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Some Comments on Output Feedback Stabilization of the Moore-Greitzer Compressor Model

Anton Shiriaev Rolf Johansson Anders Robertsson

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Abstract. This paper suggests a number of facts related to the modeling and output feedback stabilization of the Moore-Greitzer compressor model. It shows how to integrate the stall dynamics, and how to use this for further output stabilization of the model.

I. Introduction

This note is devoted to revealing some properties of a nonlinear control system – the so-called the Moore-Greitzer model [3], [2]– which has been extensively used as an approximation for describing compressor dynamics

$$\frac{d}{dt}\phi = -\psi + \frac{3}{2}\phi + \frac{1-(1+\phi)^3}{2} - 3R(1+\phi) \quad (1)$$

$$\frac{d}{dt}\psi = \frac{1}{\beta^2}(\phi - u) \quad (2)$$

$$\frac{d}{dt}R = -\sigma R^2 - \sigma R\left(2\phi + \phi^2\right), \quad R(0) \geq 0 \quad (3)$$

$$y = \psi \quad (4)$$

Here u is the control variable to be defined, y is available measurement, $\sigma > 0$. The main contribution of the note comes from the observation that the dynamics of R-variable (the stall variable) in (1)–(3) could be eliminated, and the value of $R(t)$ could explicitly be written as a function of $\phi(t)$ and $R(0)$. As a direct consequence of this fact we can suggest new sufficient conditions for output feedback stabilization of (1)–(3).

II. Main Results

Theorem 1: Given a constant $R(0) \geq 0$ and a scalar function $\phi(t)$, the corresponding solution of the differential equation (3), if exists, looks as follows

$$R(t) = \frac{R(0)\exp\left(-\sigma\int_0^t\left\{\phi^2(\tau)+2\phi(\tau)\right\}d\tau\right)}{1+\sigma R(0)\int_0^t \exp\left(-\sigma\int_0^s\left\{\phi^2(\tau)+2\phi(\tau)\right\}d\tau\right)ds} \quad (5)$$

Proof — Direct verification in (3).

The possibility to integrate the stall equation (3) can be used in a number of ways. The next statement suggests an idea of output feedback stabilization for (1)–(3) based on this integrability

Theorem 2: Assume that:

1) (Case $R(0) = 0$) The output feedback controller
$$\dot{z} = \mathcal{F}(z, y, t), \quad u(t) = \mathcal{U}(z, y, t) \quad (6)$$
makes the surge subsystem, that is (1)–(2), globally exponentially stable;
2) (Case $R(0) > 0$) The controller (6) ensures the boundedness of solutions of the closed-loop system (1)–(3), (6) and the validity of the following constraint
$$\sup_{0\leq s\leq t}\left[-\int_0^s\left\{\phi^2(\tau)+2\phi(\tau)\right\}d\tau\right] < +\infty \quad (7)$$
along a solution of (1)–(3), (6).

Then all solutions of the closed-loop system (1)–(3), (6) tend to the origin of (1)–(3) as $t \to \infty$. ∎

The condition (7) could be weakened so that the function

$$-\int_0^t\left\{\phi^2(\tau)+2\phi(\tau)\right\}d\tau \quad (8)$$

is allowed to grow to $+\infty$ but in a way that the integral of its exponential is growing faster than the exponential of this function itself. For example, if

$$-\int_0^t\left\{\phi^2(\tau)+2\phi(\tau)\right\}d\tau = k\log(1+t) \quad (9)$$

with some constant $k > 0$, then the function (5) is

$$R(t) = \frac{R(0)\exp(\sigma k)(1+t)}{1+\sigma R(0)\int_0^t \exp(\sigma k)(1+s)ds} = \frac{R(0)\exp(\sigma k)(1+t)}{1+\sigma R(0)\exp(\sigma k)\frac{(1+t)^2}{2}}$$

with $\lim_{t\to+\infty} R(t) = 0$, while the conclusions of Theorem 2 remain valid. In turn, it precludes the cases with linear growth[1]. Indeed, if (8) is growing linearly, that is

$$-\int_0^t\left\{\phi^2(\tau)+2\phi(\tau)\right\}d\tau = kt \quad (10)$$

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[1]The function in (8) cannot grow faster than linearly because the controller (6) ensures the boundedness of closed-loop system solutions.

with some constant $k > 0$, then the function (5) is

$$R(t) = \frac{R(0)\exp(\sigma k t)}{1 + \sigma R(0)\int_0^t \exp(\sigma k s)\,ds} = \frac{R(0)\exp(\sigma k t)}{1 + \sigma R(0)\frac{\exp(\sigma k t) - 1}{\sigma k}}$$

Hence, $\lim_{t\to+\infty} R(t) = k$ that corresponds to an additional equilibrium of the system (1)–(3) different from the origin.

As suggested in Theorem 2, output feedback controller design for (1)–(3) could be done firstly for the surge subsystem (1)–(2). The next statement suggests a range of dynamical output controllers for this purpose.

