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Linear pre-coding performance in measured very-large MIMO channels

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***Abstract*—Wireless communication using very-large multiple-input multiple-output (MIMO) antennas is a new research field, where base stations are equipped with a very large number of antennas as compared to previously considered systems. In theory, as the number of antennas increases, propagation properties that were random before start to become deterministic. Theoretical investigations with independent identically distributed (i.i.d.) complex Gaussian (Rayleigh fading) channels and unlimited number of antennas have been done, but in practice we need to know what benefits we can get from very large, but limited, number of antenna elements in realistic propagation environments. In this study we evaluate properties of measured residential-area channels, where the base station is equipped with 128 antenna ports. An important property to consider is the orthogonality between channels to different users, since this property tells us how advanced multi-user MIMO (MU-MIMO) pre-coding schemes we need in the downlink. We show that orthogonality improves with increasing number of antennas, but for two single-antenna users there is very little improvement beyond 20 antennas. We also evaluate sum-rate performance for two linear pre-coding schemes, zero-forcing (ZF) and minimum mean squared error (MMSE), as a function of the number of base station antennas. Already at 20 base station antennas these linear pre-coding schemes reach 98% of the optimal dirty-paper coding (DPC) capacity for the measured channels.**

I. Introduction

Multiple-antenna (MIMO) technology for wireless communications is becoming mature and has been incorporated into many advanced standards such as HSPA and LTE [1]. Basically the more antennas the transceivers are equipped with, the better performance can be obtained in terms of data rate and link reliability. The price to pay is increased complexity of hardware and signal processing at both ends. In classical point-to-point single-user MIMO systems (SU-MIMO), the multiplexing gain may disappear when the signal power is low, relative to interference and noise, or in propagation environments with dominating line-of-sight or insufficient scatterers. SU-MIMO systems also require complex and expensive multiple-antenna terminals. Practical size limitations on terminals also limit the number of antennas that can be used and thereby the achievable multiplexing gains.

To overcome these drawbacks of SU-MIMO, multi-user MIMO (MU-MIMO) with single-antenna terminals and an unlimited number of base station antennas is investigated in [2]. This approach involves MU-MIMO operation with an infinite number of base station antennas in a multi-cell environment. It is shown that all the effects of uncorrelated noise and fast fading disappear, as does the intra-cell interference, and the only remaining impediment is the inter-cell interference due to pilot contamination. All of these motivate entirely new theoretical research on signal processing, coding and network design for such very-large MIMO systems. The vision put forward in [2] is that the base station array would consist of a very large number of small active antenna units, each of which uses extremely low power.

The assumption of an unlimited number of base station antennas in [2] greatly simplifies the theoretical analysis. In a practical system, however, the number of antennas cannot be arbitrarily large due to physical constraints. From a feasibility point of view, it is reasonable to ask how large the antenna array should be. The answer depends on the propagation environment, but in general, the asymptotic results of random matrix theory can be observed even for relatively small dimensions.

The analysis in [2] assumes that inner products between propagation vectors of different users grow at a lesser rate than inner products of propagation vectors with themselves, i.e., the user channels are asymptotically orthogonal. Experimental work is clearly of great importance to investigate the range of validity of this assumption. Therefore, in the initial phase of this new research in wireless communications, we study how well the measurements resemble the theoretical results and what benefits we can obtain at very-large, but limited, number of base-station antennas.

In the present paper, we study the linear pre-coding performance in measured very-large MIMO downlink channels. We consider a single-cell environment in which a base station with a very-large antenna array serves a number of single-antenna users simultaneously. The interference between cells and pilot contamination issues are therefore not addressed in this paper. Channel measurements were done with a 128-antenna base station in a residential area. To the best of the authors' knowledge, there are no other studies performed on this type of systems, with this high a number of antennas.

