Stress and Deformation Characteristics of Concrete at High Temperatures. 1. General Discussion and Critical Review of Literature

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Yngve Anderberg
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LUND, JULY 1973

Preface

This paper is the first report from a research project carried out at Lund Institute of Technology, Division of Structural Mechanics and Concrete Construction and sponsored by the Swedish Council of Building Research. The scientific work is made by the authors under the supervision of Professor Ove Pettersson, head of the division.

The objective of the research is to provide the experimental and theoretical basis for the analysis of stresses and deformations of concrete subjected to high temperatures. This first part consists of a general discussion of the problems and a review of the literature. Primarily, this report was intended to give a background for the planning of the experimental investigation which forms the main part of the project. This experimental investigation is now in progress.

The work on this project is carried out in close cooperation between the authors. Thus this particular report has been outlined and written by the authors in close connection, though Sven Thelandersson has to a greater extent contributed to its final shape.

The authors want to thank Professor Ove Pettersson for his encouraging support and for engaging discussions about this matter. Thanks are also due to the staff of the Division of Structural Mechanics and Concrete Construction for exchange of thoughts and practical help. In particular Birgit Olsson, who typed the manuscript and Margareta Nilsson,who drawed the figures, deserve our gratitude.

1. Introduction

It has long been known, that an accurate stress analysis for a heated, loaded concrete structure is not possible to carry out without taking into account time-dependent deformations (England & Ross (1962), Hannant & Pell (1962)). Even in the case of fire exposure with short duration the time-dependent strains seem to be of considerable inportance to the magnitude of stresses, restraint forces and moments. This is due to the fact that the rate of creep and shrinkage increases rapidly with increasing temperature.

During the recent years many investigations have been performed in order to get a better knowledge of the deformation characteristics for stressed, heated concrete. This research has primarily been induced by the increasing use of prestressed concrete pressure vessels for nuclear reactors. A better knowledge of the concrete behaviour may also make it possible to increase the present working temperature of such vessels.

Despite the rather extensive research in this field our knowledge of the deformation behaviour of concrete at elevated temperatures is far from clear. The various sources of deformation are controlled by a large number of variables and the different types of deformation are not independent of each other. In addition, the strain increment in a certain moment depends on the preceding stress and temperature histories.

Since most of the tests in literature have been designed to simulate the concrete in prestressed pressure vessels, as a rule only temperatures up to about 200°C are studied. Furthermore, in most cases rather slow rates of heating are used in the tests, which are extended over long periods. In the case of fire exposure, however, we can expect more rapid heating, short durations, and essentially higher temperatures than 200°C.

This paper is intended to be a preparatory study for an investigation of the deformation behaviour of concrete, planned by the authors. The investigation will be particularly designed to simulate conditions during fire exposure i e the effect of high temperatures and rapid heating on the concrete will be studied.

The need for such knowledge has increased due to the progress made in fire research during the last years. It is now possible to calculate, with a reasonable degree of accuracy, the transient temperature fields in a concrete structure exposed to fire. Accordingly, with a better knowledge of the strength and deformation properties at high temperatures, the structural behaviour of fire exposed concrete structures could be analysed from such temperature fields.

As previously mentioned, the present knowledge on deformation behaviour is insufficient for such an analysis. Take for instance a fire-exposed column. During the course of heating thermal stresses will develop, but these stresses can not be calculated because the constitutive relationship between stress and strain is not known. Similarly, it is impossible to calculate restraint moments and forces in statically indeterminate structures subjected to temperature gradients, without taking the time-dependent deformations into account (Anderberg (1973)). Moreover, if the thermal stresses can be accurately estimated, it may contribute to the understanding of spalling effects.

This paper consists of two parts. Chapter 2 deals with the various types of deformation that can occur and the influence of different variables on the deformation as obtained in tests described in literature. In chapter 3 the problems involved in an analysis of stresses and deformations are discussed in order to provide a basis for the design of tests. In chapter 4 a summary is given and the conclusions that can be drawn are compiled.

2. Deformations in heated, stressed concrete

2.1 Components of strain

As previously mentioned, an accurate estimation of the strains in heated, stressed concrete is very difficult to make. The strain components that constitute the resulting total deformation can be related to any of the categories listed below, c.f. Birkimer et al (1969).

1) Instantaneous strain associated with external loading.

2) Thermal strains. Instantaneous strain due to change of temperature, including volume changes in the concrete due to physical and chemical reactions.

3) Shrinkage. Time-dependent strain observed on unstressed specimens, usually caused by a loss of water.

4) Creep. Time-dependent strain observed in heated, stressed concrete that cannot be otherwise accounted for.

The above components of strain are not always uniquely defined and not independent of each other. However, it is convenient to define them in relation to possible test procedures instead of the physical and chemical mechanisms behind them. The creep and shrinkage mechanisms at high temperatures, for instance, are probably different from that at normal temperatures.

The following variables affect the magnitude of the strain components.

1) Concrete mix proportions and physical nature of the constituents.

2) Age and curing of concrete prior to heating and loading.

3) Conditions determining the moisture loss from the specimens during the test.

4) Heating procedure: Temperature level, rate and duration of heating, temperature history, presence of temperature gradients.

5) External loading: Magnitude and duration of stress, stress history in relation to temperature history.

2.2 Instantaneous strain associated with mechanical load

The results of a number of investigations (Saemann & Washa (1957), Philleo (1958), Zoldners (1960), Harmathy & Berndt (1966), Cruz (1966), Davis (1967), Sullivan et al (1971)) show that the modulus of elasticity for concrete in compression decreases with increasing temperature. As an example, results obtained by Philleo (1958) for the dynamic modulus of elasticity of concrete with calcareous aggregate are shown in figure 1. We can observe that the reduction of the modulus at high temperatures is greater for

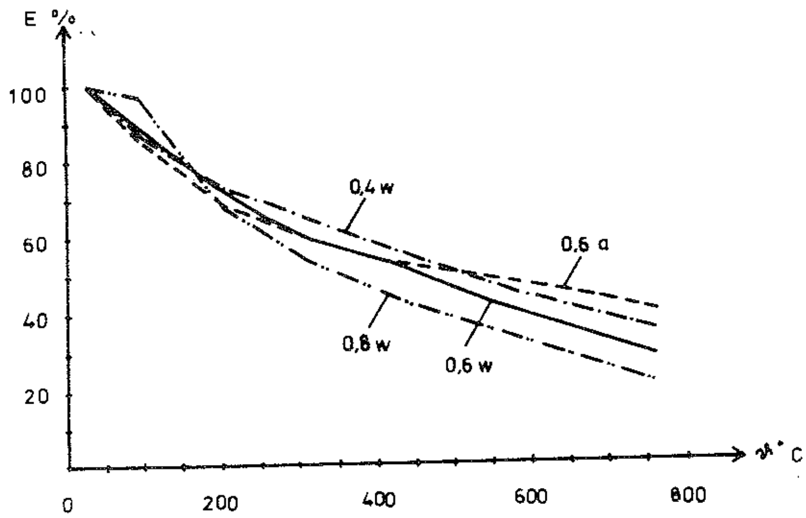


Fig 1 Variation in the short-time dynamic modulus of elasticity in a heated state, E, with the heating temperature, determined at the age of 28 days on concrete prisms, 3,8 x 5,1 x 15,2 cm in size, which were not subjected to any load during the heating period. The water-cement ratio was 0,4, 0,6 and 0,8. The test specimens were moist-cured for 28 days (curves marked "w"), or were moist-cured for 3 days, and then air-cured for 25 days at a relative humidity of the air or 50 per cent (curves marked "a"). The concrete was made with standard Portland cement, and with fine and coarse aggregate from Elgin, Illinois, U S A. From Philleo (1958)

greater w/c-ratios. Fig 2 shows the influence of aggregate type as obtained by Zoldners (1960). We can here discern a relation between the expansivity of the aggregates and the reduction of the modulus; concrete with more expansive aggregates are subjected to a greater reduction.

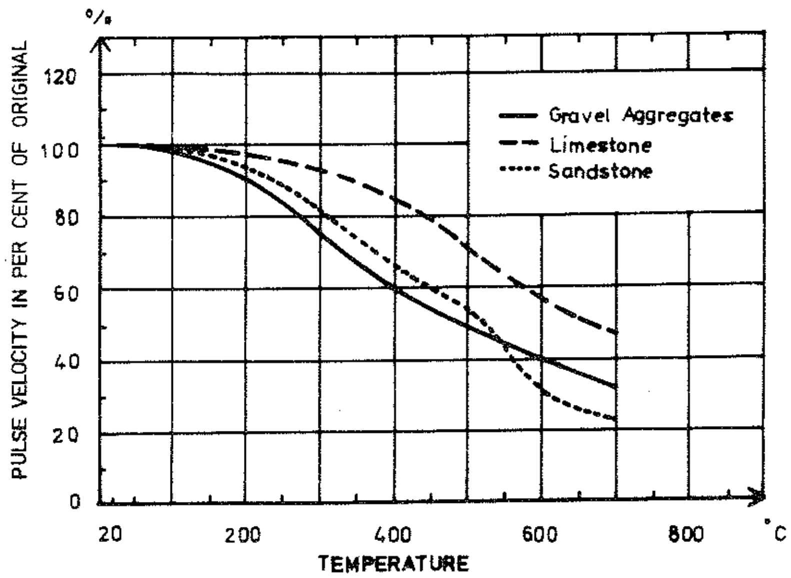


Fig 2 Variation in ultrasonic pulse velocity (i e modulus of elasticity) in per cent of original on three types of concrete after exposure to different temperature levels. Test specimens: Beams of the size 8,9 x 10,2 x 40,6 cm. From Zoldners (1960)

To sum up, it can be established that the elastic properties of concrete at high temperatures have been thoroughly studied and that the variation of the elastic modulus with temperature up to 800°C is comparatively well known.

However, when we consider the inelastic deformations, which occur for larger stress-strength ratios, the information in literature is very scanty. First, it should be notified that in normal fire engineering applications the working stresses are below the proportional limit before the fire. But when the concrete is heated above a certain level the strength decreases and the stress-strength ratio increases, which implies that the inelastic range further or later will be reached. This means that information on the stress-strain curves for high temperatures is needed for the present problem.

Harmathy and Berndt (1966) have determined stress-strain diagrams for cement paste and light-weight concrete in compression at elevated temperatures. The results for cement paste are shown in fig 3 in a 3-dimensional diagram, which gives a good impression of the change of the stress-strain curve with temperature. We can observe that the deformation at failure increases with temperature.

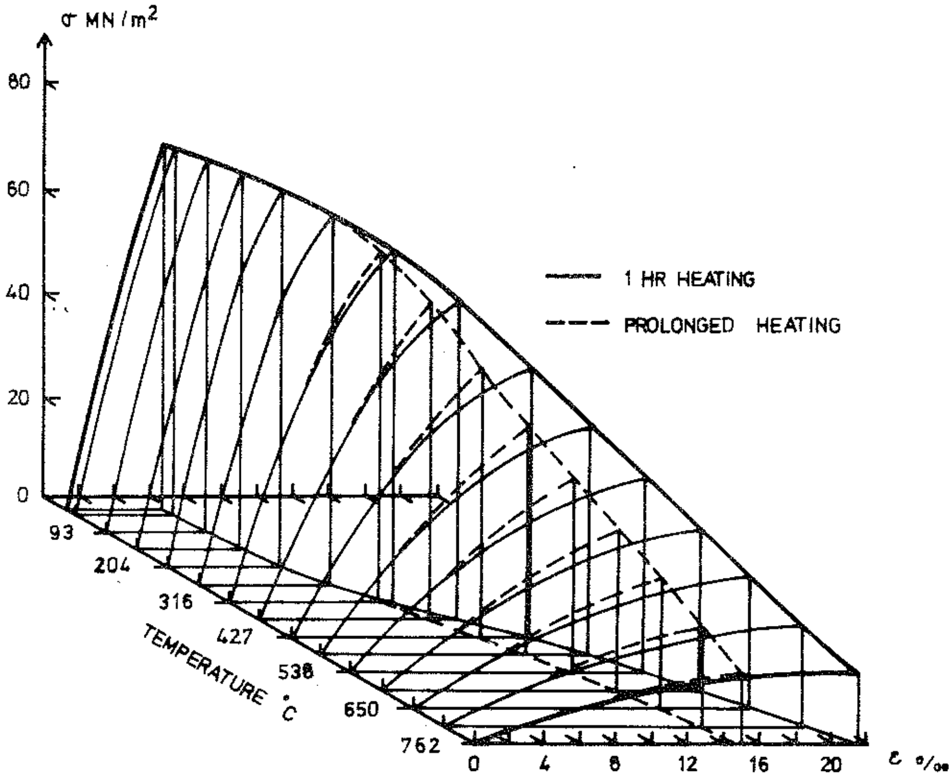
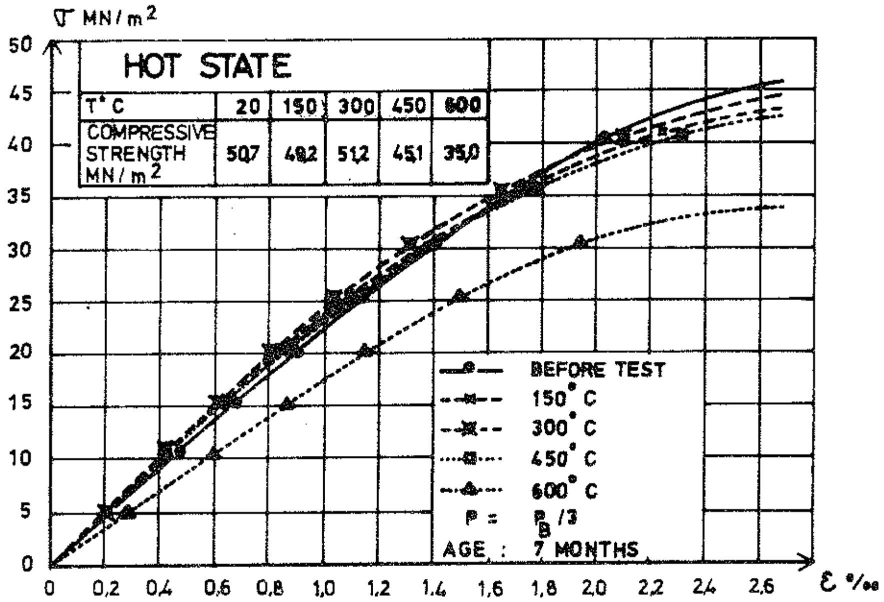


