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LARS LÖFGREN
University of Lund

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Nonseparability of Inferribility and Measurability in Quantum Mechanics as a Systema Magnum

Lars Löfgren (email: lofgren@it.lth.se)
Department of Information Theory, University of Lund, Box 118, S–221 00 Lund

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Abstract

Quantum measurement theory has a peculiar nature with its *two* processes of assertibility, by proof and by measurement, which is beyond formal logic. This two–sortedness with respect to assertibility is discussed in terms of developments of many–sorted logics, sometimes referred to as *Logica Magna*. Although useful insights are obtained this way, we argue that there is a nonseparability of inferribility and measurability that is understandable first with a shift from logic to language in its systemic conception. That is, from a *Logica Magna* to a *Systema Magnum* understood as a truly systemic whole of complementary description and interpretation processes. In introspective quantum mechanics as a *Systema Magnum*, the nonseparability sheds light on interconnections between formalism and experimentalism which, for example, helps a critical understanding of Deutsch's hypothesis of a programmable experimentability ("quantum computation").

1 Grand Systems for Complex Problems; towards a Systema Magnum

Characteristically, the complexity of an object-system, $\mathcal{O}$, is not an intrinsic property but depends on how $\mathcal{O}$ is conceived in a conceiving system, $\mathcal{C}$. For a $\mathcal{C}$ of high powers, $\mathcal{O}$ may turn out simple – but complex in a less powerful $\mathcal{C}$ admitting only very large accounts of $\mathcal{O}$. Like a large enumeration of its parts but with sparse means for expressing relations between the parts whereby the conception of $\mathcal{O}$ (for example its description) can be simplified.

As explained in [6], complexity is two-faced. On the one hand we want to increase complexities in order to reach behaviours that cannot be realized with less-complex systems. On the other hand, we want to reduce complexities in order to be able to describe the effects of the systems we construct. That is, the effects they may prove to have beside those desirable properties that make us construct them in the first place. Characteristically, these opposite tendencies do not lead to some kind of optimal complexity but to a complexity race, indicative of the evolutionary nature of scientific and technological achievements.

With the objective to be able to *communicate* ideas of large systems and of complexity, we are bound to find *language* the ultimate level of relativization – rather than logics, mathematics, physics, etc, if these are viewed as independent of language. That is, with language in its complementaristic meaning as an autonomous whole of description and interpretation processes (cf [8; 9]).

In metamathematics we find examples on developments of many-sorted logics and logics of higher order to meet the challenge from increasingly difficult problems. In [2], Beth explaines the development as a search for a *Logica Magna* (or a Grand Logic). Characteristically, in order to be able to deal with increasingly large domains of investigation, concepts of universality arize, which must be relativized in one sense or another. For example, Beth speaks of Heyting's *intuitionistic logic* as a "*Logica Magna*, as it is meant to incorporate the totality of intuitionistic mathematics". The relativization here concerns a certain constructively accessible universe. In further examples of a *Logica Magna* in [2], Beth relativizes to language. The possibility of investigating some (partial) concept of language is not considered.

The trend towards grand systems is not restricted to mathematics and logics. In a wider domain of systems involving *both formalism and experimentalism*, we find a similar trend of magnification, namely towards some large system $\mathcal{C}$. If $\mathcal{C}$ is large enough, not to need a larger category for explaining itself by communication, we will call $\mathcal{C}$ a *Systema Magnum.* To explain the concept, we will give examples both of enlargement by a mere collecting of parts which does not give increased system properties, and of magnifications into a system which is to be considered as a *Systema Magnum*.

