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Published in:
Physics Letters. Section B: Nuclear, Elementary Particle and High-Energy Physics

1968

Citation for published version (APA):
Nilsson, S. G., Möller, P., Nilsson, B., Sobiczewski, A., Szymanski, Z., & Wycech, S. (1968). Hexadecapole equilibrium distortions of nuclei. *Physics Letters. Section B: Nuclear, Elementary Particle and High-Energy Physics*, *26*(7), 418-420.

Total number of authors:
6

Volume 26B, number 7 PHYSICS LETTERS 4 March 1968

HEXADECAPOLE EQUILIBRIUM DISTORTIONS OF NUCLEI

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Received 20 January 1968

Theoretical calculations of nuclear deformed shell model equilibrium shapes in the rare earth and actinide regions have yielded hexadecapole distortions in good agreement with recent experimental shapes determined from (α,α') scattering analysis. Pairing, Coulomb and polarisation effects are included in the calculations.

In connection with a general study of the nuclear energy surface as a function of various distortion parameters we have determined equilibrium hexadecapole deformations in addition to quadrupole ones.

The nuclear potential employed consists of a generalized oscillator potential with some correction terms

$$V = \tfrac{1}{2}\hbar\omega_o(\epsilon,\epsilon_4)\rho^2(1-\tfrac{2}{3}\epsilon P_2 + 2\epsilon_4 P_4) + \\ - 2\kappa[\mathit{l}_t\cdot s - \mu(\rho^4 - \langle\rho^4\rangle_N)] ,$$

where ρ and l_t are defined in terms of "stretched coordinates". One has

$$\rho^2 = \frac{M}{\hbar}[\omega_z z^2 + \omega_\perp(x^2+y^2)] .$$

(The use of this type of coordinates assures vanishing couplings of the terms proportional to ϵP_2 between shells of different main quantum numbers N_t. Such couplings due to the $\epsilon_4 P_4$-term must be and are included, while those due to ρ^4 have so far been neglected.) An exact condition of "conservation of volume enclosed by equipotential surfaces" is imposed on the part of the potential proportional to ρ^2 with the help of the deformation dependence of the oscillator frequency $\omega_o(\epsilon,\epsilon_4)$. The subtraction of the average value of the ρ^4-term, denoted $\langle\rho^4\rangle_N$, evaluated as an average for each N_t-shell is assumed to remedy the fact that the ρ^4-term has not been included in the volume conservation condition. It has the effect of restoring the oscillator shell spacing to $\hbar\omega_o$.

As is reported elsewhere [1], this potential is found to reproduce the observed level order of heavy deformed nuclei with one set of κ and μ values for neutrons and protons, respectively.

To obtain the total energy as function of distortion the single-particle energies are then added with proper inclusion of neutron and proton pairing. Using Z proton levels and N neutron levels (thus equally many filled and empty ones) we have employed a pairing strength of $G_n = 14.0/A$ and $G_p = 19.6/A$ MeV in the actinide region and $G_n = 15.6/A$ and $G_p = 20.8/A$ MeV in the rare earth region. The choice of $\hbar\omega_o \approx 41 A^{-\frac{1}{3}}$ MeV for both neutrons and protons corresponds to a proton equivalent $(\frac{5}{2}\langle r^2\rangle)^{\frac{1}{2}}$ radius of $\approx 0.98 \times 1.2 \times A^{\frac{1}{3}}$ fm, and a neutron equivalent radius of $\approx 1.04 \times 1.2 \times A^{\frac{1}{3}}$ fm. (One might later wish to equate these radii with a choice of ω_N being different from ω_P.)

Finally, the Coulomb energy is added corresponding to a homogeneously charged equipotential surface (containing the nuclear volume) that in the spherical case corresponds to a radius of $1.2 \times A^{\frac{1}{3}}$ fm. For an approximate inclusion of

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surface diffuseness and exchange corrections see ref. 1. The actual calculation is performed with a computer program in the following way. An equipotential surface is expressed in the form

$$R = R_0(1 + a_0 + a_2P_2 + \ldots . a_{14}P_{14}) \; .$$

The Coulomb energy expansions of Nossoff [2] and Cohen and Swiatecki [3] are then employed. The total energy made up of the sum of single-particle, Coulomb, and pairing energy is then minimized with respect to ϵ and ϵ_4. This elaborate procedure is somewhat excessive at the equilibrium but called for at saddle point shapes.

