Analyticity - An Unfinished Business in Possible World Semantics

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Analyticity

An Unfinished Business in Possible-World Semantics*

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The goal of this paper is to consider how the notion of analyticity can be dealt with in model-theoretical terms. The standard approach to possible-world semantics allows us to define logical truth and necessity, but analyticity is considerably more difficult to account for. To explain this difficulty, we will first give a simplified sketch of possible-world semantics.[1] After defining necessity and logical truth, we will describe how a two-dimensional variant of that approach has been used to define the notion of *a priori*. The subsequent section is focused on analyticity. As will be suggested, the source of the difficulty in defining that notion lies in the received model-theoretical conception of a language interpretation. Intuitively, analyticity amounts to truth in virtue of meaning alone, i.e. truth solely in virtue of the interpretation of linguistic expressions. However, the received conception of a linguistic interpretation as a mapping from language to a model frame makes it impossible to keep the interpretation constant, while varying the other components of the model. To make room for analyticity, the concept of an interpretation should therefore be revised: Interpretations should be made richer in content than it has usually been assumed. As a by-product, such a revision also makes possible to provide a one-dimensional analogue of the two-dimensional account of *a priori*. We will thus be able to map out the network of formal connections between the concept of analyticity and the notions of apriority, logical truth and necessity.

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[1] The approach to be presented is a simplified version of Kripke (1963). My apologies to the reader for this rehearsal of a standard material. For a much more detailed and thorough presentation of treatments of necessity and logical truth in various versions of possible-world semantics, see Lindström and Segerberg (forthcoming), part 1 (“Alethic Modal Logic”).

1. Semantic preliminaries

We start with the notion of a model *frame*. Intuitively, a frame is a structure that comprises all the non-linguistic elements in a semantic model. More precisely, we take a frame to be a quadruple $S = <W, \omega, I, D>$, where
W is a non-empty set (the set of *possible worlds*),
ω is a designated element of W (*the actual world*),
I is another non-empty set (the set of *individual objects*)
and D is a function that to every w in W assigns a subset of I (the set of objects that *exist* in a world w).

$D(w)$ might be called *the object domain* of w. On some approaches to modal semantics, a frame of this kind is enriched with an accessibility relation on the set of possible worlds, which allows for a more sophisticated treatment of the concepts of necessity and possibility. For simplicity, though, we shall here abstain from this complication.

Now, let L be a first-order language, with a vocabulary consisting of individual variables, individual constants, predicates (unary, binary, etc.), and a collection of sentential operators and quantifiers. We take the latter group to contain at least negation, conjunction, and the universal quantifier. Other Boolean connectives and the existential quantifier are then immediately definable. Sentences of L, both open and closed, are constructed in the standard way, out of these basic building blocks.

On the received model-theoretical view, the meaning of linguistic expressions in L is given by their *interpretation* with respect to a given frame. A pair consisting of an interpretation together with the relevant frame provides a *model* for L. We let i, an interpretation of L with respect to a frame $S = <W, \omega, I, D>$, be a simultaneous mapping
(i) from variables and individual constants to objects in I,[2]
(ii) from n-ary predicates to functions that assign n-ary relations on I to possible worlds.

An interpretation is then extended to the sentences of L by means of an inductive definition. To every sentence, the extended interpretation assigns a set of worlds in W, i.e. a proposition.

[2] An assignment of objects to variables is often kept apart from the interpretation proper. (This assignment, or sometimes a pair consisting of this assignment together with the interpretation proper, is called a 'valuation'.) Given this separation, the truth of open sentences, which contain free variables, has to be relativized not just to a model (= a frame together with an interpretation, see below), but also to a particular choice of an assignment of objects to the variables. Here, however, for the sake of simplicity, we shall avoid this complication.

Intuitively, for every sentence A, the proposition $i(A)$ is the set of possible worlds at which A is true. The inductive definition goes like this:

(1) For every atomic sentence $F^n(t_1, \ldots, t_n)$, where F^n is an n-ary predicate and $t_1, \ldots, t_n$ are variables or individual constants,
$i(F^n(t_1, \ldots, t_n))$ = the set of worlds w in W such that $<i(t_1), \ldots., i(t_n)> \in i(F^n)(w)$.[3]

(2) As for the non-atomic sentences, we take i to satisfy the standard conditions for the sentential connectives and quantifiers:
$i(\neg A, w) = W - i(A)$;
$i(A \wedge B) = i(A) \cap i(B)$,
$i(\forall x\, A)$ = the set of worlds w in W such that $w \in i'(A)$ for every interpretation i' that assigns to x an object in $D(w)$ but otherwise coincides with i.[4]

If language L contains the *necessity* operator $\Box$, we take $i(\Box A)$ to equal W if $i(A) = W$, and to equal $\varnothing$ otherwise. The possibility operator $\Diamond$ is definable in terms of $\Box$, as its dual. If L contains the *actuality* operator, we let i(Actually A) equal W or $\varnothing$ depending on whether the actual world ω belongs to $i(A)$.

