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Abstract

The study illustrates how joint production theory can be applied in estimating the profitability of fractionating industrial solid wastes, a given product and the wastes produced in connection with its manufacture being regarded as a production-planning unit. Two case studies showing how the approach described can be applied both to bulk manufacturing and to the manufacture of technically complicated products are presented. The realism of this approach and the contribution it can make to optimize the separation of industrial solid waste fractions in manufacturing processes of different types, together with associated financial considerations, are discussed.

Key words - Industrial solid waste management, joint production.

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1. Introduction

Joint production theory, which among other things concerns the optimal output proportions to aim at obtaining when desirable products and wastes are jointly produced in the same process, makes frequent use of the linear programming technique. The objective of this paper is to illustrate possibilities for using joint production theory for estimates of profitability in connection with the fraction management of industrial solid waste. In line with this, a new approach to calculating the profit of wastes that are jointly produced is presented. This involves maximizing a mathematical function obtained by adding the profits (or losses) that the products and the related wastes result in, an approach that represents application mathematically of the principle of regarding each product and its related wastes as a unit in terms of production planning.

2. Methodology

In the present paper, joint production theory as it relates to profitability estimation in the separation of industrial waste fractions is reviewed. Two case studies are taken up, dealing with the applicability of the theory to two somewhat typical industrial scenarios concerning the manufacture of real products by existing companies: (1) a company manufacturing a very limited number of bulk products, all of them largely produced from the same type of raw materials, and (2) a company manufacturing many different products, most of them mechanically rather complicated. The profit maximizing method considered is applied to the two scenarios. Conclusions based on both the theoretical and the practical parts of the paper are drawn regarding the appropriateness of using joint production theory as a basis for profitability estimates in the separation of industrial solid waste fractions.

3. Joint production

Generically, pollution is a problem involving the joint outputs of economic activities concerned with production and consumption (Førsund, 1998). The various products of the joint production that takes place (including the waste that is produced) are interrelated through their having to share the capacity of the fixed equipment (Danø, 1966). In terms of operations research methodology, the problem can be stated as a general linear programming problem in the standard form that follows:

Find the values of $x_1, x_2, \ldots, x_n$ that will

maximize $z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$ (1)

subject to the following constraints:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1 \quad (2)$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m$$

where

$x_j \ge 0, j = 1, 2, \ldots, n.$ (3)

a_{mn} are constant coefficients of production

It is convenient to write linear programming problems in matrix notation. A standard problem can be written as follows:

Find a vector $x \in R^n$ that will

maximize $z = c^T x$ (4)

subject to

$Ax \le b$ (5)

$x \ge 0$

where

A is an m by n matrix

c and x are n by 1 column vectors

b is an m by 1 column vector

A vector $x \in R^n$ satisfying the constraints of a linear programming problem is called a feasible solution. A feasible solution that maximizes or minimizes the objective function of a linear programming problem is called an optimal solution. Generally, a problem in standard form can be thought of as a manufacturing problem, one in which scarce resources are allocated in a way that maximizes profit (Kolman and Beck, 1995).

4. Waste fraction separation profitability calculation

The problem can be stated in general terms for the multi-product case as follows:

Maximize $z = \sum p_j x_j - \sum q_i v_i$ (the profit function) (6)

where x_j are the outputs and v_i are the inputs

$(j = 1, 2, \ldots, n), (i = 1, 2, \ldots, m)$

subject to the following technological restrictions:

$F_k (x_1, x_2, \ldots, x_n; v_1, v_2, \ldots, v_m) = 0, (k = 1, 2, \ldots, M)$ (7)

where the production function F_k is assumed to possess continuous first and second order partial derivatives and is defined as giving the maximum amount of any product – say, x_n - that can be produced with the given technology for any feasible combination of m+n-1 independent variables $x_1, \ldots, x_{n-1}, v_1, \ldots, v_m$

and where $1 \le M \le m$

and the following non-negativity requirements:

$$x_j \geq 0,\ v_i \geq 0 \qquad (8)$$

It is assumed that the product prices p_j and the factor prices q_i are constant (in the case of fixed factors that are usually equal to zero), so that the profit function is linear (Danø, 1966).

Application of the method just described involves considering there to be different possible scenarios. A particular waste fraction is studied within a given production scenario, one which involves in part a set of different waste fractions with which various revenues and costs are associated. The outcome of the profitability analysis for a given fraction guides the decision of whether the fraction in question is to be the object of separation. Such assessments are performed again for any further fraction within the set of fractions to be examined in terms of profitability. In any given scenario, therefore, a new assessment of profits and losses is required for each further fraction considered.

