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Published in:
International Review of Business Research Papers

2007

International Review of Business Research Papers
Vol. 3 No.5 November 2007 Pp.280-296

An Examination of the Robustness of the Vector Autoregressive Granger-Causality Test in the Presence of GARCH and Variance Shifts

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The properties of the Granger-causality test in stationary and stable Vector Autoregressive (VAR) models are studied with different types of volatility processes imposed on the unconditional variance. For this test, it is examined how the size and power properties are affected by different magnitudes of GARCH processes and by structural shifts in the volatility. The study has been conducted by means of Monte Carlo simulations for different sample sizes. Our analysis reveals that substantial GARCH effects influence the size properties of the Granger-causality test, especially in small samples. The power functions of the test are usually slightly lower in the presence of GARCH disturbances compared to the case of white noise residuals. When a structural variance break is imposed, the size problem is rather severe, and the power functions are lower compared to the case with the pure GARCH-processes.

Keywords: Causality test, GARCH, Size and Power, Structural change

JEL Classification: C32

1. Introduction

In this paper, we aim to examine the properties of the Granger-causality test (Granger, 1969) in stationary and stable Vector Autoregressive (VAR) models in the presence of Generalised Autoregressive Conditional Heteroscedasticity (GARCH) effects and with an imposed structural variance break. The Granger approach to the question whether a variable say X_1 causes another variable say X_2 is to see how much of the current value of X_2 can be explained by past values of X_1. X_2 is said to be Granger-caused by X_1 if X_1 helps in the prediction of X_2, or equivalently, if the coefficients of the lagged X_1 are statistically significant in a regression of X_2 on X_1. The same is true for the opposite direction. There is also the possibility of a two-way Granger causation case; i.e. X_1 Granger causes X_2 and X_2 Granger causes X_1.

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That is, by means of the Granger-causality test we would like to study whether one variable precedes the other variable or if they are contemporaneous. Furthermore, it should be stressed that Granger causality is a necessary condition for the true existence of causality, however the correlation relationship measured by the use of the Granger (1969) causality approach is not a sufficient condition for the real causality. Nevertheless, the VAR type of Granger-causality test is very common and useful, and therefore very relevant in empirical applications.

Nowadays, it is a stylized fact that the analysis of causality is very sensitive to model specification and is almost only valid under conditions when the error terms are fairly close to white noise. Despite this, a considerable proportion of the empirical time series variables follow some type of GARCH process. These forms of time-varying volatility processes are very common in, for example, exchange rates, bond returns, commodity returns, inflation rates, interest rates, and generally in different types of financial data. The objective of such models can be to provide a volatility measure for decisions concerning risk analysis, portfolio selection, derivative pricing, or generally just to improve the efficiency of the estimated standard errors. Hence, it is important to examine the properties of this commonly used causality test in the presence of GARCH disturbances. Especially in financial studies, it is of great importance to provide accurate estimates of the persistence of the conditional variance. Furthermore, it has long been conjectured that the volatility in financial markets exhibits irregular breaks, Diebold (1986), Hendry (1986) and Lamoureux and Lastrapes (1990). More recent and related evidence is provided by Andreou and Ghysels (2002), Diebold and Inoue (2001), Ewing and Malik (2005), Granger and Hyung (1999), and Kholodilin and Yao (2006) among others, which shows that the presence of different types of breaks may also explain the findings of long memory, particularly the persistence in the volatility equation. Brooks, Clare, and Persand (2000), Gallo and Pacini (2000), Lamoureux and Lastrapes (1990), and Lastrapes (1989) argue that persistence in volatility is overestimated when standard GARCH models are applied to a series with underlying sudden changes in variance. This phenomenon is usually defined as so-called excess persistence in the variance. Engle, Ito, and Lin (1990) argue that volatilities in financial markets are often transmitted to other markets. Due to these reasons, the same event can quite possibly affect different markets at different time points and thus making it difficult to isolate those events that cause volatility breaks. There are many different suggestions for how to detect and adjust time-series models for structural breaks in the variance equation (see e.g. Cochran, Mansur and Shaffer, 2007, Ewing and Malik (2005), and Inclán and Tiao, 1994). However, all remedies essentially depend on the accurate detection of the exact position of the structural change. These methods work very well in theory, but are sometimes fairly problematic to apply in practise for certain types of real-world data sets.

In fact, it is a very complicated task to detect structural breaks in general, and it is even more complicated for variables that can be expected to follow unit root, or near unit root, processes. For instance, it can be difficult to distinguish between unit root processes and stationary processes with structural breaks. Due to these reasons it is highly relevant to study the robustness properties of the VAR Granger-causality test under different possible problems in the error process, since there is a high risk for misspecification of the underlying process for empirically applied models.

The main purpose of this paper is to investigate the properties of the Granger-causality test in the presence of (GARCH) effects and with breaks in the errors. An efficient test should have correct significance levels under the null hypothesis, irrespectively of the values of the regression and other distributional parameters. It should also have reasonable power against the class of alternative specifications under investigation, but low power against other alternatives. The study has been conducted by means of Monte Carlo simulations. The size and the power of the test have been studied by applying the test on various VAR models when the sample sizes, strength in the GARCH effects have been varied. As mentioned in the above text, the size and power of the test has also been studied when possible breaks are imposed.

The paper is arranged as follows. In the next section we present the model we analyse the design of our Monte Carlo experiment. In Section 3, we describe the results concerning the size of the test while power is analysed in Section 4. Finally, a brief summary and conclusions are presented in Section 5.

