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# Combined Linear-Viterbi Equalizers – A Comparative Study and A Minimax Design

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**Abstract:** Combined linear-Viterbi equalizer (CLVE) is a term often used for a class of digital receivers reducing the complexity of the Viterbi detector by assuming an approximate channel model together with linear pre-equalization of the received data.

We reconsider a weighted least squares design technique for CLVEs by introducing a minimax criterion for suppressing the strongest component of the residual intersymbol interference. Previously, in [2], we have studied the performance of some proposed CLVE design methods and evaluated them by simulated bit error rates. Here we investigate the performance of the minimax design and of the CLVE designs found in literature [3, 4, 5, 6] for two GSM test channels.

We also present a comparison of the CLVE designs based on a common quadratic optimization criterion for the selection of the channel prefilter and the desired impulse response.

## 1 INTRODUCTION

The Maximum Likelihood Sequence Detector (MLSD) is a procedure for estimating a sequence of bits from a sequence of channel output observables, given a model of the communication system. In the presence of *intersymbol interference* (ISI) the Viterbi algorithm (VA) provides an efficient way of computing the MLSD [7, 8, 9]. However, the VA still becomes impractical when the time spread of the ISI is large because of the exponential relation between ISI time spread and VA complexity.

The complexity of a Viterbi detector can be reduced by giving the VA an approximate channel model with a shorter time spread than that of the original channel. The class of receivers employing this technique together with linear pre-equalization of the received data

are often referred to as *combined linear-Viterbi equalizers* (CLVEs), see [1, 3, 4, 5, 6, 2]. Other classes of receivers addressing the same complexity problem can be found in *e.g.* [10, 11, 12, 13]. In this paper we focus on CLVEs.

When designing CLVEs it is often desirable to minimize the bit error rate of the receiver. The bit error probability depends on the design parameters, such as the channel model and the impulse response of the prefilter, in a complicated and non-linear way. The prefiltering of the received data perturbs the signal space and colours the channel noise. Ignoring this colouring or giving the VA an approximate channel model result in a displacement of the decision regions from their optimal locations, *cf. the residual ISI* in [14]. Instead of using the bit error rate as a design criterion, other more feasible criteria are used in CLVE design methods [1, 3, 4, 5, 6, 2].

In this paper we investigate the performance of a minimax CLVE design. Three other principal techniques for designing CLVEs [3, 4, 6] are also overviewed and compared.

## 2 THE TRANSMISSION SYSTEM MODEL

A continuous time model of a transmission system as described in [7] may, without information loss, be represented by a discrete time model as in Fig. 1. In block

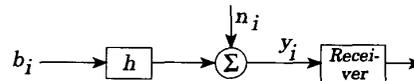


Figure 1: Discrete transmission system model.

transmission systems blocks of information are transmitted so that the decoding can be performed indepen-

dently on each block, see *e.g.* [15]. For such systems it is convenient to use the matrix formalism [16]

$$\mathbf{y} = \mathbf{H}\mathbf{b} + \mathbf{n}, \quad (2.1)$$

where the transmitted message is coded antipodally in  $\mathbf{b} \in \{-1, +1\}^N$  and the channel observables are  $\mathbf{y} \in \mathbb{R}^{(N+L-1)}$ . The time invariant channel impulse response, *i.e.* the ISI coefficients, is represented by a Toeplitz band matrix  $\mathbf{H}$ . It is a known  $(N+L-1) \times N$  matrix with the impulse response  $\mathbf{h}$ , of length  $L$ , in its columns, arranged so that matrix multiplication corresponds to convolution. The noise vector  $\mathbf{n} \in \mathbb{R}^{(N+L-1)}$  is a jointly Gaussian, zero mean random vector with a  $N(\mathbf{0}, \mathbf{R}_n)$  distribution, where  $\mathbf{R}_n$  denotes the noise covariance matrix.

### 3 CLVE DESIGN

The performance evaluation of the CLVEs in this paper is performed on block transmission systems, but historically the design methods have focused on continuous transmission systems and VAs with infinite horizon, *i.e.*  $\{b_i\}_{i=-\infty}^{\infty}$ . To relate to existing methods, we have chosen to confine the development of the *minimax* design to a criterion based on sequences of infinite length. Some remarks on CLVE design for block transmission systems can be found in section 5.