Note that since it is a question of making a choice between different products that can be produced, each product and the wastes related to it need to be regarded as a unit. It is also assumed that the quantities of various wastes related to a particular product are known and are constant per unit of time. This allows the problem to be expressed in the following way, which is adapted to the production of waste:

Find the values of $x_1, x_2, \ldots, x_n$ that will

$$\text{maximize } z = (p_1' + p_1'')\, x_1 + (p_2' + p_2'')\, x_2 + \ldots + (p_j' + p_j'')\, x_j \qquad (9)$$

where
p_j' = profit (or loss, which produces a negative value) per unit of the product j or of the input j
p_j'' = profit (or loss, which produces a negative value) from all the waste related to one unit of the product j or of the input j,

subject to the following constraints:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\leq b_1 \qquad (10)\\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\leq b_2 \\ &\text{-----} \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\leq b_m \end{aligned}$$

where

$$x_j \geq 0,\ j = 1, 2, \ldots, n \qquad (11)$$

a_{mn} are constant coefficients of production

This allows the most profitable product mix of the n products and the related wastes to be calculated, and also the total profit to be estimated by multiplying the calculated profit-maximizing amounts of the product by the marginal contribution of each of the n products and the related wastes. Mathematically, this can be expressed as follows:

Find the values of $x_1, x_2, \ldots, x_n$ that will
maximize $CM_{Tot} = (CM_1' + CM_1'')\, x_1 + (CM_2' + CM_2'')\, x_2 + \ldots + (CM_j' + CM_j'')\, x_j$ (12)

where

CM_{Tot} = total marginal contribution of the product and waste mix

CM_j' = marginal contribution per unit of the product j or of the input j

CM_j'' = marginal contribution from all the waste related to one unit of the product j or of the input j,

subject to the following constraints:

$$a_{11}\, x_1 + a_{12}\, x_2 + \ldots + a_{1n}\, x_n \le b_1 \quad (13)$$
$$a_{21}\, x_1 + a_{22}\, x_2 + \ldots + a_{2n}\, x_n \le b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m$$

where

$$x_j \ge 0,\ j = 1, 2, 3, \ldots, n. \quad (14)$$

a_{mn} are constant coefficients of production

What is new in this approach is the maximization stipulated in (9) and (12), which reflects the assumption, introduced earlier in the paper, of a product and the related wastes representing a unit.

5. Case studies

5.1 The two companies

The case studies to be presented concern two major Swedish companies. These are denoted simply as company “A” and company “B” so as to not to reveal any internal company figures that are to be kept secret. Whereas company A manufactures only a very limited number of products, all of them bulk products, largely through use of a single raw material, company B manufactures many different products that are mechanically rather complicated.

5.2 Data collection

Data collection involved interviews with the business manager and the production manager of each of the two companies so as to obtain certain economic information regarding the two companies, and the gathering of certain production statistics.

5.3 Application of the methods considered

In analyzing the data, analysis was confined to timeless production in the sense that inputs and outputs pertain to the same time period, all figures referring to the year 1998. The numerical values used are approximated. An exchange rate of \$1 = SEK (Swedish crowns) 7.70 is assumed throughout the text. To achieve the degree of simplification necessary in the examples, there is assumed to be only one kind of waste connected with the production of each of the main products. Obviously, in a concrete situation one would need to adapt the approach described to the facts at hand.

In the first case considered, there were about 7,000 tonnes of waste altogether connected with the manufacture of the two products, these being produced in the amounts of x_1 and x_2 tonnes, respectively. The aim involved can be expressed mathematically as that of maximizing the gross profit (z) for the joint production of products 1 and 2 and for the wastes connected with them. A given product and the associated waste are dealt with as a unit in production planning: This results in the following:

Find the values of x_1, x_2 that will

maximize $z = (p_1' + p_1'')\, x_1 + (p_2' + p_2'')\, x_2$ (15)

where

p_1' = per unit gross profit from product 1 = \$170

p_1'' = gross profit from the waste in connection with the manufacture of one unit of product 1 = \$20

p_2' = per unit gross profit from product 2 = \$140

p_2'' = gross profit from the waste in connection with the manufacture of one unit of product 2 = \$0,

subject to the following constraints:

$a_{11}x_1 + a_{12}x_2 \leq b_1$ (time constraint) (16)

$a_{21}x_1 + a_{22}x_2 \leq b_2$ (budget constraint)

where $x_j \geq 0$, j = 1, 2 and (17)

a_{11} = time needed to produce one tonne of product 1 = 45 seconds (s)

a_{12} = time needed to produce one tonne of product 2 = 90 s

b_1 = total time available for production = 30,000,000 a_{21} = cost of producing one tonne of product 1 = \$360

a_{22} = cost of producing one tonne of product 2 = \$260

b_2 = total budgeted costs for production = \$286 million

The calculations for company A indicate it to be best to produce 666,000 tonnes of product 1 (= x_1) and null (0) tonnes of product 2 (= x_2), yielding a gross profit of \$126.5 million (= z). In fact, company A produced approx. 650,000 tonnes of product 1 and 4,000 tonnes of product 2, yielding a gross profit of approx. \$124.1 million. The following year, product 2 was dropped from the product range due to its obvious profit-reducing impact as compared with product 1.