Theorem 3: Consider any constants γ_1–γ_4 such that the inequalities

$$\begin{array}{c} \gamma_2 > \frac{3}{2}\beta^2, \quad 1 > \gamma_1 + \frac{3}{2}\gamma_2, \quad -\frac{3}{4}\beta^2 > \gamma_3 \\ \gamma_2 > -2\gamma_3, \quad \gamma_4 > \frac{3}{2}\beta^2 \end{array} \tag{11}$$

are valid. The set of such parameters γ_1–γ_4 in (11) is not empty. Take a dynamical output controller (6) of the form

$$u = \lambda_1\psi + \lambda_2 z + \alpha_u\left(1 - (1 + c_\psi\psi + c_z z)^3\right) \tag{12}$$

$$\frac{d}{dt}z = \lambda_3\psi + \lambda_4 z + \alpha_z\left(1 - (1 + c_\psi\psi + c_z z)^3\right) \tag{13}$$

with the parameters $\lambda_2 = \gamma_1$, $\alpha_u = \gamma_3$, $c_\psi = \gamma_4$, $c_z = 1$

$$\begin{aligned} \lambda_1 &= \gamma_2 + \gamma_1\gamma_4 \\ \lambda_3 &= \frac{\gamma_4^2}{\beta^2}(\gamma_1 - 1) + \gamma_4\left(\frac{3}{2} + \frac{\gamma_2}{\beta^2}\right) - 1 \\ \lambda_4 &= \frac{3}{2} + \frac{\gamma_4}{\beta^2}(\gamma_1 - 1) \\ \alpha_z &= \frac{1}{2} + \frac{\gamma_3\gamma_4}{\beta^2} \end{aligned} \tag{14}$$

Then the closed-loop system (1), (2), (12), (13) with $R(t) \equiv 0$ is robustly globally exponentially stable. ■

Proof of Theorem 3 follows from the Circle criterion applied to the closed-loop system (1), (2), (12), (13) with two infinite sector quadratic constraints. As shown, it is valid for any parameters λ's, α's and c's of dynamical controller (12), (13) mentioned in (11), (14). More details could be found in [1]. Below we have put some additional details on properties of the dynamical feedback controllers (12)–(13) applied for the Moore-Greitzer model (1)–(3).

Fact 1: If a chosen output controller (12)–(13) ensures quadratic stability of the surge (ϕ-ψ) subsystem (1), (2), (4), then any solution of the closed-loop system (1)–(4) does not expire in finite time. ■

Fact 2: If a chosen output controller (12)–(13) ensures quadratic stability of the surge (ϕ-ψ) subsystem (1), (2), (4), then the variable R is always bounded along any solution of the closed-loop system. Furthermore, for any solution of the closed-loop system there exists a time T such that $R(t) \in [0, 1]$ for all $t \geq T$. ■

Fact 3: If a chosen output controller (12)–(13) ensures quadratic stability of the surge (ϕ-ψ) subsystem (1), (2), (4), then for any solution $\phi(t) = \phi(t, t_0, \phi_0)$, $\psi(t) = \psi(t, t_0, \psi_0)$, $R(t) = R(t, t_0, R_0)$, $z(t) = z(t, t_0, z_0)$ of the closed-loop system (1)–(4), (12)–(13) there exists a time $T_0 \geq t_0$ such that for any $T_2 > T_1 \geq T_0$ the following inequality holds

$$\int_{T_1}^{T_2} R^2(t)\left(1 + \phi(t)\right)^2 dt \leq \frac{4}{27}\left(T_2 - T_1\right) + \frac{1}{2\sigma} \tag{15}$$

Fact 4: If a chosen output controller (12)–(13) ensures quadratic stability of the surge (ϕ-ψ) subsystem (1), (2), (4), then any solution of the closed-loop system (1)–(4), (12)–(13) is bounded. ■

Fact 5: Suppose that a chosen output controller (12)–(13) ensures quadratic stability of the surge (ϕ-ψ) subsystem (1), (2), (4). Consider a solution $[\phi(t), \psi(t), R(t), z(t)]$ of the closed-loop system (1)–(4), (12)–(13) with $\lim_{t\to+\infty} R(t) = 0$, then the limit relations

$$\lim_{t\to+\infty}\phi(t) = 0, \quad \lim_{t\to+\infty}\psi(t) = 0, \quad \lim_{t\to+\infty} z(t) = 0, \tag{16}$$

hold. ■

III. Conclusions

This note has two contributions. Firstly, it is shown how to integrate the stall variable in the Moore-Greitzer compressor model, and how this could be helpful for the controller design and system stability analysis. Secondly, it suggests a new class of dynamical output controllers for the surge subsystem, and reveals their properties.

References

[1] Shiriaev A., R. Johansson and A. Robertsson. "Sufficient Conditions for Output Feedback Stabilization via the Circle Criterion," submitted to *IEEE Transactions on Automatic Control*, October, 2003.

[2] E. M. Greitzer and F. K. Moore, "A Theory of Post-stall Transients in Axial Compressor Systems: Part II-Application.", *J. Eng. Gas Turbines and Power*, 108, pp 231-239, 1986.

[3] F. K. Moore, "A Theory of rotating stall of Multistage Axial Compressors: Parts I-III", *J. Eng. Gas Turbines and Power*, 106, pp 313-336, 1984

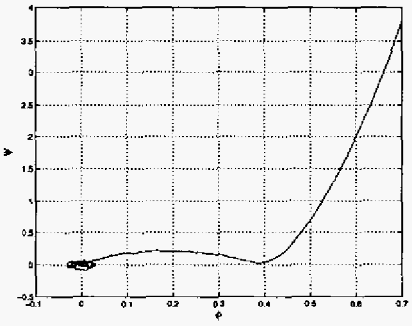

Fig. 1. Simulation of the closed-loop system [(1), (2), (12), (13)] for $R_0 = 0$ with measurement noise added. (The solution converges exponentially to the origin in the noise free case).