A design model is presented in Fig. 2, *cf.* [6] where  $q$  is the *desired impulse response* (DIR), *i.e.* the channel model given to the VA. If the time delay, modelled by the filter  $d$ , is zero, the system is equivalent to the one found in [3]. Since this paper only considers causal channel impulse responses, the introduction of the time delay offers the possibility of placing the energy of the DIR in an arbitrary position. This possibility is accounted for in [3, 4, 5] by allowing the channel impulse response anti-causal components.

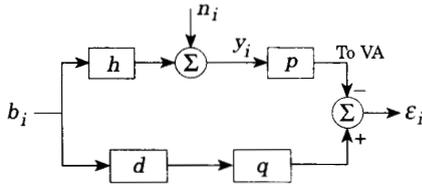


Figure 2: Discrete design model for infinite sequences.

The error  $\varepsilon_i$ , in Fig. 2, can be expressed as

$$\varepsilon_i = [b * (h * p - d * q) + n * p]_i, \quad (3.2)$$

where  $*$  denotes convolution. By using  $\|\mathbf{x}\|_{\mathbf{M}}^2 := \mathbf{x}^T \mathbf{M} \mathbf{x}$ , the variance of  $\varepsilon_i$  is given by

$$E\{\varepsilon_i^2\} = \|\mathbf{H}\mathbf{p} - \mathbf{D}\mathbf{q}\|_{\mathbf{R}_b}^2 + \|\mathbf{p}\|_{\mathbf{R}_n}^2, \quad (3.3)$$

where  $\mathbf{p}$  and  $\mathbf{q}$  are vectors containing the impulse response of the prefilter  $p$  and the DIR  $q$  respectively, and  $\mathbf{R}_b$  and  $\mathbf{R}_n$  are covariance matrices for the transmitted sequence  $b$  and the noise  $n$ .  $\mathbf{H}$  and  $\mathbf{D}$  are Toeplitz band matrices such that the multiplications  $\mathbf{H}\mathbf{p}$  and  $\mathbf{D}\mathbf{q}$  describe convolution. Since the length of  $h * p$  is greater than the length of  $d * q$ , the subtraction  $\mathbf{H}\mathbf{p} - \mathbf{D}\mathbf{q}$  in (3.3) necessitates that the size of  $\mathbf{D}$  is chosen such that the dimensions agree. The residual ISI is defined by this difference, as

$$\text{residual ISI} := \mathbf{H}\mathbf{p} - \mathbf{D}\mathbf{q}, \quad (3.4)$$

*i.e.* as the ISI that is not accounted for in the VA, *cf.* [14].

#### 3.1 A SELECTION OF PREFILTER DESIGNS

In this section we compare design methods for  $\mathbf{p}$  and  $\mathbf{q}$ . The methods are due to Falconer and Magee [3], Fredricsson [4] and Ödling *et al.* [6], where the last one is a weighted least squares method inspired by [17].

By introducing a weighting matrix  $\mathbf{W}$  in (3.3) selective weighting of the residual ISI becomes possible. This is used to formulate the design criterion

$$J(\mathbf{p}, \mathbf{q}) := \|\mathbf{W}(\mathbf{H}\mathbf{p} - \mathbf{D}\mathbf{q})\|_{\mathbf{R}_b}^2 + \|\mathbf{p}\|_{\mathbf{R}_n}^2. \quad (3.5)$$