Fig 3 The stress-strain relationship in compression of hydrated Portland cement (w/c = 0,33) at elevated temperatures.
The stress-strain-temperature surface is formed by average curves from scattered data.
Heating procedure: The length of time from the beginning of the heating to the beginning of the compression test was either 1 h or a longer period of time, which amounted to 4 or 24 hours (prolonged heating). Thus the dotted surface averages the test values both from 4 and 24 hours heating.
Test specimens: Cylinders, 4,4 cm in diameter, 8,9 cm long.
From Harmathy & Berndt (1966)

Fischer (1970) has measured stress vs. strain for concrete in compression at high temperatures. In fig 4 is shown the stress-strain diagrams obtained on specimens heated to different tempe-



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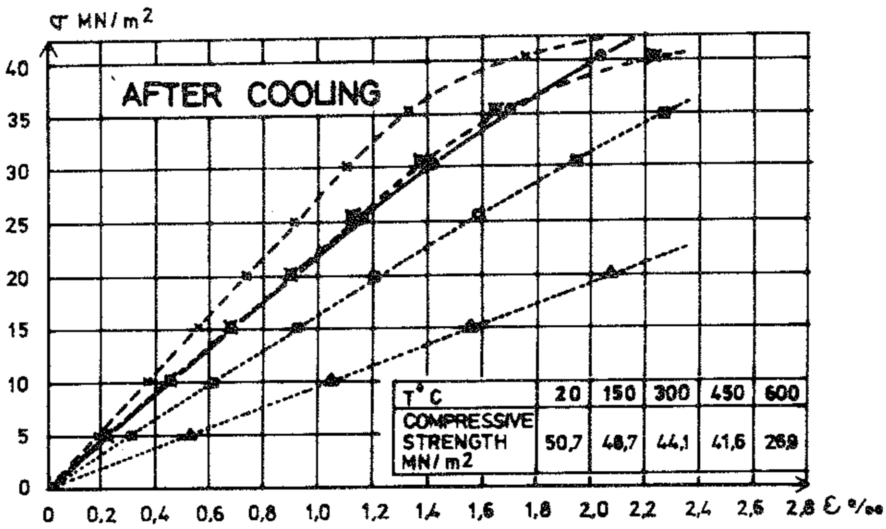

4b.

Fig 4 The stress-strain relationship in compression of concrete (made with standard Portland cement and with quartz aggregate in proportions 1:5,1 and w/c = 0,6) at elevated temperatures when loaded during the heating to 1/3 of the original ultimate load

a) hot state

b) after cooling down to room temperature

Loading procedure: The test specimens were loaded during the whole temperature treatment but unloaded immediately before the compression test.

Heating procedure: Rate of heating 1,9^{o}C·min^{-1} and stabilization for 3 hours at each temperature level before the compression test. Rate of cooling 1,9^{o}C·min^{-1}.

Test specimens: Cylinders, 50 mm in diameter, 70 mm long.

From Fischer (1970)

ratures and loaded during heating to 1/3 of the original ultimate load. The tests were performed in a hot state (fig a) as well as after cooling to room temperature (fig b). We can see that when the specimens are tested hot, the stress-strain diagram is not affected by temperatures up to 450°C. This must be due to the load imposed during heating; if the specimens are unstressed during the heating we can expect a larger influence of temperature on the deformation characteristics. This is indicated in fig 5, also from Fischer (1970), where comparison is made

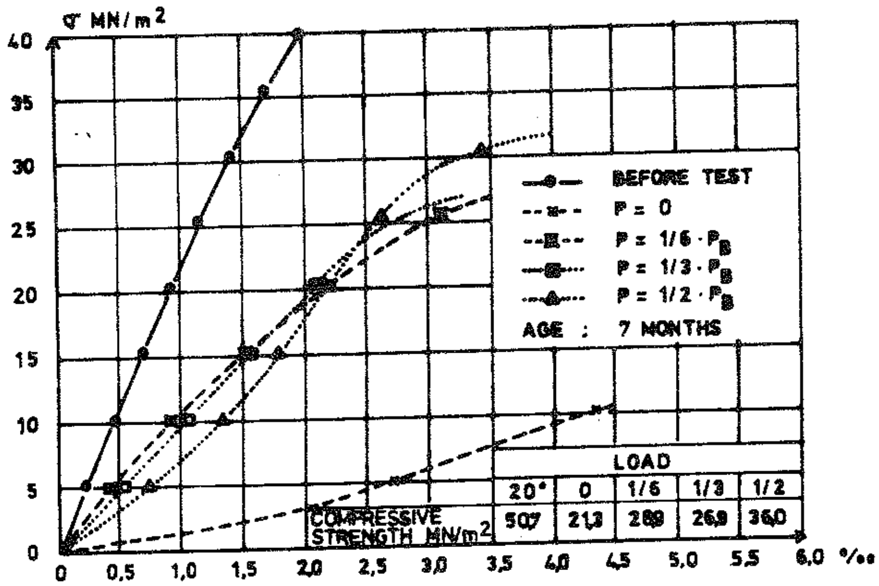

Fig 5 The stress-strain relationship in compression of concrete (made with standard Portland cement and with quartz aggregate) after cooling down to room temperature. During the heating to 600°C various stress levels were applied.
(P=0, 1/6, 1/3, 1/2 x P_B)
P_B = Original ultimate load
Heating procedure: See fig 4.
Test specimens: See fig 4
From Fischer (1970)

between stress-strain diagrams obtained after cooling on specimens heated to 600°C and subjected to different compressive stress levels during heating. It can be seen that specimens which were unloaded during heating exhibited a quite different stress-strain relation than the loaded ones. The magnitude of the load, however, seems to have small influence on the deformations.

In view of the test results shown above, we can establish that the previous load history has a considerable influence on the deformability of the concrete.

2.3 Thermal strain

A comprehensive review of the thermal expansion behaviour of concrete at high temperatures has been made by Zoldners (1971), and extensive data from thermal expansion measurements on concrete can be found in the literature.

The magnitude of the thermal expansion strongly depends on the type of aggregate used in the concrete. As a rule, the thermal expansion increases with increasing content of quartz in the rock used for aggregate. Zoldners gives extensive data on thermal expansion properties of different rocks.

The proportions between aggregate and cement paste also affect the thermal expansion. This can be observed in fig 6 which shows the volume changes during heating for limestone, concrete with limestone aggregate and five mortars with cement: sand ratios ranging from 1:5 to 1:1 - tests by Harada taken from Zoldners (1971).

When concrete is heated, the aggregate particles tend to expand while the cement paste - above about 150°C - tends to shrink. The result of this "thermal incompatibility" is - for normal, unstressed concrete - an expansion almost equal to that of the pure aggregate as shown in fig 6. This means that tensile strains of such a magnitude that some form of cracking occurs, are imposed on the cement paste. If the ratio between cement paste and aggregate increases, then the thermal strain deviates more and more from that of pure aggregate and finally the cement paste prevails, which results in a contraction, as seen in fig 6.

If an external compressive stress is imposed on the concrete during heating, it will act like a "prestress". Consequently, when the concrete is heated, the cracking in the cement paste

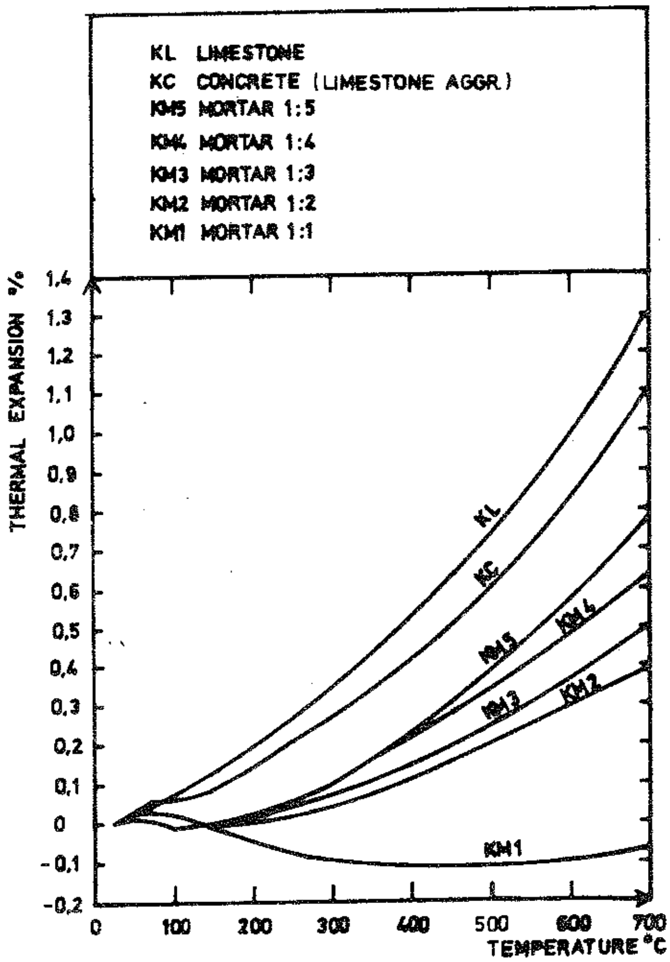

Fig 6 Thermal expansion of mortar (various mix), concrete and limestone at elevated temperatures. Mortar mix varies with the ratio between cement and sand.
From Harada, taken from Zoldners (1971)

is delayed or prevented and the thermal expansion will be reduced. This effect, which has been explained by Fischer (1970), cannot be directly observed in tests, because simultaneous time-dependent strains can not be eliminated.

It is rather clear that there is a close interconnection between this reduction of the thermal expansion and the influence of an applied load on the stress-strain relation, as described in the previous section and illustrated in fig 5. If the specimen is loaded during heating, the destruction of the internal structure is partly prevented, the thermal expansion is reduced and the strength and elastic modulus remain relatively high. If, on the other hand, the specimen is not loaded during heating, the thermal expansion is larger, more cracks will appear in the internal structure and the strength and elastic modulus decrease more.

Accordingly, when a loaded specimen is heated, the continuous "softening" with increased temperature results in a compressive strain component, which gives a reduction of the thermal expansion compared to the "free" thermal expansion of unstressed specimens. This "softening" or increased deformability probably corresponds to that determined from the stress-strain curves obtained on specimens unstressed during heating. This is quite consistent with the fact that specimens which are stressed during heating exhibit smaller deformability than unstressed specimens.

Weigler and Fischer (1968) have measured the thermal expansion of specimens under compressive stress and part of their results is shown in fig 7. The specimens were heated to 600°C at a rate of $120^{\circ}C \cdot h^{-1}$, maintained at 600°C for 3 hrs and then cooled down at the same rate to room temperature. It can be seen that the effect of the imposed stress on the thermal expansion is considerable and for a load level equal to half of the original ultimate load the expansion is replaced by a contraction. This effect is partly caused by short time creep and partly by the instantaneous reduction of the thermal expansion due to the load as explained above.

In the thermal dilatation, when measured on specimens, volume changes due to the formation of new mineral phases will be automatically included. For instance, if rocks containing quartz are used as aggregate, the quartz inversion at 573°C, which is accompanied by a large expansion, strongly affects the thermal dilatation measurements. This can be observed in fig 7 as a large increase in thermal strain immediately below 600°C.

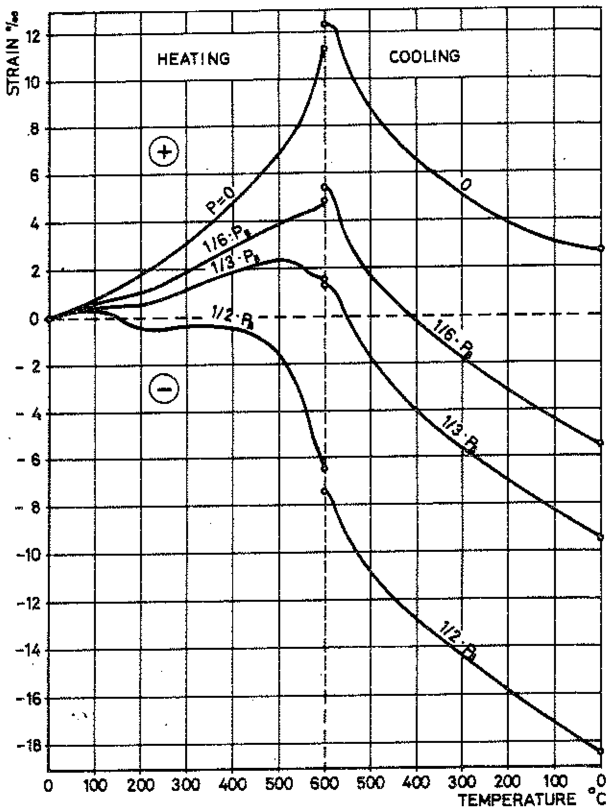

Fig 7 Thermal expansion of concrete specimens subjected to various compressive stress levels at a heating and a cooling rate of $120^oC \cdot h^{-1}$. The temperature was kept at 600^oC for 3 hours. The concrete consisted of standard Portland cement combined with an aggregate of the type quartz, in proportions 1:5,1 and w/c = 0,6. The tests were made at the age of 7 months.
Test specimens: Cylinders, 50 mm in diameter, 70 mm long.
Original ultimate load, $P_B = 50.7\ MN \cdot m^{-2}$
From Fischer (1970)

2.4 Shrinkage

When measuring the thermal expansion, drying shrinkage will normally also be included. Due to the shrinkage, thermal expansion measurements will depend on the initial moisture content and also on the rate of drying. Shrinkage can be avoided if the specimens are sealed against moisture loss during the test. This is very difficult to accomplish for temperatures above 100^oC because of the high steam pressure that will arise. Another way to avoid the shrinkage is to dry out the specimens before the test.