In Sec. II we describe our system model and define a number of measures. In Sec. III we describe the measurement setup and the residential-area environment where the measurements are performed. As a basis for our comparison of systems with more or less base station antennas we describe a number of pre-coding schemes in Sec. IV – both the linear zero-forcing (ZF) and minimum mean squared error (MMSE) pre-coders, as well as the optimal dirty-paper coding (DPC). System performance is then evaluated in Sec. V for different

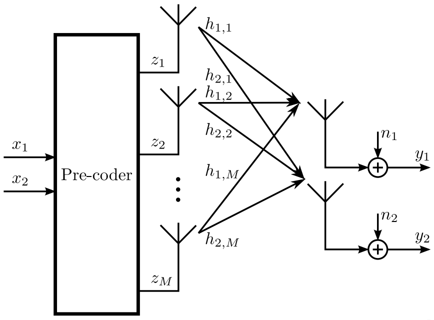

Fig. 1. System model of a MU-MIMO system with an M-antenna base station and two single-antenna users.

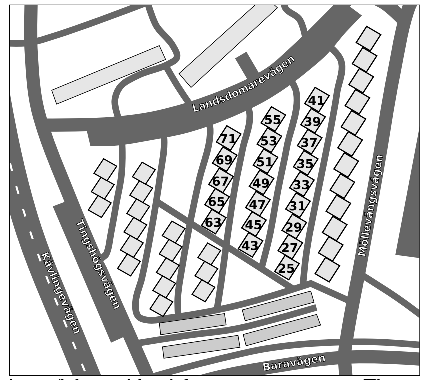

Fig. 2. Overview of the residential measurement area. The numbers indicate specific houses.

number of antennas and we show how the low-complex linear pre-coders perform in relation to the optimal DPC scheme. Finally, we summarize our contributions and draw conclusions in Sec. VI.

II. System description

We consider the downlink of a single-cell MU-MIMO system: the base station is equipped with M antennas, and serves K single-antenna users. The total transmit power is constrained to an average of P_{t}. The composite received $K\times 1$ vector $\boldsymbol{y}$ at the users can be described as

$$\boldsymbol{y} = \sqrt{\rho}\boldsymbol{H}\boldsymbol{z} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{H}$ is a composite $K \times M$ channel matrix, $\boldsymbol{z}$ is the transmitted vector across the M antennas, and $\boldsymbol{n}$ is a noise vector with unit variance. The variable ρ contains the transmit energy and channel energy so that the total power in $\boldsymbol{H}$ is K and $\boldsymbol{z}$ satisfies $\mathbb{E}\left\{\|\boldsymbol{z}\|^2\right\} = 1$. The $M\times 1$ transmit vector $\boldsymbol{z}$ contains a pre-coded version of the $K\times 1$ data symbol vector $\boldsymbol{x}$. Through pre-coding at the transmit side we have

$$\boldsymbol{z} = \boldsymbol{U}\boldsymbol{x}, \tag{2}$$

where $\boldsymbol{U}$ is a $M \times K$ pre-coding matrix including power allocation to the data symbols. The vector $\boldsymbol{x}$ comprises data symbols from an alphabet $\mathcal{X}$, and each entry has unit average energy, i.e. $\mathbb{E}\left\{|x_k|^2\right\} = 1,\ k = 1, 2, \ldots, K$. Taken together, the energy constraints on $\boldsymbol{x}$ and $\boldsymbol{z}$ yield an energy constraint on $\boldsymbol{U}$: $\mathrm{Tr}\left(\boldsymbol{U}^H\boldsymbol{U}\right) = 1$, where $\mathrm{Tr}\left(\cdot\right)$ is the trace-operator and $(\cdot)^H$ denotes the Hermitian transpose.