In the sequel, both the channel input and the channel noise are assumed white and stationary, hence  $\mathbf{R}_b = \mathbf{I}$  and  $\mathbf{R}_n = \sigma_n^2 \mathbf{I}$ . Completing the square in (3.5) gives

$$J(\mathbf{p}, \mathbf{q}) = \|(\mathbf{p} - \mathbf{p}_o)\|_{\mathbf{A}}^2 + \|\mathbf{q}\|_{\mathbf{B}}^2, \quad (3.6)$$

where

$$\begin{aligned} \mathbf{A} &= \mathbf{H}^T \mathbf{W}^T \mathbf{W} \mathbf{H} + \sigma_n^2 \mathbf{I}, \\ \mathbf{B} &= \mathbf{D}^T \mathbf{W}^T (\mathbf{I} - \mathbf{W} \mathbf{H} \mathbf{A}^{-1} \mathbf{H}^T \mathbf{W}^T) \mathbf{W} \mathbf{D}, \\ \mathbf{p}_o &= \mathbf{A}^{-1} \mathbf{H}^T \mathbf{W}^T \mathbf{W} \mathbf{D} \mathbf{q}. \end{aligned}$$

In the presence of noise, the matrices  $\mathbf{A}$  and  $\mathbf{B}$  are positive definite, so the minimum of (3.6), with respect to  $\mathbf{p}$ , is obtained if

$$\mathbf{p} = \mathbf{p}_o = \mathbf{A}^{-1} \mathbf{H}^T \mathbf{W}^T \mathbf{W} \mathbf{D} \mathbf{q}, \quad (3.7)$$

with a residual error of  $\|\mathbf{q}\|_{\mathbf{B}}^2$ .

The most straightforward design approach is to assume  $\mathbf{W} = \mathbf{I}$ , *i.e.* a uniform weighting of the residual ISI, and to find the global minimum of (3.6) by using

(3.7) in combination with finding a  $\mathbf{q}$  that minimizes  $\|\mathbf{q}\|_{\mathbf{B}}^2$  under some constraint *e.g.*  $\|\mathbf{q}\| = 1$ . This is done by Falconer and Magee in [3], where the prefilter obtained is

$$\mathbf{p}_{FM} = \mathbf{A}^{-1} \mathbf{H}^T \mathbf{D} \mathbf{q}_{FM}, \quad (3.8)$$

and the DIR

$$\mathbf{q}_{FM} = \text{the normalized eigenvector corresponding to the smallest eigenvalue of } \mathbf{B}. \quad (3.9)$$

There are other methods that do not minimize (3.6), but still often render a lower probability of bit error than the method above. One such method was presented by Fredricsson in [4]. For the purpose of preserving the similarity to the expressions in the original reference, we present his result in the Fourier domain:

$$P_F(f) = \frac{Q_F(f) H^*(f)}{|H(f)|^2 + \sigma_n^2} \quad (3.10)$$

and

$$|Q_F(f)|^2 = \frac{|H(f)|^2 + \sigma_n^2}{\sigma_n^2}. \quad (3.11)$$

Equation (3.10) corresponds to (3.8) of Falconer and Magee. The DIR, however, is determined by spectrum matching the left hand side to the right hand side of equation (3.11) given a length constraint on  $q$ . This matching was addressed by Beare in [5] where an adaptive algorithm was presented.

A weighted least squares (WLS) design approach for  $\mathbf{p}$  and  $\mathbf{q}$  was presented by Ödling *et al.* in [6]. The DIR in this design method is assigned an exact copy of the corresponding positions of  $\mathbf{H}\mathbf{p}$ , thus giving the VA a correct channel model for those positions, *i.e.*

$$\mathbf{q}_{WLS} = \mathbf{D}^T \mathbf{H} \mathbf{p}_{WLS}. \quad (3.12)$$

The prefilter  $\mathbf{p}_{WLS}$  is chosen by modifying (3.7) to

$$\mathbf{p}_{WLS} = \mathbf{A}^{-1} \mathbf{H}^T \mathbf{W}^T \mathbf{W} \mathbf{D} \frac{\mathbf{q}}{q_l}, \quad (3.13)$$

where the position  $l$  is chosen as a suitable index in  $\mathbf{q}_{WLS}$ . By choosing the weighting matrix in (3.13) as

$$\mathbf{W} = \text{diag}(1 \dots 10 \dots 0.1 \ 0 \dots 0.1 \dots 1), \quad (3.14)$$

pos.  $k$

where the zeroes coincide with the DIR position and  $k$  is chosen so that it coincides with the  $l$ :th position of  $\mathbf{q}$ . This ensures that the trivial, all zero, solution for  $\mathbf{p}_{WLS}$  is avoided and that the energy in  $\mathbf{H}\mathbf{p}_{WLS}$  is concentrated to the DIR interval. Equation (3.13) can now be simplified to the form in which it was presented by Ödling *et al.*

$$\mathbf{p}_{WLS} = \mathbf{A}^{-1} \mathbf{H}^T \delta_k \quad (3.15)$$

where  $\delta_k$  is a vector with 1 in position  $k$  and zeroes elsewhere.

