2, N-2n) distribution under the hypotheses that $a_n = c_n = 0$. V_n denotes the loss function for the model of order n. The loss function is decreased significantly on the 5 % level if this test quantity is greater than 3.0. A straight-forward use of this test for real data is not possible, probably because of violated assumptions, e.g. structure errors.

Table 3 shows that a fourth order model is appropriate for series 2, a second order model for series 5, and a first order model for series 3. The roots of the polynomials A(q) for series 2, n=4, are given in Table 4.

Roots of A(q)	Roots of C(q)
0.766 ± 0.052i	0.601
-0.867 ± 0.498i	-0.846 ± 0.490i

Table 4. Roots of the polynomials A(q) and C(q) for series 2, n=4.

There is one factor almost common to the two polynomials. If the accuracy of the parameter estimates is considered, it is questionable if the roots $-0.867 \pm 0.498\, i$ and $-0.846 \pm 0.490\, i$ can be distinguished or should be considered as a common factor. The loss function reduction is, however, clearly significant. The same factor appears also in several of the other experiments when higher order models are analysed. Thus it seems actually to be a characteristic of the data, probably due to some weak resonance phenomenon in the measuring or recording devices. Considering this the remaining model is of second order. From the values of the parameter λ(Table 3) we can see that even for series 2, most of the explainable variation of the output is gained already for the first order model. For series 5 on the other hand the second order model is clearly indicated and this is in accordance with the power spectra given in Fig. 3 in Section 3. For most of the series model orders of one or two were satisfactory.

A study of residuals and of autocorrelation functions seems to show that there is very often a resonance frequency of about 0.3 - 0.5 Hz. The power of this peak is rather small and the mode is sometimes almost hidden in the heavy noise. For many series it has been found in higher order models, particularly if the data have been prefiltered with the filter $(1 - q^{-k})$. Notice that rather short parts of the series were used in order to eliminate slow modes; resonance frequencies of about 0.01 Hz and lower and trends. Because of this fact perhaps a third order model with predetermined structure, one time constant plus an oscillation, should be used for further studies. The prefiltering problem must also be considered more carefully.

Another remarkable observation was made. The model

$$(1 - q^{-1})\, y(t) = \varepsilon(t) \qquad (7)$$

is almost as good as the model

$$(1 + a_1\, q^{-1})\, y(t) = (1 + c_1\, q^{-1})\, \varepsilon(t) \qquad (8)$$

for many series, cf Table 5.

Series	Model (7)	Model (8)
1	10.58	10.47
2	44.28	42.31
3	11.50	10.84
4	543.76	508.22
5	362.97	353.29

Table 5. Loss function values for series 1 - 5 for the different models (7) and (8).

This observation is discussed at some length in [5], where the same phenomenon was observed for a number of series of economic data. For predictive purposes the models are almost equal. However, if it is possible to pose any physiological meaning to the parameter a_1 as has been tried in Section 3, the differences between the models are important. Different values for the parameter a_1 then indicate different behaviour of the control system.

7. CONCLUSIONS

Some typical properties of time series from experiments with the human balancing system have been found by maximum likelihood identification of the series. For some diseases the total variance of the signal and the time constant of the first order model can to some extent be used as characteristics of the balancing system. Results consistent with common neuro-physiological findings are e.g. the small time constants in the models of the patients suffering from sensory ataxia. It might be emphasized that large individual differences, however, exist. The properties of such time series have been studied by other methods and the results are also consistent with results published earlier.

The study indicates that the gross characteristics of the system can qualitatively and to some extent quantitatively be recognized e.g. from an estimate of the power spectrum. Parametric models seem to give only small further information. They might be applicable when a better understanding of the system is achieved, so that a model with a structure, predetermined of physiological reasons, can be used. The slow modes of half a minute or more must also be taken care of by filtering the data by more elaborate methods than used here. It is difficult to make so long experiments that the slow modes can be modelled accurately.

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