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A New LMI-Based Procedure for the Design of Robust Damping Controllers for Power Systems

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***Abstract*— This paper presents a new procedure for the design of controllers to damp electromechanical oscillations in power systems. The new procedure is derived from a previously developed methodology, which can simultaneously satisfy a number of practical requirements for this type of controller, such as robustness, decentralization, zero steady-state gain and output feedback structure. However, the original methodology eventually yielded controllers with high gain in the electromechanical frequency range (due to the formulation of the control design as a feasibility problem). Controllers with lower gains (desired for practical reasons) are provided by this new procedure, which formulates the first stage of the design algorithm as an LQR problem. The tests show that the designed controllers are able to provide adequate damping for both local and inter-area modes.**

I. Introduction

SINCE the early 60's, the presence of sustained, low frequency electromechanical oscillations in power systems has been a major problem for engineers and researchers in this field. These oscillations usually place restrictions in the amount of power that can be transferred across strategic transmission lines. For this reason, the use of controllers to adequately damp these oscillations is often required, as a cost effective alternative to building new lines.

The most used procedure for designing such controllers was developed in the late 60's [1], and has been improved since then [2], [3]. This procedure is based on the classical control technique of phase compensation, applied over a simplified, linearized power system model, know as the Heffron-Phillips (HP) model. Controllers designed by this methodology are called Power System Stabilizers (PSSs), and this technique is referred to as the classical PSS design.

There are some major disadvantages associated to this classical procedure. First of all, as the operating point of the power system drifts away from the nominal one, the performance of the classical PSS degrades. The highly nonlinear behavior of the system contributes to worsen this problem. Besides, the oversimplification of the system dynamics involved in the HP model discards some important modes of oscillation, such as the inter-area modes. To deal with this problem, a procedure called tuning is employed *a posteriori* in the classical PSS design. After the controller is designed, its parameters are empirically adjusted, aiming to maximize the controller phase margin in the frequency range of interest. Due to its empirical nature, the success of this procedure strongly depends on the designer's experience and knowledge of the system.

To avoid these drawbacks of the classical PSS design, several alternative methodologies have been proposed in the past few years, most of them employing recently developed robust control techniques [4], [5]. However, such methodologies rarely come to the field, with a few exceptions (such as [6] or [7], for example), because they cannot simultaneously satisfy all the practical requirements of the oscillation damping problem.

The methodology presented in [8] (which will be referred to as the original methodology, in this paper) was developed having this consideration in mind. The simultaneous fulfilling of the most important practical requirements of the problem was the motivation for the development of the design procedure. However, due to its formulation of the controller design as a feasibility problem, the original methodology eventually provided controllers with high gain in the electromechanical frequency range.

This paper presents a new procedure, based on the original methodology, where the controller gain issue was treated as an LQR problem in the first stage of the design. With this new feature, controllers with smaller gains (comparing to the ones provided by the original methodology) are obtained. Such smaller gains are necessary to avoid controller interactions with non-modeled dynamics, such as torsional or interplant modes.

The presentation of the new procedure is structured as follows: section II brings a summary of the main features of the original methodology; section III explains in detail how the controller gain problem was approached in the new procedure; the complete design algorithm is given in section IV; finally, section V presents some results obtained with the controller tests, and section VI brings a few concluding remarks.

II. Summary of the Original Methodology

The main features of the original methodology are summarized in this section. This presentation is intended as a succinct explanation of the main points of the methodology. A more rigorous and detailed treatment of each of these points, as well as the original algorithm, can be found in [8] and [9].

Since the fulfilling of the practical requirements was the driving idea behind the development of the original procedure, this explanation will follow a comprehensive sequence oriented by the requirements, instead of explaining the algorithm flowchart itself.

A. Output Feedback Structure and Zero Steady-State Gain

The implementation of controllers with state feedback structure in power systems faces some difficulties, most of them related to the need of a common angular reference for the rotor angles. For this reason, the first requirement to be considered is a dynamic output feedback structure for the controllers, which avoids the cited drawback.

After a linearization around an operating point, the power system can be modeled by:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{1}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{2}$$

In (1)-(2), $\mathbf{x} \in R^n$ is the state vector (in this work, composed by the rotor angles, the rotor speeds, the quadrature axis voltages and the field voltages), $\mathbf{u} \in R^p$ is the input vector (containing the stabilizing signals) and $\mathbf{y} \in R^q$ is the output vector (composed by the derivatives of the rotor speeds of the selected machines, in this particular work). All variables here denote deviations with respect to an equilibrium point.

An output feedback controller for (1)-(2) can be described by

$$\dot{\mathbf{x}}_c = \mathbf{A}_c\mathbf{x}_c + \mathbf{B}_c\mathbf{y} \tag{3}$$

$$\mathbf{u} = \mathbf{C}_c\mathbf{x}_c \tag{4}$$

where $\mathbf{x}_c \in R^n$ is the state vector of the controller. The closed loop connection of (1)-(2) and (3)-(4) is given by

$$\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{A}}\tilde{\mathbf{x}} \tag{5}$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{B}\mathbf{C}_c \\ \mathbf{B}_c\mathbf{C} & \mathbf{A}_c \end{bmatrix} \tag{6}$$

and $\tilde{\mathbf{x}} \in R^{2n}$ is a state vector containing both the states of the plant (1)-(2) and the controller (3)-(4), respectively. With these definitions, the problem of stabilizing (1)-(2) with a controller of type (3)-(4) can be solved by finding matrices $\mathbf{A}_c$, $\mathbf{B}_c, \mathbf{C}_c$ and $\tilde{\mathbf{P}} > 0$ such that

$$\tilde{\mathbf{A}}^T\tilde{\mathbf{P}} + \tilde{\mathbf{P}}\tilde{\mathbf{A}} < 0 \tag{7}$$

As pointed out in [9], the definition of the rotor speed derivatives as the output variables is sufficient to guarantee zero gain in steady-state conditions. Moreover, these derivative terms can be included in the controller transfer function, so the actual measured variables will be the rotor speeds, rather than accelerations.

