Stability Problems of Adaptive Robot Control ad modum Slotine and LI

Johansson, Rolf

1988

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Stability Problems of Adaptive Robot Control
ad modum Slotine and LI

Rolf Johansson

Department of Automatic Control
Lund Institute of Technology
August 1988
Title and subtitle
Stability Problems of Adaptive Robot Control ad modum Slotine and Li.

Abstract
This paper presents criticism of stability properties of an algorithm of Slotine and Li (1987). It is shown that the arguments of Slotine and Li for claims on global asymptotic stability are not sufficient. A counterexample is formulated and is verified by simulation.

Key words

Classification system and/or index terms (if any)

Supplementary bibliographical information

ISSN and key title

<table>
<thead>
<tr>
<th>Language</th>
<th>Number of pages</th>
<th>ISBN</th>
</tr>
</thead>
<tbody>
<tr>
<td>English</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

The report may be ordered from the Department of Automatic Control or borrowed through the University Library 2, Box 1010, S-221 03 Lund, Sweden, Telex: 33248 lubbis lund.
STABILITY PROBLEMS OF ADAPTIVE ROBOT CONTROL
ad modum SLOTINE AND LI

Rolf JOHANSSON
Department of Automatic Control
Lund Institute of Technology
Box 118, S-221 00 Lund
Sweden
Phone +46 46-108791

Abstract

This paper presents criticism of stability properties of an algorithm of Slotine and Li (1987). It is shown that the arguments of Slotine and Li for claims on global asymptotic stability are not sufficient. A counterexample is formulated and is verified by simulation.
1. Introduction

Robotic systems intended for autonomous operation need an ability of adaptation to new and rapidly changing operating conditions. In such a situation various automatic control methods become important. Craig et al [3] applied ideas of model reference adaptive control and developed a regulator and stability proofs. Slotine and Li [13] approached the problem in a similar way but with weaker assumptions. They presented a regulator that is linear in the parameters and without any requirement of acceleration measurement.

2. Problem statement

Slotine and Li (1987) recognized the problem with acceleration measurement and the matrix inversion and tried to solve this problem without acceleration. Their technical innovation makes use of the skew-symmetric system matrix properties and thereby eliminates the problems of measurement and computation. Stability properties are however not quite satisfactory with respect to position errors. Elimination of steady-state errors is not guaranteed in their fundamental algorithm. The authors attempt to modify the algorithm (Slotine and Li 1987; sec. 2.2.2) to obtain stability but then make formal errors. They formulate a Lyapunov function candidate containing a linear combination of velocity and position error state vectors \( s = \dot{\theta} + \Lambda \dot{\eta} \). The suggested Lyapunov function candidate is not a function of the complete state vector and there is thus a 'forgotten' subspace of the state. A formal requirement is that the Lyapunov function is a function of all state vector components and not only a subset thereof. A counterexample to the claim of global asymptotic stability in (Slotine and Li 1987) can be formulated as follows:

**Example 1**

Define with the notation of Slotine and Li (1987) the transformed state vector

\[
s = \dot{\theta} + \Lambda \dot{\eta}, \quad \Lambda = \Lambda^T > 0
\]

with the associated Lyapunov function candidate

\[
V_s = s^T H(q)s
\]

Introduce also the functions

\[
s_\perp = -\Lambda \ddot{\eta} + \dot{\eta}; \quad V_\perp = s_\perp^T H(q)s_\perp
\]
Figure 1. Simulation of the example from Slotine and Li ([3]; app. 1) with $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ and non-zero initial conditions. The upper graph shows the “Lyapunov function” $V_x$. The lower graph shows $V_{\perp}$. All graphs vs. time [s].

Slotine and Li (1987) show correctly that $V_x(t)$ and $s(t)$ converge to zero as the time $t$ increases. However, the state vector $s_{\perp}$ orthogonal to $s$ is not represented in the function $V_x$. Simulations shows that $V_{\perp}(t)$ develops irregularly with time also when $s$ is very small, see Fig 1. Sometimes it slowly tends towards zero for non-zero initial conditions. In some simulations of the example, however, $V_{\perp}$ remains constant and rather large.

3. Conclusions

We have shown that the suggested Lyapunov function candidate is not formally correct. Moreover, the derivative of the suggested Lyapunov function candidate negative definite only with respect to a subset of the state vector components. Neither is it negative definite with respect to parameter errors. The stability problems have been verified by simulation.

The arguments presented by Slotine and Li (1987) for a claim on global asymptotic stability are thus not valid.
4. References