### 3.2 A MINIMAX DESIGN

The criterion of (3.5) has the total energy of the weighted residual ISI as one part. It can be discussed if also the distribution of the residual ISI is of any importance for the probability of bit error of the resulting receiver. Here we investigate an approach that takes into account the maximum absolute value of the residual ISI, thereby shaping the distribution of the same. Let us use this *minimax* approach and solve

$$\mathbf{p}_{MM} = \arg \min_{\mathbf{p}} \left\{ \max_i J'_i(\mathbf{p}) \right\}, \quad (3.16)$$

where  $J'_i(\mathbf{p})$  is the  $i$ :th element in the vector

$$J'(\mathbf{p}) = |\mathbf{W}(\mathbf{H}\mathbf{p} - \delta_k)|^2 + \sigma_n \|\mathbf{p}\|^2. \quad (3.17)$$

The expression  $|\mathbf{W}(\mathbf{H}\mathbf{p} - \delta_k)|$  denotes the vector of the absolute values of each element in  $\mathbf{W}(\mathbf{H}\mathbf{p} - \delta_k)$ . The DIR  $\mathbf{q}_{MM} = \mathbf{D}^T \mathbf{H} \mathbf{p}_{MM}$ , the weighting matrix  $\mathbf{W}$  and  $\delta_k$  are chosen as in the WLS design method.

## 4 SIMULATIONS

To evaluate the performances of the CLVEs for the different prefilter and desired impulse response design methods described in section 3, we have simulated the block transmission system of section 2. We have used two GSM test channels [18], the *Typical Urban area channel* (TU) and the *Rural Area channel* (RA).

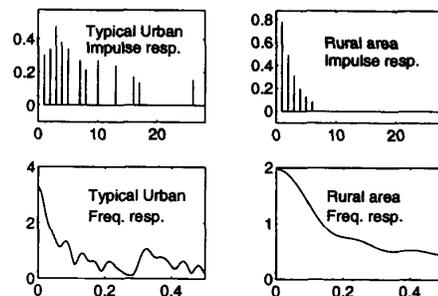


Figure 3: Impulse and frequency responses of the GSM Typical Urban channel (left) and Rural Area channel (right).

When implementing the minimax design, we have used a sequential quadratic programming method provided by the "minimax" routine of the MATLAB™ Optimization Toolbox™ [19] to solve (3.16), starting with an initial prefilter  $\mathbf{p}$  given by (3.13).

To determine the prefilter  $\mathbf{p}$  and the DIR  $\mathbf{q}$  of Fredricsson, we have solved equation (3.11) by means

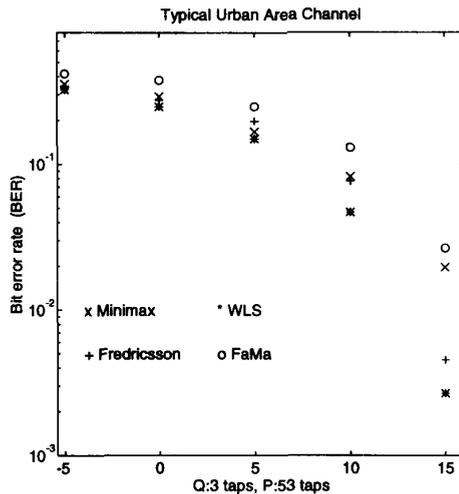


Figure 4: Bit error rate for the GSM Typical Urban channel.

of a least squares fit of the spectrum coefficients of  $q$  to the right hand side of (3.11). The DIR  $q$  were then calculated analytically from these coefficients. The RA channel has a smooth frequency response and is comparatively easy to match. The matching of  $q$  to the TU channel is more difficult due to the spectral dips of this channel (see Fig. 3). As a consequence, at high SNRs, the number of points used in the least squares fit might have to be adjusted in order to obtain real coefficients.