To facilitate analytical derivation of pre-coders and their performance, we will assume that the number of users is $K = 2$. The input-output relation of the channel for this two-user case is shown in Fig. 1. The Gram matrix associated with $\boldsymbol{H}$ can be expressed as

$$\boldsymbol{G} \triangleq \boldsymbol{H}\boldsymbol{H}^H = \begin{bmatrix} 1+g & \delta \\ \delta^* & 1-g \end{bmatrix}, \tag{3}$$

where g denotes the power imbalance between the two user channels, and δ is a factor measuring the correlation between the two channels. Since we can permute the rows of $\boldsymbol{H}$ at will, we can without loss of generality assume that $0 \le g < 1$. The correlation between the channels to the two users can be expressed as $|\delta|/\sqrt{1-g^2}$. Further, we require $|\delta| < \sqrt{1-g^2}$ in order to have a positive definite matrix $\boldsymbol{G}$.

III. Measurement scenario

The channel measurements were carried out in a residential area north of Lund city center, Sweden. Fig. 2 shows an overview of the measurement area, where the numbers indicate specific houses. The measurements were originally performed with the aim of studying channel properties for residential femto-cell systems [3]. However, the large receive array with 128 antenna ports also enables this study of very-large MIMO channels. The receive antenna array was placed upstairs in house 63, which is shown at street level in Fig. 3. This array is a cylindrical patch array having 16 dual polarized antennas in each circle and 4 such circles stacked on top of each other, giving in total 128 antenna ports. The left part of Fig. 4 shows this large antenna array. The diameter is 29.4 cm and the height is 28.3 cm. The distance between adjacent antennas is about 6 cm, half a wavelength at the 2.6 GHz carrier frequency used. The transmit antenna array was placed indoors and outdoors at different positions, therefore indoor-to-outdoor-to-indoor and outdoor-to-indoor channels were measured. The right part of Fig. 4 shows the transmit array which consists of a planar patch array having 2 rows of 8 dual polarized antennas, giving in total 32 antenna ports. The outdoor-to-indoor channels are selected for very-large MIMO study, as we consider the scenario in which the users are outdoors around the base station. The outdoor measurement positions were (to the west of) houses 29, 33, 37, 41, 43, 45, 47, 49, 51, and 53, respectively. The measurement data was recorded with the RUSK LUND channel sounder at a center frequency of 2.6 GHz and a signal bandwidth of 50 MHz. At each measurement position, the transmit antenna was moved along a 5-10 m straight line parallel to the direction of antenna array back-plane.

For this very-large MIMO study, we extract the measurement data to form MU-MIMO channels. The first antenna in the 32-antenna transmit array is selected to represent a single-antenna user terminal. Through all the measurement positions, we can have several different users. In this paper we concentrate on the two-user case, where two different positions

Fig. 3. Street level view of the measurement area, house 63, in which the receive antenna array was positioned.

Fig. 4. Receive antenna array (left) and transmit antenna array (right).

are selected randomly. The receive antenna array positioned in house 63 represents an indoor base station.

IV. Pre-coding schemes

In this section we derive closed form expressions for DPC capacity and linear pre-coding sum rates for two-user case, using the system model in Sec. II.

A. Dirty-paper coding

The optimal sum rate in the downlink of a MU-MIMO system can be achieved by the interference pre-subtraction coding technique called dirty-paper coding (DPC), as long as the transmitter has perfect side information about the additive interference at the receiver [4]. The optimal DPC capacity for the two-user case is given as

$$C_{\mathrm{DPC}} = \max_{P_1,P_2} \log_2 \det\left(\boldsymbol{I} + \rho \boldsymbol{H}^H \boldsymbol{P} \boldsymbol{H}\right), \tag{4}$$

where $\boldsymbol{P}$ is a 2×2 diagonal matrix for power allocation with P_1 and P_2 on its main diagonal. The DPC capacity is maximized by optimizing over the power allocation under constraint that $P_1+P_2=1$. By substituting the Gram matrix $\boldsymbol{G}$ in (3) into (4), we find the optimal power allocation as