No systematic study of shrinkage at elevated temperatures has yet been made. The reason for this is probably that it is difficult to make relevant tests when elevated temperatures are involved. If for example the tests should be made at constant temperature, the temperature level as such is less significant than the way in which the temperature influences the moisture state. Since the shrinkage is very closely connected to the moisture situation and occurs due to drying, the results will be strongly affected by the conditioning before the testing temperature is reached.

A better way of investigating the shrinkage effect is to study in what way the thermal dilatation is affected by the initial moisture content. For instance, by comparing predried, air cured and saturated specimens, the shrinkage strain can be readily deduced.

2.5 Creep

2.5.1 General considerations

Creep of concrete in the temperature range up to 200°C has been studied in a number of investigations, most of them being motivated by the use of prestressed concrete for nuclear reactor pressure vessels. Hence, comparatively low rates of heating and long durations have been used, while in the case of fire we are interested in the creep behaviour within the first hours from heating begins. Many investigators have not accounted for the creep strains within the first hours from loading; they are rather thinking in terms of days than in hours. Concrete structures subjected to fire are also heated very rapidly and subjected to considerably higher temperatures than 200°C.

Furthermore, most of the tests have been done at steady state conditions; i e at constant temperature and in a stabilized moisture state. In fire applications the temperatures as well as the moisture state change comparatively quickly, which leads to a radically different creep behaviour.

Nevertheless, the experience gained in such "long term" creep tests at elevated temperatures described in literature could be very valuable to the present problem.

In the temperature range up to 150°C the evaporable water plays the main role in the mechanisms controlling creep behaviour. Consequently, variables controlling moisture content, moisture migration and moisture loss from the specimens are also significant for the magnitude of creep. The conditions fixing the moisture loss in tests are usually considered in that the specimens can be either completely sealed against moisture loss or unsealed. The sealed specimens then approximate to the situation of concrete in the interior of massive structures.

2.5.2 Tests on sealed specimens at constant temperature

Creep of sealed specimens has been reported by various authors (England & Ross (1962), Nasser & Neville (1965), Arthanari & Yu (1967), Browne (1968), Hannant (1968), Ruetz (1968), Maréchal (1970)). Due to experimental difficulties such tests are usually limited to the temperature range up to 100°C.

It is generally agreed that sealed specimens don't shrink, i e that no significant deformation will occur in an unstressed specimen at constant temperature.

Several authors have found a steady increase in creep with the temperature for sealed specimens in the range 20-80°C. For instance, fig 8 shows the results from Hannant (1968), with specimens cured for 5 months under water and then 1 month in a sealed condition before testing. We can see that the creep is accelerated for temperatures near 100°C.

In tests by Maréchal (1969, 1970), see fig 11, curve 3, also a steady increase in creep was observed up to 95°C, the rate of increase however decreasing with temperature. These specimens had been cured for one year in 20°C and 95% RH. The difference

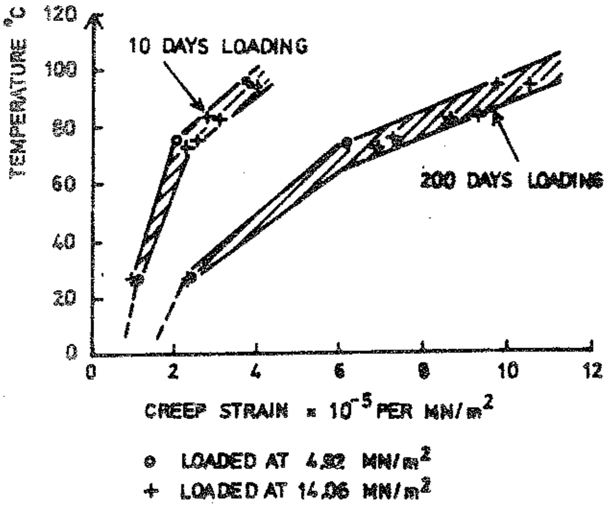

Fig 8 Creep strain per unit uniaxial stress of sealed specimens, related to temperature.
Heating procedure: Rate of heating $10^{o}C\cdot h^{-1}$
The load was applied 24 h after the start of heating in all tests.
Concrete mix: Sulphate resisting Portland cement with plasticizer, course aggregate of limestone, maximum grain size 9.5 mm, and sand. Cement: aggregate ratio = = 1:4.5 and w/c = 0.47.
Curing procedure: Water curing for 5 months and then sealed 1 month before test.
Test specimens: Cylinders, 105 mm in diameter, 305 mm long.

between Maréchal's and Hannant's tests, in which similar pretest curing conditions were used, was that Maréchal's specimens were heated very slowly and then maintained at the temperature for 15 days before loading, while in Hannant's tests the total time elapsed from heating commenced until the load was applied only amounted to 1 day. The somewhat different temperature dependence may be explained by the fact that the degree of hydration of the cement paste is increased due to heating. This increase depends on the temperature level and the heating procedure, i e the time under temperature before loading. As a matter of fact, the effect of continued hydration during elevated temperature creep tests is considerable. Ruetz (1966) has investigated this effect in creep tests on cement paste. In fig 9 is shown the creep as a function of temperature for sealed specimens treated in different ways. Curve 1 corresponds to saturated sealed specimens heated to respective test temperature and then loaded, while curve 2

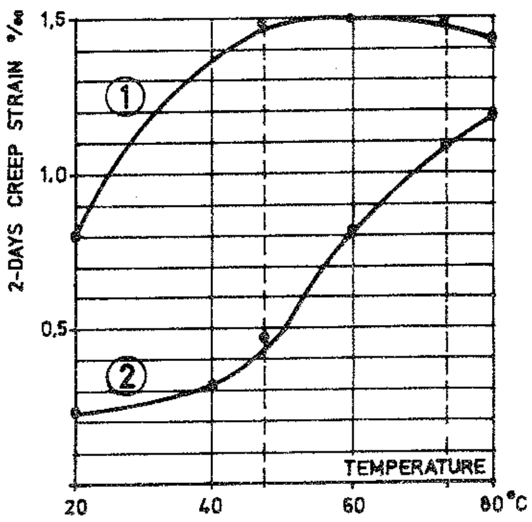

Fig 9 Creep strain during 48 hours of saturated, sealed cement paste specimens, related to temperature.
Curve 1. Specimens heated to respective test temperature and then stressed for 48 hours.
Curve 2. Specimens exposed to 80°C for 72 hours without loss of water, and then tested as those of curve 1.
Test specimens: Cylinders, 1.7 cm in diameter and 6 cm long. w/c = 0.30.
From Ruetz (1966).

shows the creep for specimens treated as those of curve 1 except that they have been exposed to 80°C for 72 hours before testing. In the latter case the increase in hydration takes place before the test and for all temperatures, which implies that curve 2 more shows the "true" temperature influence. We can see from the figure that the behaviour is notably different in the two cases.

2.5.3 Tests on unsealed specimens at constant temperature

When trying to simulate concrete, from which the moisture loss is prevented only partly or not at all - which applies to the majority of normal building structures - several difficulties arise. The creep rate depends not only on temperature and moisture content, but is also influenced by the change per se of moisture content and temperature.

In the literature many tests aimed at determining the effect of temperature on creep on unsealed specimens are accounted for. In practically all cases the testing procedure has been the following. The specimens are heated at the same rate until the actual test temperature is reached. Then, after a certain period of time, during which the temperature is stabilized, the constant load is applied and the strains are measured. The strains of unloaded specimens treated in the same way are simultaneously measured. The difference between the strains of the loaded and the unloaded specimens then constitutes the creep strain.

This method of testing involves certain problems. When moist specimens are treated according to the above procedure, the moisture situation in the specimens at start of and during the creep tests will be different for different test temperatures. Since the creep rate besides temperature also strongly depends on moisture content and rate of moisture change, the creep in such tests will be influenced by the initial moisture content, the time of heat treatment before loading and the geometry of the specimens. Therefore, in many such tests the temperature level per se is not significant for the creep rate, but serves rather as a condition determining the moisture situation.

This is illustrated in fig 10, from Hannant (1968), which shows the weight loss with the time at 3 different testing temperatures. The heating started 1 day after removal from water. In Hannant's tests the load was applied 1 day after the start of heating. Hannant concludes that creep tests under these conditions are "of little relevance to conditions occuring in practice".

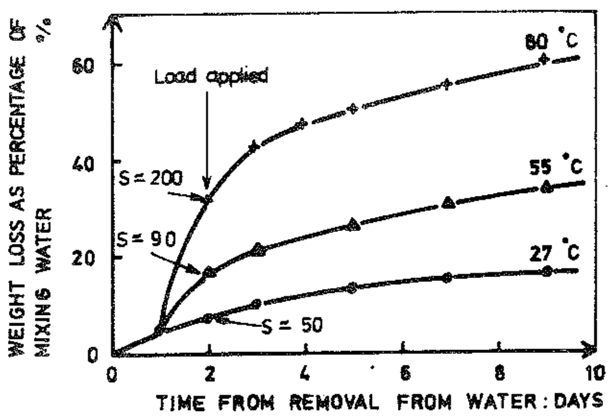

Fig 10 Moisture loss after removal from curing tank of unsealed concrete cylinder specimens. Heating commenced after 24 hours and loading was applied after 48 hours.
S = Approximate shrinkage strain x 10^{-6}.
Size of specimens and concrete mixture: see fig 8
From Hannant (1968)

Maréchal (1970) has made comprehensive creep tests on unsealed specimens at stabilized temperature. A special feature in his tests was the considerably long time from heating commenced until the load was applied, which implied that the desorption of evaporable water from the specimens had decreased to a very low rate when the load was applied. This means that the specimens at the start of the creep tests were almost in moisture equilibrium with the environment and that the main part of the shrinkage had already occured. Furthermore, Maréchal made tests on initially dry specimens as well as sealed specimens for comparison. His results are shown in fig 11, where 3 cases are compared,

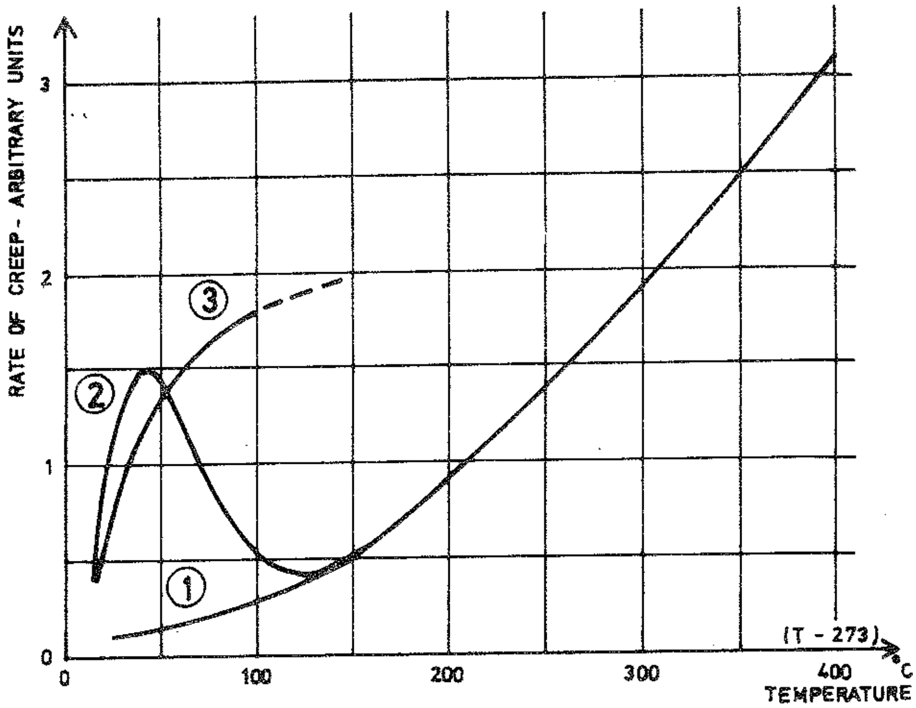

Fig 11 . Variation in rate of creep related to elevated temperatures for three types of concrete specimens viz.
1 Specimens predried at 105°C
2 Saturated specimens tested unsealed
3 Saturated specimens tested in a sealed condition
All specimens were cured in 95% RH, 20°C for 1 year before the test.
The heating rate was 0,25°C/h and the test temperature was maintained for 15 days before the load was applied.
Concrete mix: Portland cement and quartz aggregate. The proportions were not given in the reference.
Test specimens: Prisms, 70 x 70 x 280 mm.
From Maréchal (1970)

viz. specimens predried at 105°C, initially saturated specimens tested unsealed and saturated specimens tested in sealed condition. All specimens were cured in 95% RH, 20°C for 1 year before the test. The specimens were slowly heated at a rate of 0,25°C/h and maintained at the testing temperature for 15 days before the load was applied. The creep was obtained after subtraction of shrinkage and the creep rate is represented as the slope of the creep curve, when plotted against log time, c.f. fig 19.