A *model* is a pair, (S, i), where S is a frame and i is an interpretation of L with respect to S. Given the notion of a model, we can specify what it means for a sentence to be true. Consider any model $M = (S, i) = (<W, \omega, I, D>, i)$ and any world w in W. Truth *at a world* in a model is defined as follows:

A sentence A is *true at* a world w in a model $M = (<W, \omega, I, D>, i)$ iff $w \in i(A)$.

Truth *simpliciter* is truth at the designated world of the model. Just like truth at a world, truth is a property of a sentence relative to a model.

A is *true* in a model $M = (<W, \omega, I, D>, i)$ iff $\omega \in i(A)$.

Similarly, necessity ('metaphysical' necessity, as opposed to the logical one) is a property of a sentence relative to a model. It is defined as truth at all the worlds in the model.

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[3] For simplicity, we haven't assumed that identity is a logical constant of L. If we did, then atomic identity sentences, of the form $t_1 = t_2$, would be interpreted as follows:
$i(t_1 = t_2)$ equals W or $\varnothing$ depending on whether $i(t_1) = i(t_2)$.

[4] It is philosophically problematic that the truth condition for a quantified sentence, on a given interpretation, requires us to consider *other* interpretations of the language. This problem is avoided, however, if the assignment of objects to variables is kept apart from the interpretation proper.

A is *necessary* in a model $M = (<W, \omega, I, D>, i)$ iff $i(A) = W$.

Clearly, if the language contains the necessity operator $\Box$, then

A is necessary in a model M iff $\Box A$ is true in M.

Let Σ be the class of all frames that we take to be admissible and let $\Delta(\Sigma)$ be the set of all the models of L that are constructible on the basis of Σ.[5] That is, $M \in \Delta(\Sigma)$ iff for some $S \in \Sigma$ and for some interpretation i of L with respect to S, $M = (S, i)$. In what follows, the phrases such as "all models" should always be understood as referring to the models that can be built up from Σ.

We can now define logical truth as truth in all models:

A is *logically true* relative to Σ iff for all models $M \in \Delta(\Sigma)$, A is true in M.

Since the set of logical truths depends on the choice of Σ, we might say that Σ determines the logic of L.

Logical truth, if defined as above, should be distinguished from truth at all worlds in all models, i.e. from necessary truth in every model. The latter notion might be called *logical necessity*.

A is *logically necessary* relative to Σ iff for all models $M \in \Delta(\Sigma)$, A is necessary in M.

At a cursory glance, it might seem that logical necessity is coextensive with logical truth. Or, at least, it might seem so if we assume – as we shall do in this paper – that the set Σ of admissible frames is closed under arbitrary variations of the designated world:

Closure: If $<W, \omega, I, D> \in \Sigma$, and $w \in W$, then $<W, w, I, D> \in \Sigma$.

This closure condition on Σ might seem to imply that truth in all models entails necessary truth in all models. For if a sentence is *not* necessary in a model $M = <W, \omega, I, D> \in \Delta(\Sigma)$, i.e. if it is not true at some world w in M, then – it might seem – this sentence will not be true in the model M' that we obtain from M by letting w be the designated world, instead of ω. By Closure, the latter model belongs to $\Delta(\Sigma)$. However, if language L contains such resources as

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[5] If the idea of a possible world is taken very literally, then it is arguable that the set of possible worlds should be the same in all admissible frames and the objects that exist in any given world should also be the same. Thus, in this limiting case, two admissible frames could only differ in the choice of the actual world. But modal logicians seldom take their semantics so literally. Normally, possible worlds and their object domains are allowed to vary from one admissible frame to another.

the actuality operator, these appearances are misleading. To see this, consider any sentence *A* that is true at the designated world ω in *M* but false at some world *w* in that model. Then the material conditional "Actually $A \rightarrow A$" is false at *w* in *M*: Its consequent is false at that world, but its antecedent is true, because *A* is true at ω. Nonetheless, this conditional becomes true in the model *M'* that we obtain from *M* by letting *w* be the designated world. For in *M'*, in contrast to *M*, the antecedent of the conditional will be false at *w* if its consequent is false at that world. As a matter of fact, the conditional in question is logically true (= true in all models). However, as we have just seen, it is not logically necessary. It is a logical truth, but not a necessary one, that a state of affairs obtains if it actually obtains.