$$(P_1)_{\mathrm{DPC}}^{\mathrm{opt}} = \begin{cases} \frac{1}{2} + \frac{g}{\rho\left(1-g^2-|\delta|^2\right)}, & |\delta|^2 \leq \delta_{\mathrm{th}} \\ 1, & |\delta|^2 > \delta_{\mathrm{th}}, \end{cases} \tag{5}$$

where $\delta_{\mathrm{th}} = 1-g^2-2g/\rho$. The corresponding DPC capacity becomes

$$C_{\mathrm{DPC}} = \begin{cases} \log_2\left[1+\rho+\frac{\rho^2\left(1-g^2-|\delta|^2\right)^2+4g^2}{4\left(1-g^2-|\delta|^2\right)}\right], & |\delta|^2 \leq \delta_{\mathrm{th}} \\ \log_2\left[1+\rho\left(1+g\right)\right], & |\delta|^2 > \delta_{\mathrm{th}}. \end{cases} \tag{6}$$

If $|\delta|^2$ is higher than a certain threshold δ_{th}, all power will be allocated to the user with the stronger channel, and the DPC capacity becomes the same as the single-user transmission rate

$$C_{\mathrm{SU}} = \log_2\left[1+\rho\left(1+g\right)\right]. \tag{7}$$

Although optimal sum rate can be achieved, DPC is far too complex to be implemented in practice. We hence take the optimal DPC capacity as a benchmark for the sum rates achieved by the linear pre-coding schemes, ZF and MMSE, which are of more practical interest.

B. Linear pre-coding schemes

The pre-coding matrix $\boldsymbol{U}$ can be decomposed as

$$\boldsymbol{U} = \frac{1}{\sqrt{\gamma}} \boldsymbol{W} \sqrt{\boldsymbol{P}}, \tag{8}$$

where $\boldsymbol{W}$ represents a particular linear pre-coding algorithm, $\boldsymbol{P}$ is the power allocation matrix, and γ is used to normalize the total transmit power in $\boldsymbol{z}$ to unity. Therefore, from $\mathrm{Tr}\left(\boldsymbol{U}^H\boldsymbol{U}\right)=1$, the power normalization factor γ should be

$$\gamma = \mathrm{Tr}\left(\boldsymbol{P}\boldsymbol{W}^H\boldsymbol{W}\right). \tag{9}$$

ZF pre-coding scheme. ZF pre-coding eliminates the interference by transmitting the signals towards the intended user with nulls in the "direction" of other users. The ZF pre-coder is given as

$$\boldsymbol{W}_{\mathrm{ZF}} = \boldsymbol{H}^{\dagger}, \tag{10}$$

where $\boldsymbol{H}^{\dagger} = \boldsymbol{H}^H\left(\boldsymbol{H}\boldsymbol{H}^H\right)^{-1}$ is the pseudoinverse of the channel matrix $\boldsymbol{H}$. Using ZF pre-coding, the signal model becomes

$$\boldsymbol{y} = \sqrt{\frac{\rho}{\gamma}\boldsymbol{P}}\boldsymbol{x} + \boldsymbol{n}. \tag{11}$$

Since perfect nulling makes this scheme interference free, the sum rate can be calculated as

$$C_{\mathrm{ZF}} = \max_{P_1,P_2} \sum_{i=1}^{2} \log_2\left(1+\frac{\rho P_i}{\gamma}\right), \tag{12}$$

subject to $P_1+P_2=1$. By substituting the ZF pre-coder and power allocation matrix into (9), and letting $P_2=1-P_1$ we obtain the normalization factor

$$\gamma = \frac{1+g-2P_1 g}{1-g^2-|\delta|^2}. \tag{13}$$

Inserting this γ into (12), we find the optimal power allocation

$$(P_1)_{\mathrm{ZF}}^{\mathrm{opt}} = \begin{cases} \frac{1}{2}\left[1+g+\frac{2g\left(1-g^2\right)}{\rho\left(1-g^2-|\delta|^2\right)+2g^2}\right], & |\delta|^2 \leq \delta_{\mathrm{th}} \\ 1, & |\delta|^2 > \delta_{\mathrm{th}}. \end{cases} \tag{14}$$