2. Methodology and Experimental Design

Consider the following data-generating process (DGP) which consists of a two dimensional time series generated by a stable VAR(p) process:

$$X_t = A_1 X_{t-1} + \ ... \ + \mathrm{A_p} X_{t-p} + \varepsilon_t \, , \tag{1}$$

where $\varepsilon_t = \left(\varepsilon_{1t}, ..., \varepsilon_{kt}\right)'$ is a zero mean independent white noise process with nonsingular covariance matrix Σ_ε and, for j = 1, ... , k, $\mathrm{E}\left|\varepsilon_{jt}\right|^{2+\tau} < \infty$ for some τ > 0. The order p of the process is assumed to be known. Let $\alpha_p = \mathrm{vec}\left[\mathrm{A}_1, \cdots, \mathrm{A_p}\right]$ be the vector of the true parameters, where vec[.] denotes the vectorization operator that stacks the columns of the argument matrix. Now, suppose that we are interested in testing q independent linear restrictions:

$$H_o: \mathrm{R}\alpha_p = \mathrm{s} \quad \mathrm{vs.} \quad H_1 : \mathrm{R}\alpha_p \neq \mathrm{s} \, , \tag{2}$$

where q and s are fixed (q x 1) vectors and R is a fixed [q x $k^2(p)$] matrix with rank q. The process $\{X_t\}$ is generated by the VAR(p) process in (1), with the $\hat{A}_i$ (i = 1, … p) the Ordinary Least Squares (OLS) estimators and $\hat{\alpha}_p$ the $\left[k^2(p)\right]$dimensional vector, consisting of the $k^2(p)$ elements of $\hat{\alpha} = vec\left[\hat{A}_1, \cdots, \hat{A}_p\right]$ Then:

$$T^{1/2}\left(\hat{\alpha}_p - \alpha_p\right) \Rightarrow N\left(0, \Sigma_p\right), \tag{3}$$

where $\Rightarrow$ denotes weak convergence in distribution and the $[k^2(p)$ x $k^2(p)]$ covariance matrix Σ_p is non-singular. The α_p is the $\left[k^2(p)\right]$ dimensional vector of the true parameters.

Moreover given a consistent estimator $\hat{\Sigma}_p$, then the Wald test of the null hypothesis in (2):

$$\lambda_w = T(R\hat{\alpha}_p - s)'(R\Sigma_p R')^{-1}(R\hat{\alpha}_p - s) \tag{4}$$

has an asymptotic $\chi^2(q)$-distribution under the null hypothesis. With X_t portioned in (m) and (k-m) dimensional sub vectors X_t^1 and X_t^2, and A_i matrices portioned conformably, X_t^2 does not Granger-cause the X_t^1 if the following hypothesis is true:

$$H_0 = \mathrm{A}_{12,\mathrm{i}} = 0 \quad \text{for} \quad \mathrm{i} = 1, \cdots, p\text{-}1 . \tag{5}$$

The error components $\left(\varepsilon_{1t}, \varepsilon_{2t}\right)'$ in (1) and (2) are generated by GARCH(1,1) models, i.e.,

$$\begin{gathered} \varepsilon_{it} = h_{it}\upsilon_{it} \quad i = 1,2 \\ \upsilon_{it} \sim \text{ i.i.d., } \mathrm{E}(\upsilon_{it}) = 0, \mathrm{E}(\upsilon_{\mathrm{it}}^2) = 1 \\ h_{it}^2 = \gamma_i + \phi_i h_{it-1}^2 + \varphi_i \varepsilon_{it-1}^2 \end{gathered} \tag{6}$$

and $\mathrm{Cov}\left(\varepsilon_{1t}\ \varepsilon_{2t}\right) = 0$. The condition for finite variance is $\phi_i + \varphi_i < 1$ and the condition for finite fourth moment is $3\phi_{i1}^2 + 2\phi_i\varphi_i + \varphi_i^2 < 1$. Furthermore, if $\gamma_i > 0$ and $\phi_i + \varphi_i < 1$, then the unconditional variance of the ε_i exist and equals $\sigma_{\varepsilon i}^2 = \left(\gamma_i / 1 - \phi_i - \varphi_i\right)$. In addition to the above constraints, the following stationarity conditions must be satisfied: $\phi_i < 1$, $\varphi_i < 1$, and since it is well known that there is a considerable risk that the conditional heteroscedasticity may produce negative values it is necessary that $h_{it}^2 > 0$ is obtained for all t. Furthermore, note that when $\phi = \varphi = 0$, the ε_{it} is reduced to iid white noise.

The size and power of the causality test is also examined in the presence of breaks in the errors in (1). $\varepsilon_t = \left(\varepsilon_{1\mathrm{t}},\ \varepsilon_{2\mathrm{t}}\ \right)'$ are iid with covariance matrix Σ_ε, where

$$\Sigma_\varepsilon = \begin{pmatrix} \sigma_{1t}^2 & 0 \\ 0 & \sigma_{2t}^2 \end{pmatrix}. \tag{7}$$

The effects of neglected changes in volatility are examined through the following experiment, where simultaneous and identical changes in volatility are imposed, $\sigma_{1t}^2 = \sigma_{2t}^2 = \sigma_b^2 \text{ for } \leq T/2$ and $\sigma_{1t}^2 = \sigma_{2t}^2 = \sigma_a^2 \text{ for} > T/2$; In this study, we selected σ_b^2 = 1 and σ_a^2 = 0.1.

To illustrate and study the effects of a GARCH(1,1)-process with and without a break in the errors on the Granger-causality test in a stable VAR(1) system, we apply Monte Carlo methods. We calculate the *estimated size* by simply observing how many times the null is rejected in repeated samples under conditions where the null is true. To judge the reasonability of the results we use an approximated 95% confidence interval for the actual size (π):

$$\hat{\pi} \pm 2\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{N}} \tag{8}$$

where $\hat{\pi}$ is the estimated size and *N* is the number of replications.