As regards actuality, an alternative treatment of that notion is available within a *two-dimensional* approach to possible-world semantics. On that approach, frames no longer feature designated worlds. They are just triples that consist of a set of worlds, a set of individual objects and a function that assigns object domains to worlds. To compensate for this removal of a designated world, interpretations are correspondingly enriched. Thus, an interpretation *i* of language *L* with respect to such a reduced frame $<W, I, D>$ is indexed by worlds in *W*, with all worlds in *W* serving as possible indices. In other words, *i* is now a function that to every possible world in *W* assigns a mapping from *L* to $<W, I, D>$. For each index $w \in W$, the mapping i_w behaves in the same way as *i* has been defined above, with one exception. The interpretation of the actuality operator is different:

i_w(Actually A) equals W or $\varnothing$ depending on whether $w \in i_w(A)$.

Thus, actuality is an explicitly indexical notion on this approach. The index determines the semantic 'perspective', so to speak, i.e. the point of view from which semantic evaluation is being made. Note that predicates and other linguistic expressions might well be given different semantic values from different world perspectives. Thus, if F is a predicate, and the model contains two worlds w and v, it may well be the case that $i_w(F) \neq i_v(F)$. Consequently, for two predicates F and G, $i_w(F)$ might equal $i_w(G)$, without this identity obtaining from the perspective of some other world in the model. For a standard example, think of "water" and "H_2O". From the perspective of our world, these two expressions receive the same interpretation. But from the perspective of a 'Twin Earth'-world, in which the transparent drinkable liquid that flows in rivers and lakes has a different chemical composition from the one we are used to, the semantic values of "water" and "H_2O" will not coincide.

Truth on this approach becomes a two-dimensional concept. Instead of just being true (or false) at a world, a sentence now is true (or false) *at* a world *from* a ceratin world-perspective:

A sentence A is true *at* a world w *from* the perspective of a world v in a model $M = (<W, I, D>, i)$ iff $w \in i_v(A)$.

A sentence A is then said to be *logically true* iff for every model and for every world w in that model, A is true at w in that model from w's own perspective. As in the one-dimensional approach, the two-dimensional account also has room for a notion that is stronger than logical truth. If two-dimensional logical truth is defined as above, as truth in every model at every world from the perspective of that world itself, then this notion is weaker than *logical necessity*, i.e. necessary truth in every model from the perspective of every world in that model. (Note that this amounts to truth in every model at every world from every world's perspective.) While the conditional "Actually $A \rightarrow A$" is logically true, it does not satisfy the stronger requirement.

The two-dimensional approach has been found useful to explicate the notion of apriority. It has been suggested that

A sentence A is *a priori* iff A is true at every world from that world's own perspective.

That is,

A is *a priori* in $M = (<W, I, D>, i)$ iff for every $w \in W$, $w \in i_w(A)$.

Note that, on this suggestion, logical truth in two-dimensional semantics is the same as apriority in all models.

It is not clear whether this account of a priori is satisfactory. Apriority is fundamentally an *epistemic* notion: Intuitively, a sentence is a priori iff it is enough to know what it means in order to know that it is true. It is somewhat unclear how this essentially epistemic character of the notion of a priori is accounted for in the two-dimensional approach. The answer that has been suggested by David Chalmers (2004, and forthcoming) is that the different perspective worlds might be seen as different *epistemic* possibilities that are compatible with what is a priori knowable about the actual world.[6] A sentence is therefore a priori iff it is actually true

[6] As a matter of fact, Chalmers takes epistemic possibilities to be 'centered' worlds, i.e. triples consisting of a possible world, an individual and a time. The time and the individual specify the location within a possible world from which the semantic evaluation is being made. Such localization is primarily needed for the semantic evaluation of indexical sentences. Here, we ignore this complication.

from every possible perspective.[7] But even if this suggestion is workable, we shall here abstain from going two-dimensional. As we shall see, a concept of a priori that is essentially equivalent to the one sketched above can be defined without leaving the one-dimensional framework. Therefore, in what follows, we continue to work with one-dimensional semantics, with frames that contain a designated world.