In the temperature range 20-100°C the presence of the evaporable water causes a considerable increase in creep compared to that of dry specimens. The creep rate for the initially saturated unsealed specimens at 50°C was about the same as the creep rate for sealed specimens at the same temperature. Above 50°C however, the creep rate of the unsealed specimens decreases rapidly. Evidently, at these temperatures, extraction of moisture before the creep test is so large that the rate of creep approaches the curve for predried specimens.

If the specimens are more rapidly heated and if the load is applied within a short time after the start of heating, then desorption takes place during the creep test and the results can be radically changed. In fig 12 the influence of different test procedures is illustrated by a comparison between Marèchal's tests and tests by England & Ross (1962). The results are presented in terms of specific creep in order to make it possible to compare creep strains under different stresses. The stresses used in the tests are given in the text of the figure and it can be seen that they are of the same order of magnitude for both test series. In all the tests the creep after 60 days under load was chosen.

In the tests performed by England & Ross the specimens were only 10 days old at the start of the tests and they were heated only one day before loading. We can observe that the specific creep in this case is approximately equal to that of Marèchal's test at 50°C, but at higher temperatures the creep is considerably larger. This is apparently due to continued drying followed by increased creep in England & Ross's tests.

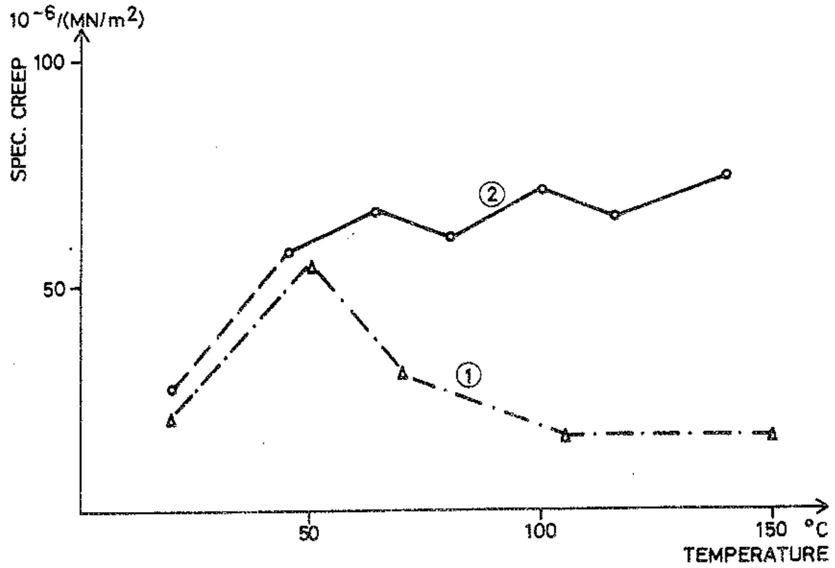

Fig 12 Variation in specific creep of unsealed concrete specimens, related to elevated temperatures. Comparison between different test procedures.
① Maréchal (1970). Pretest curing and heating as in fig 11. Stress 5 $MN \cdot m^{-2}$
② England & Ross (1962). Curing: 3 days under water and then in 90% RH, 17°C until the test at 10 days. Stress: 7 $MN \cdot m^{-2}$.

An interesting contribution to the discussion of these problems has been given by Birkimer et al (1969). These authors are well aware of the importance of a more complete picture of the strain behaviour. In fig 13 is shown results from their tests, illustrating the influence of different temperature and loading histories on creep behaviour. The explanation of the curves is given in the figure text. It can be seen that for specimens which are loaded before heating (curve E) the thermal expansion is cancelled and a further contraction takes place with the time, while for specimens which are loaded after the temperature 121°C is reached (curve D), the final deformation is quite different from curve E. The figure as a whole shows clearly that the three parameters stress, temperature and time is far from independent of each other when their influence on deformation behaviour is concerned.

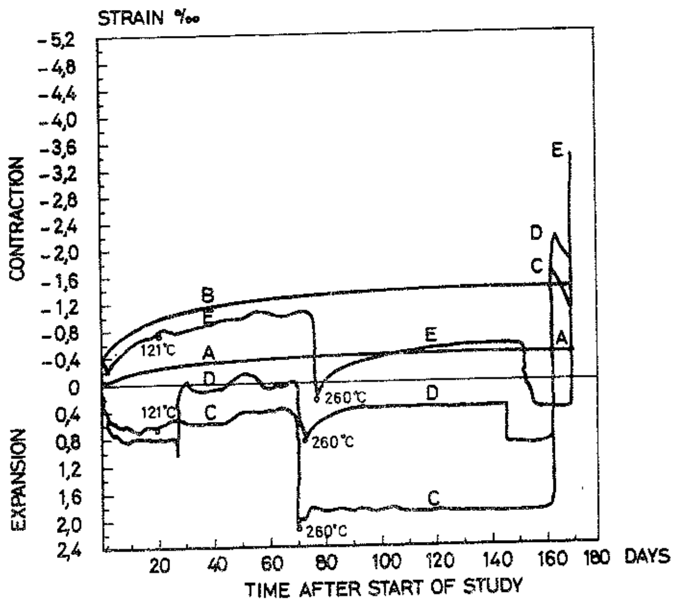

Fig 13 Time-dependent strains in gravel concrete.
Curve A: Specimens neither heated nor stressed.
Curve B: No heating. Load level = 40% x $\sigma_{ult}^{20^oC}$
Curve C: First heating to 121°C. After 70 days the temperature increased to 260°C and after 160 days the cooling started. No load.
Curve D: First heating to 121°C. After 28 days a load = = 40% x $\sigma_{ult}^{20^oC}$ was applied and after 70 days temperature increased to 260°C. The load was removed after 145 days. Cooling after 160 days.
Curve E: A load = 40% x $\sigma_{ult}^{20^oC}$ was applied from the beginning. Then, heating to 121°C and after 75 days heating to 260°C. The load was removed after 150 days. Cooling begun after 165 days. From Birkimer et al (1968).

2.5.4 Effect of drying and temperature change

It is a well known fact that concrete during drying will creep far more rapidly than concrete in moisture equilibrium with the environment. In fig 14 from Ruetz (1968) a comparison is made between the 6-day creep strains for cement paste specimens in moisture equilibrium during the test (basic creep) and specimens which were drying during the test (sorption creep). On the abscissa is indicated the moisture content during the test and at the end of the test respectively. Ruetz also found that the creep increases with the rate of drying and creep rates more than 10 times larger than the corresponding basic creep were observed.

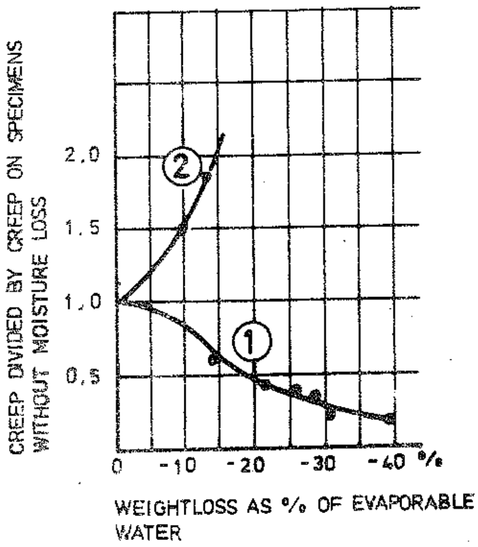

Fig 14 6-days creep strain for cement paste as influenced by drying.
(1) Specimens were dried to equilibrium before the creep test (basic creep).
(2) Specimens were drying during the creep test (sorption creep). In this case the creep strain is plotted against the moisture loss at the end of the 6-day period.
w/c = 0.30
Test specimens: Cylinders, 17 mm in diameter, 60 mm long.
From Ruetz (1968).

Another effect, which is of importance in this connection, is that the creep seems to increase due to temperature change. Hansen and Eriksson (1966) have measured time-dependent deflections of cement mortar beams, loaded in flexure and subjected to various temperature conditions. All the beams were cured for 28 days in water at 20°C and also stored in water throughout the test, in this way moisture conditions being constant and equal in all tests. The creep deflection vs. time is shown in fig 15 for 4 different cases. Curve 1 shows the creep of beams which were first loaded and then heated from 20°C to 40°C at a rate of 120°C·h^{-1} then maintained at 40°C for the rest of the test. The specimens of curve 2 were treated in the same way except that the rate of heating was 2°C·h^{-1}. In the third case the specimens were heated to 40°C at a rate of 2°C·h^{-1} and then slowly cooled to 20°C. Then the beams were loaded and again heated (2°C·h^{-1}) to 40°C and maintained there during the rest of the test. Curve 4 shows the creep of specimens which were first heated to 40°C at a rate of 2°C·h^{-1} and then loaded. The temperature was held constant at 40°C during the test.

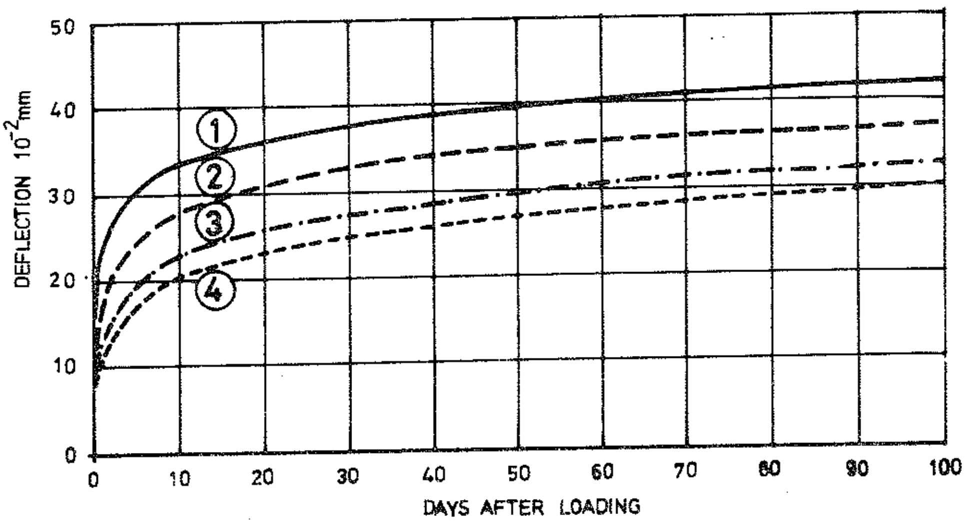

Fig 15 Long-time deflection of cement mortar beams loaded in water and exposed to various rates of temperature change before and after load application.
Curve 1: Beams were first loaded and then heated from 20°C to 40°C at a rate of $120^{\circ}C \cdot h^{-1}$.
Curve 2: The same as in curve 1 but the rate of heating was $2^{\circ}C \cdot h^{-1}$.
Curve 3: Beams were heated to 40°C at a rate of $2^{\circ}C \cdot h^{-1}$ and then slowly cooled to 20°C. Beams were after that loaded and heated again in the same way as for curve 2.
Curve 4: Beams were first heated to 40°C at a rate $2^{\circ}C \cdot h^{-1}$ and then loaded.
Test beams: 20 x 50 x 400 mm, made from standard Portland cement and sand from glacial origin in proportions 1:1.18. w/c = 0.37.
Loading: Simply supported beams loaded in flexure by two symmetrical point-loads giving a constant moment over a length of 280 mm. The deflection is referred to the constant moment zone.
From Hansen et al (1966)

Comparing curves 2 and 4, we can conclude that the temperature change in itself gives an increased creep. We can also see that the creep increases with the rate of heating. Furthermore, if the specimens are subjected to one temperature cycle before loading the creep will be reduced.

To sum up, changes in temperature and moisture conditions generally tend to increase creep. The creep also increases with the rate of change of these conditions. The question if these conclusions also hold at higher temperatures than 40°C remains to be answered.

2.5.5 Creep under rapid heating to high temperatures

As previously mentioned, most of the above described test results from existing literature have been obtained on specimens which have been slowly heated and then subjected to the actual temperature during a long period of time. The question is how the influence of free moisture should be considered when concrete is rapidly heated to high temperatures. And how is the creep behaviour affected by temperatures in the range above 200°C?

It can be established that the dimensions of normal concrete structures, that are of interest in connection with fire, are such that the moisture is free to disappear i e they are most accurately approximated with unsealed specimens, though it must be remembered that the moisture migration depends very much on geometry.