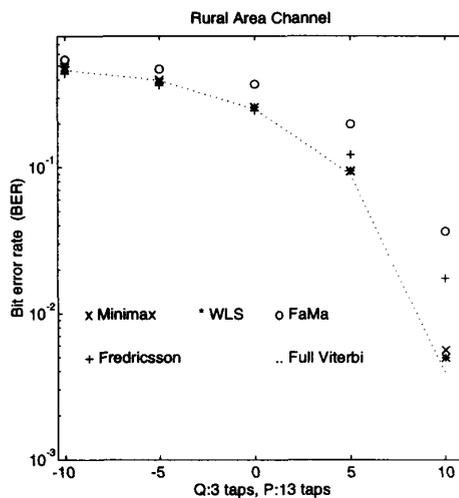


Figure 5: Bit error rate for the GSM Rural area channel.

Simulation results are shown in Fig. 4 and 5 for

CLVE designs by the Falconer and Magee method [3] ('o'), the WLS method [6] ('\*'), the Minimax method ('x') and the method of Fredricsson [4] ('+'). The simulations on the RA channel are also compared with the full complexity Viterbi decoder [7] (dotted line in Fig. 5). Such a comparison was not feasible in simulations on the TU channel, because of this channel's length. In all cases we have chosen the length of the desired impulse response to 3 coefficients and the block length to 50 bits. The prefilter length is set to one coefficient more than twice the ISI, *i.e.* 13 coefficients for the RA channel and 53 for the TU channel.

## 5 CONCLUSIONS AND FUTURE WORK

The basic idea in most design methods for CLVEs is reflected by the criterion  $J(p, q)$ . The main difference between the methods presented in the literature [3, 4, 5, 6] lies in the way the DIR  $q$  is chosen. Falconer and Magee find the global minimum of the criterion under the constraint  $\|q\| = 1$ . The WLS method and the method proposed by Fredricsson has slightly different approaches where the subsequent processing of data by the VA is taken into consideration when applying the criterion. Fredricsson uses a projection on the minimum distance error sequence giving an *effective signal to noise ratio* [7] tailored for the VA. The WLS method ensures that the DIR given to the VA is a true replica of the total system impulse response in the corresponding time interval.

In our simulations the WLS receiver shows a superior performance and is indeed close to the full complexity Viterbi decoder. The receiver proposed by Fredricsson performs well on the TU channel, which is slightly surprising considering the spectral shape of this channel. The results in the presented simulations agree with applicable observations in [4, 5] and in our earlier investigations [6, 2].

The new method based on the minimax design was introduced as an attempt to shape the distribution of the residual ISI in a fashion favourable to the VA. Being similar to the WLS method it gives a truncated version of the total system impulse response as a DIR to the VA, but instead of minimizing the criterion  $J(p, q)$  it suppresses the largest residual ISI coefficient. The performance is almost up to par with the WLS receiver, which indicates a potential for the concept of shaping the residual ISI.

The hitherto discussed methods for CLVE design are derived for continuous transmission systems. However, many contemporary and future communication systems are of block transmission type, *e.g.* the cellular

telephone systems of Europe (GSM), Japan (JDC) and the USA (ADC). To our knowledge, there are today no CLVE design methods that take advantage of the structure of such systems. An increased understanding of the properties of block transmission systems could result in improved receivers with respect to bit error probability as well as reduced implementation complexity and cost. In CLVEs developed for block transmission systems this could be reflected by the time-invariant linear prefiltering being replaced by *e.g.* a general matrix multiplication, in order to utilize the "edge" effects at the block boundaries for performance improvement. The noise correlation due to the prefilter  $p$  is another important issue in connection with CLVEs, *cf.* [16]. This is recognized by Fredricsson and Beare, but not considered in the other described methods.

We regard the above issues as key components in the development of new CLVEs.

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