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Iterative Optimal Control of Liquid Slosh in an Industrial Packaging Machine

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Abstract

This paper presents an iterative approach to optimal control when modeling errors are present. The problem considered is movement of open containers containing liquid in an industrial packaging machine. The goal is to move as fast as possible without too much slosh. There is no measurement of the slosh available in the machine, therefore open loop control via the acceleration reference is the only possibility to control the slosh. However it is possible to measure the slosh in an experimental testbed. This can be used to offline determine the acceleration reference. An algorithm for iterative optimal control is derived that uses experimental data to refine the solution of the optimal control problem.

1. Introduction

This paper deals with a control problem common in machines for packaging fluids. The operation of the packaging machine can be divided into three independent sub tasks: folding, filling and sealing. These tasks are performed simultaneously on three different packages. The operation of the machine is as follows: The folded package is placed in a holder which carries the package through the machine. The movement of the package is stepwise and the same movement is applied in every step and on every package in the machine, the number of steps between the subtasks depend on the machine type. The time needed to produce one package is the sum of the time it takes to fill one package, which is the slowest of the subtasks, and the time it takes to move the package one step.

The package contains liquid when it is moved between the filling station and the sealing station. Acceleration of the package induces motion of the liquid in the package, this is referred to as slosh or liquid vibration. The amount of slosh depends on how the package is accelerated, the geometry of the package and the properties of the fluid. If there is too much slosh, the liquid might splash out of the package and contaminate the machine or onto the sealing surfaces. This can result in packages that are not properly sealed and possibly not airtight.

The movement of the packages is controlled by a servo system that controls the position of the packages. The motion is specified as an acceleration reference which is integrated to obtain a velocity and a position reference. The only measurement available to the servo system is the position of the packages. Therefore, the only way to control the slosh is open loop via the acceleration reference. If a model of the slosh is available the acceleration reference can be calculated using optimal control techniques. However, this requires a very accurate model to be successful.

In [12] a linear model of the slosh phenomenon is presented, this model works well for small surface oscillation amplitudes. A nonlinear model is presented in [9], comparisons with experimental data have shown that this model only works well for small surface oscillation amplitudes. A review of the slosh modeling problem can be found in [5]. Solutions to the movement problem are presented in [2, 3, 4, 13] where the allowed maximum slosh is small and the linear model works well. In the case considered in this paper and in [5, 6, 7] the allowed maximum slosh is relatively large and the linear model does not fully describe the slosh.

The traditional way to handle model uncertainty is feedback. In this case it is not possible to use direct feedback in the control loop since there is no slosh measurement device in the packaging machine. The slosh can however be measured in an experimental testbed. Experiments have shown that the response to an acceleration reference is very repeatable, see [5]. The goal is to find an open loop acceleration reference that gives the desired behavior. The control error from the experiment can be used to modify the acceleration reference to be used in the next experiment. The procedure is repeated until the desired behavior has been obtained. This methodology is called iterative learning control (ILC), see [1, 10].

2. Problem formulation

The problem is to find an open loop acceleration reference that moves the package the distance d, with zero velocity at the start and in the end, as fast as possible while the surface elevation is less than s_{max} at the walls of the package. Since the package is moved several steps, the surface elevation constraint should not be violated if the acceleration reference is repeated. This can be achieved by ensuring that the slosh is in the same state at the beginning of each step. The natural choice of initial state of the slosh is that the liquid is at rest, since this is approximately the state after the package has been filled. The slosh should therefore be zero at the beginning and in the end of the movement.

In [5, 6, 7] optimal control techniques are used to solve the minimum time problem and the minimum energy problem using a simple linear model. Experimental evaluation shows that the minimum energy approach works reasonably well, but for faster movements the difference between the slosh predicted by the linear model and the measured slosh is large.

Since the response to an acceleration reference is very repeatable iterative learning control (ILC) is applicable on this problem.