One major disadvantage of this approach is the fact that equation (7) is bilinear in the matrix variables $\mathbf{A}_c$, $\mathbf{B}_c$ and $\mathbf{C}_c$. In subsection II-C, a different formulation is presented, allowing the solution of this problem with a set of Linear Matrix Inequalities (LMIs).

B. Robustness with Respect to Variations in the Operating Conditions

A power system is subjected to daily, unpredictable variations in its operating point. In this sense, it is important that the damping controllers behave with similar effectiveness, irrespective of the operating conditions. However, classical controllers are designed based on a linearized model, and therefore their performances are guaranteed only in a small neighborhood of their nominal equilibrium points. To extend this performance guarantee, this methodology employs a technique called polytopic modeling.

The polytopic model uses a set of L models (1)-(2), which can be obtained, for example, from the load variation curves of the respective power system. The models in this set constitute the vertices of a convex set, called polytope. Expressing the closed loop connections (as shown in item a)) by $\tilde{\mathbf{A}}_i, i = 1,\ldots,L$, the methodology will then search for matrices $\mathbf{A}_c$, $\mathbf{B}_c, \mathbf{C}_c$ and $\tilde{\mathbf{P}} > 0$ such that

$$\tilde{\mathbf{A}}_i^T\tilde{\mathbf{P}} + \tilde{\mathbf{P}}\tilde{\mathbf{A}}_i < 0 \tag{9}$$

As a benefit, given by the convex structure of the polytopic set, the controller described by matrices $\mathbf{A}_c, \mathbf{B}_c$ and $\mathbf{C}_c$ will stabilize all the linear models contained in this set [10] (which might correspond to intermediate operating points of the power system, not considered in the design). This characteristic of the polytopic model provides the desired robustness to the controllers given by the original methodology.

C. Decentralized Structure

In large power systems, it is common to find electrically coupled plants separated by large geographical distances. This fact turns unfeasible most of the proposals for centralized controllers with fast dynamics, due to the difficulties for implementing reliable remote feedback links. For this reason, decentralization becomes another practical requirement, which was treated by the original methodology.

Decentralization constraints can be easily handled in formulation (1)-(4) with the imposition of block diagonal structures of appropriate dimensions for matrices $\mathbf{A}_c, \mathbf{B}_c$ and $\mathbf{C}_c$. These block diagonal structures will ensure that the controller of a particular generator will be based only on its own input and output.

As mentioned in subsection II-A, the presented formulation involves bilinear matrix inequalities. This fact has discouraged many research proposals based on the cited formulation for several years. Recently, however, a new parameterization, introduced in [11] and extended to the power system damping problem in [8], allowed the solution of this problem with a set of LMIs.

To obtain this new formulation from equation (7), it is necessary to define the following partitions:

$$\widetilde{\mathbf{P}} = \begin{bmatrix} \mathbf{X} & \mathbf{U} \\ \mathbf{U}^T & \mathbf{X}_c \end{bmatrix}, \widetilde{\mathbf{P}}^{-1} = \begin{bmatrix} \mathbf{Y} & \mathbf{V} \\ \mathbf{V}^T & \mathbf{Y}_c \end{bmatrix} \qquad (10)$$

New variables are also defined:

$$\mathbf{L} = \mathbf{C}_c\mathbf{V}^T; \mathbf{F} = \mathbf{U}\mathbf{B}_c; \mathbf{M} = \mathbf{V}\mathbf{A}_c^T\mathbf{U}^T \qquad (11)$$

$$\mathbf{P} = \mathbf{Y}^{-1}; \mathbf{S} = \mathbf{Y}^{-1}\mathbf{M} \qquad (12)$$

It is possible to show [11] that, with these new variables, equation (7) can be rewritten in the equivalent form

$$\begin{bmatrix} \overline{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\overline{\mathbf{A}} & \mathbf{P}\mathbf{A} + \overline{\mathbf{A}}^T\mathbf{P} + \mathbf{C}^T\mathbf{F}^T + \mathbf{S} \\ \mathbf{A}\mathbf{P} + \mathbf{P}\overline{\mathbf{A}} + \mathbf{F}\mathbf{C} + \mathbf{S}^T & \mathbf{A}^T\mathbf{X} + \mathbf{X}\mathbf{A} + \mathbf{F}\mathbf{C} + \mathbf{C}^T\mathbf{F}^T \end{bmatrix} < \mathbf{0} \qquad (13)$$

where $\overline{\mathbf{A}} = \mathbf{A} + \mathbf{B}\mathbf{C}_c$, with $\mathbf{C}_c = \mathbf{L}\mathbf{Y}^{-1}$ given *a priori* (see the design algorithm in section IV) by the solution of

$$\mathbf{Y}\mathbf{A}^T + \mathbf{A}\mathbf{Y} + \mathbf{B}\mathbf{L} + \mathbf{L}^T\mathbf{B}^T < 0 \qquad (14)$$

One can see that the new equations (13) and (14) are now LMIs in the matrix variables $\mathbf{Y}, \mathbf{L}, \mathbf{X}, \mathbf{F}, \mathbf{S}$ and $\mathbf{P}$. This fact allows the application of LMI solvers to seek for solutions of the problem, from which the controller matrices are obtained. These solvers have already given good results for other similar control problems.