In quantum mechanics with its difficult measurement problem, and problems of how to interpret the quantum formalism, we have examples of large systems. Like the so called "many-worlds" interpretation where the world, at each measurement, splits into simultaneously existing parallel worlds. Or, the less extreme "many-points-of-view" interpretation where, even mutually exclusive, points of view of the world are collected together with their superpositions. Both interpretations maintain that not only subatomic particles and measuring instruments, but also tables, cats, people, oceans, and stars are quantum mechanical systems whose states evolve according to linear quantum mechanical equations (process **2**, section 4). Although such grand collections may resolve, or remove, certain problems inherent in the assumption of an external observer, like the "collapse of the wave function", it does so at the expense of communicability. There is no communication between the worlds while remaining split. Compare as well a claim, made in the many-worlds interpretation, that the mathematical formalism defines its own interpretation. This is contrary to the linguistic complementarity. For some further comments we refer to section 7. If a conceiving system $\mathcal{C}$ is to be considered as a *Systema Magnum*, we will troughout require a systematization of its grandeur admitting communicability in some language – whereby language, in its complementaristic conception, becomes the ultimate reference frame.

As another example of enlargement, consider Bohr's proposals for the necessity of using *two* languages for a proper account of quantum mechanical measurements. One language, L_m, for the measuring instruments and their readings and another, L_q, for the quantum phenomena. L_m is a fragment of our ordinary, natural, language with suitable application of the terminology of classical physics. Bohr argued the necessity of this language L_m on the belief that it admitted a nonambiguous communication of an experiment, its measuring instruments and their results. Without that, an experiment would seem to loose its intended meaning, namely to admit a communicable full account of truth-by-measurement.

This proposal with its appeal to language, even two of them, may at first appear quite compatible with our intensions. At second thought, however, it appears surprising, because if *two* languages really can be *used*, say by a group of communicating physicists, this would mean that the two languages, while being used, are parts of *one* larger many-sorted language. Except, of course, if "use" is understood in a "split brain" sense with no intercommunication between the halfs (compare as well the many-worlds interpretation). But a physicist, or a group of physicists which share one and the same language which allows them to communicate, is not in that situation. Of course, the physicists may not fully understand each other, in particular if they reason about some difficult linguistic phenomenon like measurement. But the point is that they can communicate – in the shared language – eventually to conclude in incompleteness results for quantum measurement theories.

Of course, at the time of Bohr's proposal for two languages in the late 20's, neither Gödel's incompletenss results for formal languages, nor Tarski's incompleteness results concerning the definability of truth, were known, much less the later to come complementaristic conception of language. What is remarkable, however, is Bohr's early insight into his *primary view of complementarity* as a tension between definability and measurability – which may well be considered a step towards quantum mechanics as a *Systema Magnum*, a system consisting of formalism as well as experimentalism without trying to reduce the one to the other within the system. This is not to be confused with the plea for two languages (with language in its modern conception). Two languages are never to be considered complementary, except possibly in some elementary set-theoretic sense. Proper relations between languages are instead, for example, translation and reduction. The linguistic complementarity refers to a tension within a language between describability and interpretability. In [9] we have argued Bohr's primary view of complementarity as a case of the linguistic complementarity for a quantum measurement language.

The self-referential nature of the difficult quantum measurement problem will gradually force us to enlarge the conceptual apparatus, not by sacrificing language as prerequisite for communicability but, on the contrary, by moving from logics to language in its wholistic systemic conception, or from a *Logica Magna* to a *Systema Magnum* in the form of language in its complementaristic conception. As explained in [8; 9], there are various related ways of looking at *complementarity in language*:

(i) as descriptional incompleteness: in no language, its interpretation process can be completely described in the language itself;

(ii) as a tension between describability and interpretability within a language;

(iii) as degrees of partiality of self–reference (introspection) within a language: complete self–reference within a language is impossible;

(iv) as a principle of "nondetachability of language".

2 The TWO Assertibility Processes in Quantum Measurement Theory

Inferribility by proof in a theory, and measurability by physical measurement processes, are two distinct methods of *assertibility.* In quantum measurement *language* both do indeed occur. However, in attempts to describe quantum measurement in a *theory* in the language, like in a formal quantum measurement theory, only inferribility by proof is available as method of assertibility. Hence the problem arizes to simulate, in a sense of reducing, assertibility by measurement in terms of assertibility by proof.