3. Iterative learning control

In the standard formulation of ILC a reference trajectory is given and the input is modified to make the output follow the reference trajectory. In this problem the reference trajectory is the solution of an optimal control problem. Since the reference trajectory is generated using a model of the system the trajectory will be non optimal and even infeasible for the real system if there are model errors. Therefore an alternative approach will be used where data from the experiments are used when solving the original optimal control problem in an iterative procedure. The approach presented is similar to the work presented in [11].

Another iterative solution to this specific problem which is more similar to the standard ILC formulation is presented in [8].

4. Derivation of iterative optimal control algorithm

As the underlying optimal control problem the minimum energy optimal control problem in [5, 6, 7] will be used. The original optimal control problem is given by

$$\begin{aligned}
&\min_{u(t)} \int_0^T u^2(t)\, dt \\
&\text{subject to: } |u(t)| \le u_{max} \\
&\qquad y^1(t) \le s_{max},\ y^2(t) \le s_{max} \\
&\qquad y^1(0) = y^1(T) = \dot{y}^1(0) = \dot{y}^1(T) = 0 \\
&\qquad y^2(0) = y^2(T) = \dot{y}^2(0) = \dot{y}^2(T) = 0 \\
&\qquad x(0) = \dot{x}(0) = \dot{x}(T) = 0 \\
&\qquad x(T) = d
\end{aligned}$$

where $u(t)$ is the acceleration reference, $y^1(t)$ and $y^2(t)$ is the surface elevation on the backward and forward walls of the container, $x(t)$ is the position of the container, T is the movement time and d is the movement distance. The hard constraints on the slosh at time T will give some difficulties in the iterative procedure and therefore the following optimal control problem will be considered instead

$$\begin{aligned}
&\min_{u(t)} \rho \int_0^T u^2(t)\, dt + \int_T^{T+\tau} [y^1(t)]^2 + [y^2(t)]^2\, dt \\
&\text{subject to: } |u(t)| \le u_{max} \\
&\qquad u(t) = 0 \text{ for } t \ge T \\
&\qquad y^1(t) \le s_{max},\ y^2(t) \le s_{max} \\
&\qquad y^1(0) = \dot{y}^1(0) = y^2(0) = \dot{y}^2(0) = 0 \\
&\qquad x(0) = \dot{x}(0) = \dot{x}(T) = 0 \\
&\qquad x(T) = d
\end{aligned} \tag{1}$$

The acceleration reference is discretized with sampling period h such that $T = nh$ and $T + \tau = mh$. $G(q)$ is a linear discrete time model of the slosh such that

$$y^1(t) = G(q)u(t), \quad y^2(t) = -G(q)u(t)$$

where $y^1(t)$ and $y^2(t)$ is the surface elevation on the backward respectively the forward side of the container. Introduction of the vectors

$$\begin{aligned}
U &= [\, u(0) \quad u(h) \quad \dots \quad u((n-1)h)\,]^T \\
Y^1 &= [\, y^1(h) \quad y^1(2h) \quad \dots \quad y^1(mh)\,]^T \\
Y^2 &= [\, y^2(h) \quad y^2(2h) \quad \dots \quad y^2(mh)\,]^T
\end{aligned}$$

and the matrix

$$G = \begin{bmatrix} g(h) & 0 & \dots & 0 \\ g(2h) & g(h) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g(nh) & g((n-1)h & \dots & g(h) \\ \vdots & \vdots & & \vdots \\ g(mh) & g((m-1)h & \dots & g((m-n)h) \end{bmatrix}$$

where $g(t)$ is the pulse response of $G(q)$ gives

$$Y^1 = GU, \quad Y^2 = -GU$$

The position and velocity of the container is given by

$$x(t) = \underbrace{\frac{h^2(q+1)}{2(q-1)^2}}_{P(q)} u(t), \quad \dot{x}(t) = \underbrace{\frac{h}{q-1}}_{V(q)} u(t)$$

this gives

$$\begin{bmatrix} \dot{x}(T) \\ x(T) \end{bmatrix} = \underbrace{\begin{bmatrix} v(nh) & v((n-1)h) & \dots & v(h) \\ p(nh) & p((n-1)h) & \dots & p(h) \end{bmatrix}}_{C} U$$

where $v(t)$ and $p(t)$ are the pulse responses of $V(q)$ and $P(q)$.

