A note on: an empirical comparison of forgetting models

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A Note on "An Empirical Comparison of Forgetting Models"

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***Abstract*—In the above paper, Nembhard and Osothsilp (2001) empirically compared several forgetting models against empirical data on production breaks. Among the models compared was the learn–forget curve model (LFCM) developed by Jaber and Bonney(1996). In previous research, several studies have shown that the LFCM is advantageous to some of the models being investigated, however, Nembhard and Osothsilp (2001) found that the LFCM showed the largest deviation from empirical data. In this commentary, we demonstrate that the poor performance of the LFCM in the study of Nembhard and Osothsilp (2001) might be attributed to an error on their part when fitting the LFCM to their empirical data.**

***Index Terms*—Comparative study, empirical, forgetting, learning, learn–forget curve model (LFCM), production breaks.**

I. Introduction

Learning and forgetting processes have received increasing attention by researchers and practitioners in the field of production and operations management for the last two decades. A handful of theoretical, experimentally, and empirical mathematical forgetting models have been developed, with no unanimous agreement among researchers and practitioners on the form of the forgetting curve. One set of research has focused on modeling forgetting mathematically (e.g., [3], [4], and [6]). Other researchers have focused on comparing models of forgetting with experimental data collected from laboratory experiments (e.g., [2] and [5]). Finally, some researchers have compared models of forgetting curves with empirical data from real-life settings (e.g., [1] and [8]).

In the above paper, Nembhard and Osothsilp [9] evaluated 14 forgetting models on an empirical dataset consisting of assembling car radios. Among the models fitted to empirical data were the learn–forget curve model (LFCM) developed by Jaber and Bonney [6], and the recency model (RC) developed by Nembhard and Uzumeri [10].

In [9], the models were ranked on efficiency, stability, and balance. Efficiency was measured as the mean absolute deviation (MAD), where lower MAD indicates higher efficiency. On this measure, the RC model provided the lowest and the LFCM the highest. The standard deviation of absolute deviations (STDAD) indicates the stability of the model. On this measure, the RC model was shown to have the highest stability (lowest STDAD) among the 14 models, where the LFCM registered the lowest stability (highest STDAD). The final measure, the balance of predictions for each model is the ratio of the number of under predictions to the number of over predictions. On this measure, the LFCM performed worse than the RC model but better than the other models. In summary, the overall performance of the LFCM was the worst.

In this paper, we fit the LFCM model to the empirical data provided by Nembhard and Osothsilp [9, Fig. 1, p. 285]. The results indicate that the LFCM performed better than that of Nembhard and Osothsilp [9] reported. This might be attributed to an inappropriate estimation of the LFCM parameters.

The remainder of this article is organized as follows. In Section II, we present a brief mathematics of the LFCM and RC models. In Section III, we fit the learning-forgetting model of interest to empirical data and discuss results. Section IV is for concluding remarks.

II. Mathematics of the Learning–Forgetting Models

This section presents the mathematics for the LFCM [6] and RC [10] models. Both models assume that learning conforms to Wright's learning curve [11], or that the time to produce a unit is a power-function of the number of units produced

$$T_x = T_1 x^{-b} \tag{1}$$

where T_x is the time to produce the xth unit, T_1 is the time to produce the first unit, x is the cumulative number of units produced, and b is the learning curve constant $(0 < b < 1)$.

A. Mathematics of the LFCM

Jaber and Bonney [6] suggested a power-form forgetting curve, with its exponent computed as

$$f_i = \frac{b(1-b)\log(u_i + n_i)}{\log\left(1 + \frac{D}{t(u_i+n_i)}\right)} \tag{2}$$

where $0 \le f_i \le 1$, n_i is the number of units produced in cycle i up to the point of interruption, and u_i denotes the cumulative units of experience remembered at beginning of cycle i during the $i-1$ previous cycles (note that in i production cycles there are $i-1$ production breaks), where $u_i \le \sum_{h=1}^{i} n_h = x$, with x being the cumulative production by the end of cycle i. Let B_i denote the break length that occurs following production of unit n_i in cycle i, where n_i is an integer valued. The parameter D is the time when total forgetting occurs. Denote $t(u_i + n_i)$ as the time to produce unit $u_i + n_i$, and b is the learning curve constant in (1). $t(u_i + n_i)$ is computed from (1) as

$$t(u_i + n_i) \cong \int_0^{u_i+n_i} T_1 x^{-b}\,dx = \frac{T_1}{1-b}(u_i + n_i)^{1-b}. \tag{3}$$

The number of units produced at beginning of cycle $i+1$ is given from Jaber and Bonney [6] as

$$u_{i+1} = (u_i + n_i)^{\left(1+\frac{f_i}{b}\right)} y_i^{-\frac{f_i}{b}} \tag{4}$$

where $u_1 = 0$, and y_i is the number of units that would have been accumulated, if production was not ceased for B_i units of time. y_i is computed from (3) as

$$y_i = \left\{\frac{1-b}{T_1}\left[t(u_i + n_i) + B_i\right]\right\}^{\frac{1}{(1-b)}}. \tag{5}$$