The two methods of assertibility, by proof and by measurement, in a quantum measurement language L can certainly be roughly conceived, provided that L is sufficiently well shared. Attempts at a deeper understanding reveals measurement to be interpretation in L. Thus the problem of simulating measurability by provability is nothing else than trying to describe, or formalize, in a quantum measurement theory T, its own intended interpretation. According to the linguistic complementarity, this cannot be done completely. Some undescribable properties of L will have to be referred to, and the assumption that these are accessible in L as a shared background knowledge may well turn out problematic.

Now, how complete must an assertion process be in order to be acceptable as such? Do we need to assert that an alleged assertion really is an assertion. Do we need to prove that an alleged proof really is a proof. Do we need to convince ourselves by measurement that a proposed measurement process really is a measurement process. Or, could it be inherent in the concept of assertibility that such meta-reflections, leading to infinite regress, are not necessary.

In accepting a certain method of assertion, we do indeed express a faith in it which we do not doubt – otherwise a regress is unavoidable. The faith may be due to a feeling of "obviousness". Compare Descartes' Cogito. But when a method of assertion is made the object of investigation, like in quantum measurement theory which investigates measurability, or in formal meta-mathematics which investigates provability, the problem of regress is lurking and it becomes less obvious how to trunkate the regress.

Quantum measurement theory, with its *two* methods of assertion, by proof and by measurement, becomes interesting both from the point of view of *Logica Magna* and, even more so, from a perspective of *Systema Magnum.*

With respect to *Logica Magna*, we may attempt to deal with the two sorts of assertibility in terms of a *many-sorted* logical system (cf [2; 14]).

Sometimes an n-sorted system, T_n, can be reduced to a conventional one-sorted system, $T_1^{(n)}$. One then introduces sort-predicates (type-predicates) $S_i(x)$, where i runs from 1 to n, on the variable x (instead of using one variable for each sort). Wang [14] considers a case where it is possible to define a translation between T_n and $T_1^{(n)}$ such that a statement of T_n is provable in T_n if and only if its translation is provable in $T_1^{(n)}$. And, furthermore, such that there is a primitive recursive method by which, given a statement of T_n and a proof for it in T_n, we can find a proof in $T_1^{(n)}$ for its translation in $T_1^{(n)}$; and conversely.

The quantum mechanical case, with its two sorts of assertibility, appears too complicated to allow a reduction of this kind (cf end of section3).

3 Further on Bohr's Measurement Requirement: Are there Well-Formed-Formulas for Basic Measurement Sentences?

Usually quantum measurement theory is formulated to contain Schrödinger's equation (referred to as process **2** in [11]) describing the autonomous development of a quantum mechanical system (while not being measured upon), together with a description of the measurement (process **1** in [11]). Implicit is here a distinction between *measurement propositions* and *inferred propositions.* The distinction obviously has a methodological significance, for without it one would not know when to apply process **1**, or **2**.

A way of formalizing the distinction is to introduce sort-predicates $S_i(s)$ on the sentences s of the quantum mechanical language (or on their Gödel numbers). For example, such that $S_1(s)$ means that s is a *basic measurement sentence*, meaning that s can be asserted (true or false) by a direct measurement. And such that $S_2(s)$ means that s is not a basic measurement sentence, but a sentence whose assertion needs inferences of a less constructive type (than a direct measurement). The language may contain further higher types corresponding to sentences understood as metareasonings about **1** and **2**; examples are provided in [9].

Let us assume that quantum measurement theory can be formalized in a quantum mechanical language as a 2-sorted theory T_2 which can be reduced to a 1-sorted theory $T_1^{(2)}$. This implies that its sentences can be effectively decided as to sorts. For example, there are *well-formed-formulas*, abbreviated wff's, for its basic measurement sentences.

We regard the quest for wff's for basic measurement sentences as a modern way of making understandable Bohr's early request for experimentability. Namely, that descriptions of experiments, including measurement instruments and measurement results, should be unambiguously interpretable (which Bohr believed could be accomplished by requiring the use of "plain language", or language of ordinary shared experience).