In view of the discussions in section 2.4.4, we can expect a large creep during that period when the concrete is heated and moisture is extracted. This is sustained by creep tests made by Thelandersson (1972) in pure torsion. In fig 16 a comparison is made between the angular deformations obtained if on the one hand, the load is applied before heating, and on the other hand, the load is applied after that the temperature is reached . The temperature time curve is the same in both cases and is also shown in the figure. As we can see the deformations are much larger during that period when the temperature is changing than at constant temperature. It should be noted that in torsion tests the angle of twist is not affected by neither thermal strains nor shrinkage strains. A reservation must be done for the possibility that steam pressure during the heating period might have affected the creep rate. The steam pressure gives hydrostatic tensile stresses in the material and thus increases the tensile stress level which leads to an increased creep rate. However, if the steam pressure should have any significant effect the creep rate should be largest during that period when the water is vaporized. Since, the tests at higher temperatures did not exhibit such a behaviour, the influence of the steam pressure seems to be less important. The conclusion must be that the creep under

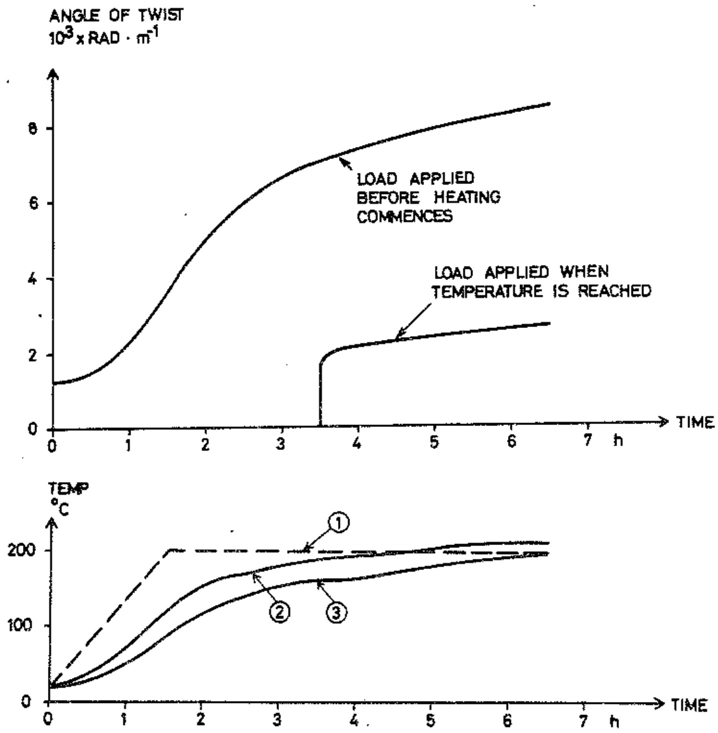

Fig 16 Influence of the time at which the load is applied on creep behaviour in torsion. The load is equal to 30% of ultimate load at room temperature. In the figure is also shown the temperatures in the test. Cylindrical specimens, 15 cm in diameter.
Curve 1: Desired furnace temperature.
Curve 2: Temperature inside specimen, 10 mm from the surface.
Curve 3: Temperature inside specimen, 72 mm from the surface.
Concrete mix: Portland cement and quartz aggregate in proportions 1:4.65.
w/c = 0.55
From Thelandersson (1972)

heating is significantly higher than the creep at constant temperature.

In literature a few other creep tests at high temperatures can be found. All those tests were performed at constant stabilized temperature. In fig 11 from Maréchal we can observe that the rate of creep increases steadily from 150°C to 400°C.

It should be born in mind that beoynd 150°C the dehydration of the hardened cement paste starts and non-evaporable water is lost. Fig 17 shows this water loss after a prolonged heating period at temperatures up to 260°C. It should be noted that in

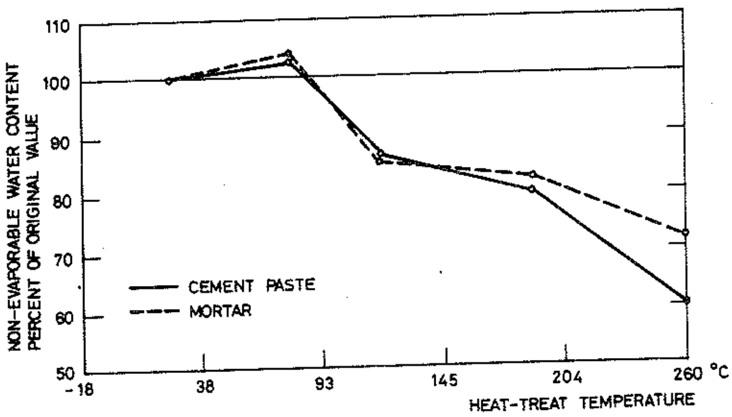

Fig 17 Effect of prolonged heat exposure on the nonevaporable water content of cured Portland cement and mortar. From Lankard et al (1971)

Maréchal's tests the concrete was subjected to the testing temperature for a long time before the load was applied, which implies that the non-evaporable water content probably was stabilized before the test. Different results might be expected if the loss of the non-evaporable water takes place during the creep test.

The test results by Cruz (1968) shown in fig 18, indicate similar temperature dependence as that found by Maréchal. We can see that the deformation under load increases monotonically with the temperature. In these tests the specimens were heated at a rate of 5.5°C/min until the test temperature was reached, and then the temperature was held constant throughout the test. The load was applied about 1 h after that the test temperature was reached. In addition, it must be noted that in Cruz's tests the creep strains were not corrected for shrinkage and, accordingly, a comparison with Maréchal's results may be misleading.

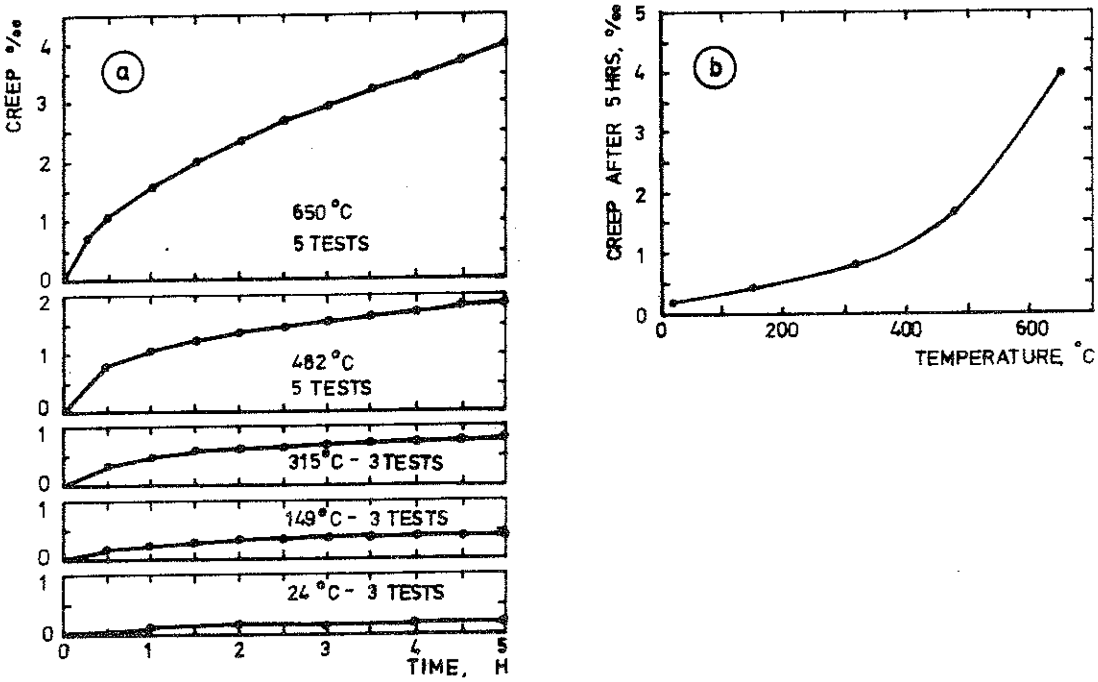

Fig 18 Creep of carbonate aggregate concrete at high temperatures.
a) Creep vs time.
b) 5 hours creep vs temperature
Heating procedure: Rate of heating $5.5^{\circ}C \cdot min^{-1}$.
The creep test started 1 hour after the test temperature was reached.
Concrete mix: Sand and gravel aggregate and Portland cement in proportions 7:1. w/c = 0.56.
Curing: Moist for 7 days and then until test at 28 days in air at 50% RH and 20°C.
$\sigma_{ult}^{20^{\circ}C} = 29\ MN \cdot m^{-2}$
Applied stress = $0.45 \cdot \sigma_{ult}^{20^{\circ}C}$

Specimens: Cylinders, 50 mm in diameter, 100 mm long.
From Cruz (1968)

2.5.6 Mathematical formulation of creep behaviour

In the previous sections the influence of temperature on the magnitude of creep has been discussed on a phenomenological level. In this section the time and stress dependence for creep at high temperatures will be discussed. Mathematical formulas for the temperature dependence given in literature will also be accounted for.

Numerous mathematical expressions have been introduced to describe the time variation for creep of concrete. Neville (1970) mentions the following cathegories:

Power expressions	$\varepsilon_c \sim t^B$ or $\log \varepsilon_c \sim \log t$
Exponential expressions	$\varepsilon_c \sim \exp(-f(t))$
Hyperbolic expressions	$\varepsilon_c = \frac{t}{A+Bt}$
Logarithmic expressions	$\varepsilon_c \sim \log t$

where ε_c = creep strain

t = time

A, B = constants depending on stress and temperature

The most widely used time function for the creep of concrete at elevated temperatures is the logarithmic one, which implies that the creep plotted against log t forms a straight line. Among others, Maréchal (1970), Browne (1968) and Lewis et al (1968) have suggested expressions of that type. Results from Maréchal plotted in a semilogarithmic diagram is shown in fig 19. Maréchal writes

$$\varepsilon_c = b \cdot \log t + \varepsilon_o \qquad (2:1)$$

where b is a constant equal to the slope of the straight line and depending on stress, temperature and age of concrete and ε_o is the creep at the time t=1.

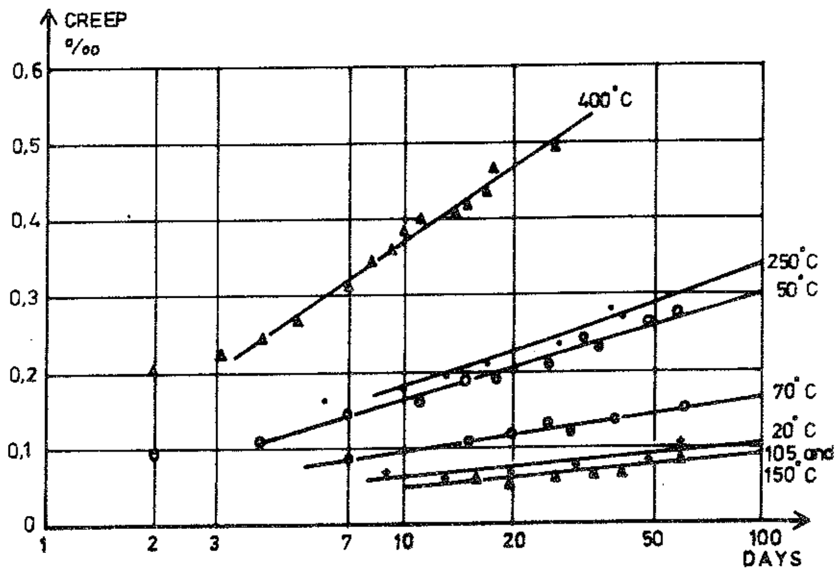

Fig 19 Creep of quartz aggregate concrete at elevated temperatures.
All specimens were cured in 95% RH, 20°C 1 year before the test.
The heating rate was 0.25°C/h and the test temperature was maintained for 15 days before the load was applied.
Test specimens: Prisms, 70 x 70 x 280 mm of quartz aggregate and Portland cement. Proportions were not given in the reference.
Applied stress 5 MN/m^2, which corresponds to 14% of ultimate load at 20°C.
From Marêchal (1970)

The logarithmic expression can also be used in another form,

$$\varepsilon_c = b \cdot \log(t + 1) \qquad (2:2)$$

which has the advantage that ε_c is defined and equal to zero when t=0. Then it is also possible to make a well-defined distinction between instantaneous and time-dependent strain as shown principally in fig 20, from Lewis et al (1968). The total strain is plotted against log(t + 1) and forms a straight line, except for very short times. The intersect with the axis t = 0 defines an initial strain component which can include some short-time creep. If desired, a modified elastic modulus E'less than the ordinary elastic modulus E can be introduced as shown in the figure. In that case the total strain can be written

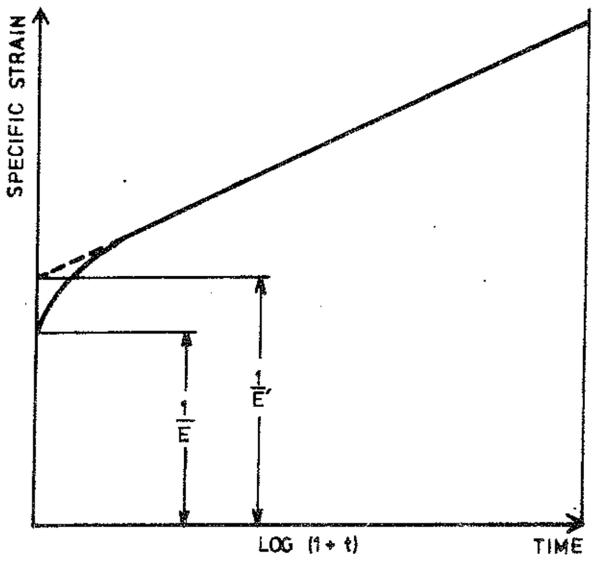

Fig 20 Typical time-deformation curve for sealed concrete. From Lewis et al (1968)

$$\varepsilon_{tot} = \frac{\sigma}{E'} + b \cdot \log(t + 1) \tag{2:3}$$

where b depends on stress, temperature and age of concrete.