Now the continuous time optimal control problem in (1) can be translated into the following discrete time optimal control problem

$$\begin{aligned} \min_U \; & U^T(\rho I + 2G_2^T G_2)U \\ \text{subject to: } & -U_{max} \leq U \leq U_{max} \\ & G_1 U \leq S_{max} \\ & -G_1 U \leq S_{max} \\ & CU = \begin{bmatrix} 0 \\ d \end{bmatrix} \end{aligned} \tag{2}$$

where $G_1 = G_{1:n,1:n}$ and $G_2 = G_{(n+1):m,1:n}$. This is a standard quadratic programming problem.

Now assume that an experiment has been performed with the input U_k and the measurements Y_k^1 and Y_k^2 have been obtained, where k denotes the iteration number. In the next iteration the input $U_{k+1} = U_k + \delta U$ will be applied. A prediction of the outputs Y_{k+1}^1 and Y_{k+1}^2 is then

$$\begin{aligned} \hat{Y}_{k+1}^1 &= Y_k^1 + G\delta U = Y_k^1 + G(U_{k+1} - U_k) \\ \hat{Y}_{k+1}^2 &= Y_k^2 - G\delta U = Y_k^2 - G(U_{k+1} - U_k) \end{aligned}$$

The optimal control problem in (2) for U_{k+1} given U_k, Y_k^1 and Y_k^2 gives the following quadratic program

$$\begin{aligned} \min_{U_{k+1}} \; & U_{k+1}^T(\rho I + 2G_2^T G_2)U_{k+1} + \\ & 2(Y_k^1 - Y_k^2 - 2G_2 U_k)^T G_2 U_{k+1} \\ \text{subject to: } & -U_{max} \leq U_{k+1} \leq U_{max} \\ & G_1 U_{k+1} \leq S_{max} - Y_k^1 - G_1 U_k \\ & -G_1 U_{k+1} \leq S_{max} - Y_k^2 + G_1 U_k \\ & CU_{k+1} = \begin{bmatrix} 0 \\ d \end{bmatrix} \end{aligned} \tag{3}$$

The iterative optimal control (IOC) algorithm is now given by (3) and the initial values $U_0 = 0$, $Y_0^1 = 0$ and $Y_0^2 = 0$.

5. Simulation results

The iterative optimal control procedure in (3) is evaluated using simulations on both a linear and a nonlinear process model. The nonlinear process model is chosen to mimic some of the nonlinear behavior experienced in reality, see [5], but there is no direct physical meaning of the model.

The model used in the iterative optimal control algorithm is given in continuous time by the transfer operator

$$G(p) = \frac{a}{2g} \frac{\omega_m^2}{p^2 + 2\zeta_m \omega_m p + \omega_m^2}$$

with $\zeta_m = 0$ and $\omega_m = \sqrt{\frac{g\pi}{a} \tanh \frac{b\pi}{a}} = 21.0$ rad/s where $a = 0.07$ m is the package width and $b = 0.2$ m is the liquid depth, see [5]. The system is sampled with sampling period $h = 0.01$ s giving the discrete time transfer operator $G(q)$ and the matrix G.

The movement time is $T = 0.46$ s, the movement distance is $d = 0.2$ m and the extra time for the penalty on the residual slosh is $\tau = 0.2$ s. The maximum allowed acceleration is $a_{max} = 10$ m/s^2 and the maximum allowed surface elevation is $s_{max} = 0.035$ m.

The simulations are performed in the Matlab/Simulink environment and the Matlab function quadprog is used to solve the quadratic program.