2. Analyticity

All of the above is more or less standard. But what about *analyticity*? It appears that the approach we have sketched positively hinders defining this notion. Here is the reason why. Intuitively, a sentence is analytic iff it is true in virtue of its meaning alone.[8] In the semantics we consider, the meaning of linguistic expressions is given by their interpretation. The truth of an analytic sentence should depend solely on the interpretation and on nothing else. It is therefore natural to suggest the following model-theoretical explication of the notion of analyticity:

(DefAn) A sentence is *analytic* on a given interpretation iff
it is true in all models that involve this interpretation.

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[7] An objection might be that such a view unduly restricts epistemic possibilities. A sentence $\Box$*A* is true at w (from the perspective of a world v) iff (from that perspective) *A* is true at all the worlds in the model, Now, if $\Box$ stands for metaphysical necessity, then the worlds in the model must all represent different metaphysical possibilities. But what is a priori possible (i.e. possible for all that can be known a priori) need not be possible in this metaphysical sense. In other words, some epistemic possibilities might not be present among metaphysically possible worlds.

To avoid this objection, one might have to give up the idea that all possible worlds are metaphysically possible. Instead, one could introduce a specific ‘metaphysical’ accessibility relation among the worlds in a frame. This move would relativize metaphysical necessity to worlds: What is metaphysically necessary at one world need not be so at another. A sentence of the form $\Box$*A* would then be true at *w* (from the perspective of v) in a model *M* iff (from that perspective) *A* is true at all the worlds in *M* that are accessible from *w*. This relativization of necesssity is of course standard in possible-world semantics. Then the epistemic possibilities that are metaphysically *im*possible relative to w would be representable by those worlds in the model that are *in*accessible from w.

Along with Frank Jackson (see, for example, Jackson 1998), Chalmers is probably the most eloquent modern proponent of the two-dimensionalist semantics. Early proposals on two-dimensionalist lines can be found in the work from the 70-ies and early 80-ies by Segerberg (1973), Evans (1977), Stalnaker (1978), Kaplan (1978) and Davies & Humberstone (1980). Stalnaker has nowadays become a critic of two-dimensionalism; see Stalnaker (2004). Another critic is Soames (2005).

[8] This conception of analyticity goes back at least to Quine (1951), p. 20f: “Kant conceived of an analytic statement as one that attributes to its subject no more than is already conceptually contained in the subject. This formulation has two shortcomings: it limits itself to statements of subject-predicate form, and it appeals to a notion of containment which is left at a metaphorical level. But Kant's intent, evident more from the use he makes of the notion of analyticity than from his definition of it, can be restated thus: a statement is analytic when it is true by virtue of meanings and independently of fact.”

However, a moment of reflection shows that DefAn just doesn't make sense given the above sketched approach to semantics. A model, remember, consists of a frame and an interpretation, with the latter being a mapping from the language to the frame in question. An interpretation takes linguistic expressions and assigns to them entities that either belong to the frame or are constructible from the frame's components. Therefore, it does not make sense to say that such a mapping is retained while the model otherwise is allowed to vary. To keep the mapping constant, we need in the first place to keep constant the range of the mapping, which means we need to keep constant the frame itself, from which the entities are drawn that are assigned to linguistic expressions. But then there is nothing left in the model that can be varied![9]

How should this problem be solved? We should not, I think, give up DefAn. It does seem right to explicate analyticity as truth in all models under a fixed interpretation. Instead, the very concept of an interpretation should be re-interpreted. It must make sense to say that an interpretation can remain unchanged as we move from one model to another, with different underlying frames. To be analytic, a sentence must be true just in virtue of linguistic meaning and *not* in virtue of the elements of the model that are external to language. It must therefore be possible to vary the frame without changing the linguistic interpretation.[10]

Here, then, is a new proposal that implements this idea. We will now think of an interpretation as a mapping that to each linguistic expression in L assigns a function from frames in Σ to their components or to the entities that are constructible from these components. In other words, we now interpret the language not just with respect to *one* frame, but with respect to a whole *class* of frames. Thus, let $\boldsymbol{i}$ be an interpretation of L along these lines. (We use the bold $\boldsymbol{i}$ to signal that this interpretation is richer in content than an

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[9] To be sure, we could keep a given mapping constant while varying the designated world of the frame. But if analyticity were defined as truth in all the models that differ from a given model at most in the choice of the designated world, then all sentences that are necessarily true would come out as analytic on (DefAn), as long as they do not involve such locutions as "actually" or "as a matter of fact" (i.e. as long as they do not explicitly refer to what actually is the case). Thus, even necessary aposteriori truths such as "Morning Star = Evening Star" or "water = H_2O" would come out as analytic. Clearly, this is unacceptable.