Instead of setting up such equations for a single system model, it is possible to write them for all the vertices of a polytopic power system model. The transformation of each vertex equation (9) to its equivalent LMIs (13) and (14) allows the development of a robust design methodology.

D. Minimum Damping Guarantee

The most used index to evaluate the small-signal stability of a power system is the minimum damping ratio among all modes of oscillation. The final performance evaluation of the controllers in such systems is carried out by inspection of their effects over this minimum damping, calculated for a number of operating conditions. However, in the classical PSS design, there is no guarantee that a certain overall damping will be achieved. If this criterion is not met, the controllers must be redesigned and rechecked, in a trial-and-error process that incorporates some heuristics from the designer's experience (the tuning process).

One of the great advantages of the original methodology over the classical PSS design is the possibility to include an overall minimum damping ratio as a design objective, so the designed controllers can automatically guarantee a satisfactory small-signal stability index. This feature of the original methodology eliminates the need for a trial-and-error process.

In this work, the performance criteria are defined based on the concept of *D-stability* [12]. According to this concept, a matrix $\widetilde{\mathbf{A}}_i$ is *D-stable* if all of its eigenvalues are contained in the region shown in figure 1, called *region D* (which contains all modes with $\zeta \le \zeta_0$, where ζ stands for the damping ratio and ζ_0 is a predefined minimum value). In this case, *D-stability* of the matrix $\widetilde{\mathbf{A}}_i$ can be assured by the existence of a matrix $\widetilde{\mathbf{P}} > 0$ satisfying

$$\begin{bmatrix} \operatorname{sen}\theta\left(\widetilde{\mathbf{A}}_i^T\widetilde{\mathbf{P}} + \widetilde{\mathbf{P}}\widetilde{\mathbf{A}}_i\right) & \cos\theta\left(\widetilde{\mathbf{A}}_i^T\widetilde{\mathbf{P}} - \widetilde{\mathbf{P}}\widetilde{\mathbf{A}}_i\right) \\ \cos\theta\left(\widetilde{\mathbf{P}}\widetilde{\mathbf{A}}_i - \widetilde{\mathbf{A}}_i^T\widetilde{\mathbf{P}}\right) & \operatorname{sen}\theta\left(\widetilde{\mathbf{A}}_i^T\widetilde{\mathbf{P}} + \widetilde{\mathbf{P}}\widetilde{\mathbf{A}}_i\right) \end{bmatrix} < \mathbf{0} \qquad (15)$$

where θ is the angle defined in figure 1.

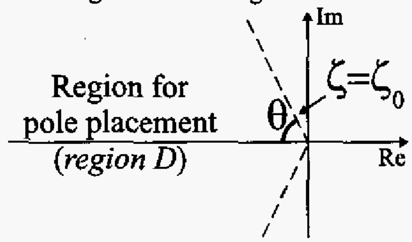

Figure 1. Minimum damping region for pole placement.

Recalling the notion of a polytopic model, if we set up one equation in the form (15) for each vertex system in a polytope, we can ensure a minimum damping ratio for all modes of all system models contained in this polytope. So, using this formulation, it is possible to guarantee that, once the controllers are found, the desired small-signal stability criterion is automatically met.

III. Minimization of the Feedback Gain

As mentioned earlier, the original methodology formulates the controller design as a feasibility problem, and LMI solver is applied to find a feasible solution to this problem. Since any controller satisfying all requirements provides a feasible solution, there is no way to predict or control which solution will be provided by the solver. Eventually, controllers with high gains in the low frequency range are provided. However, high gain controllers have a bigger probability of interacting with non-modeled dynamics, such as torsional modes (torsional interactions along the generator shaft) or interplant modes (different units of the same facility oscillating against each other).

Modeling these dynamics would significantly increase the computational burden of the design, since it would involve the addition of many new state variables to the system model. The alternative, then, is to modify the original methodology aiming to obtain the controllers with smaller gains that still satisfy all practical requirements.

A careful look at sections II-A and II-C reveals some insights of the original methodology. In equations (1)-(2) and (3)-(4), it is possible to see that matrix $\mathbf{C}_c$ acts as a state feedback matrix, generating the input signal for the plant from the controller states. This matrix is obtained as a solution of equation (14), which comes from a particular formulation of the state feedback problem. In other words, the first stage of the design procedure can be considered as a conventional state feedback design, while the second stage can be viewed as the design of the controller dynamics (defined by matrices $\mathbf{A}_c$ and $\mathbf{B}_c$), to generate estimates of the plant states.

Control effort and controller gain are closely related. When solving the well-known Linear Quadratic Regulator (LQR) problem, the designer seeks for a state feedback matrix that minimizes the control effort, given an arbitrary initial state $\mathbf{x}_0$ [10]. So, this idea can be adapted to the original methodology, generating the new procedure, in which the first stage now seeks for a matrix $\mathbf{C}_c$ that would minimize the control effort in a state feedback framework.