5.1 Linear process model

The linear process model is given in continuous time by the transfer operator

$$P(p) = \frac{1.1a}{2g} \frac{\omega_p^2}{p^2 + 2\zeta \omega_p p + \omega_p^2}$$

with $\omega_p = 0.9\omega_m$ and $\zeta = 0.01$. The measurements are given by

$$\begin{aligned} y_k^1(t) &= P(p)u_k(t) + w_k^1(t) \\ y_k^2(t) &= -P(p)u_k(t) + w_k^2(t) \end{aligned}$$

where w_k^* is white noise with a standard deviation in the same range as the measurement noise found in the real measurements of the surface elevation.

The influence of the control cost ρ is examined. Figure 1 shows the acceleration reference and the surface elevation for four different values of ρ between 10 and 0.01 when the iteration has converged. The figure shows that the influence on the acceleration reference is very small. The influence on the surface elevation is only seen in the residual slosh, that is for $T > 0.46$. Figure 2 illustrates that the surface elevation for $T > 0.46$. The figure shows that the

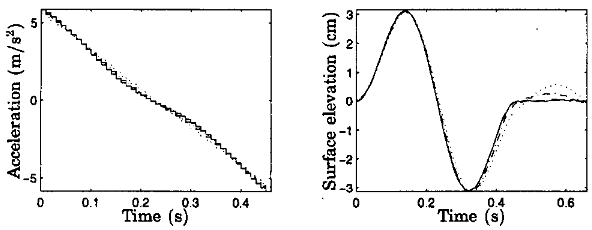

Figure 1 Simulated surface elevation on the backward side of the container $y^1(t)$ (right) and the corresponding acceleration reference (left) for different values of ρ when the iteration has converged, $\rho = 10$ (dotted), $\rho = 1$ (dash-dotted), $\rho = 0.1$ (dashed) and $\rho = 0.01$ (solid). There is only a small influence on the acceleration reference and the only difference is found in the residual slosh.

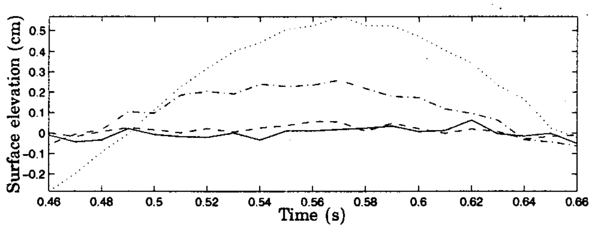

Figure 2 Simulated surface elevation on the backward side of the container $y^1(t)$ for different values of ρ when the iteration has converged, $\rho = 1$ (dash-dotted), $\rho = 0.1$ (dashed) and $\rho = 0.01$ (solid). The residual slosh is much smaller for $\rho = 0.1$ than for ρ equal to 1 or 10, but the difference between $\rho = 0.1$ and $\rho = 0.01$ is very small.

difference between $\rho = 0.1$ and $\rho = 0.01$ is very small and that the residual slosh is much larger for $\rho = 1$ and $\rho = 10$. This indicates that $\rho = 0.1$ is a suitable choice.

Figure 3 shows the predicted cost (the cost from the solution of the quadratic program) and the actual cost (the cost calculated from the data after the experiments) for ten iterations with $\rho = 0.1$. The figure shows that the actual cost and the predicted cost are very close after six iterations and that the actual cost decreases rapidly during the first iterations.

Figure 4 shows the surface elevation on the backward side of the container for iterations one to ten. With the linear process model the surface elevation on the forward side is the same as on the backward side but with opposite sign except for the noise. The figure shows that there is a large amount of residual slosh in the first iteration but the amount of residual slosh is decreased as more iterations are performed. This lead to the conclusion that the algorithm is successful in finding a suitable acceleration reference.