[10] The idea that analyticity should be understood as truth under all frame variations, with the interpretation ("valuation") kept fixed, was to my knowledge first put forward in Stig Kanger's doctoral dissertation, Kanger (1957), section 7.3, and in Kanger (1970 [1957])), section 4. (The latter essay was originally published as a privately distributed pamphlet. It was then re-published in Hilpinen (1970).) It should be pointed out, though, that for Kanger a frame was just a set of (individual) objects. On his approach, which predated possible-world semantics, analyticity was therefore understood as truth in all object domains under a fixed interpretation. For a thorough discussion of Kanger's views, see Lindström (1998).

interpretation in the old sense.) Then, for every frame $S = <W, \omega, I, D>$ in Σ,

(i) if t is a variable or an individual constant of L, $\boldsymbol{i}(t)(S) \in I$,

(ii) if F^n is an n-ary predicate of L, $\boldsymbol{i}(F^n)(S)$ is a function that to each world w in W assigns an n-ary relation on I.

As previously, this interpretation is extended to sentences of L by means of an inductive definition. For any sentence A of L, $\boldsymbol{i}(A)(S)$ is a subset of W. The inductive definition looks as follows:

(1) For each atomic sentence $F^n(t_1, \ldots, t_n)$, and for every frame S,

$\boldsymbol{i}(F^n(t_1, \ldots, t_n))(S)$ = the set of worlds w in S such that $<\boldsymbol{i}(t_1)(S), \ldots, \boldsymbol{i}(t_n)(S)> \in \boldsymbol{i}(F^n)(S)(w)$.

(2) The conditions for the sentential operators and for the universal quantifier are adjusted in the corresponding way.

Note that, if $\boldsymbol{i}$ is interpreted on these lines, then for predicates and sentences we can think of $\boldsymbol{i}$ as specifying what might be called their *ultraintensions*. While the intension of an n-ary predicate in a frame S is a function that determines that predicate's extension for each world in S, its ultraintension determines its intensions in the different frames. For sentences, the situation is analogous: The intension of a sentence A in S is the set of worlds in S in which A is true, while the ultra-intension of S determines its intensions in every frame. (For a further elucidation of this notion of an ultraintension, see below.)

Note also that our new notion of interpretation could be constructed in a different, but essentially equivalent way. Thus, instead of letting an interpretation be a function that to every expression in L assigns an ultra-intension (= a mapping from frames to intensions), we can think of it as a function from frames to the 'old-style'-interpretations of L. On this reading, an interpretation $\boldsymbol{i}$ is a function that to every frame $S = <W, \omega, I, D>$ in Σ assigns a mapping, $\boldsymbol{i}(S)$, (i) from variables and individual constants in L to objects in I, and (ii) from n-ary predicates in L to functions from worlds in W to n-ary relations on I. This mapping determines, in the standard way, a mapping from sentences in L to subsets of W. $\boldsymbol{i}(S)$ thus behaves as an interpretation in the old sense of this term. In other words, an interpretation in the new sense can be seen as a function that to each frame assigns an interpretation in the old sense.[11]

[11] The two readings of an interpretation $\boldsymbol{i}$, as a function from Σ to the 'old style' interpretations and as an assignment to linguistic expressions of functions from frames to intensions, are equivalent in the following

While the notion of an interpretation is revised on our proposal, the definition of a model remains the same, mutatis mutandis: A model is a pair (S, $\boldsymbol{i}$) that consists of a frame belonging to Σ and an interpretation re-interpreted along the new lines. As before, $\Delta(\Sigma)$ is the set of all models that can be built up on the basis of Σ. The definitions of truth at a world in a model, of truth in a model and of logical truth are adjusted accordingly.

What about analyticity? Well, now the answer is easy. While logical truth is truth in all models, analyticity is truth in all models that are based *on a given interpretation*, i.e. in all models that involve this interpretation. In other words, analyticity is understood in accordance with (DefAn).

A sentence A is *analytic* on an interpretation $\boldsymbol{i}$ iff for all $S \in \Sigma$, A is true in (S, $\boldsymbol{i}$),
i.e., iff for all $S = \langle W, \omega, I, D \rangle$ in Σ, $\omega \in \boldsymbol{i}(A)(S)$.