In [10], where we have argued the need for a many-sorted quantum measurement language both in order to understand von Neumann's formulation [11] and in order to understand a recent double-prism experiment, we have indicated severe difficulties of a formalism allowing wff's for the basic measurement sentences. This points at a *completeness problem* for a quantum measurement *language*, which is fundamental for completeness problems for quantum measurement *theories* (which are the more frequently discussed problems).

4 Nonseparability of Measurability and Inferribility

Consider again von Neumann's distinction in [11] between two parts of a quantum measurement theory, a distinction which is maintained also in many modern formulations:

1 "the projection postulate" describing the measurement process;
2 "the Schrödinger equation" describing the autonomous evolution of the system.

The Schrödinger equation, together with all subsumed mathematics, describes the quantum system wile no measurements (as under 1) are made. *Measurements* are said to interfere with the otherwise non-observed system. But the making of *inferences* is assumed not to interfere with the description 2 or with the whole description 1+2.

This assumption of non-interfering inferences, an assumption which we usually always make, turns out nontrivial, and in fact problematic, in a context like the above were we face the problem of simulating measurement by inference.

The phenomenon of interfering inferences is easy to understand for broadly conceived inferences, like inductive inferences. What is remarkable, however, is that also inferences in pure formal theories, like its rules if inference, also exhibit interference when investigated at depth.

By the linguistic complementarity we know that no theory can completely describe all of its own rules of inference. This implies the existence of inferences which interfere with the description – provided that the rules of inference are made objects for description in the theory. Even though the theory may be partially introspective, it cannot completely describe its rules of inference, i.e., these cannot be isolated as non-interfering objects.

After these preliminary remarks on both measurement and inference as interfering processes, or else as processes which each lead to regress, let us trace them down individually to levels where they may be trunkated. This will shed light on their nonseparability.

Ultimately, inferribility by proof in a formal system has a dynamic, or processual, property which is irreducible to the static theorems of the system. In Gödel's early definition of a *formal system* he refered to the processual and constructive nature of a rule of inference in terms of a "finite procedure" which he at the time considered sufficiently clear on an intuitive level. After Turing's conception of computing machines, Gödel replaced "finite procedure" in the definition of a formal system with the Turing machine concept. Thereby, he was able to trunkate the regress in proving that an alleged proof really is a proof, by saying that a proof is a proof if it can be checked to be so by a Turing machine. He argued in [5] the concept of computability as an *absolute* epistemological concept.

However, recognizing Turing computation as an interpretation of a program in a programming language, we see how the proof regress *continues* into the regress involved in proving a program correct with respect to its intended interpretation. Compare the problem whether the computer based proof of Appel and Haaken of the four-color theorem, "which no mathematician has seen", should be accepted as proof or not. Again, compare the difficulties in judging the alleged proof of Taylor and Wiles of Fermat's last theorem as a proof.

We have seen how both inferribility and measurability, in a sufficiently introspective system, come to interfere with the system. Gödel attempts to trunkate the inference regress by accepting automata, in the form of Turing machines, as sufficiently clear. What this amounts to is the assumption that a (programming) language is shared whereby questions of ambiguity are resolved by appeal to shared but not described properties. Bohr attempts to trunkate the measurement regress, by assuming a shared language where experiments and measuring apparata can be unambiguously communicated by appeal to shared but not described properties.

It is in this common reference to a shared language (involving both descriptions and interpretations), which is sufficiently "simple" to allow nonambiguous understandings, even if beyond description, that the two processes of assertibility become tied. More precisely, the two reference languages are inseparable in their identical appeal to "constructivity" as explained in [10]. Inferences may be of various degrees of constructivity. But the lowest degree of an inference is identical with the constructivity of a basic measurement sentence.

5 Computability Confinements on von Neumann's Operator Constructions for Measurables

As we have explained in [10], von Neumann's basic quantum theory of measurement [11] is underexposed with respect to the essential constructive nature of measurements (cf section 3). Which may seem reasonable since [11] was published already in 1932 well before the Turing-Church-Post conceptions of computability in 1936. In [11] von Neumann describes how to construct new measurement operators (observables), $F(A)$, from old, A, while considering F arbitrary with no explicitly made computability conditions.