The resulting sum rate for ZF pre-coding becomes

$$C_{\mathrm{ZF}} = \begin{cases} \log_2\left[\frac{\left(2+\rho\left(1-g^2-|\delta|^2\right)\right)^2}{4\left(1-g^2\right)}\right], & |\delta|^2 \leq \delta_{\mathrm{th}} \\ \log_2\left[1+\frac{\rho\left(1-g^2-|\delta|^2\right)}{1-g}\right], & |\delta|^2 > \delta_{\mathrm{th}}. \end{cases} \tag{15}$$

The ZF interference cancellation has significant signal power penalty if the two user channels are highly correlated. From (15) we can see that the capacity goes to zero when the channel correlation is high (low orthogonality), i.e. when $|\delta|$ approaches $\sqrt{1-g^2}$.

MMSE pre-coding scheme. MMSE pre-coding can trade interference suppression against signal power efficiency. The optimal MMSE pre-coder is given by [5]

$$\boldsymbol{W}_{\mathrm{MMSE}} = \boldsymbol{H}^H \left(\boldsymbol{H}\boldsymbol{H}^H + \alpha \boldsymbol{I}\right)^{-1}, \tag{16}$$

where $\alpha = K/\rho$, or in our case, $\alpha = 2/\rho$. At high SNRs (α small) the MMSE pre-coder approaches the ZF pre-coder, while at low SNRs (α large) the MMSE pre-coder approaches the matched filter (MF) pre-coder. In [6], the power allocation, i.e. the matrix $\boldsymbol{P}$, is also considered in minimizing the mean square error, but in this paper we optimize $\boldsymbol{P}$ to get the maximal sum rate.

The equivalent signal model of MMSE pre-coding scheme can be written as

$$\boldsymbol{y} = \sqrt{\frac{\rho}{\gamma}}\boldsymbol{G}\left(\boldsymbol{G} + \alpha\boldsymbol{I}\right)^{-1}\sqrt{\boldsymbol{P}}\boldsymbol{x} + \boldsymbol{n}, \tag{17}$$

where the normalization factor γ is

$$\begin{aligned}\gamma \;=\;& \frac{2P_1 g\left(\alpha^2 - 1 + g^2 + |\delta|^2\right)}{\left[(1+g+\alpha)\left(1-g-\alpha\right) - |\delta|^2\right]^2} \\ &+ \frac{(1+g+\alpha)^2\left(1-g\right) - (1+g+2\alpha)\,|\delta|^2}{\left[(1+g+\alpha)\left(1-g-\alpha\right) - |\delta|^2\right]^2}. \end{aligned} \tag{18}$$

Therefore, the two signal branches can be expressed by parameters ρ, g, δ and power allocations P_1 and P_2. We can calculate the SINR and then obtain the sum rate of the MMSE pre-coding scheme, subject to $P_1 + P_2 = 1$, as

$$C_{\mathrm{MMSE}} = \max_{P_1, P_2} \sum_{i=1}^{2} \log_2(1 + \mathrm{SINR}_i), \tag{19}$$

where

$$\mathrm{SINR}_1 = \frac{\rho P_1\left[(1+g)\left(1-g+\alpha\right) - |\delta|^2\right]^2}{\rho P_2 \alpha^2 |\delta|^2 + \gamma\left[(1+g+\alpha)\left(1-g+\alpha\right) - |\delta|^2\right]^2} \tag{20}$$

and

$$\mathrm{SINR}_2 = \frac{\rho P_2\left[(1-g)\left(1+g+\alpha\right) - |\delta|^2\right]^2}{\rho P_1 \alpha^2 |\delta|^2 + \gamma\left[(1+g+\alpha)\left(1-g+\alpha\right) - |\delta|^2\right]^2}. \tag{21}$$