Clearly, logical truth entails analyticity so defined. More exactly, logical truth = analyticity on all interpretations. It should also be clear that analyticity on a given interpretation does not entail logical truth: A sentence may be analytic on one reading without being true on other readings; it need not be true in models in which its individual terms and/or its predicates are interpreted in a different way. Sentences such as “All bachelors are married” or “Whatever is red, is coloured” provide obvious examples. On the intended interpretation, the extension of “red” is a subset of the extension of “coloured” in every world in every frame. But on other interpretations of these predicates, this connection between their extensions need no longer hold.

As for the relation between analyticity and necessity, none of these properties entails the other. That analyticity does not entail necessity is clear, since not even logical truth entails necessity. As we have seen, a logically true sentence such as “Actually $A \rightarrow A$” may not be necessary in a model. For an example of a contingent analytic sentence that is not a logical truth, think of Kripke’s “The standard metre in Paris is one meter long”. For Kripke, this was an illustration of a contingent a priori truth,[12] but the sentence in question could just as well be seen as analytic, if the meaning of “one metre” is taken to be specified by reference to the actual length of the Parisian standard. Admittedly, according to Kripke, the descriptive phrase

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sense: For each expression e of L and each frame in S, $\boldsymbol{i}(S)(e)$ on the former reading equals $\boldsymbol{i}(e)(S)$ on the latter reading.

[12] See Kripke (1980), pp. 54ff.

“the length of the standard metre in Paris” does not give the meaning of “one metre”. The former is merely a reference fixer for the latter, since it does not designate rigidly: In different possible worlds it might pick out different lengths. However, “the *actual* length of the standard metre in Paris” is a rigid designator, which means that the problem of rigidity can be circumvented. Nevertheless, Kripke himself had a different view of the matter: He had no problems with contingent a priori but his strict conception of linguistic meaning made him reject the possibility of contingent analyticity:

> I am presupposing that an analytic truth is one which depends on *meanings* in the strict sense and therefore is necessary as well as *a priori*. If statements whose *a priori* truth is known via the fixing of a reference are counted as analytic, then some analytic truths are contingent; this possibility is excluded in the notion of analyticity adopted here. (Cf . Kripke 1980, p. 122, fn. 63, his italics.)

On this point, my position is different: It allows for analytic truths that are not necessary. However, in the approach I present, nothing hinders introducing a notion that corresponds to Kripke’s more demanding understanding of analyticity: We can say that a sentence A is *analytically necessary* on an interpretation $\boldsymbol{i}$ iff, for every S in Σ, A is necessary in (S, $\boldsymbol{i}$).[13]

Nowadays, the official definitions of length units usually refer to fundamental physical values instead, such as the distance actually travelled by light per unit of time. Nevertheless, these definitions also give rise to contingently analytic sentences, if we allow for possible worlds that violate physical laws or possible worlds in which physical constants have other values than in our world (say, worlds in which the speed of light is different).

So, analyticity does not entail necessity. Nor does the opposite entailment hold: The necessity of a sentence in a model M = (S, $\boldsymbol{i}$) does not entail its analyticity on interpretation $\boldsymbol{i}$. To see why a sentence A that is necessary in M need not be analytic, note that, as we vary the frame, the range of possible worlds may expand. If the designated world in the new model represents such a new possibility, then A might well be false in that world, which would make it false in the new model. Another kind of cases in which the necessity of A in M = (S, $\boldsymbol{i}$) is compatible with its falsity in $M’$ = ($S’$, $\boldsymbol{i}$) has to do with the role of the designated world in determining the semantic value of linguistic expressions. While a sentence such as “Water is H_2O” is necessary in a model in which the term “water” picks out H_2O in the designated world, this sentence will be false (and necessarily so) in a model in which the designated world is like Putnam’s Twin Earth, with a different chemistry from ours, but with the same

[13] Below, more will be said about the relationship between analyticity and necessity.

phenomenal appearance. For that choice of the designated world, the term “water” picks out a different chemical substance. The reason for this difference is that that the analytical criterion for ‘water’ relies on phenomenal characteristics. ‘Water’ is whatever exhibits these characteristics in the actual world. On Twin Earth, this criterion picks out, say, XYZ, instead of H_2O.