The equilibrium values so obtained refer to the shapes of the total nuclear field. Compared to the quadrupole distortions obtained from the measurements of charge quadrupole moments the ϵ-values are generally low by 10% although the general trends with A are very well reproduced [4]. (Note that no ad hoc adjustments of levels belonging to lower shells has been introduced in the present calculations.) On the other hand the ϵ-distortions obtained for the nuclear field from a detailed analysis of (α,α') scattering experiments are in good agreement with our results [5]. This might be taken possibly to indicate a larger excentricity of the proton distribution than of the neutron distribution in the nucleus. Detailed results will be published elsewhere [6].

We wish to emphasize here mainly the results as to the hexadecapole deformations. To the left in fig. 1 we exhibit the ϵ_4 (and ϵ) distortions obtained when only couplings due to $\rho^2 Y_{40}$ within each oscillator N_t-shell is considered. The inclusion of the couplings between shells N_t and $N_t \pm 2$ leads to larger values of $|\epsilon_4|$ by more than a factor of two (fig. 1, right). This is a direct measure of the hexadecapole polarization being of the same order of magnitude as the quadrupole polarization.

In figs. 2 and 3 we have expressed the results for the rare earth region in terms of β_4, where β_4 is the Y_{40} component in the radius vector expansion of the nuclear surface. Comparison is made in fig. 2 with the β_4-values obtained from the (α,α') analysis [5]. The agreement is excellent, and it should be noted that the inclusion of the higher polarisation terms directly coupling shells N_t with $N_t \pm 4$, would tend to lower the theoretical (solid) curve additionally. The dashed line corresponds to theoretical calculations [5] where only diagonal effects of the field are con-

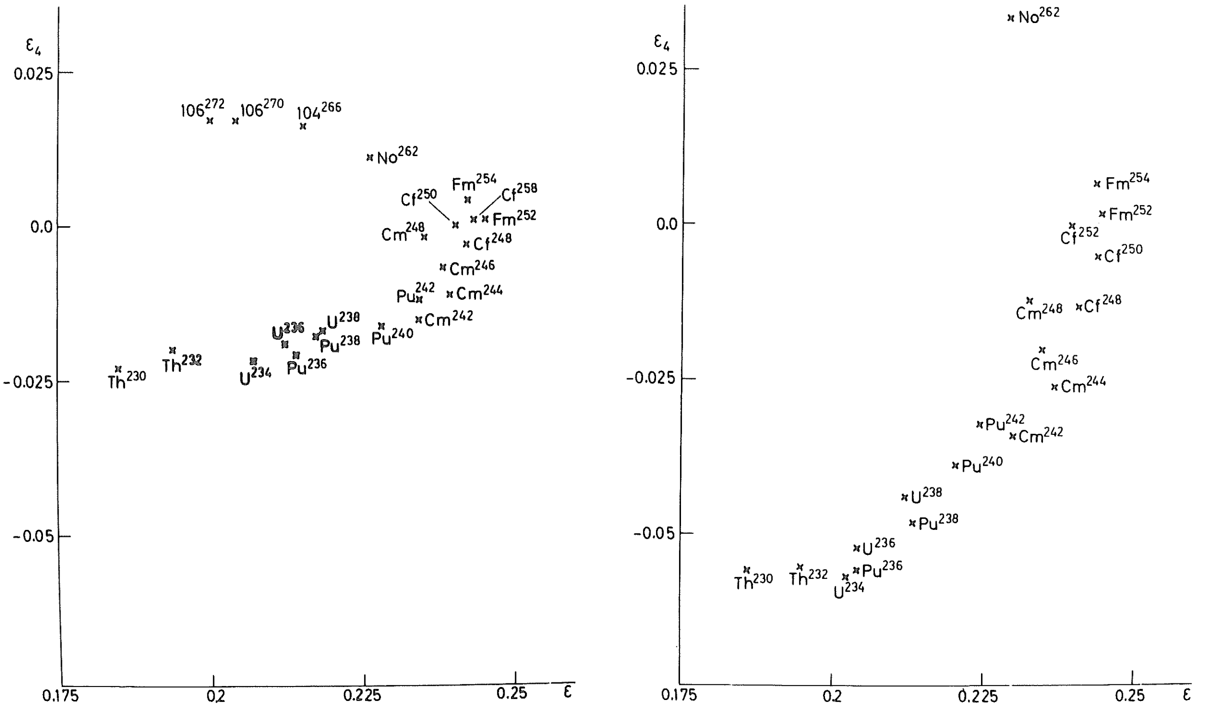

Fig. 1. Equilibrium $\epsilon(Y_{20})$ and $\epsilon_4(Y_{40})$ deformations in the actinide region with coupling between shells N_t and $N_t \pm 2$ neglected (left) and included (right).

