Nonlinear Income Taxes and the Utility Possibility Set

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Abstract

We consider nonlinear pricing policies that are designed by a social welfare maximizer who operates under a non-negative profit requirement. In our two-type economy, we characterize the set of all feasible nonlinear pricing policies and the frontier of the utility possibility set. Our results provide a link between distortion in consumption and individual, as well as, social welfare.

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1 Introduction

The price of a good is said to be nonlinear if the unit price not is constant but depends on how much the consumer buys. Examples include the price of electricity and telecommunication services. The theory of nonlinear pricing has received considerable attention in the literature, see e.g. Mussa and Rosen (1978), Maskin and Riley (1984) and Sharkey and Sibley (1993) among others. In most studies, the social welfare maximizer or the profit-maximizing monopolist, who designs the tariff, is supposed to know the distribution of consumer types but he is, by assumption, unable to tell consumer types apart. As a consequence, the nonlinear prices must be restricted by a set of incentive-compatibility constraints. A direct consequence of the introduction of these constraints is that consumption, in most cases, is distorted. For example, a well-known result in the literature states that, under certain assumptions, every consumer except the largest user is served at an inefficient level of output (the "no-distortion-on-the-top" result), see e.g. all of the above cited papers. In a recent article, Weichenreider (2004) demonstrated that this standard result may also appear in a contestable market and, moreover, that stable "distortion-on-the-top", as well as, stable first-best equilibria are possible. These results have previously been recognized in a noncontestable market by e.g. Sharkey and Sibley (1993). It is, however, also well established that depending on how consumption is distorted, net utility for the various consumer types are affected differently, see e.g. Cremer and Gahvari (2002) or Bernard and Wittwer (2002). Hence, it is of interest to investigate the frontier of the utility possibility set (UPS, henceforth). In this note we address this question and characterize the frontier of the UPS in a two-type consumer economy. Our results provide a link between distortion in consumption and individual, as well as, social welfare.

This note is organized as follows. In section 2, we specify the basic model. The results are found in Section 3. Section 4 concludes the note. All proofs appear in the Appendix.

2 The Model

The economy consists of two differing consumer types and one publicly owned (natural) monopoly. Consumer $i = 1, 2$ has preferences over consumption-outlay bundles $x_i = (q_i, t_i)$, where q_i denote type i's consumption of a perfectly divisible good (e.g. water) and t_i is a monetary transfer from consumer i to the publicly owned firm. We assume that consumer i's preferences can be represented by a quasi-linear utility function: $u_i(x_i) = \phi_i(q_i) - t_i$, where $\phi_i(q_i)$ is supposed to be continuous and (at least) twice differentiable with: $\phi_i(0) = 0$, $\phi_i'(q_i) > 0$ and $\phi_i''(q_i) < 0$. We shall also make the sorting assumption that marginal willingness-to-pay for a given quantity is increasing in type, i.e.:

$$\phi_2'(z) > \phi_1'(z) \text{ for all } z > 0. \tag{1}$$

The cost of producing output $q \equiv q_1 + q_2$ is given by $C(q) = \beta q + F$, where $\beta > 0$ is the constant marginal cost of production and F is a fixed cost. Consumption is said to be first-best for agent i if: $u_i'(q_i^*) = \beta$. The social planner is supposed to operate under a non-negative profit requirement. Moreover, he is supposed to be unable to tell consumers apart and, therefore, his objective is to:

$$\max_{q_i,t_i}\{\alpha_1(\phi_1(q_1) - t_1) + \alpha_2(\phi_2(q_2) - t_2)\} \tag{2}$$

subject to:

$$\phi_1(q_1) - t_1 \geq 0, \tag{3}$$

$$\phi_2(q_2) - t_2 \geq 0, \tag{4}$$

$$\phi_1(q_1) - t_1 \geq \phi_1(q_2) - t_2, \tag{5}$$

$$\phi_2(q_2) - t_2 \geq \phi_2(q_1) - t_1, \tag{6}$$

$$t_1 + t_2 \geq \beta(q_1 + q_2) + F. \tag{7}$$

In the above specification, α_1 and α_2 represents the welfare weights and we shall suppose, without loss of generality, that $\alpha_1 + \alpha_2 = 1$. These weights are chosen to correspond with the social planner's redistributive objectives. Constraints (3) and (4) are the individual rationality constraints, which must hold since each consumer type can voluntarily choose not to consume the good. We will refer to conditions (5) and (6) as the "upward IC constraint" and the "downward IC constraint", respectively. These constraints guarantee that the consumer types pick the bundles that are designed for them. Condition (7) is represents the non-negative profit requirement.