Calculations regarding the waste at company B concerned metal cutting chips stemming from the production of three differently sized products produced in the amounts of x_1, x_2 and x_3 units, respectively. The aim involved here can be expressed mathematically as that of maximizing the gross profit (z) for the joint production of products 1, 2 and 3 and for the wastes connected with them. A given product and the associated waste are dealt with as a unit in production planning: This results in the following:

Find the values of x_1, x_2, x_3 that will

maximize $z = (p_1' + p_1'')\, x_1 + (p_2' + p_2'')\, x_2 + (p_3' + p_3'')\, x_3$ (18)

where

p_1' = per unit gross profit from the smallest product = \$500

p_1'' = gross profit from the chip-waste in connection with the manufacture of one unit of the smallest product = \$0

p_2' = per unit gross profit from the medium-sized product = \$1,700

p_2'' = gross profit from the chip-waste in connection with the manufacture of one unit of the medium-sized product = \$1

p_3' = per unit gross profit from the largest product = \$1,600

p_3'' = gross profit from the chip-waste in connection with the manufacture of one unit of the largest product = \$3,

subject to the following constraints:

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \le b_1$ (time constraint) (19)

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \le b_2$ (budget constraint)

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \le b_3$ (chip waste constraint)

where $x_j \ge 0$, j = 1, 2, 3 and (20)

a_{11} = time needed to produce one unit of the smallest product = 3,500 s

a_{12} = time needed to produce one unit of the medium-sized product = 3,500 s

a_{13} = time needed to produce one unit of the largest product = 3,600 s

b_1 = total time available for production = 34,000,000 s

a_{21} = cost of producing one unit of the smallest product = \$1,800

a_{22} = cost of producing one unit of the medium-sized product = \$3,600

a_{23} = cost of producing one unit of the largest product = \$6,300

b_2 = total budgeted costs for production = \$36 million

a_{31} = chip waste from production of one unit of the smallest product = 48 kg

a_{32} = chip waste from production of one unit of the medium-sized product = 88 kg

a_{33} = chip waste from production of one unit of the largest product = 127 kg

b_3 = total chip waste production due to resource use efficiency targets and production planning budget targets = 636,000 kg

The calculations for company B indicate it to be best to produce no (0) units of the smallest product (= x_1), 7,227 units of the medium-sized product (= x_2) and no (0) units of the largest product (= x_3), yielding a gross profit of $12.3 million (= z). According to the calculations, the company would thus fare best financially if it restricted its manufacture to that of the medium-sized product. What company B in fact produced were 6,105 units of the smallest product, 2,505 units of the medium-sized product and 940 units of the largest product, yielding a gross profit of approx. $8.8 million.

6. Discussion and conclusions

The paper illustrates how joint production theory can be applied in estimating the profitability of fractionating industrial solid wastes. In both case studies, the calculated product mix is somewhat extreme, since the conclusion implied in each case is that only one kind of product should be produced. This is due, however, to the maximization principle inherent in the linear programming algorithm employed, which takes no account of the increase in business contacts which a more diversified product range can result in or of corporate product-diversification strategies adapted to customer orders. Such considerations can and should be expressed mathematically as additional, possibly non-linear constraints that can help provide as adequate a basis as possible for management decisions. In the present paper, only elementary constraints are employed, so that the degree of simplification needed to make the examples easy to grasp can be achieved. However, a more in-dept study can be done in the future. In both case studies, nevertheless, the product mix calculated to be best results in an increase in gross profit as compared with the gross profit for the product mixes currently produced. This implies the approach suggested to be a promising one, since the profit maximization issue is of paramount importance to corporate well being. As the case studies also show, the approach described can be applied both to bulk manufacturing and to the manufacture of technically complicated products. Linear programming can thus be shown, if handled properly, to be a very useful tool generally in the preparing of decisions. The present findings indicate the potential usefulness of a joint production theory approach to waste management, one that attempts to optimize financial aspects of the industrial fractionation of solid wastes.

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