The Monte Carlo experiment has been performed by generating data according to the model defined by (1) and (2),

$$y_t = \begin{bmatrix} 0.02 \\ 0.03 \end{bmatrix} + \begin{bmatrix} 0.5 & 0.3 \\ T^{-1/2}\lambda & 0.5 \end{bmatrix} y_{t-1} + \varepsilon_t , \tag{9}$$

If λ= 0, y_{1t} is Granger-non-causal for y_{2t} and if $\lambda \neq$ 0, y_{1t} causes y_{2t}. Therefore, we use the λ= 0 to study the size of the test.

We simulate three GARCH versions with a) high persistence, HP, (0.01,0.09,0.9), b) medium persistence, MP, (0.05, 0.05, 0.9) and c) low persistence, LP, (0.20,0.05,0.75). Finally, simultaneous and identical changes in the ε_t are imposed. The processes includes a constant term and we fit a VAR(1) :

$$y_t = v + A_1 y_{t-1} + \varepsilon_t . \tag{10}$$

For each model we perform 10 000 replications and use three different nominal sizes, namely 1%, 5% and 10%. However, We use simple graphical methods, developed and illustrated by Davidson and MacKinnon (1998), which are based on the empirical distribution function of the P-values and are easy to interpret. The P value plot is used to study the size and the Size-Power curves to study the power of the test. Furthermore, to judge the reasonability of the results we use a 95% confidence interval for the actual size (π) as:

$$\pi_0 \pm 2\sqrt{\frac{\pi_0(1-\pi_0)}{N}} , \tag{11}$$

where *N* is the number of replications. Results that lie between these bounds will be considered satisfactory.

Several factors are expected to affect the size and power properties of causality tests. We, therefore, have investigated samples typical for small, medium, large and very large sizes. For each time series 20 pre-sample values are generated with zero initial conditions, and with net sample sizes of T = 50, 100, 200, 500, 1000. Table 1 shows the different parameters of our Monte Carlo design. The number of replications per model is 10 000 for the size, and 1000 for the power of the test. The calculations were performed using GAUSS 6.0.

Table 1 - Monte Carlo parameters of the GARCH effects

Level of Persistence	Λ	γ	ϕ	φ
High	0	0.01	0.09	0.90
Medium	0	0.05	0.05	0.90
Low	0	0.20	0.05	0.75
High	2	0.01	0.09	0.90
Medium	2	0.05	0.05	0.90
Low	2	0.20	0.05	0.75

In the following Figures 1 and 2, we show, by simulated data, how these processes look like. Figure 1 shows two stable VAR(1) models with GARCH errors and their respective residuals for sample sizes of 150 and 500 observations, respectively, while Figures 2, shows two stable VAR(1) models with breaks in the errors and for the same sample sizes.