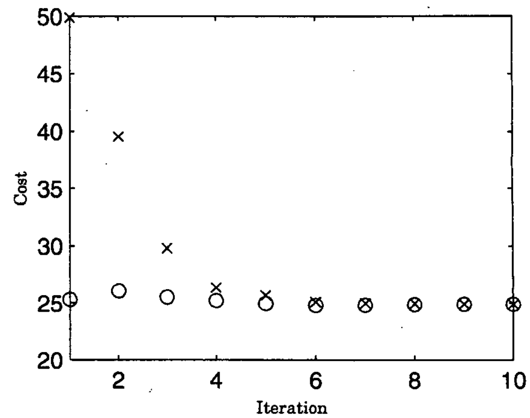

Figure 3 The actual cost ($\times$) and the predicted cost ($\circ$) for iterations one to ten with the linear process model. The actual cost decreases very fast in the first iterations and after six iterations the actual cost and the predicted cost are very close.

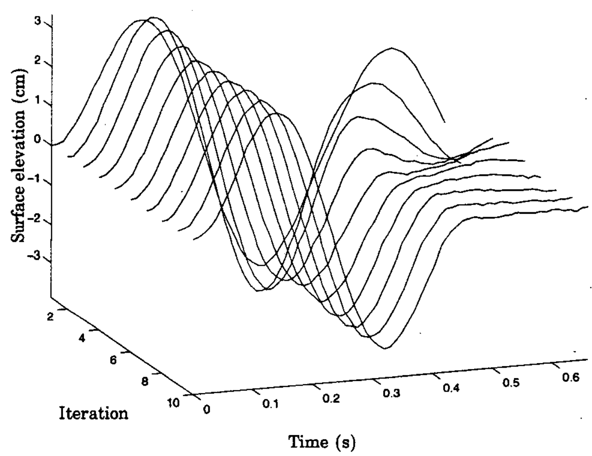

Figure 4 Simulated surface elevation on the backward side of the container $y^1(t)$ for iteration one to ten with the linear process model. There is a large amount of residual slosh in the first iteration but the amount of residual slosh is decreased as more iterations are performed. Note that the surface elevation is the same on the forward side but with opposite sign except for the noise. The method is successful in finding a suitable acceleration reference.

The resulting acceleration reference after ten iterations is very close to the acceleration reference obtained if the optimal control problem is solved using the process model $P(p)$. This indicates that the iterative optimal control algorithm gives the actual optimal solution even if the actual process model is unknown.

5.2 Nonlinear process model

The nonlinear process model is given in continuous time by the state space description

$$\begin{aligned}
\dot{x}_1(t) &= -2\zeta\omega_m x_1(t) - \omega_m^2 \frac{\tanh 30x_2(t)}{30} + \frac{a\omega_m^2}{2g}u_k(t) \\
\dot{x}_2(t) &= x_1(t) \\
y_k^1(t) &= \frac{4}{5}x_2(t) + 7x_2^2(t) + w_k^1(t) \\
y_k^2(t) &= -\frac{4}{5}x_2(t) + 7x_2^2(t) + w_k^2(t)
\end{aligned}$$

where ζ, ω_m, a and w_k^* are the same as earlier in the paper. The term $\frac{\tanh 30x_2(t)}{30}$ gives an amplitude dependent oscillation frequency. The quadratic term in the output equations give an asymmetric oscillation where the peaks are higher than the crests are deep. These are behaviors that have been observed in real slosh, see [5], they can also be motivated mathematically, see [9]. This model is only designed to mimic these nonlinear phenomena and has not been derived from first principles.

Figure 5 shows the predicted cost and the actual cost for ten iterations with $\rho = 0.1$. The figure shows that the actual cost decreases very fast as more iterations are performed and that the actual cost and the predicted cost are close after five iterations. Figure 6 and Figure 7 show the surface elevation on the backward and the forward side of the container for iterations one to ten. The figures show that there is a large amount of residual slosh in the first iteration but as more iterations are performed the amount of residual slosh is decreased. This shows that the method is successful in finding a suitable acceleration reference.

With the nonlinear process model it is hard to calculate the actual optimal solution and no comparison has been made corresponding to the comparison made with the linear process model.