In this operator construction, there are two ideas of construction merging (cf [10]). On the one hand, a mathematical construction of new operators (observables) from old by arbitrarily choosen functions F (which also determine the corresponding spectra, i.e., the possible measurement outcomes). And, on the other hand, a construction of corresponding measuring instruments. According to the basic idea behind experimentalism – compare section 3 on Bohr's measurement requirement – the latter construction must be effective, which ought to reflect back on the mathematical F-construction. It becomes necessary to impose on F some sort of computability condition – which in von Neumann's theory of measurement does not arize from its basic conception – or else quantum measurement would loose its basic nature as a method of assertibility.

6 Programmability Confinements on Experimentability; Deutsch's "Quantum Computation"

In Deutsch's view [4] of a universal "quantum computer", supposed to be programmable to perform any quantum mechanical measurement experiment, we seem to have a somewhat similar but independent view of how to confine quantum measurement experiments to meet with the requirement of communicability of experimentability. Namely, in terms of Deutsch's view of programmability of physical experiments which he argues on the assumption that there exists a universal "quantum computer" $\mathcal{Q}$ which can be programmed to perform any physical experiment.

In comparing Deutsch's view of experimentability, as being programmable, with our computability confinement on von Neumann's operator constructions $F(A)$, we find differences.

(a) Deutsch's view is more general in the sense that it allows constructions on *several* variables (operators).

(b) Deutsch's universal "quantum computer" $\mathcal{Q}$ is intended as interpreter of programs for measuring experiments. Unlike a universal Turing machine $\mathcal{T}$, which is an interpreter of programs for computation, $\mathcal{Q}$ is not a computer. It does not realize computations but quantum measurements which are in general indeterministic. Only from the outside (allowing computation processes), a statistical determinism may be *inferred* from repeated runs of $\mathcal{Q}$.

(c) The question of the realizability, and principal existence, of $\mathcal{Q}$ is by far less developed than the question of the existence of a universal computer $\mathcal{T}$ (cf [8]).

7 From Physical to Systemic Realism

Experimentability involves a *physical realism* whereby the interpretation (model) of the physical theory reflects a reality in which the measurement processes are realized as intended. In the case of measurements which may interfere with the the system measured upon, as in quantum theory, the underlying physical realism admits inferences that do not interfere with the theory of the physical system..

As we have seen, in sufficiently introspective systems, physical or not, also inferribility – even beyond direct measurability – can interfere with the system inferred upon. In such a more complex, or more introspectice, case of a *Systema Magnum*, a further kind of realism is subsumed. Namely, a *systemic* or *linguistic realism* as explained in [8]. Here a reality may be inhabited also by linguistic phenomena beyond measurement phenomena. It is assumed to exist in a sense that is tied with both measurability and inferribility. Characteristically, in attempting to describe this reality, we find its describability low, in agreement with the linguistic complementarity. The corresponding interpretability can then be made sufficiently wide to admit conceptions of independence and of an existing reality – which can only be vagely described in terms of noninterfering inferences. Attempts at a sharper description of the reality will result in interfering inferences, whereby the conception of its independence will be perturbed.

A totally independent reality would not allow experiments, nor communicable descriptions. A reality conception, in conformity with our experience of how we do interact with the real world, must not be one of total isolation, but one allowing interactions in making experiments as well as inferences.

In this broad perspective, let us recall Deutsch's view of a programmable experimentability. It obviously assumes a quantum reality which is fragmentable in the sense that it allows constructive interactions to form measuring experiments which realize measurements according to "programs" with structures that allow them to be communicated much like programs in an ordinary programming language. Concerning such a reality assumption, let us quote Shor [13]: "The most difficult obstacles [for realizing a "quantum computer"] appear to involve the decoherence of quantum superpositions through the interaction of the computer with the environment, and the implementation of quantum state transformations with enough precision to give accurate results after many computation steps. Both of these obstacles become more difficult as the size of the computer grows, so it may turn out to be possible to build small quantum computers, while scaling up to machines large enough to do interesting computations may present fundamental difficulties."