Browne and Blundell (1969) have suggested that a power expression could be used at high temperatures. This can be written

$$\varepsilon_c = a \cdot t^n \tag{2:4}$$

where a and n are constants. If the creep strain is plotted against time in a log-log diagram, the results will be straight lines having a slope equal to n and intersecting the axis t = 1 at a. Browne and Blundell have shown that results from creep tests made by themselves and others agree with the above expression.

The influence of the stress on the rate of creep is most often supposed to be linear up to a certain stress-strength ratio. At normal temperatures this upper limit has been estimated to values ranging from 0.30 to 0.75 (Neville (1970)). Above this level the strain rate increases more than linear with stress as shown in fig 21 from tests by Gvozdev, taken from Neville (1970).

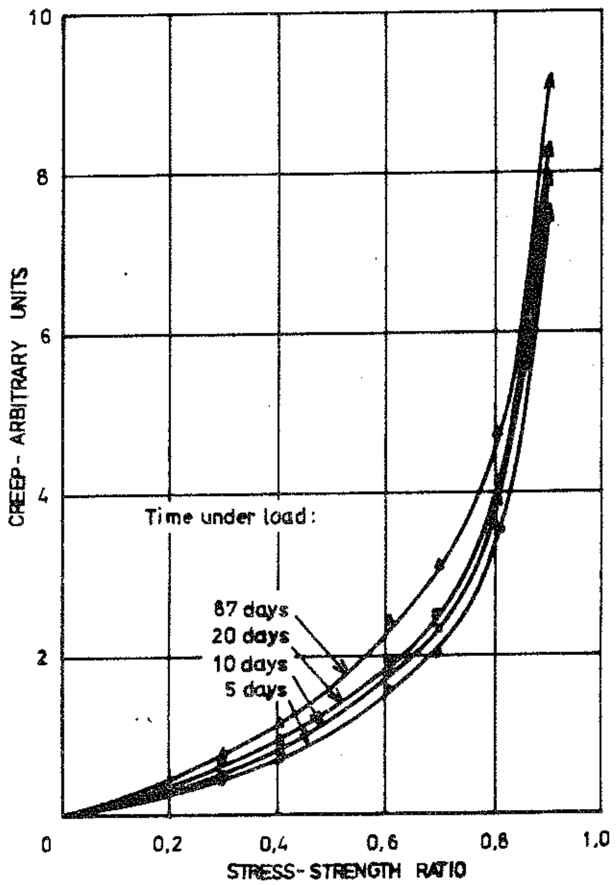

Fig 21 Relation between creep and stress-strength ratio for some Russian concretes at normal temperature. From Gvozdev (1966), but taken from Neville (1970)

Because the working stresses usually fall within the linear region, many scientists assume a linear relationship in that they work with the specific creep, which is the creep strain divided by the actual stress. This is also done in many investigations dealing with creep at elevated temperatures, and some test results in the temperature range 20-100°C support this assumption (Nasser and Neville (1965)). But, since the strength changes with temperature, the question arises whether the absolute stress or the ratio between stress and strength is the most significant parameter. The change of strength is not so important for temperatures up to 100 or 200°C, but when higher temperatures are considered it might be better to define a "specific" creep in terms of the stress-strength ratios at the respective temperatures. At higher temperatures the stress-strength ratio will be large and the relation between strain rate and stress will become non-linear for stress values lying within the linear region at normal temperatures. This means that we are interested in creep at high (relative to the strength) stresses at high temperatures and that the linear relationship then is not sufficient to describe the stress dependence.

In literature the following expressions have been suggested for describing the stress dependence of the rate of creep for concrete over the whole stress range.

$$\left.\begin{array}{l}\dot{\varepsilon}_c \sim \sin h\,(c\cdot\sigma) \\ \\ \text{and} \\ \\ \dot{\varepsilon}_c \sim \sigma^n\end{array}\right\} \qquad (2{:}5)$$

where c and n are constants. The first one has been deducted theoretically by Freudenthal in 1950 and the second is an empirical formula. Both of them fits well with test data at normal temperatures, although the power expression is more widely used for the sake of convenience.

At high temperatures sufficient data on the influence of stress level on the rate of creep are lacking, especially at large stress-strength ratios. Maréchal (1970), in his comprehensive study of elevated temperature creep, used 3 different stress levels 5, 10 and 15 MN/m^2 corresponding to about 12, 24 and 36% of normal temperature strength for porphyry concrete. Maréchal proposed that for constant temperature and time the strain rate $\dot{\varepsilon}_c$ should be proportional to

$$\sigma^{\frac{\alpha}{kT}}$$

where T = absolute temperature, k = Boltzmann's constant and α is another constant. In fig 22 the strain rate expressed as the factor b in eq (2:1) is plotted against σ for different temperatures in a log-log diagram. As seen, parallell straight lines are obtained which implies that the exponent $\frac{\alpha}{kT}$ is independent of temperature. $\frac{\alpha}{kT}$ was found to be approximately 0.82, which

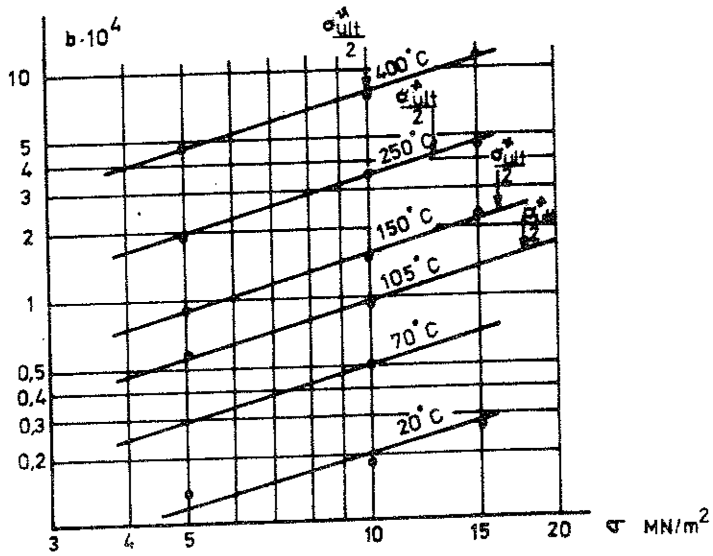

Fig 22 Creep rate of porphyry concrete as a function of σ at different temperature levels in a log-log scale. In the figure is indicated one half of the strength at respective temperatures ($1/2\ \sigma^{*}_{ult}$) From Maréchal (1970)

means that the strain rate increases almost linearly with the stress. Thus Maréchal's results can be interpreted as

$$\dot{\varepsilon}_c \sim \sigma^n \tag{2:6}$$

where n is constant and independent of temperatures up to 400°C.

The influence of temperature on the rate of creep is often estimated assuming that the creep can be treated as a flow of a newtonian liquid. Newton's formula is

$$\dot{\gamma} = \frac{\tau}{\eta} \tag{2:7a}$$

where $\dot{\gamma}$ = shear strain rate

τ = shear stress

η = coefficient of viscosity

For normal stresses we can write

$$\dot{\varepsilon} = \frac{\sigma}{\lambda} \tag{2:7b}$$

where $\dot{\varepsilon}$ = strain rate

σ = normal stress

λ = Trouton's coefficient of viscous traction

Assuming incompressible flow, Poisson's ratio equals 0,5 and the coefficient of viscous traction λ is equal to 3η.

The viscosity of a liquid is related to temperature in the following manner

$$\ell n\ \eta \sim \frac{1}{T} \qquad (2:8a)$$

where T = absolute temperature.

Accordingly,

$$\ell n\ \lambda \sim \frac{1}{T} \qquad (2:8b)$$

The above interrelations have been used to predict the temperature sensitivity of creep in different materials. For metals and later also for concrete and cement paste this has been formulated in terms of Arrhenius' activation energy equation

$$\dot{\varepsilon}_c \sim e^{-\frac{\Delta H}{RT}} \qquad (2:9)$$

where ΔH = activation energy for creep

R = gas constant

T = absolute temperature

This relationship agrees with the creep experiments by Maréchal (1970) on predried specimens. In fig 23 is shown the plot of log $\dot{\varepsilon}_c$ against $\frac{1}{T}$ for 3 different stress levels. $\dot{\varepsilon}_c$ is expressed as the slope of the creep vs log time curve. As seen from the figure the data fit with straight lines, their slope determining the activation energy ΔH.

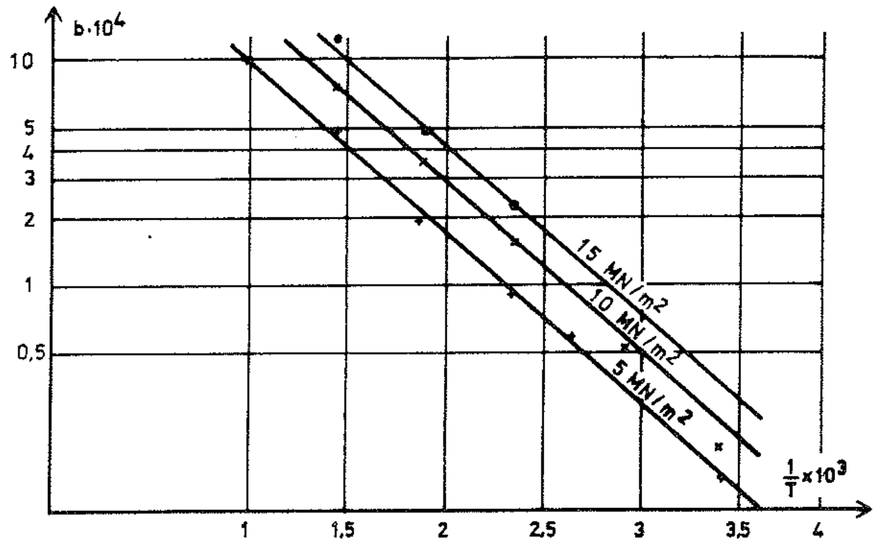

Fig 23 Creep rate of porphyry concrete as a function of $\frac{1}{T}$ at different load levels.
From Maréchal (1970)

However, Maréchal's data on the creep of initially saturated specimens is not in conformity with the relation (2:9) as far as the temperature range up to 100°C is concerned, which means that the influence of moisture have to be superimposed on the "basic" temperature dependence, cf fig 11.

It is interesting to compare these data with creep tests on cement paste made by Ruetz (1966). He studied the temperature range 20-80°C and in fig 24 some of the data are plotted in the same way as Maréchal's data in fig 23, viz. log $\dot{\varepsilon}$ vs $\frac{1}{T}$.

Curve 1, corresponding to predried specimens, is a straight line and the activation energy calculated by Ruetz from its slope amounts to 15500 J/mole. Maréchal found for concrete with 5 different types of aggregate values of ΔH in the range between 13400 and 16300 J/mole, i e of the same order of magnitude as for cement paste. Ruetz notifies that the value of ΔH for free water obtained from the relation between viscosity and temperature is about the same as that for cement paste (ΔH_{water} = = 14600 J/mole). According to Ruetz this indicates that the creep process takes place in the water layers between gel particles.

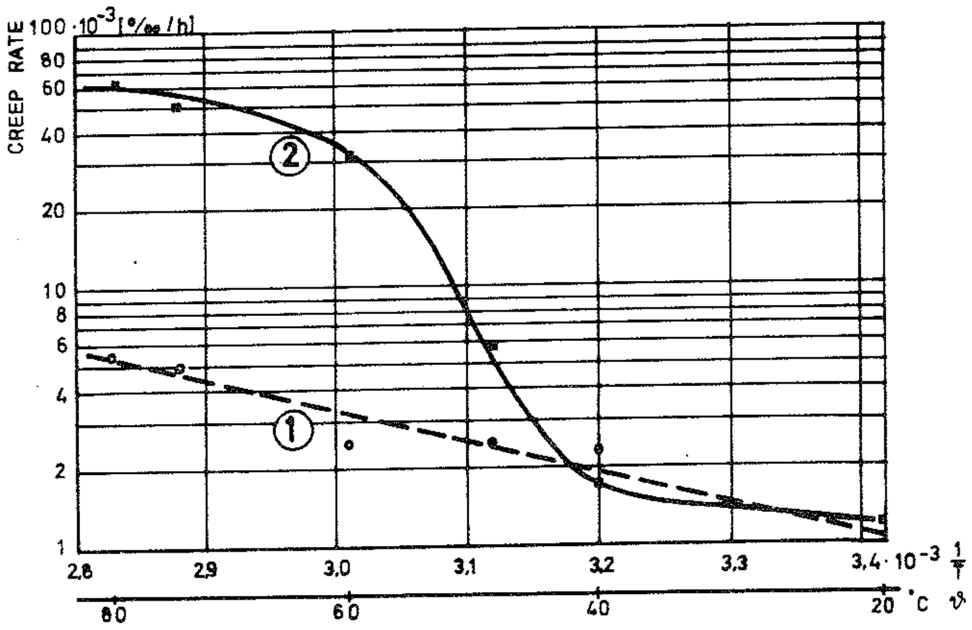

Fig 24 Creep rate of cement paste as a function of $\frac{1}{T}$.
Curve 1: Predried specimens
Curve 2: Initially containing evaporable water.
Age of specimens: 28 days. w/c = 0.30.
Test specimens: Cylinders, 17 mm in diameter, 60 mm long.
From Ruetz (1966)

Curve 2 in fig 24 corresponds to cement paste specimens initially containing evaporable moisture, all other things being equal to curve 1. The presence of evaporable moisture increases the rate of creep and the plot of log $\dot{\varepsilon}_c$ against $\frac{1}{T}$ is no longer a straight line.