An additional difficulty is the fact that the initial state (consisting of deviations from an equilibrium point) is not usually known in advance. Nevertheless, the LQR problem can be extended to the case where $\mathbf{x}_0$ is known to belong to an ellipsoid $\Phi = \{\phi^T \mathbf{W} \phi \leq 1\}$, with a proper definition of matrix $\mathbf{W}$. This extended LQR problem can then be formulated as [10]:

minimize λ
subject to

$$\begin{bmatrix} \lambda \mathbf{I} & \mathbf{W}^{-1/2} \\ \mathbf{W}^{-1/2} & \mathbf{Y} \end{bmatrix} > 0 \tag{16}$$

$$\begin{bmatrix} \mathbf{YA}^T + \mathbf{AY} + \mathbf{BL} + \mathbf{L}^T\mathbf{B}^T & \mathbf{L}^T \\ \mathbf{L} & -\mathbf{I} \end{bmatrix} < 0 \tag{17}$$

So, adding a couple of restrictions in the form (16)-(17) for each vertex system $\widetilde{\mathbf{A}}_i, i = 1,\ldots,L$, and replacing the feasibility solver by a linear objective minimization solver, it is possible to add a control effort minimization feature to the first stage of the procedure. Our results indicate that this approach is effective in reducing the controller gain in the desired frequency range.

The next section rearranges all the previously explained concepts in an algorithm, constituting the proposed new design procedure.

IV. Algorithm of the New Design Procedure

The following general stages and steps constitute the algorithm of the new design procedure:

STAGE I

Step I-1: Build equations of type (16) and (17) for each vertex system $\widetilde{\mathbf{A}}_i, i = 1,\ldots,L$;

Step I-2: Choose a value for the desired minimum damping ratio ζ_0 and calculate $\theta = \arccos\zeta_0$;

Step I-3: With the calculated value of θ, build one equation of type (15) for each vertex system $\widetilde{\mathbf{A}}_i, i = 1,\ldots,L$, and transform them to their equivalent LMIs in the form (14);

Step I-4: By a minimization of λ, find $\mathbf{Y}$ and $\mathbf{L}$ that satisfy all equations in the form (14), (16) and (17);

Step I-5: Calculate the controller matrix $\mathbf{C}_c = \mathbf{LY}^{-1}$.

STAGE II

Step II-1: Calculate $\overline{\mathbf{A}}_i = \mathbf{A}_i + \mathbf{B}_i\mathbf{C}_c$ for $i = 1,\ldots,L$;

Step II-2: With the same θ calculated in step I-2, build one equation of type (15) for each vertex system $\widetilde{\mathbf{A}}_i, i = 1,\ldots,L$ and transform them to their equivalent LMIs in the form (13) (using the respective $\overline{\mathbf{A}}_i$);

Step II-3: Find $\mathbf{X}$, $\mathbf{F}$, $\mathbf{S}$ and $\mathbf{P}$ satisfying all equations in the form (13);

Step II-4: Calculate the controller matrices $\mathbf{B}_c = (\mathbf{P} - \mathbf{X})^{-1}\mathbf{F}$ and $\mathbf{A}_c = (\mathbf{P} - \mathbf{X})^{-1}\mathbf{P}^{-1}\mathbf{SP}$.

The calculations in steps I-5 and II-4 follow from the definitions given in equations (10)-(12). Matrices $\mathbf{A}_c$, $\mathbf{B}_c$ and $\mathbf{C}_c$ define all the damping controllers for the system. Each local controller can be then converted to the form of a transfer function.

V. Tests and Results

To test the ability of the controllers to provide damping for both local and inter-area modes, a benchmark system (where both types of modes are clearly distinguishable) was used. This system can be viewed in figure 2, and complete data for its model can be obtained from [13].

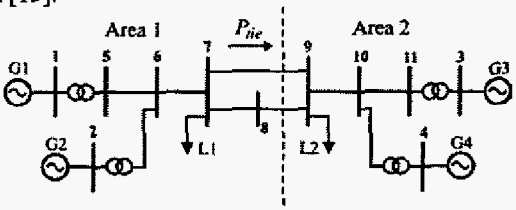

Figure 2. Diagram of the test system.

Changes of ±10% in the load levels L1 and L2 were used to vary the operating point of the system, generating the vertices of the polytopic model for this system. In all the operating points, generator 3 was considered as an infinite bus, to provide an angular reference for the system. This procedure generated the vertex systems shown in table I, where the base case was also taken as a vertex. In this table, P_{tie} represents the real power flowing across the double-circuit line 7-9, in MW. Bus 8 is fictitious, and was included to facilitate the simulation of a short circuit in the middle point of the lower circuit of line 7-9.