3 The Characterization Results

We first state some basic properties of the solution to the maximization problem.

Lemma 1 *The solution to the maximization problem have the following properties:* (i) $q_2 > q_1$ (ii) *The individual rationality constraint for type* 2 *is always non-binding.* (iii) *The non-negative profit constraint is always binding.* (iv) *The upward and the downward IC constraints cannot be simultaneously binding.* (v) *There exists at most one nonlinear outlay schedule where net utility for type 1 is zero.*

We next characterize the set of all feasible nonlinear pricing policies and the frontier of the utility possibility set. Note also that since at most one nonlinear pricing policy where the individual rationality constraint for type 1 is active exists, by Part (v) of Lemma 1, it will be useful to know the properties of the outlay schedules in the case when the individual rationality constraint for type 1 is inactive.

Proposition 1 *If the individual rationality constraint for type 1 is inactive, then the downward IC constraint binds if $\alpha_1 > \frac{1}{2}$. Moreover, the first-order conditions are given by:*

$$\phi_1'(q_1) = \beta + (2\alpha_1 - 1)(\phi_2'(q_1) - \phi_1'(q_1)), \tag{8}$$

$$\phi_2'(q_2) = \beta. \tag{9}$$

From the sorting condition (1), it follows that consumption for type 1 always is less than the first-best quantity in the case when the social planner has a bias in favor of type 1, i.e. when $\alpha_1 > \frac{1}{2}$. The reason for this is that the social planner tries to increase net utility for type 1 by using type 2 to cover the costs through the balanced-budget equation. However, to increase funding for type 2 and at the same time respect the incentive compatibility constraints, the planner must make the consumption-outlay pair offered to type 1 sufficiently unattractive for type 2. This task is achieved by distorting consumption for type 1. In the case when the social bias in favor of type 1 is absolute (i.e. when $\alpha_1 = 1$), consumption for type 1 is fully distorted. This utility pair is marked with A in Figure 1. Note next that if α_1 is decreased with a small $\varepsilon > 0$ (so that $\alpha_1 - \varepsilon > \frac{1}{2}$), total utility increases since q_1 increase by condition (8). However, net utility for type 1 decreases and, therefore, net utility for type 2 must increase. Hence, the utility pairs that correspond to $\alpha_1 \in]\frac{1}{2}, 1]$ are located along the curve AB in Figure 1, where utility pair B is approached when $\alpha_1 \to \frac{1}{2}$.

Proposition 2 *If the individual rationality constraint for type 1 is inactive and $\alpha_1 = \alpha_2 = \frac{1}{2}$, then $q_i = q_i^*$ for $i = 1, 2$.*

Note next that if consumption is first-best for both consumer types, then incentive compatibility and budget-balance are respected if:

$$\phi_2(q_1^*) - \phi_1(q_1^*) \leq t_2 - t_1 \leq \phi_2(q_2^*) - \phi_1(q_2^*), \tag{10}$$

$$t_1 + t_2 = \beta(q_1^* + q_2^*) + F. \tag{11}$$

Since consumption is constant and utility functions are quasi-linear, it follows directly that if the above restrictions are satisfied, the frontier of the UPS must be linear and have a slope equal to minus one. This is also indicated in Figure 1, where all utility pairs that satisfy the above restrictions are located along the line BC. Note also that at utility pair B and C, the downward and the upward IC constraint are binding, respectively. Both of the incentive constraints are non-binding at the utility pairs located between the pairs B and C.

Proposition 3 *If the individual rationality constraint for type 1 is inactive, then the upward IC constraint binds if* $\alpha_2 > \frac{1}{2}$. *Moreover, the first-order conditions are given by:*

$$\phi_1'(q_1) = \beta, \tag{12}$$

$$\phi_2'(q_2) = \beta - (2\alpha_2 - 1)(\phi_2'(q_2) - \phi_1'(q_2)). \tag{13}$$

From the sorting condition (1), it follows that consumption for type 2 always is larger than the first-best quantity in the case when the social planner has a bias in favor of type 2. The intuition behind this result is exactly the same as in the case when consumption is distorted for type 1. Moreover, we see that net utility for type 2 is maximized when when q_2 is fully distorted. The utility pairs that correspond to $\alpha_2 \in]\frac{1}{2}, 1]$ are located along the curve CDE in Figure 1.