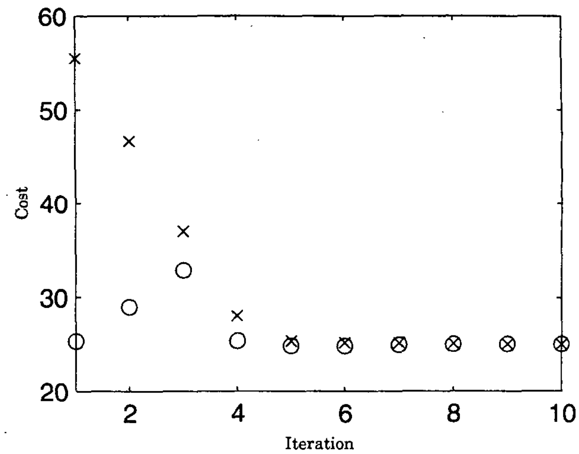

Figure 5 The actual cost (×) and the predicted cost (○) for iterations one to ten with the nonlinear process model. The actual cost decreases very fast and after five iterations the actual and the predicted cost are very close.

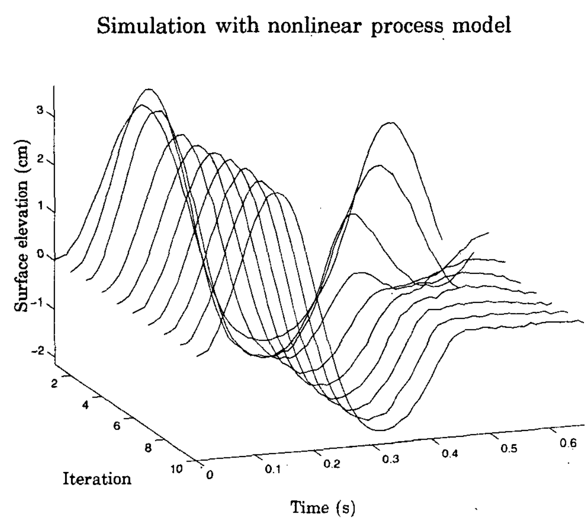

Figure 6 Simulated surface elevation on the backward side of the container $y^1(t)$ for iteration one to ten with the nonlinear process model. There is a large amount of residual slosh in the first iteration but the amount of residual slosh is decreased as more iterations are performed. The method is successful in finding a suitable acceleration reference.

6. Conclusions

An iterative approach to optimal control has been developed. The resulting procedure uses quadratic programming to refine the acceleration reference based on data from the previous iteration. A linear model of the system is used to predict the output in the next iteration.

The method is evaluated in simulations using both a linear and a nonlinear process model. The simulations give insight into how to choose the control cost and show that the method is successful in finding a suitable acceleration reference both with the linear and the nonlinear process model.

Future work include experimental evaluation in the industrial testbed and analysis of convergence properties of the method proposed.

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Simulation with nonlinear process model

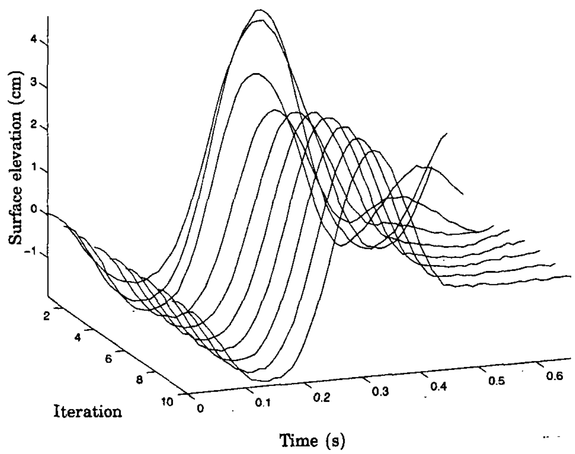

Figure 7 Simulated surface elevation on the forward side of the container $y^2(t)$ for iteration one to ten with the nonlinear process model. There is a large amount of residual slosh in the first iteration but the amount of residual slosh is decreased as more iterations are performed. The method is successful in finding a suitable acceleration reference.

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