It should be noted that the discussions above refer to creep under stabilized temperatures and moisture states. Under these circumstances the temperature influence can be described with the Arrhenius' expression, eq (2:9), with some modifications due to the moisture content in the range below 100°C.

If the temperature is varying with time a method originally suggested by Dorn (1954) for the creep of metals may be applied. Dorn combined temperature and time into one single parameter, called temperature compensated time Θ. Then the rate of creep at constant stress could be presented as a unique function of

Θ only. The temperature compensated time is based on the Arrhenius' activation energy equation (2:9) and is derived from

$$\Theta = \int_0^t e^{-\frac{\Delta H}{RT}} \cdot dt$$

where t is time coordinate.

The use of a temperature compensated time postulates however that the creep rate at a certain temperature is the same whether the temperature is changing or not. This is not the case for concrete as has been shown in section 2.5.4. Accordingly, the Dorn theory in its original form can probably not be used for predicting creep of concrete under variable temperatures.

3. The possibility of predicting stresses and deformations in a fire-exposed concrete structure

3.1 General considerations

As seen from the previous review of literature the deformation behaviour of concrete at elevated temperatures is extremely complex, and it seems to be very difficult to establish any theory that is capable to give a general description of this behaviour. Therefore, we have to make clear to ourselves the special conditions occuring under fire exposure, and then, possibly, we can find solutions for this more limited problem.

In view of the complexity of the behaviour, we also have to consider the practical application of our data at this stage. This means that the aim of the planned investigation should be rather to provide basic data for solving important practical problems than to give a complete "scientific" description of the behaviour. For that reason, a brief outline is given below (sections 3.3, 3.4) of two suggested methods for theoretical calculations. The description of these methods will serve as a basis for the establishment of definitions for the strain components and for the outline of the test program.

3.2 The characteristics of fire exposure

The fire exposure is characterized by one cycle of rapid heating to high temperatures and a subsequent cooling phase. Hence, every point inside a concrete mass will undergo a similar cycle, with the maximum temperature depending on its distance from the fire exposed surface. The transient temperature fields can in principal be calculated for arbitrary geometry and temperature exposure.

The maximum gas temperature in a fire amounts to 600-1200°C, cf Magnusson & Thelandersson (1970).

Ordinarily, if the structure is not too massive, the temperatures will cause an extraction of the main part of the evaporable water during the course of the fire. In addition, large quantities of the non-evaporable water will also disappear.

Another characteristic feature of structural fire engineering problems is that the external mechanical loading on a structure usually is constant during the whole fire exposure. Failure occurs when the load-bearing capacity due to the fire exposure has decreased to the same level as the actual load. Before failure the internal stresses are continually redistributed under the influence of imposed thermal, creep and shrinkage strains as well as changing deformation properties. For statically determinate members the stress redistribution takes place within the sections, while the distribution of moments, normal and shear forces remains unchanged. In statically indeterminate structures, on the other hand, the redistribution involves also moments and forces.

3.3 Limit state approach

For an internal point in a fire exposed structure the stress can vary almost arbitrarily. In some parts of the concrete structure the stresses will increase due to imposed thermal strain while in other parts the stresses may decrease, according to the equilibrium conditions. However, if the stress tends to be higher than the proportional limit in some point, the strain in this point will increase more than linear and hence a further increase in stress tends to be prevented.

In this way the state of stress successively approach the limit state where failure occurs. If the material in the structure were perfectly plastic, the load-bearing capacity could be uniquely estimated by inserting the yield stresses into the relevant equilibrium equation. This is valid even if the yield stresses vary with temperature. According to plastic theory the stress distribution in the limit state is completely independent of imposed thermal, creep and shrinkage strains. Such strains will influence the stress distribution before the limit state is reached, but the strains will be automatically cancelled as the limit state is approached.

Although concrete is not a perfectly plastic material it seems to have a considerable "deformation capacity" at high temperatures. We have seen that the rate of creep increases rapidly with the temperature and the creep strains will tend to facilitate the plastic behaviour. Existing test results show that the instantaneous strain under stress increases with the temperature.

In cases where sufficient deformation capacity exists an appropriate calculation of the load-bearing capacity is comparatively simple. The only data needed for such a calculation is the ultimate stress as a function of temperature. Unfortunately the ultimate stress at high temperatures is not uniquely defined; it will depend on the method of testing. When the strength is measured at stabilized temperature the result will be different if the specimens are stressed during the heating or not. Another method of testing, which probably gives the best results for this purpose, is to apply the load from the beginning, and then heat the specimen at a constant rate of heating until the specimen fails. This is a method widely used for testing of ceramic materials for high-temperature use. From such tests we can get information on the deformations and hence it might be possible to estimate the deformation capacity i e to examine in which cases the plastic theory is valid. One problem with such tests is that a certain temperature gradient is present during the whole test. Therefore, it is difficult to know if thermal stresses will influence the result. The fact that the temperature varies inside the specimen implies that the temperature at failure will not be uniquely defined. But despite these problems it ought to be possible to obtain the necessary data for the purpose mentioned above.

It should be noted here that the plastic theory becomes far more complicated when the stresses are two- or three-dimensional. In this case some yield condition has to be found, and it is very difficult to do this for concrete. Although many important building structures can be characterized as one-dimensional stress problems, we must consider the thermal stresses, which are of a three-dimensional nature. However, we may hope that these thermal stresses remain at such a low level that the uniaxial yield stresses will not be changed too much.

3.4 Step-by-step method

If we are interested in the total behaviour of the structure under fire exposure we have to make some kind of successive calculations of the stresses and strains from time to time using small time steps. In this case a much more detailed knowledge of the strain behaviour is required, in order that the necessary constitutive relationships between stresses and strains shall be established.

In the following the method of step-by-step calculations is illustrated by a simple example. The main problem of finding the appropriate constitutive relationships is the same in this simple application as in more complicated cases.

Consider a short, circular concrete column, centrically loaded with a load P. The column is axisymmetrically exposed to fire i e inside the column there is a transient, axisymmetrical temperature field (fig 25). We want to calculate the development to strains and stresses during the fire exposure from the initial stage with uniformly distributed stresses until the moment when failure occurs. The equilibrium equation is easily written

$$\int_A \sigma_x dA = P \tag{3:1}$$

where σ_x = the normal stress in the axial direction and
A = area of cross section.

The compatibility conditions in this case give

$$\varepsilon_x = \bar{\varepsilon} = \text{constant} \tag{3:2}$$

where ε_x = normal strain in the axial direction
$\bar{\varepsilon}$ = overall strain of the column.

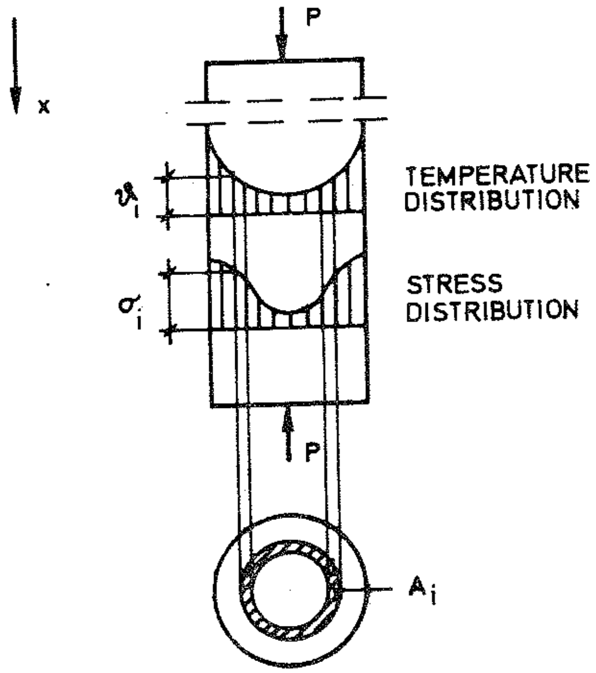

Fig 25 Schematic temperature and stress distributions in a centrically loaded, axisymmetrically fire-exposed concrete column at an arbitrary instant t.

It follows from eq (3:2) that this is a plane strain problem.

Eq (3:1) and (3:2) together with the appropriate constitutive relations give the solution of the problem.

We assume that the calculation has proceeded until the time t and that σ_x, ε_x, $\bar{\varepsilon}$ and the temperature ϑ is known at t. Then we want to calculate the increments $\Delta\sigma_x$, $\Delta\varepsilon_x$ and $\Delta\bar{\varepsilon}$ of these quantities between the times t and t + Δt. The temperature ϑ is assumed to be known at t + Δt; the equation of heat conduction can be solved simultaneously or the temperature fields can be included in the initial data. The cross section is divided into n small elements, within which the temperature and stress are uniform (fig 25). The equilibrium equation at the time t will be written

$$\sum_n \sigma_i A_i = P \tag{3:3}$$

where σ_i = stress in element i

A_i = area of element i

In difference form:

$$\sum_{n} \Delta\sigma_i \, A_i = 0 \qquad (3:4)$$

$\Delta\sigma_i$ = stress increment in element i.

The total strain increment $\Delta\varepsilon_{x,i}$ in element i is divided into two parts, c f Gustaferro (1971)

$$\Delta\varepsilon_{x,i} = \Delta\varepsilon_{I,i} + \Delta\varepsilon_{\sigma,i} \qquad (3:5)$$

where ε_I = "imposed" strain, including thermal, creep and shrinkage strains, and

ε_σ = "stress-related" strain, i e that part of the strain, which directly depends on the stress.

The imposed strain increment $\Delta\varepsilon_I$ from t to t + Δt must be predicted from the stress, strain and temperature values at the time t and the temperature at the time t + Δt. The quantity ε_I can be separated into thermal strain ε_ϑ and viscous strain ε_v. Then, the thermal strain is supposed to be stress and time invariant and depends only on temperature. The viscous strain depends in a complex manner on practically all involved parameters.

At a given temperature the stress-related strain ε_σ depends only on stress and stress history; it is independent of time. This component has to be subdivided into reversible, elastic strain ε_e and irreversible, plastic strain ε_p. Some kind of interrelationship must be established so that a given value of $\Delta\varepsilon_\sigma$ can be translated into a stress increment $\Delta\sigma$.

The method of calculation is as follows. $\Delta\varepsilon_{I,i}$ is determined for each element. A value of $\Delta\bar{\varepsilon}$ is chosen on trial (for example $\Delta\bar{\varepsilon}$ = = 0) and inserted into eq (3:5)

$$\Delta\varepsilon_{\sigma,i} = \Delta\bar{\varepsilon} - \Delta\varepsilon_{I,i} \qquad i = 1 \ldots n \qquad (3:6)$$

From the n values of $\Delta\varepsilon_{\sigma,i}$ we get n values for $\Delta\sigma_i$. Inserting these values into the left hand side or eq (3:4), we get the resultant of the stress increments which shall be equal to zero. If this resultant falls within a certain small interval around zero, the chosed value on $\Delta\bar{\varepsilon}$ is deemed correct, otherwise a new value has to be picked. Some criteria has to be found, after which the new value is chosen. This procedure is repeated until the correct value on $\Delta\bar{\varepsilon}$ is found, and hence all stresses and strains at the time $t + \Delta t$ are known. Then, the whole procedure is repeated for the next time step Δt, and so on.

If this limited problem can be solved, the method can easily be extended to many other problems. The equilibrium and compatibility equations will be different but the constitutive relationships are the same. Complications will arise if the stresses and strains are of a two- or three-dimensional nature. The discussion of this matter in the previous section is also valid in this case.

However, before the above described method can be applied, we need basic input data which have to be obtained from tests on concrete specimens. A successful application of the step-by-step method requires the solution of the following problems.

1) Separate, in a unique and well-defined manner, the total strain into "imposed" and "stress-related" components.

2) Find a theoretical description of the imposed strain component, taking into account all relevant parameters and their previous histories. This is essentially a question of analysing the creep behaviour.

3) Describe theoretically the stress-related strain vs stress, for different temperatures and loading histories. Arbitrary stress variation under changing temperature must be considered.

These problems will be discussed in more detail in section 3.5.

3.5 Definitions of the strain components

Starting from the concepts "imposed" and "stress-related" strains, we will discuss the problems of establishing consistent definitions of the strain components. The aim should be to find generally valid theoretical descriptions of the strain components.

3.5.1 Stress-related strain

The stress-related strain component is independent of time and is characterized by a stress-strain relation. The stress-strain relation for concrete depends on the temperature, which means that we have a set of stress-strain curves, one for each temperature, as indicated schematically in fig 26. Such curves can be described with suitable analytical expressions, which then should give the stress-related strain as a function of stress and temperature. Complications arise if a change in stress or temperature corresponds to an unloading, because some part of the strain is not reversible. This will be discussed in some detail in what follows. It will be assumed that the elastic part of strain is determined by the initial slope of the stress-strain curve and that the remaining part of strain is irreversible. This remains still to be proved for concrete at high temperatures.