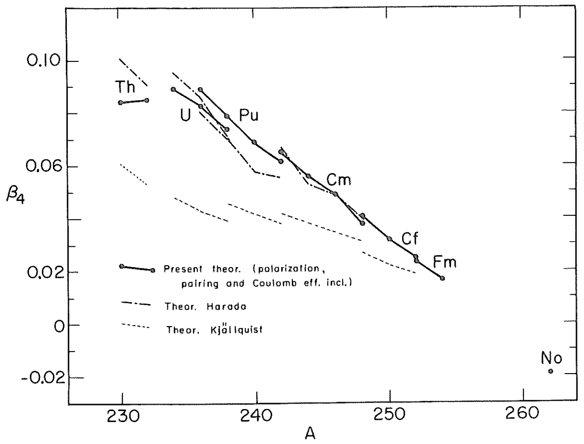

Fig. 2. Hexadecapole deformations β_4 in the rare earth region compared with experimental and theoretical values of ref. 5. Recent reevaluation of the data has resulted in a small increase in the experimental Sm β_4 values and a small decrease in the Yb and Hf values. The theoretical β_4-values for W and Os are somewhat uncertain as they are based on too small a number of gridpoints.

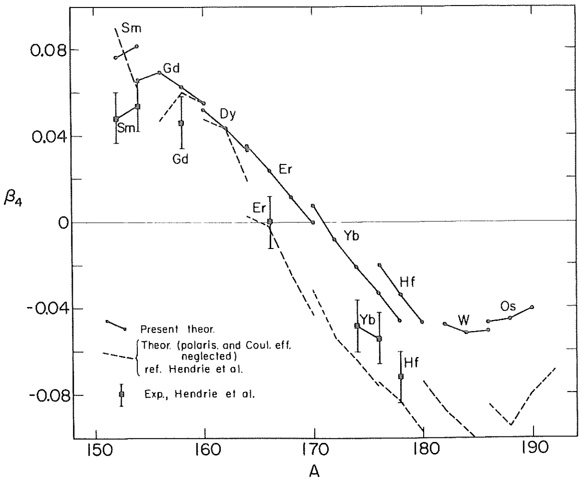

Fig. 3. Hexadecapole deformations β_4 in the actinide region compared with the theoretical calculations of refs. 7 and 8.

sidered and pairing, Coulomb, and polarisation effects neglected. The theoretical curve is in the latter case adjusted to agree with the experimental ^{168}Er value. The agreement between the different theoretical methods appears to indicate that polarization and Coulomb effects are fairly well balanced by the opposing effect of pairing. The basic trend of the Y_{40} distortions reflects directly on the properties of the un-filled shell orbitals of which the first, $n_z \approx N_t$, orbitals favor positive Y_{40} distortions, i.e., negative ϵ_4 and large β_4 (some β_4 being contained in the pure spheroid shape). On the other hand, the $n_z \approx \frac{1}{3}N_t$ to $\frac{2}{3}N_t$ orbitals, filled later, favor negative Y_{40} distortions, leading to larger, eventually positive, ϵ_4-values and correspondingly decreasing β_4-values.

The same trends are common to the rare-earth and actinide regions. In the actinide region covered by fig. 3, no experimental β_4-values are available for comparison, as, e.g., the α-branching analysis is presently not sensitive enough to decide uniquely between possible hecadecapole shapes. We exhibit, however, for comparison the theoretical equilibrium distortions obtained by Kjällquist-Marken [7] and Harada [8]. These authors exclude pairing, Coulomb, and polarisation effects and differ from each other by their respective employment of "asymptotic" and "exact" deformed single-particle wave functions.

We are grateful to Drs. Hendrie et al., for access to their data prior to publication.

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