Table I. Characteristics of the vertex systems

Load Level	P_{tie}	Inter-area Mode	ζ [%]
Base Case	391	-0,0068±j1,9310	0,0035
L1+10%, L2 -10%	262	-0,0144±j2,2050	0,0065
L1-10%, L2 -10%	360	-0,0126±j2,1349	0,0059
L1-10%, L2+10%	515	0,0008±j1,4456	-0,0006
L1+10%, L2+10%	420	-0,0012±j1,6765	0,0007

The algorithm given in section 4 was applied to the polytopic model described in Table I, with $\zeta_0 = 5\%$ chosen as the required minimum damping ratio. To define the ellipsoids of initial conditions, the following limits were specified for the deviations with respect to the equilibrium values, expressed in the state variables:

$$\Delta\delta_i \le \mathbf{0.1745} \text{ rad} \quad ; \quad \Delta\omega_i \le \mathbf{0.01} \text{ rad/s} \qquad (18)$$

$$\Delta E'_{qi} \le \mathbf{0.1} \text{ p.u.} \quad ; \quad \Delta E_{FDi} \le \mathbf{2.5} \text{ p.u.} \qquad (19)$$

The whole design process took approximately 4 minutes and 17 seconds (in a computer equipped with a Pentium III 750 MHz processor and 64 MB of RAM) and yelded the following transfer functions for the robust damping controllers (called RDCs from now on):

$$\mathrm{RDC}_7(s) = 224.14 \frac{s}{(s+0.12)} \frac{(s+155.30)(s^2+38.03s+398.50)}{(s+157.40)(s+91.95)(s+14.95)}$$

$$\mathrm{RDC}_2(s) = -733.26 \frac{s}{(s+0.11)} \frac{(s+133.60)}{(s+88.71)} \frac{(s-25.15)(s+20.83)}{(s^2+170.90s+11400)}$$

$$\mathrm{RDC}_4(s) = -2292.14 \frac{s}{(s+0.19)} \frac{(s+176.60)}{(s+48.42)} \frac{(s-29.58)(s+21.17)}{(s^2+702.10s+198200)}$$

The evaluation of the design results started with a linear analysis of the closed loop system. The eigenvalues of the closed loop vertex systems, plus other 12 intermediate operating conditions (combing variations of ±2.5%, ±5% and ±7.5% in both loads), were calculated and plotted in figure 3. Vertex system poles are highlighted in red. The transversal lines represent the loci of the modes with $\zeta_0 = 5\%$. It can be seen that the required minimum damping was achieved in all operating conditions considered in this test.

To evaluate the performance improvement given by the designed controllers, classical PSSs were tuned (according to the guidelines given in [2]) for the test system. Equal PSSs were used for all generators, due to the symmetric structure of this system. The transfer function of the tuned PSSs is given in equation (20), and figure 4 presents a comparison between the Bode diagrams of the designed RDC (in blue) and the tuned PSS (in green) for generator 4.

$$\mathrm{PSS}(s) = 40.00 \frac{s}{(s+0.10)} \frac{(s+2.86)}{(s+3.45)} \frac{(s+2.86)}{(s+3.45)} \qquad (20)$$

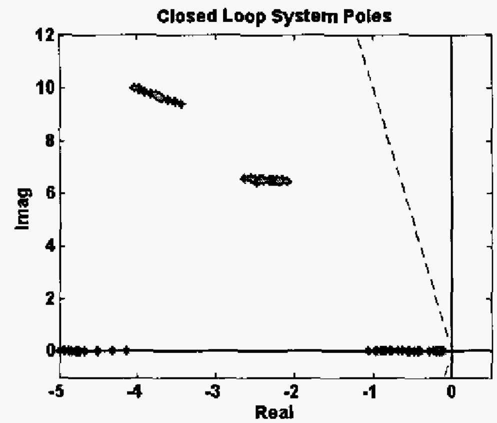

Figure 3. Poles of the test system (closed loop).

The Bode diagram of an RDC designed by the original methodology (identified as ORDC, in red) is also shown in figure 4. It can be seen that the gain of the RDC designed by the new procedure is smaller. It must be remembered, however, that the RDCs have to fulfill some robustness requirements, so their gains cannot be arbitrarily reduced in the minimization process.

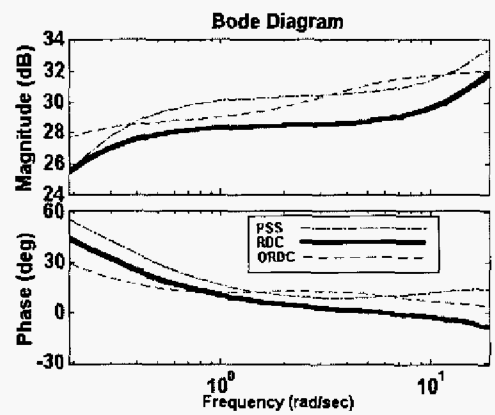

Figure 4. Comparison of controller Bode diagrams.

Nonlinear simulations were then carried out to validate the results of the linear analyses. Figure 5 shows a comparison between the responses of the system, when controlled by classical PSSs or by the proposed RDCs. A three-phase short circuit was applied to bus 8, in t=2s, and isolated from the system by the protection relays in t=2.032s. After 200ms, the short circuit is removed and the lines are reconnected to the system in t=2.232s. The small duration of this perturbation ensures that the operating point of the system does not drift too far away from its original equilibrium. In this simulation, the system was operating with +7.5% of power in both loads, with respect to the base case levels (which defined the nominal operating point for the PSS design). Although the performances of the system with PSSs and RDCs are quite similar, it can be seen that the RDCs provide faster stabilization of the rotor speeds. It must also be remarked that the RDCs were designed at once by the new procedure, while the design of the PSSs required the trial-and-error process involved in the tuning procedure.

Other tests were carried out to evaluate the performance of the RDCs in different operating points, giving similar satisfactory results. All these tests were not shown here due to the limitation of space.