Note, finally, that the observations in Propositions 1-3 are all based on the situation when the individual rationality constraint for type 1 is inactive. However, as illustrated in Figure 1, the individual rationality constraint for type 1 is active at utility pair D. Hence, $u_1 \leq 0$ for all utility pairs that are located along the line DE with $u_1 = 0$ only at utility pair D. Suppose now that utility pair D is generated by setting $\alpha_1 = \widetilde{\alpha}_1 < 1$. In this case, the individual rationality constraint for type 1 is binding for all $\alpha_1 \in [0, \widetilde{\alpha}_1]$ and non-binding for all $\alpha_1 \in]\widetilde{\alpha}_1, 1]$. Two important conclusions can be drawn. Firstly, the solution to the maximization problem is the same for all $\alpha_1 \in [0, \widetilde{\alpha}_1]$. Secondly, it is not possible to generate the utility pairs that are located along the line DE that are different from D by solving the maximization problem from Section 2 since individual rationality is violated. Hence, in the situation that we have illustrated in Figure 1, the frontier of the UPS is given by the curve $ABCD$. We shall end this paper with a numerical example that illustrate the results of the paper.

[**FIGURE 1 - ABOUT HERE**]

Example 1. Assume that the marginal and the fixed costs are given by $\beta = 1$ and $F = 11$, respectively, and that: $u_i = \theta_i\sqrt{q_i} - t_i$, where $\theta_1 = 5$ and $\theta_2 = 7$. The frontier of the UPS is given by all utility pairs (u_1, u_2) that are located along the curve $ABCD$ in Figure 1. More explicitly, the net utilities at the marked utility pairs are given by $A = (1.75, 4.75)$,

$B = (1.25, 6.25)$, $C = (0.25, 7.25)$ and $D = (0, 7.45)$. In this example, the individual individual rationality constraint for type 1 is binding for all $\alpha_1 \in [0, 0.387]$ and, therefore, the utility pair that corresponds to the solution for every $\alpha_1 \in [0, 0.387]$ is marked by D in Figure 1. ■

4 Conclusions

In this paper, we have characterized the frontier of the UPS and, moreover, demonstrated how consumption is distorted along this frontier. Our results regarding distortion support the results in e.g. Sharkey and Sibley (1993) and Weichenrieder (2004). Our characterization of the frontier of the UPS, as well as, how consumption is distorted depending on the explicit choice of the welfare weights has, to the very best of my knowledge, not been presented in the literature before.

Appendix

In this Appendix, we prove the results of this paper. The maximization problem from Section 2 leads to the Lagrangian:

$$\begin{aligned} L &= \alpha_1(\phi_1(q_1) - t_1) + \alpha_2(\phi_2(q_2) - t_2) + \lambda_1(\phi_1(q_1) - t_1) + \gamma_1(\phi_1(q_1) - t_1 - \phi_1(q_2) + t_2) \\ &\quad + \gamma_2(\phi_2(q_2) - t_2 - \phi_2(q_1) + t_1) + \theta(t_1 + t_2 - \beta(q_1 + q_2) - F) \end{aligned}$$

where $\lambda_1 \geq 0$, $\gamma_i \geq 0$ and $\theta \geq 0$. We have the following first-order conditions:

$$(\alpha_1 + \lambda_1 + \gamma_1)\phi_1'(q_1) - \gamma_2\phi_2'(q_1) - \theta\beta = 0, \tag{14}$$

$$(\alpha_2 + \gamma_2)\phi_2'(q_2) - \gamma_1\phi_1'(q_2) - \theta\beta = 0, \tag{15}$$