Figure 1 - VAR(1) with GARCH errors

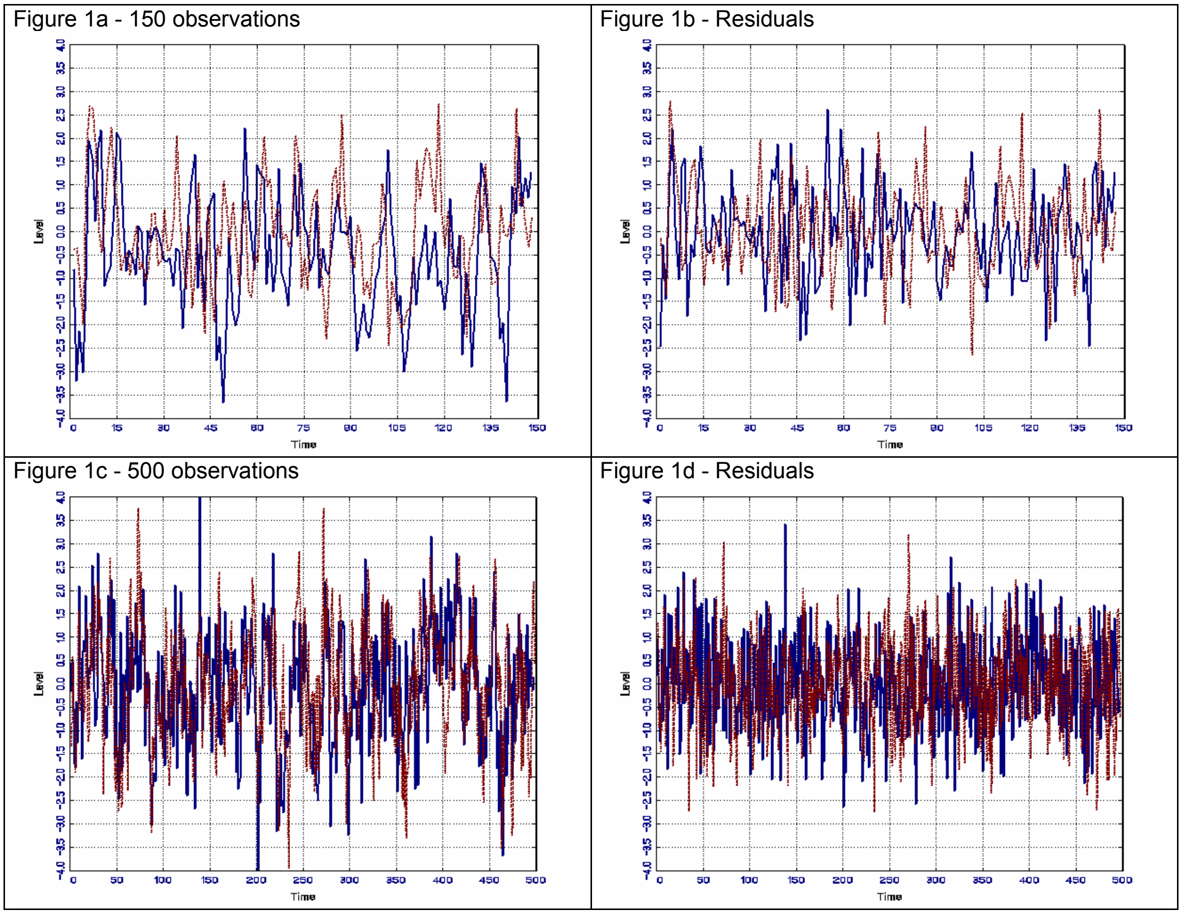


Figure 1a - 150 observations

Figure 1b - Residuals

Figure 1c - 500 observations

Figure 1d - Residuals

Figure 2 - VAR(1) with a break in the errors

Figure 2a - 150 observations

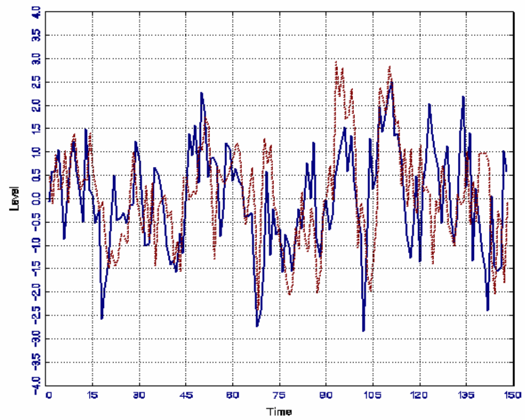


Figure 2b - Residuals

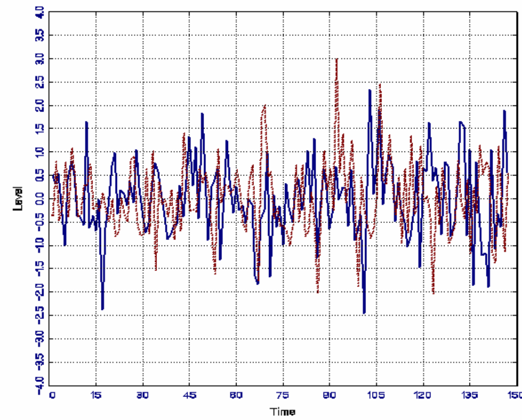


Figure 2c - 500 observations

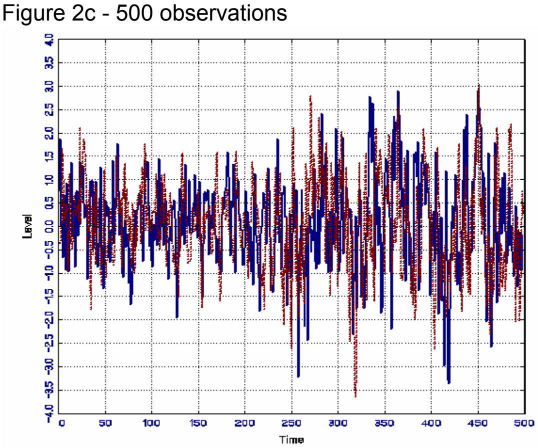
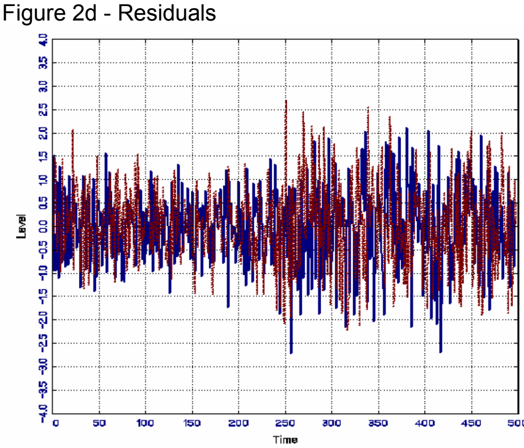


Figure 2d - Residuals



3. Results of the Size of the Test

In this section, we present the most important results of our Monte Carlo experiment concerning the size of the test. Regarding the P value plots, under the condition when the distribution used to compute the p_s is correct, each of the p_s should be distributed as uniform (0,1) and therefore the resulting graph should be close to the 45^o line as in Figure 3 in the Appendix.

Looking at Tables 2a-e in the Appendix, we can see that the calculated sizes of the test over estimate the nominal sizes in all situations more or less regardless whether there exist low, medium or high GARCH effects. This is the case when we study a small sample of 50 observations. This is also confirmed when we observe the P-value plots in Figure 3, in the Appendix, in which we only present the size when white noise and high GARCH effects are imposed. Here we see that in both cases the test over rejects the size, but that the calculated sizes still lay near to the 95% confidence interval for nominal size with a slightly higher over rejection when the high GARCH magnitudes are present.

When the sample size increases to 100 observations, as is illustrated Table 2b and Figure 3b, the properties of the test become better but there is still some over rejection present. When the sample size is increased to 200 observations the test performs well in all cases except for the case with high GARCH effects. In this case the test slightly over rejects the nominal size, as can be seen in Table 2c. Figure 3c shows that the over rejection is more severe for larger nominal sizes. The same is also true when the sample size is equal to 500 observations, as is illustrated in Table 2d and Figure 3d in the Appendix. In a very large sample, i.e. 1000 observations in Table 2e and Figure 3e, the test performs satisfactorily in almost all situations, however with one exception in the case when a high GARCH effect is present.

In situations when breaks are also imposed, the test is shown to over estimate the nominal size in almost all situations, and this over estimation turn out to be higher when the sample size is increased. This is also confirmed from the P-value plots in Figure 4, in the Appendix.

In general, we could not find the over rejection to be that severe even in the where high GARCH effect is imposed. The test is consistent and converges slowly to its nominal size as the sample size increases. On the other hand, the test has not shown to be consistent when the breaks are imposed.