A comment like this of Shor may be further understood while distinguishing between a "quantum computer" as a realization of a quantum measuring experiment, which it is intended to be, and as a computer, which it is not. That brings us right back to our problem of how to distinguish between a measurement, and an inference from measurements, which is itself not a strict measurement. What F-constructions (sections 5 and 6) preserve strict measurability?

This question would seem equivalent to the question if there is a program for $\mathcal{Q}$ which makes it realize F as a measuring experiment. A kind of complication here, may be illuminated in terms of a paradox suggested by Peres in [12].

Peres' reasoning starts out from the (Einstein-Podolsky-Rosen) *EPR criterion of reality*:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

In the spirit of this criterion, Peres suggests a further construction of elements of reality. It takes support in the von Neumann operator construction $F(A)$ (see section 5), now extended as follows. If operators A and B commute, quantum mechanics allows us in principle to measure both of them simultaneously together with the constructed operator $F(A, B)$, for any F (suitably confined; cf section 5). Therefore, Peres argues, if commuting operators A and B correspond to elements of reality, i.e., are predictable with certainty by noninterfering measurements, it is tempting to say that any $F(A, B)$ also corresponds to an element of reality.

Now, against this, Peres describes a spin system, like in Bohm's version of the EPR-experiment with two spin $\frac{1}{2}$ particles, prepared with opposite spin and travelling far apart. For the system, Peres defines operators A and B, and a function F, which satisfy the above condition for measurability – but where quantum theory proves $F(A, B)$ not to be measurable.

Let us try to understand this paradox as follows. First, neither in the EPR-criterion nor in the $F(A, B)$ construction, there is a clear distinction between measurability and inferribility. To predict with certainty from a measurement (without disturbane), is an inference in the theory which itself is beyond direct measurment. The question is then what F-constructions are real in admitting nondisturbing, certain, predictions.

In terms of Deutsch's philosophy, we would have the answer that $F(A, B)$ is measurable if there is a program which makes $\mathcal{Q}$ realize it. As Deutsch points out in [4], "quantum computers raise interesting problems for the design of programming languages". It would seem that we have one here. A programming language with interpretations defined by $\mathcal{Q}$ must obviously be consistent with quantum measurement theory. The conclusion would then be either that there is no program for $F(A, B)$ or that the inference that $F(A, B)$ is nondispersive (predictable with certainty) cannot be drawn in the programming language. Or, again, that the EPR-criterion of reality is not applicable.

The question of programmability of $\mathcal{Q}$ obviously must connect both to quantum theory and to its interpretability (in models of quantum reality). It may well be more complicated than the corresponding question for programmability of a universal Turing machine (with its comparatively well understood degrees of constructivity). It will have to refer to the deep introspective level where measurability and inferribility become nonseparable. (In [1], Albert connects to introspective possibilities of quantum automata, explained with reference to a many-points-of-view interpretation of quantum mechanics with its relaxed view of communicability.)

8 Conclusions

Fragmentation is what we use in description as well as in perception and conceptualization. Every description, even a whole descriptive theory, is a description of something, not of everything. Were it not for the remarkable property of nature that it seems to allow fragmentation, every attempt at describing nature would fail (according to systems thinking of Chew).

Now, failures at complete descriptions are constantly being revealed, in domains of logic and metamathematics as well as physics. Accordingly, explanations are sought in terms of phenomena of nonfragmentability, such as intrinsic linguistic dependence, nondetachability of language, entanglements or nonseparabilities. In our view, phenomena of nonseparability are best understood, not only as fragmented in disciplinary thinking like in quantum accounts of phenomena of entanglement (see d'Espagnat [3] for a good account of nonseparability here), but in genuine systems thinking. Where language, as an autonomously nonfragmentable whole, provides a required wider frame of reference.

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