$$-\alpha_1 - \lambda_1 - \gamma_1 + \gamma_2 + \theta = 0, \tag{16}$$

$$-\alpha_2 - \gamma_2 + \gamma_1 + \theta = 0. \tag{17}$$

Proof Lemma 1. (i) Adding conditions (5) and (6) yields: $\phi_2(q_2) - \phi_2(q_1) \geq \phi_1(q_2) - \phi_1(q_2)$. By the sorting condition (1), this equality can only be satisfied if $q_2 \geq q_1$. However, if $q_2 = q_1$, Pareto efficiency is violated. To see this, note that if $q_1 = q_2$, it follows, from the balanced-budget rule, that: $t_1 = t_2 = \beta q_1 + \frac{F}{2}$. But this schedule is Pareto dominated by the schedule where consumption is first-best for both types and $t_i^* = \beta q_i^* + \frac{F}{2}$ since $q_i^* = \arg\max_{q_i}\{\phi_i(q_i) - \beta q_i - \frac{F}{2}\}$ and $q_i \neq q_i^*$ for $i = 1$ and/or $i = 2$ by condition (1). Hence, pooling violates Pareto efficiency. (ii) Note first that $u_1 \geq 0$ in an optimal solution.

From this observation, the sorting condition (1) and the downward IC constraint (6), it then follows that: $\phi_2(q_2)-t_2 \geq \phi_2(q_1)-t_1 > \phi_1(q_1)-t_1 \geq 0$. (iii) Adding the first-order conditions (16) and (17) yields: $\theta = \frac{1}{2}(1+\lambda_1) > 0$. Hence, the non-negative profit constraint must bind in optimum. (iv) From Part (i) of this lemma, we know that $q_2 > q_1$ in an optimal solution, but conditions (5) and (6) can only be simultaneously binding when $q_2 = q_1$. (v) Suppose the opposite, i.e. that there exist two different utility pairs, $\overline{u} = (\overline{u}_1, \overline{u}_2)$ and $\widetilde{u} = (\widetilde{u}_1, \widetilde{u}_2)$, that both are generated by solving the maximization problem, where $\overline{u}_1 = \widetilde{u}_1 = 0$. But since $\overline{u} \neq \widetilde{u}$, by assumption, $\overline{u}_2 > \widetilde{u}_2$ or $\overline{u}_2 < \widetilde{u}_2$, so utility pair $\overline{u}_2$ or $\widetilde{u}_2$ cannot be Pareto efficient. ■

Proof Proposition 1. If constraint (3) is inactive, then $\lambda_1 = 0$. But then $\theta = \frac{1}{2}$, by Part (iii) of Lemma 1, so we can rewrite the first-order condition (16) to: $\alpha_1 - \frac{1}{2} = \gamma_2 - \gamma_1$. Since $\alpha_1 > \frac{1}{2}$, by assumption, and the upward and the downward IC constraints cannot be simultaneously binding, by Part (iv) of Lemma 1, it follows that: $\alpha_1 - \frac{1}{2} = \gamma_2$ and $\gamma_1 = 0$. Using these facts and the first-order conditions (14) and (16), we get the first-order conditions in the proposition. ■

Proof Proposition 2. If constraint (3) is inactive, then $\lambda_1 = 0$. But then $\theta = \frac{1}{2}$, by Part (iii) of Lemma 1, and $\theta = \alpha_1 = \alpha_2$, by assumption, so we can rewrite the first-order conditions (16) and (17) to: $0 = \gamma_2 - \gamma_1$ and $0 = \gamma_1 - \gamma_2$, respectively. Since the upward and the downward IC constraints cannot be simultaneously binding, by Part (iv) of Lemma 1, it follows that: $\gamma_1 = \gamma_2 = 0$. Using these facts and the first-order conditions (14)-(17), it follows directly that consumption is first-best for both consumer types. ■

Proof Proposition 3. If constraint (3) is inactive, then $\lambda_1 = 0$. But then $\theta = \frac{1}{2}$, by Part (iii) of Lemma 1, so we can rewrite the first-order condition (17) to: $\alpha_2 - \frac{1}{2} = \gamma_1 - \gamma_2$. Since $\alpha_2 > \frac{1}{2}$, by assumption, and the upward and the downward IC constraints cannot be simultaneously binding, by Part (iv) of Lemma 1, it follows that: $\alpha_2 - \frac{1}{2} = \gamma_1$ and $\gamma_2 = 0$. Using these facts and the first-order conditions (15) and (17), we get the first-order conditions in the proposition. ■

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Figure 1. The Frontier of the Utility Possibility Set