4. Analysis of the Power of the Test

Using Monte Carlo experiments, the power of the Granger-causality test is evaluated in the following segment of the paper. The power of the test is analysed for the sample sizes of 50, 100, 200, 500 and 1000 observations. The power functions have been calculating for the test in the case of white noise errors and under different GARCH effects, and with and without structural variance breaks. For the cases of white noise, low persistence and medium persistence of the GARCH, the power functions exhibit fairly similar properties. Based on this fact and since we could not find any noticeable differences in the performances of the test between these combinations regarding the size properties, we only show and compare the power functions of the white noise and the high GARCH cases. The power functions of the test, when breaks are imposed, are also compared to the cases that are characterised by white noise.

The estimation of the power functions are conducted by calculating the rejection frequencies in 1000 replications by changing the λ_i-coefficients, in Equation (9), to 2. So-called Size-Power Curves are created in order to illustrate the estimated power functions against the nominal size. These estimated power functions of the test are compared graphically and presented in Figures 5-9 in the Appendix.

As expected, the power functions are shifted upwards as the sample size increases. We observe lower power when the samples are small, while higher power functions are observed when the samples are large. A closer examination of the figures shows, that most frequently, the power functions are slightly lower in the case of the GARCH residuals (the dashed lines) than for the cases of white noise. The power functions are even slightly lower in the situations when breaks are imposed.

5. A Brief Summary and Conclusions

The size and power properties of the Granger-causality test in stable Vector Autoregressive (VAR) models are analysed in the presence of different magnitudes of Generalised Autoregressive Conditional Heteroscedasticity (GARCH) disturbances, and with and without structural breaks in the (unconditional) variance. The study is conducted by the use of Monte Carlo simulations. The size is analysed by executing 10 000 replications for each model with 1%, 5%, and 10% nominal size. The power properties are examined based on 1000 replications per model and for different alternative hypotheses. In the experimental design the GARCH effects are defined as low, medium, or high magnitudes, for different numbers of observations.

The size results are illustrated both in the form of tables and by P-value graphs. It is found that the Granger-causality test slightly over rejects the nominal sizes especially in small samples and in the presence of high GARCH effects. When the sample size increases and when the GARCH effects are not too high, we observed less over rejection. The Granger-causality test is observed to converge slowly to its nominal size as the sample size increases and is therefore statistically consistent. However, in the presence of structural breaks in the variance the test suffers from severe over rejection of the true null hypothesis. In the presence of structural breaks in the variance the test is inconsistent since this over rejection increase when the sample size increases.

For illustrative purposes, the power functions are merely presented graphically. As is expected, an increasing sample size leads to higher power functions that are shifted upwards.

For the most part, the power functions of the Granger-causality test are slightly lower in the presence of GARCH errors compared to the cases of white noise residuals, and the power is also slightly lower in the cases when structural breaks are imposed in the variances.

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APPENDIX

Table 2a - Size of the test for 50 observations

Nominal	White Noise	GARCH(1,1)			Break
		LP	MP	HP	
0.01	0.0160	0.0151	0.0156	0.0152	0.0337
0.05	0.0642	0.0643	0.0658	0.0668	0.1010
0.10	0.1169	0.1222	0.1231	0.1225	0.1654

Table 2b - Size of the test for 100 observations

Nominal	White Noise	GARCH(1,1)			Break
		LP	MP	HP	
0.01	0.0133	0.0126	0.0126	0.0141	0.0410
0.05	0.0584	0.0579	0.0578	0.0593	0.1159
0.10	0.1093	0.1069	0.1051	0.1087	0.1857

Table 2c - Size of the test for 200 observations

Nominal	White Noise	GARCH(1,1)			Break
		LP	MP	HP	
0.01	0.0112	0.0119	0.0119	0.0146	0.0431
0.05	0.0546	0.0528	0.0527	0.0584	0.1206
0.10	0.1056	0.1036	0.1054	0.1109	0.1889

Table 2d - Size of the test for 500 observations

Nominal	White Noise	GARCH(1,1)			Break
		LP	MP	HP	
0.01	0.0095	0.0107	0.0108	0.0141	0.0453
0.05	0.0558	0.0544	0.0535	0.0639	0.1218
0.10	0.1068	0.1031	0.1038	0.1121	0.1934

Table 2e - Size of the test for 1000 observations

Nominal	White Noise	GARCH(1,1)			Break
		LP	MP	HP	
0.01	0.0096	0.0083	0.0084	0.0150	0.0458
0.05	0.0476	0.0479	0.0496	0.0628	0.1292
0.10	0.0979	0.1034	0.0997	0.1183	0.2030

Figure 3 - P-value plots HP (GARCH)