When total forgetting occurs, we have $u_{i+1} = 0$. However, from (4), $u_{i+1} \to 0$ as $y_i \to +\infty$; or alternatively, as $B_i \to +\infty$, where all the other parameters in (4) are nonzero positive values. The intercept of the forgetting curve is determined as

$$\hat{T}_{1i} = T_1(u_i + n_i)^{-(b+f_i)}. \tag{6}$$

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The time to produce the first unit in cycle i is predicted from (1) as

$$\widetilde{T}_{1i}^{\mathrm{LFCM}} = T_1(u_i + 1)^{-b} \tag{7}$$

B. Mathematics of the RC

Like the LFCM, the RC model has the capability of capturing multiple breaks. Nembhard and Uzumeri [10] modified the three hyperbolic learning functions of Mazur and Hastie [7] by introducing a measure they termed recency of experiential learning R. For each unit of cumulative production x, Nembhard and Uzumeri [10] determined the corresponding recency measure R_x by computing the ratio of the average elapsed time to the elapsed time of the most recent unit produced. Nembhard and Osothsilp [9] suggested that R_x could be computed as

$$R_x = 2\frac{\sum_{i=1}^{x}(t_i - t_0)}{x(t_x - t_0)} \tag{8}$$

where x is the accumulated number of produced units, t_x is the time when units x is produced, t_0 is the time when the first unit is produced, t_i is the time when unit i is produced, and $R_x \in (1, 2)$. Altering (1), the performance of the first unit after a break of length B_i is computed as

$$\widetilde{T}_{1i}^{\mathrm{RC}} = T_1\left(xR_x^{\alpha}\right)^{-b} \tag{9}$$

where α is a fitted parameter that represents the degree to which the task is forgotten. However, Nembhard and Uzumeri [10] and Nembhard and Osothsilp [9] did not provide evidence to how (8) was developed, or the factors affecting α.

III. Fitting to Nembhard and Osothsilp's Empirical Data

In this section, we fit the LFCM model to the empirical data provided by Nembhard and Osothsilp [9, Fig. 1, p. 285]. Nembhard and Osothsilp [9] states that "the relationship between the production time per unit versus cumulative production, as illustrated in Fig. 1, reveals the effects of forgetting by the increase in production times following breaks." They further define a production unit to consist of 20 consecutive radios [9, p. 285], where [9, Fig. 1] shows 5500 units of cumulative production, meaning 110 000 radios. This implies that the data in Fig. 1 hold information of all subjects. These data were collected from a six-month period of final inspections of car radios. For a more precise description of the data collection, see Nembhard and Osothsilp [9].

The LFCM was fitted to the same empirical data. The fit was made by minimizing the value of the MAD by adjusting $T_1 = 0.102, b = 0.130$, and $D = 500\,069\,082$. The seven break times B_i $(i = 1, 2, \ldots, 7)$ with lengths of 3.7 days, 6.5 days, 11.6 days, 3.8 days, 4.7 days, 3.7 days, and 11.2 days, occurred after producing 540, 2640, 2680, 2940, 3860, 4260, and 4440 units, respectively [9, Fig. 1, p. 285]. The LFCM registered a MAD of 0.102, which is slightly smaller (i.e., 7%) than the RC $(\mathrm{MAD} = 0.110)$ and considerably smaller (i.e., 93%) than LFCM $(\mathrm{MAD} = 1.62)$ values reported by Nembhard and Osothsilp [9, Table 3, p. 288]. The fit for the LFCM in Nembhard and Osothsilp [9] is clearly unreasonably large for any dataset with similar amplitude, because it deviates approximately fifteen times more compared to our results. To test the robustness of the prediction of the LFCM, we conducted a simulation experiment where the performance data and the length of production breaks corresponding to the data in [9, Fig. 1] are randomized to produce additional sample data. These sample data were fitted to both the LFCM and the RC models. Results indicate the MAD values of the LFCM were found to be consistent with those of the RC model. It follows that the LFCM performs better than, or at least as good as, any other forgetting models investigated by Nembhard and Osothsilp [9].

The corresponding STDAD, and the ratio of number of under predictions to over predictions were, respectively, 0.075, and 43% to 57% (43:57). These values were smaller than those reported by Nembhard and Osothsilp [9, Table 3, p. 288] for both the RC (0.098; 42:58) and the LFCM (1.60; 13:87) models.

Both the LFCM and the RC models use constant learning slopes for all episodes when fitting these models to the data [9, Fig. 1, p. 285]. Nembhard and Osothsilp [9] used the fitted learning slope of each episode in the calculation rather than assuming a constant slope [9, p. 287]. Carrying on such an assumption would require a fundamental modification of the LFCM, which Nembhard and Osothsilp [9] apparently did not do. Thus, this may leave us to conclude that using the LFCM with the assumptions carried by Nembhard and Osothsilp [9] would have most probably contributed to error in fitting of the LFCM.

IV. Conclusion

In this note, we empirically compared the LFCM with available empirical data from Nembhard and Osothsilp. Contrary to the Nembhard and Osothsilp [9] finding, the LFCM performed better than the recency model. The result of Nembhard and Osothsilp [9] might be attributed to an error on their part when fitting the LFCM to their empirical data. This error is most probably the result of their assumption of varying learning slopes rather than a constant learning slope as advocated by the LFCM.

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