Closed form expressions of optimal power allocation and maximized sum rate can be reached but are far too long and complicated to be given here, but in the case of $g = 0$, a simple expression of the sum rate is obtained as

$$C_{\mathrm{MMSE}}|_{g=0} = 2\log_2\left[1 + \frac{\rho}{2}\left(1 - \frac{\rho}{\rho+2}|\delta|^2\right)\right]. \tag{22}$$

V. Performance comparison

Using the closed form sum rate expressions above, we first study how the DPC capacity and linear pre-coding sum rates are affected by correlation and power imbalance between user channels. We then let the number of the base station antenna grow large, both for measured channels and simulated i.i.d. Gaussian channels, to see to what degree a realistic propagation environment decorrelates the user channels. Finally, we compare the linear pre-coding sum rates with the DPC capacity as the number of antennas increases.

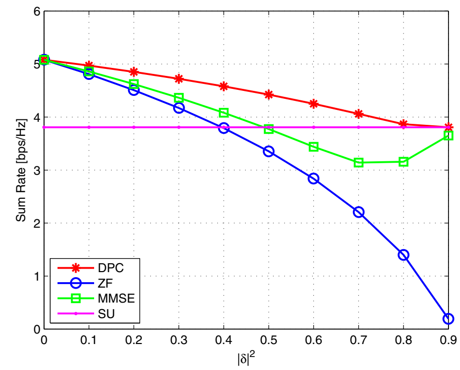

Fig. 5. Sum rates for DPC, ZF, MMSE and single user transmission versus channel correlation-related factor $|\delta|^2$ when $\rho = 10$ dB, $g = 0.3$.

A. Numerical evaluation

It can be seen from the expressions above that if the channel correlation approaches to zero, i.e. $|\delta| \approx 0$, ZF and MMSE pre-coding sum rates become equal to the DPC capacity,

$$C_{\mathrm{ZF,MMSE,DPC}} = \log_2\left[\frac{\left(2 + \rho\left(1 - g^2\right)\right)^2}{4\left(1 - g^2\right)}\right]. \tag{23}$$

If the channel correlation grows very high, i.e. $|\delta|$ approaches $\sqrt{1-g^2}$, signal power would only be transmitted over the stronger user channel, and the other user would get zero capacity. In that case, C_{MMSE} and C_{DPC} become equal to single user transmission rate in (7), while C_{ZF} tends to zero.

Fig. 5 shows the DPC capacity, linear pre-coding sum rates and single-user transmission rate as functions of the correlation-related factor $|\delta|^2$ for $\rho = 10$ dB and $g = 0.3$. We can see that the gap between DPC capacity and linear pre-coding sum rates becomes smaller when $|\delta|^2$ decreases. Eventually the linear pre-coding sum rates are the same as the DPC capacity when $|\delta|^2 = 0$, i.e, when the two user channels are orthogonal. When the channel correlation grows high, ZF capacity decreases rapidly to zero and the DPC capacity decreases to single-user capacity. It is interesting to notice that MMSE sum rate decreases first and in fact becomes lower than the single-user capacity, but then increases after $|\delta|^2$ reaches a certain value, e.g. around 0.7-0.8 in this figure. By investigating the power allocation for MMSE, we find the power is only transmitted to the stronger user channel when $|\delta|^2 > 0.7$, hence, as the correlation gets higher, the MMSE pre-coding eventually approaches the single-user transmission.

The channel power imbalance factor g also has an effect on the capacity. Basically, as g grows, the channel power difference becomes large and thus the channel correlation $|\delta|/\sqrt{1-g^2}$ grows. Consequently, the ZF sum rate decreases rapidly while the MMSE sum rate and DPC capacity decrease first and then both become the same as single-user capacity. Furthermore, the DPC capacity and linear pre-coding sum rates are low when g is large. Hence, in order to have higher capacity, users with small channel power differences should be served at the same time according to some grouping strategies.