As things actually stand, expressions such as “water” and “H_2O” are necessarily co-extensive, which means that they have the same intension. However, they are not synonymous. Obviously, analyticity and synonymy are closely related notions. In our semantics, synonymy can be defined as the identity of ultraintensions. It is then easily seen that predicates F and G are synonymous iff the sentence “$\forall x\ (F(\mathrm{x}) \leftrightarrow G(\mathrm{x})$” is analytic. Similarly, of course, for the synonymy of sentences: Sentences A and B are synonymous (i.e. have the same ultraintension) iff the equivalence $A \leftrightarrow B$ is analytic. One might say, therefore, that ultraintensions correspond in our approach to something like Frege’s *Sinne*. As for the term “ultraintension”, I have chosen it here to signal the connection between the notion we are after and the so-called ‘ultraintensional’ sentential operators. While in the context of intensional operators (such as $\Box$ or Actually) co-intensional sentences are freely substitutable *salva veritate,* the corresponding free substitution in the context of ultraintensional operators (such as ‘S believes that …”) requires not just co-intensionality but synonymy.

3. Analytic and apriori

What is the relationship between analytic and a priori? Given the new concept of interpretation, it is easy to provide a definition of a priori that is essentially equivalent to the two-dimensional account of that notion. On the two-dimensional account, as we remember, a sentence A is a priori in a model iff for every world w in that model, A is true at w from w’s own perspective. Changing perspective amounts to changing the actual world in the model. On the current account, we can therefore express the same idea without going two-dimensional. Instead, to determine whether a sentence is a priori, we consider all models that differ from a given one only with respect to the designated world. Thus, all the other components of the frame *and* the interpretation remain the same. Apriority is then defined as truth is all such variants of a given model:

A sentence A is *a priori* in $M = (<W, \omega, I, D>, \boldsymbol{i})$ iff for every $w \in W$, A is true in $(<W, w, I, D>, \boldsymbol{i})$.

Note that this definition presupposes the closure condition we have imposed on frames. For the definition to work, Σ must be closed under variation of the designated world.

It immediately follows that analyticity (on a given interpretation) entails apriority (in any model that involves this interpretation).[14] This is as it should be, of course. On the other hand, the opposite entailment does not hold: A sentence may be a priori in a model (S, $\boldsymbol{i}$) without being analytic on $\boldsymbol{i}$. It may be false in some other model based on the same interpretation, if the frame of that model differs from S in other ways than just with respect to the designated world. Thus, on this approach, apriority does not entail analyticity: There is at least in principle room for 'synthetic a priori'.[15]

4. Conceptual map

We arrive at the following map of entailment relations:
logical truth $\Rightarrow$ analyticity on an interpretation i $\Rightarrow$ apriority in a model (S, $\boldsymbol{i}$) $\Rightarrow$ truth in a model (S, $\boldsymbol{i}$).

The necessity analogues of these notions are related accordingly:

logical necessity $\Rightarrow$ analytic necessity on an interpretation i $\Rightarrow$ apriori necessity in a model (S, $\boldsymbol{i}$) $\Rightarrow$ necessity in a model (S, $\boldsymbol{i}$).

Clearly, entailments hold not only within each chain but also between the chain: Each member of the latter chain entails the corresponding member of the former. Logical necessity entails logical truth, analytic necessity entails analyticity, and so on.

What about potential entailments going in the other direction, from the members of the first chain to their necessity analogues in the second chain? Obviously, no such entailments hold in the general case. As we have seen, not even logical truths need to be necessary. However, things are different for a special class of sentences which we might call 'perspective-independent', as their truth value in different worlds does not depend on the perspective from which they are evaluated. That is, their truth value at worlds does not depend on the choice of the designated world. To put it formally:

[14] In fact, as is easily seen, analyticity on a given interpretation is equivalent with apriority in all models that involve this interpretation.

A sentence A is *perspective-independent* in a model $M = (<W, \omega, I, D>, \boldsymbol{i})$ iff

for all $w, v \in W$, A is true at w in $(<W, \omega, I, D>, \boldsymbol{i})$ iff A is true at w in $(<W, v, I, D>, \boldsymbol{i})$.[16]

A sentence A is *strictly perspective-independent* on an interpretation $\boldsymbol{i}$ iff it is perspective-independent in every model based on $\boldsymbol{i}$.

A sentence A is *logically perspective-independent* iff it is strictly perspective-independent on every interpretation.

It can be shown that for perspective-independent sentences in a given model, apriority entails apriori necessity. Similarly, for strictly perspective-independent sentences, analyticity entails analytic necessity. And, for logically perspective-independent sentences, logical truth entails logical necessity.