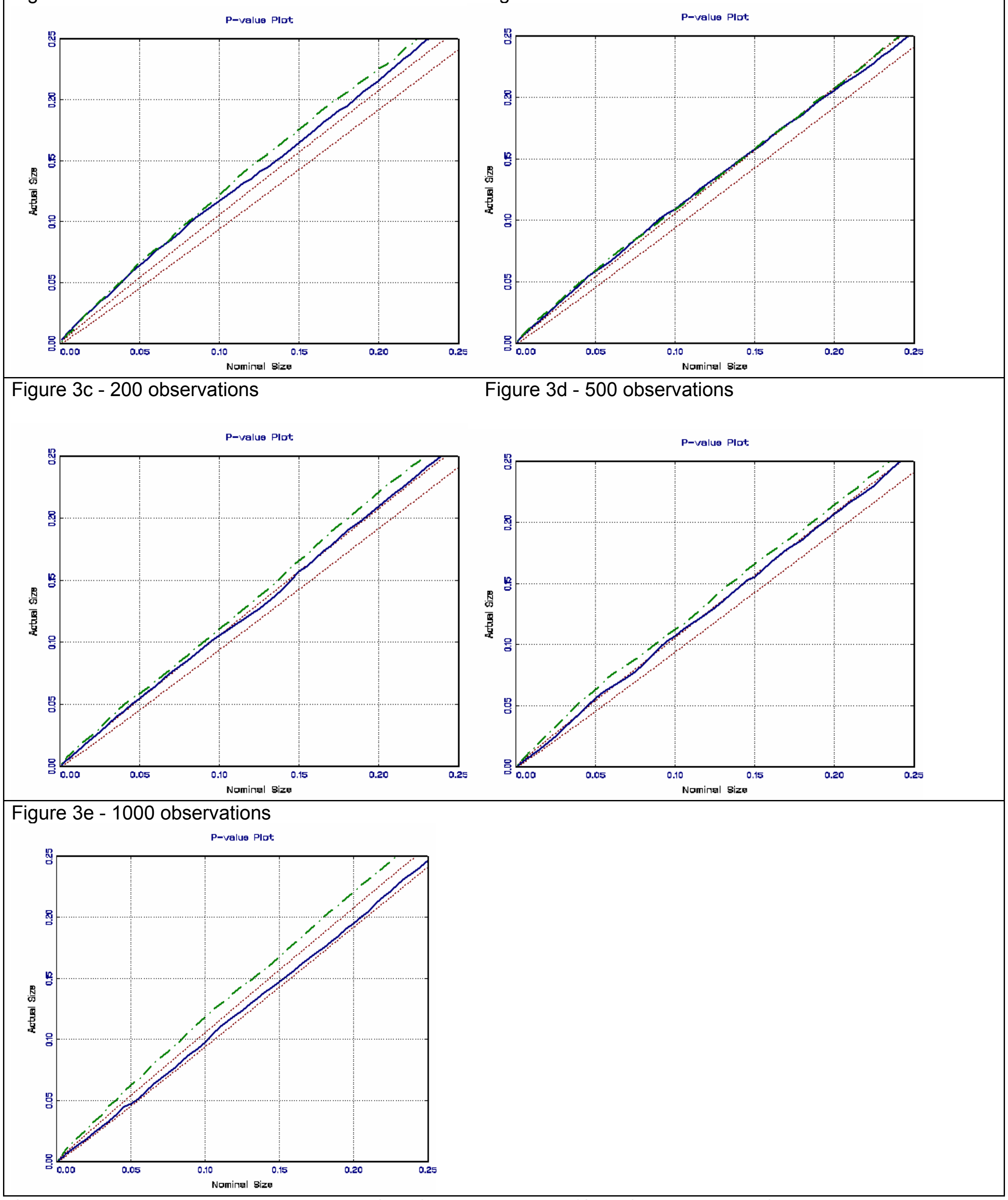
Figure 3a - 50 observations

Figure 3b - 100 observations

Figure 3c - 200 observations

Figure 3d - 500 observations

Figure 3e - 1000 observations



Solid lines = White noise. Dot dash line = GARCH. Dot lines = 95% confidence interval for nominal size.