B. Measured channels

As the number of base station antennas M increases, one hopes that the two user channels become less and less correlated. The ideal scenario would be that if the two users are spatially separated enough, the channels could be approximately orthogonal, i.e. $|\delta|^2$ approaches zero. In that case the DPC capacity could be achieved by linear pre-coding schemes. Here we verify whether it is true or not that the correlation decreases as M goes large for measured physical channels. Simulated i.i.d. Gaussian channels with the same dimension and channel power imbalance as the measured channels are used as reference.

We select one representative group of user positions - the two users are positioned outside house 49 and 53 respectively. Fig. 6 plots the average channel correlation as a function of M. For each M, the averaging is performed over time and frequency, and also over different groups of antennas since the cylindrical structure of the array may cause receive power imbalances over the antennas. From Fig. 6 we can see that the channel correlation is higher in the measured channels than in the i.i.d. Gaussian channels. This is because the two user positions are close and probably have common scatterers that make the channels similar [3]. However, in both i.i.d. Gaussian and measured channels, the average channel correlation decreases as M increases. This suggests that the very-large array can decorrelate the user channels. Then we compare the linear pre-coding sum rates with the DPC capacity. Fig. 7 shows the ratio of average linear pre-coding sum rates and average DPC capacity as M grows. The transmit SNR here is set to 20 dB and the total transmit power is kept unchanged. With the increase of M, the ratios for ZF and MMSE in both i.i.d. and measured channels is close to one.

We notice that the channel correlation in Fig. 6 decreases fast as the number of base station antennas increases from 2 to 8. Correspondingly in Fig. 7 the ratios of linear pre-coding sum rates and DPC capacity grow very rapidly as the number of antennas increases. When the number of antennas increases to more than 20, the channel correlation as well as the sum rate ratios saturate. At 20 base station antennas the linear pre-coding sum rates already reach 98% of DPC capacity. This shows that the optimal DPC capacity can be achieved by linear pre-coding schemes at a relatively limited number of base station antennas.

VI. Summary and Conclusions

In this paper, linear pre-coding performance is studied for measured very-large MIMO downlink channels. We find that the user channels, in the studied residential-area propagation environment, can be decorrelated by using reasonably large antenna arrays at the base station. With linear pre-coding, sum rates as high as 98% of DPC capacity were achieved for two single-antenna users already at 20 base station antennas. This shows that even in realistic propagation environments and with a relatively limited number of antennas, we can see clear benefits with using an excessive number of base station antennas.

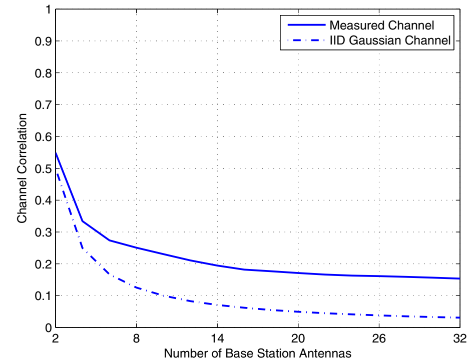

Fig. 6. Average channel correlation $|\delta|^2/\left(1-g^2\right)$ as a function of the number of base station antennas. The two users are outside house 49 and 53 respectively.

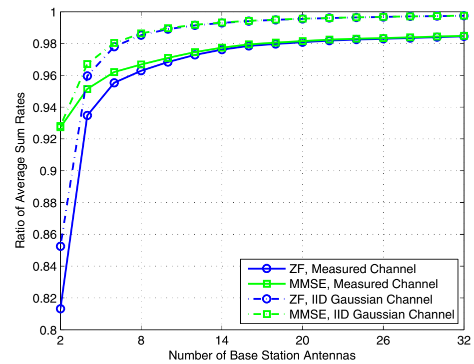

Fig. 7. Ratio of average linear pre-coding sum rate and DPC capacity as a function of the number of base station antennas. The two users are outside house 49 and 53 respectively.

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