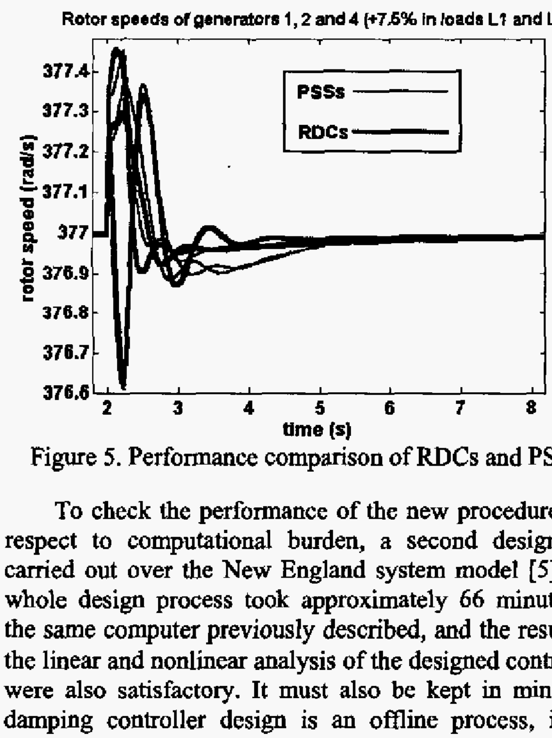

Figure 5. Performance comparison of RDCs and PSSs.

To check the performance of the new procedure with respect to computational burden, a second design was carried out over the New England system model [5]. The whole design process took approximately 66 minutes, in the same computer previously described, and the results of the linear and nonlinear analysis of the designed controllers were also satisfactory. It must also be kept in mind that damping controller design is an offline process, in the system operational and expansion planning stage. So, a large computational time is allowed, as long as it is fits in the planning schedule.

VI. Conclusions

This paper presented a new procedure for designing controllers to damp electromechanical oscillations in power systems. The new procedure results from an improvement over a previously developed methodology, which was able to fulfill several practical requirements of the oscillation damping problem. However, this previous methodology eventually provided controllers with high gains, which are undesirable for practical reasons. The new procedure includes a control effort minimization stage, resulting in smaller gains in the frequency range of interest. This is achieved via LMI optimization, with an extended formulation of the LQR problem.

The obtained results showed that the new procedure is effective in providing controllers able to satisfy all the required specifications with smaller gains, when compared with the original methodology. A comparison with classical PSSs also showed that the performance of the designed controllers is satisfactory. Moreover, the trial-and-error process involved in the tuning of classical PSSs is not necessary with the new procedure, which constitutes one of the main advantages of this approach. Other improvements for the procedure are under research. Among them, the application of this procedure over reduced order models of real-sized systems is the main focus, so the procedure can be effectively used in the field.

References

[1] F. P. DeMello and C. Concordia, "Concepts of Synchronous Machine Stability as Affected by Excitation Control", *IEEE Trans. Power App. Sys.*, PAS-88 (4), 1969, pp. 316-329.

[2] E. V. Larsen and D. A. Swann, "Applying Power System Stabilizers", *IEEE Trans. Power App. Sys.*, PAS-100 (6), 1981, pp. 3017-3033.

[3] F. P. DeMello, L. N. Hannet, and J. M. Undrill, "Practical Approaches to Supplementary Stabilizing from Accelerating Power", *IEEE Trans. Power App. Sys.*, PAS-97 (5), 1978, pp. 1515-1522.

[4] G. Boukarim, S. Wang, J. Chow, G. Taranto and N. Martins, "A Comparison of Classical, Robust, and Decentralized Control Designs for Multiple Power System Stabilizers", *IEEE Trans. Power Sys.*, 15 (4), 1999, pp. 1287-1292.

[5] G. J. Rogers, *Power System Oscillations*, Kluwer Academic Publishers, Norwell, 2000.

[6] H. Quinot, H. Bourlès, and T. Margotin, "Robust Coordinated AVR+PSS for Damping Large Scale Power Systems", *IEEE Trans. Power Sys.*, 14 (4), 1999, pp. 1446-1451.

[7] I. Kamwa, R. Grondin, and Y. Herbert, "Wide-Area Measurement Based Stabilizing Control of Large Power Systems – A Decentralized/Hierarchical Approach", *IEEE Trans. Power Sys.*, 16 (1), 2001, pp. 136-153.

[8] R. A. Ramos, L. F. C. Alberto, and N. G. Bretas, "A New Methodology for the Coordinated Design of Robust Decentralized Power System Damping Controllers", *IEEE Trans. Power Sys.*, 19 (1), 2004, pp. 444-454.

[9] R. A. Ramos, L. F. C. Alberto, and N. G. Bretas, "Decentralized Output Feedback Controller Design for the Damping of Electromechanical Oscillations", *Int. Journal of Elec. Power and Energy Sys.*, 26 (3), 2004, pp. 207-219.

[10] S. Boyd, L. El Gahoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.

[11] M. C. de Oliveira, J. C. Geromel, and J. Bernussou, "Design of Dynamic Output Feedback Decentralized Controllers via a Separation Procedure", *Int. Journal of Control*, 73 (5), 2000, pp. 371-381.

[12] M. Chiali and P. Gahinet, "H-infinity Design with Pole Placement Constraints: An LMI Approach", *IEEE Trans. Automatic Control*, 41 (3), 1996, pp. 358-367.

[13] P. Kundur, *Power System Stability and Control*, McGraw-Hill, New York, 1994.