Figure 4 - P-value plots when breaks in the errors are imposed

Figure 4a - 50 observations

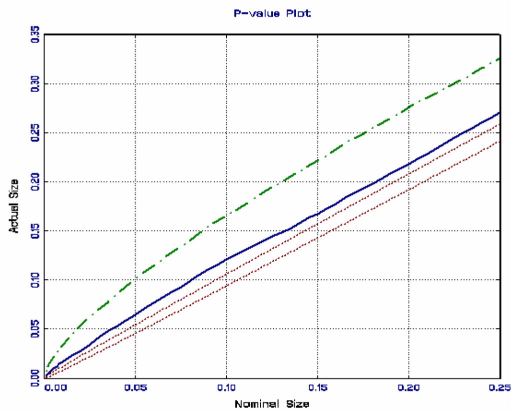


Figure 4b - 100 observations

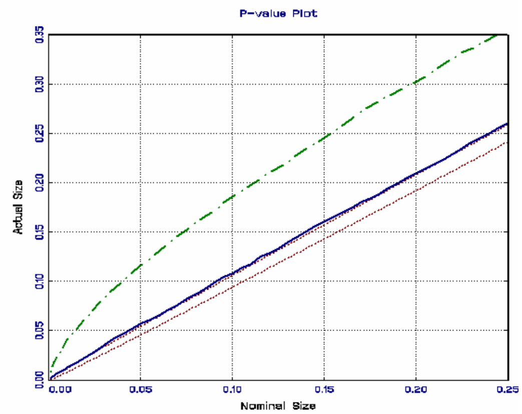


Figure 4c - 200 observations

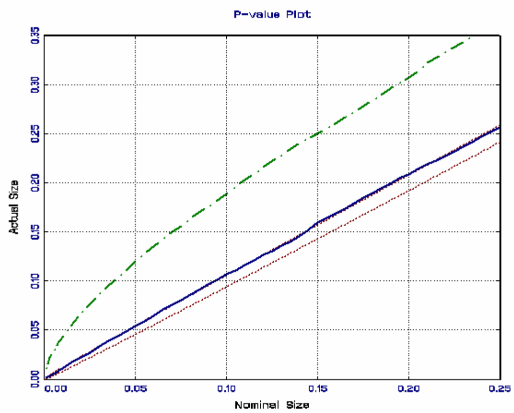


Figure 4d - 500 observations

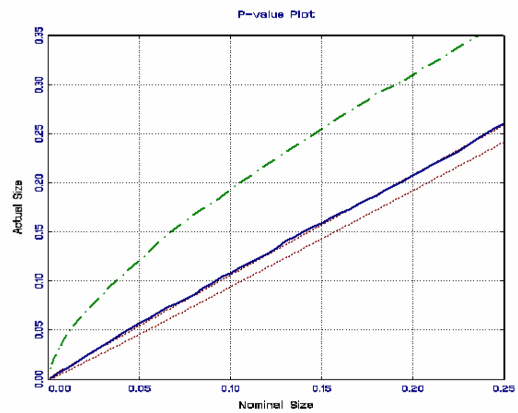
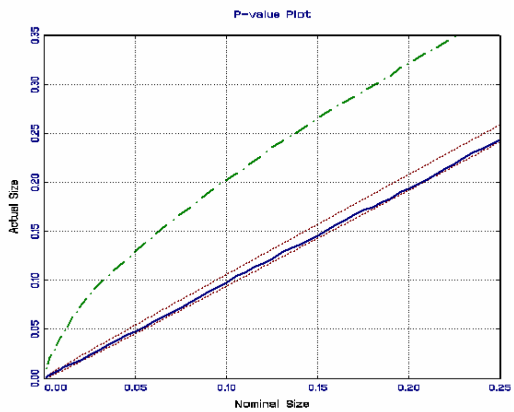


Figure 4e - 1000 observations



Solid lines = White noise. Dot dash line = GARCH. Dot lines = 95% confidence interval for nominal size.

Figure 5 - Power–Size plots of the Granger-causality test for 50 observations

Figure 5a - VAR(1) GARCH-WN

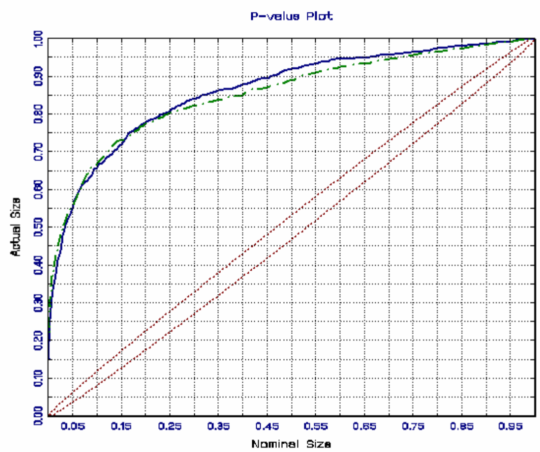
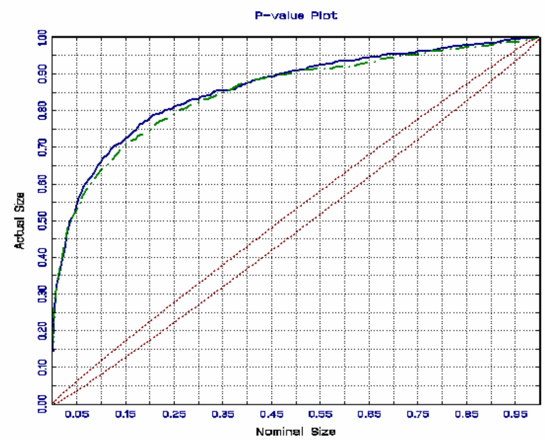


Figure 5b - VAR(1) Break-WN



Solid lines = White noise. Dot dash line = GARCH. Dot lines = 95% confidence interval for nominal size.

Figure 6 - Power-Size plots of the Granger-causality test for 100 observations

Figure 6a - VAR(1) GARCH-WN

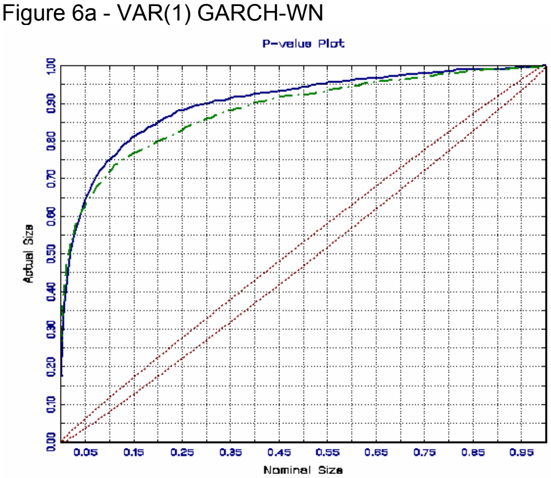
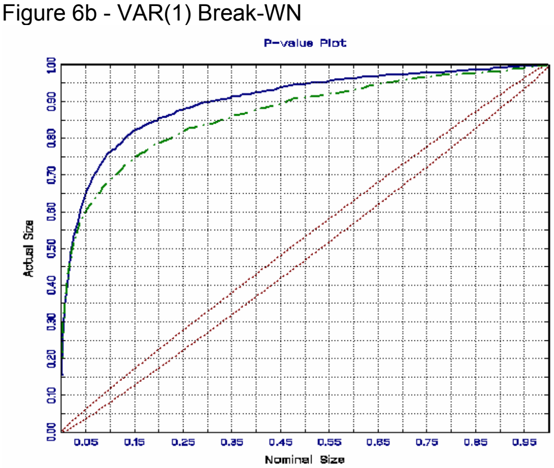


Figure 6b - VAR(1) Break-WN



Solid lines = White noise. Dot dash line = GARCH. Dot lines = 95% confidence interval for nominal size.