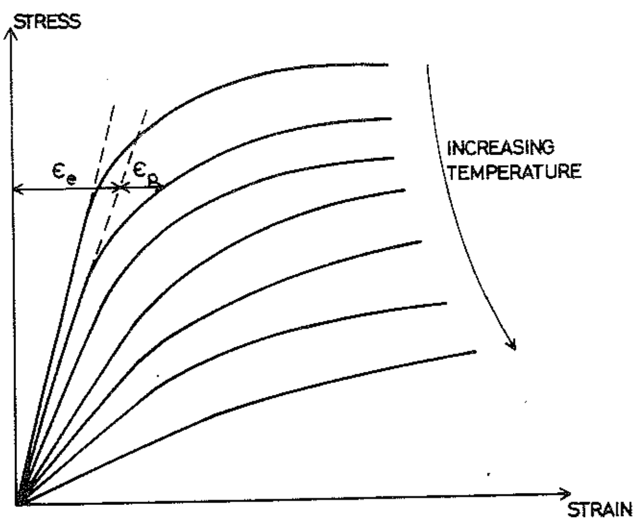

Fig 26 Schematic stress-strain curves for concrete at elevated temperatures.
ε_e = elastic strain, ε_p = plastic strain

Let us consider the case when the temperature increases from ϑ_1 at the time t_1 to ϑ_2 at the time t_2, corresponding to two different stress-strain curves indicated in fig 27. At the time t_1 the stress is σ_1, corresponding to a strain ε_1 (point A in the figure), which can be divided into two parts ε_{e1} and ε_{p1}. If the stress σ_1 remains constant between t_1 and t_2, or if it increases to σ_2, the strain at the time t_2 will be easily determined from the new stress-strain curve at point B and C respectively. But if the stress decreases we must distinguish between the elastic part and the plastic part of the strain. The plastic strain, which is irreversible can not obtain a value smaller than ε_{p1}.

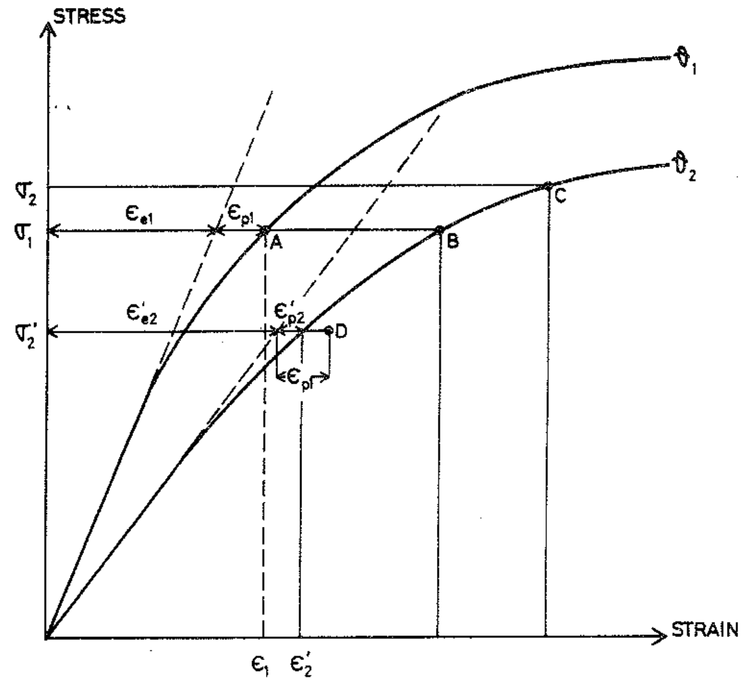

Fig 27 Stress-strain relations under changing temperature and stress.

A B The temperature increases from ϑ_1 to ϑ_2 under constant stress.

A C The temperature increases to ϑ_2 and the stress increases to σ_2.

A D The temperature increases to ϑ_2 and the stress decreases to σ_2'.

Thus, if the decreased stress-level σ_2' at the time t_2 corresponds to a point on the stress-strain curve, where the plastic part ε_{p2}' is less than ε_{p1}, then the total strain will be $\varepsilon_{e2}' + \varepsilon_{p1}$

(point D in figure 27) instead of ε_2', which corresponds to the actual stress-strain curve. Generally, we can represent the stress-related strain ε_σ as the sum of one elastic part ε_e and one plastic part ε_p, both of which are functions of the stress σ and the temperature ϑ. The elastic part can be assumed to be

$$\varepsilon_e = \frac{\sigma}{E(\vartheta)} \tag{3:7}$$

where $E(\vartheta)$ is the elastic modulus which varies with the temperature ϑ. The plastic part can also be written as a function of ϑ and σ, obtained from the set of stress strain relations:

$$\varepsilon_p = \varepsilon_p(\vartheta, \sigma) \tag{3:8}$$

This is valid as long as no decrease in ε_p takes place. If the stress σ and temperature ϑ are changed to $\sigma + \Delta\sigma$ and $\vartheta + \Delta\vartheta$ respectively the plastic part is changed to $\;_p + \;_p$. Then we get

$$\Delta\varepsilon_p = \max(0,\ \varepsilon_p(\vartheta + \Delta\vartheta, \sigma + \Delta\sigma) - \varepsilon_p(\vartheta, \sigma)) \tag{3:9}$$

which is the condition of irreversibility.

The above description of the stress-related strain postulates stress-strain relations which are uniquely determined by the temperature. According to test results in literature (see section 2.2) the stress-strain curve obtained at a constant high temperature is different depending on the presence of stress during heating or not. This variation in the stress-strain relation with the stress history corresponds to the higher strength that concrete specimens exhibit when they have been subjected to compressive stress during heating. Accordingly, the stress history influences the stress-related strain as well as the ultimate stress. It is very difficult to take this influence into account in a theoretical description of the stress-related strain and it seems that ordinarily the assumption of one unique stress-strain relation for each temperature has to be maintained. This means that some stress-strain relation has to be chosen as a basis for calculating the stress-related strain. What is nearest to hand is to use a stress-strain relationship which is directly obtained by testing specimens at different temperatures, with or without stress during heating. But if data from specimens being unstressed during heating are used, the ultimate stresses might be underestimated when compressive stresses are present.. On the other hand, if we use the stress-strain curves obtained on specimens which have been stressed during heating, difficulties will arise in defining the thermal dilatation; it will not be correct to put together free thermal expansion with stress-related strain defined in this way, cf. section 2.2. Furthermore, it should be remembered that in the real case the mechanical load is present the whole time and the load-bearing capacity successively decreases with temperature until failure occurs. This type of failure is of a long-term nature and probably occurs at a lower stress level than the corresponding short-time strength.

3.5.2 Imposed strain

The imposed strain consists of two components, thermal strain and another part which will be termed viscous strain.

$$\varepsilon_I = \varepsilon_\vartheta + \varepsilon_V \tag{3:10}$$

The thermal strain must be independent of time and stresses and hence be a unique function of temperature. If the free expansion of an unstressed specimen is measured during heating, a relation between temperature and thermal strain is obtained. This relation, however, is not unique, not even for the same concrete mixture and type of aggregate. Since shrinkage of the concrete takes place simultaneously the expansion of an unstressed specimen depends on moisture content and rate of heating. It is not clear from literature how large these influences are compared to the other strain components. If they are small they can be included in the thermal strain component, but if the shrinkage effects are of some magnitude they have to be treated together with the viscous strains.

The viscous part of strain is time-dependent and is also the most difficult of the components to predict. At constant temperature and constant stress the viscous strain appears as a creep deformation. Tests of this type has been thoroughly reviewed in section 2.4. It is also clear from section 2.4, however, that test results at constant, stabilized temperature can not be used for predicting the creep strain under variable temperature when moisture is extracted from the concrete. Therefore, we must study the deformations under varying temperature in order to examine the influence of drying and heating on the viscous deformations. Under these conditions thermal and stress-related strains will develop together with the viscous strain, which then will be determined as the total strain ε_x minus the stress-related and the thermal strains.

$$\varepsilon_v = \varepsilon_x - \varepsilon_\sigma - \varepsilon_\vartheta \qquad (3:11)$$

Accordingly, the viscous strain determined in such tests will depend on the definition of the thermal and stress-related components. In other words, the viscous strain constitutes that part of the strain that can not be accounted for otherwise.

The primary parameters determining the viscous deformation is time, stress and temperature. The influence of each one of these has been discussed in section 2.5.6. When considering the combined effect of these parameters the effect of the stress and temperature histories must be included in some way. The first task will be to find a way to describe viscous strain under constant stress but varying temperature. In doing this, the effect of drying and heating on the viscous deformations must be included. If this is possible to acquire, the next step will be to find some superposition principle to take into account variations of stresses.

It is necessary to perform tests to get such a picture of the viscous strain behaviour. The tests must be made under constant as well as varying temperatures and stresses. Then, hopefully, it will be possible to fit some kind of theoretical description of the viscous strain with the test results.

4. Summary and conclusions

When concrete structures are subjected to high temperatures, as in the case of fire, large temperature gradients are induced. These gradients give rise to thermal stresses in the concrete which are superimposed on the stresses due to mechanical load. These stresses can not be calculated, however, because we know too little about the deformation properties at high temperatures. Inelastic and time-dependent deformations must be taken into account and the knowledge of this type of deformations at high temperatures are very insufficient.

In this paper the problems involved in making a theoretical stress analysis of concrete structures at high temperatures are formulated and discussed to som extent. The information available in literature on deformation properties of concrete at elevated temperatures is reviewed from the point of view of high temperatures and rapid processes of heating, i e the conditions characterizing fire exposure. Possible methods of calculation are briefly outlined and the needs of data for applying these methods are accounted for. The paper is part of a research project, which will also comprise a comprehensive test series for studying the deformation behaviour of concrete at higher temperatures.

Relatively much information was found in the literature regarding creep at temperatures below 200°C, while practically no studies of time-dependent deformations have been performed at higher temperatures. The results in literature clearly showed that the influence of moisture content, moisture change and moisture migration is very significant at least at lower temperatures. It was established that the deformations became much larger under changing temperature and moisture state than under stable conditions. This is a parallell to the distinction between basic creep and sorption creep which is a well-known feature for creep at normal temperatures.

As regards the instantaneous strains the most important aspect is the marked influence of the previous stress history. Concrete which has been subjected to stress during heating exhibits different strength and accordingly different stress-strain relation compared with concrete which have been unstressed during heating. It is emphasized that the inelastic part of the stress-strain curve is very important for the present problem.

The possibility of applying an ultimate load approach on concrete at high temperatures is briefly discussed. The main question in this context is whether the deformability of heated concrete is large enough for the redistribution of stresses to take place. Another aspect is the definition of the ultimate stress, since, in tests, this quantity has been found to depend on the previous stress history. It is suggested that the ultimate stress might be determined from tests, where the specimens are first loaded to certain stress levels and then heated until failure occurs.

The problem of calculating the complete stress and deformation behaviour of a fire exposed concrete structure can be tackled by using a stepwise procedure. In this case a very thorough knowledge of the constitutive relations between stresses and strains including time-dependent behaviour is needed. The most complicated problem is to find consistent definitions of strain components, which together must constitute the total strain and which are readily obtainable from the given parameters. It is suggested that the total strain should be divided into a "stress-related" component, which is directly related to the actual stress, and an "imposed" strain component which includes thermal and viscous strains. The "stress-related" part is characterized by stress-strain relations at different temperatures. A distinction must be made between the reversible and irreversible parts of this strain. The construction of such stress-strain curves requires experimental information which is not available at the present time.

The imposed strain consists of thermal strain and a second component which in this paper is termed viscous strain. This name is not strictly justified, but was chosen because the main part of that component probably consists of irreversible time-dependent strain. Our knowledge of these properties is insufficient at the moment. It should be emphasized that the viscous strain in general depends on the definitions on the other strain components. In tests under varying temperature it can never be measured separately. The viscous strain is the most complex part of the total deformation, since it depends in a complicated way on time, temperature and stress and on the history of the latter two parameters. It seems to be rather significant compared to the other components, and it is necessary to make experimental studies and find a way to predict this component of strain.

To sum up, an appropriate analysis of concrete structures subjected to high temperatures can only be done if the deformation behaviour can be understood. In this paper we have put forward some principal methods for such an analysis and explained what kind of information is lacking. This has served as a basis for the planning of an experimental investigation, which is now in progress.

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Main symbols

Letters

A	Area of cross section	m^2
A_i	Area of element no i	m^2
a	Coefficient	
B	Dimensionless coefficient	-
b	" "	-
C	Coefficient	
E	Modulus of elasticity	$MN \cdot m^{-2}$
E'	Modified modulus of elasticity	$MN \cdot m^{-2}$
ΔH	Activation energy for creep	$J \cdot mole^{-1}$
k	Boltzmann's constant	$J \cdot {}^oK^{-1}$
n	Coefficient	-
R	Gas constant	$J \cdot mole^{-1} \cdot {}^oK^{-1}$
t	Time	s
Δt	Time interval	s
T	Absolute temperature	${}^oK$
P	Load	N
P_B	Ultimate load at normal temperature	N

Greek letters

α	Coefficient	J
ε	Strain	-
$\dot{\varepsilon}$	Strain rate	s^{-1}
$\Delta\varepsilon$	Strain increment during time interval Δt	
$\bar{\varepsilon}$	Total strain	
λ	Trouton's coefficient of viscous traction	$MN \cdot s \cdot m^{-2}$
σ	Stress	$MN \cdot m^{-2}$

$\sigma_{ult}^{20^\circ C}$	Ultimate strength at 20°C	$MN \cdot m^{-2}$
σ_{ult}^{ϑ}	Ultimate strength at ϑ °C	$MN \cdot m^{-2}$
τ	Shear stress	$MN \cdot m^{-2}$
γ	Shear strain	
$\dot{\gamma}$	Shear strain rate	s^{-1}
θ	Temperature-compensated time	s
ϑ	Temperature	°C
η	Coefficient of viscosity	$MN \cdot s \cdot m^{-2}$

Subscripts

c	Creep
e	Elastic
I	Imposed
i	In element no i
n	In element no n
v	Viscous
x	In axiel direction
ϑ	Thermal
σ	Stress-related