Observation:

(i) For every model M and every perspective-independent sentence A in M,
if A is apriori in M, then $\Box A$ is apriori in M.

(ii) For every interpretation $\boldsymbol{i}$ and every strictly perspective-independent sentence A on $\boldsymbol{i}$,
if A is analytic on $\boldsymbol{i}$, then $\Box A$ is analytic on $\boldsymbol{i}$.

(iii) For every logically perspective-independent sentence A,
if A is logically true, then $\Box A$ is logically true.

Proof of (i): Suppose that A is both perspective-independent and apriori in $M = (<W, \omega, I, D>, \boldsymbol{i})$. Consider any model M' that differs from M only by the choice of the designated world. To establish that $\Box A$ is apriori in M, we need to show that $\Box A$ is true in M'. Consider therefore any world w in M'. We have to show that (i) A is true at w in M'. By perspective-independence of A in M, (i) will be the case iff (ii) A is true at w in M, which in its turn will be the case iff (iii) A is true at w in $M'' = (<W, w, I, D>, \boldsymbol{i})$. But (iii) must be the case if A is apriori in M.

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[15] Note, however, that the notions of apriority and analyticity will coincide in scope in the limiting case in which admissible frames are allowed to differ only with respect to the designated world.

[16] The following is a seemingly stronger, but in fact equivalent definition:
A sentence A is *perspective-independent* in a model $M = (<W, \omega, I, D>, \boldsymbol{i})$ iff
for all $w, v, u \in W$, A is true at w in $(<W, v, I, D>, \boldsymbol{i})$ iff A is true at w in $(<W, u, I, D>, \boldsymbol{i})$.

Proof of (ii): Since strict perspective-independence on *i* equals perspective-independence in all models based on ***i***, and since analyticity on ***i*** coincides with apriority in all models that are based on ***i***, (ii) immediately follows from (i).

Proof of (iii): Since logical perspective-independence equals strict perspective-independence on every interpretation, and since logical truth coincides with analyticity on every interpretation, (iii) immediately follows from (ii).

The notion of perspective-independence is thus quite important. On the traditional view, apriori truths, analytical truths and logical truths are all necessary. This view is wrong as a general claim, but it is right in the area of perspective-independent sentences.

5. Loose ends

As was suggested above, the standard two-dimensional account of a priori might be criticized. It is not clear whether it adequately captures the fundamentally epistemic character of a priori. The same problem arises, of course, for our one-dimensional analogue of the two-dimensional account. For this reason, connections between analyticity and a priori require further investigation.

In this paper I haven't dealt with Quine's famous criticism of analyticity (cf. Quine 1951). I don't think my account would help to allay his worries regarding the meaningfulness of this notion. Another problem I haven't dealt with concerns the relationship between my account of analyticity and the well-known post-Kantian explications of that concept. I have in mind such proposals as Frege's, according to whom a sentence is analytic if it is derivable from logical truths plus definitions,[17] or Carnap's, for whom analyticity consists in derivability from logical truths together with the 'meaning postulates' of the language.[18] My own proposal, I suppose, is more or less co-extensive with Carnap's. From the perspective of this paper, his meaning postulates might be understood as specifications of interrelations between the ultraintensions of different linguistic expressions. But this is, of course, not the way he

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[17] Frege (1960 [1884]), p. 4: "The problem becomes, in fact, that of finding the proof of the proposition, and of following it up right back to the primitive truths. If, in carrying out this process, we come only on general logical laws and on definitions, then the truth is an analytic one, bearing in mind that we must take account also of all propositions upon which the admissibility of any the definitions depends. If, however, it is impossible to give [such a] proof […], then the proposition is a synthetic one."

[18] See Carnap (1956), pp. 222-9. (There, Carnap reprints as a supplement his paper on meaning postulates, Carnap (1952), in which he first presented this idea.)

treats these postulates himself. Carnap solves the problem of analyticity by *fiat*, so to speak. He simply assumes an inventory of meaning postulates, i.e. a list of sentences that specify all the analytic but non-logical connections between linguistic expressions. This seems to me rather unsatisfactory, not only because such an account of analyticity is too far removed from the level of semantics (Carnap himself, however, would not agree with me on this point), but also because for rich languages their meaning postulates might well be too complex and numerous to allow codification. Be that as it may, a more detailed comparison between my own account of analyticity and other accounts of that notion, Carnap's included, must await another occasion.

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