Figure 7 - Power-Size plots of the Granger-causality test for 200 observations

Figure 7a - VAR(1) GARCH-WN

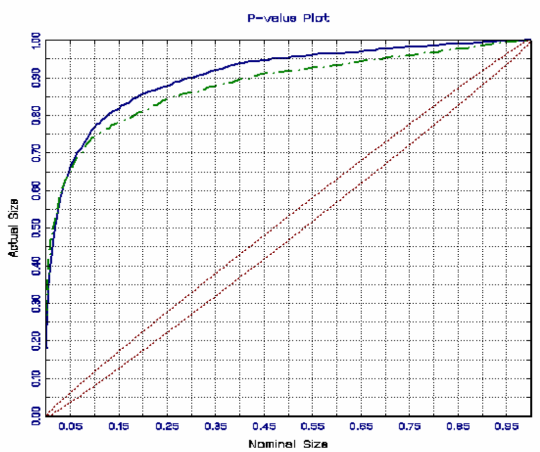
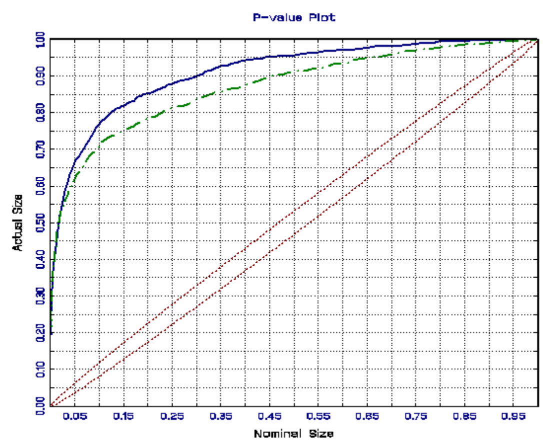


Figure 7b - VAR(1) Break-WN



Solid lines = White noise. Dot dash line = GARCH. Dot lines = 95% confidence interval for nominal size.

Figure 8 - Power-Size plots of the Granger-causality test for 500 observations

Figure 8a - VAR(1) GARCH-WN

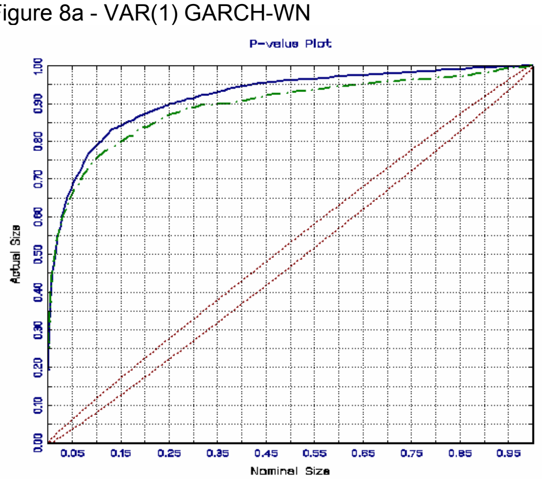
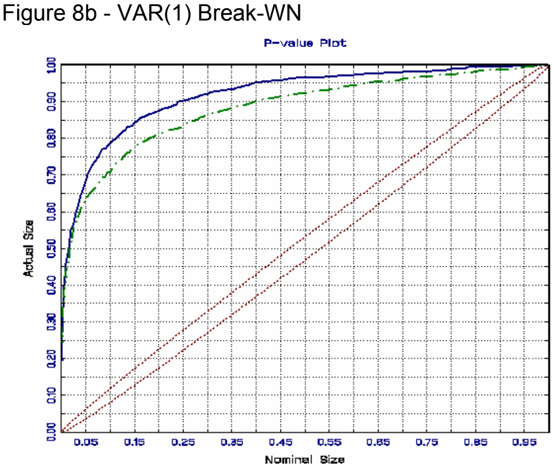


Figure 8b - VAR(1) Break-WN

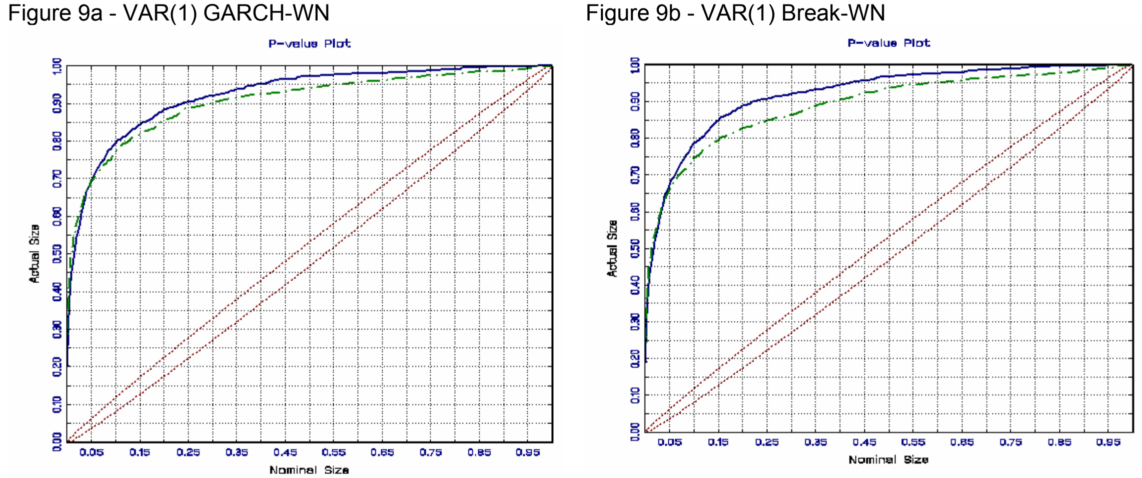


Solid lines = White noise. Dot dash line = GARCH. Dot lines = 95% confidence interval for nominal size.

Figure 9 - Power-Size plots of the Granger-causality test for 1000 observations

Figure 9a - VAR(1) GARCH-WN

Figure 9b - VAR(1) Break-WN



Solid lines = White noise. Dot dash line = GARCH. Dot lines = 95% confidence interval for nominal size.