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Constrained Iterative Learning Control of Liquid Slosh in an Industrial Packaging Machine

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Abstract

This paper considers the problem of moving a container with liquid without too much slosh in an industrial packaging machine. There is no measurement of the slosh available in the machine to use for control, it is however possible to measure the slosh in a testbed. The problem is to determine the shape of the open loop acceleration reference that is programmed in the motion control system that controls the movement of the containers. In this paper the acceleration reference is calculated using iterative learning control where the results of an experiment are used to refine the acceleration reference to be used in the next iteration.

1. Introduction

This paper deals with a control problem common in machines for packaging fluids. The operation of a packaging machine can be divided into three independent sub tasks: folding, filling and sealing. These tasks are performed simultaneously on three different packages. The operation of the machine is as follows: The folded package is placed in a holder which carries the package through the machine. The movement of the package is stepwise and the same movement is applied in every step and on every package in the machine, the number of steps between the subtasks depend on the machine type. The time needed to produce one package is the sum of the time it takes to fill one package, which is the slowest of the subtasks, and the time it takes to move the package one step.

The package contains liquid when it is moved between the filling station and the sealing station. Acceleration of the package induces motion of the liquid in the package, this is referred to as slosh or liquid vibration. The amount of slosh depends on how the package is accelerated, the geometry of the package and the properties of the fluid. If there is too much slosh, the liquid might splash out of the package and contaminate the machine or onto the sealing surfaces. This can result in packages that are not properly sealed and possibly not airtight.

The movement of the packages is controlled by a servo system that controls the position of the packages. The motion is specified as an acceleration reference which is integrated to obtain a velocity and a position reference. The only measurement available to the servo system is the position of the packages. Therefore, the only way to control the slosh is open loop via the acceleration reference. If a model of the slosh is available the acceleration reference can be calculated using optimal control techniques. However, this requires a very accurate model to be successful.

In [14] a linear model of the slosh phenomenon is presented, this model works well for small surface oscillation amplitudes. A nonlinear model is presented in [12], comparisons with experimental data have shown that this model only works well for small surface oscillation amplitudes. A review of the slosh modeling problem can be found in [6]. Solutions to the movement problem are presented in [2, 3, 4, 15] where the allowed maximum slosh is small and the linear model works well. In the case considered in this paper and in [6, 8, 9] the allowed maximum slosh is relatively large and the linear model does not fully describe the slosh.

The traditional way to handle model uncertainty is feedback. In this case it is not possible to use direct feedback in the control loop. It is however possible to measure the slosh in an the experimental testbed. Experiments have shown that the response to an acceleration reference is very repeatable, see [6]. The goal is to find an open loop acceleration reference that gives the desired behavior. The control error from the experiment can be used to modify the acceleration reference to be used in the next experiment. The procedure is repeated until the desired behavior has been obtained. This methodology is called iterative learning control (ILC) see [1, 13].

2. Problem formulation

The problem is to find an open loop acceleration reference that moves the package the distance d, with zero velocity at the start and in the end, as fast as possible while the surface elevation is less than s_{max} at the walls of the package. Since the package is moved several steps, the surface elevation constraint should not be violated if the acceleration reference is repeated. This can be achieved by ensuring that the slosh is in the same state at the beginning of each step. The natural choice of initial state of the slosh is that the liquid is at rest, since this is approximately the state after the package has been filled. The slosh should therefore be zero at the beginning and in the end of the movement.

In [6, 8, 9] optimal control techniques are used to solve the minimum time problem and the minimum energy problem using a simple linear model. Experimental evaluation shows that the minimum energy approach works reasonably well, but for faster movements the difference between the slosh predicted by the linear model and the measured slosh is large.

Since the response to an acceleration reference is very repeatable Iterative Learning Control (ILC) is well suited for this problem.

3. Iterative learning control

This paper describes methods to increase the performance using Iterative Learning Control. A reference trajectory and initial acceleration reference is determined using the methods described in [6, 8, 9].

The usual way to implement ILC is to use the following updating formula for the input signal $u_k(t)$

$$u_{k+1}(t) = Q(q)(u_k(t) + L(q)e_k(t)) \tag{1}$$

where $Q(q)$ and $L(q)$ are linear filters, not necessarily causal, and $e_k(t) = r(t) - y_k(t)$ is the control error.

In this application of ILC the input signal $u(t)$ is the acceleration reference. The specifications on the movement gives the following constraints on $u(t)$

$$\int_0^T u(t)\, dt = 0\,, \qquad \int_0^T \int_0^t u(s)\, ds dt = d \tag{2}$$

where T is the movement time. These constraints make it hard to choose the filters $Q(q)$ and $L(q)$ in (1). Therefore an alternative approach will be used which will result in linear time-varying filters.

An alternative approach is presented in [7] where the underlying optimal control problem is solved in each iteration using data from the experiments to refine the solution.

4. Derivation of ILC algorithm

The following algorithm is similar to algorithms presented in [1, 5, 10].

With the methods in [6, 8, 9] a reference $r(t)$ for the surface elevation on the backward side of the package and an initial acceleration reference $u_0(t)$ is obtained. The surface elevation is measured on the backward and on the forward side of the package with respect to the direction of motion giving the measurements $y^1(t)$ and $y^2(t)$. The model used when calculating $r(t)$ gives a symmetric surface elevation, hence $-r(t)$ is the surface elevation on the forward side of the package.

The acceleration reference and the surface elevation reference is sampled with sampling period h such that $T = nh$. The surface elevation reference is augmented zeros giving $m > n$. This is to make it possible to penalize the residual slosh after the movement.

The problem is to find $\delta u_k(t)$ in $u_{k+1}(t) = u_k(t) + \delta u_k(t)$ that minimizes the error in the next iteration $e_{k+1}(t) = r(t) - y_{k+1}(t)$.

Suppose that the slosh is approximately described by the linear discrete time transfer operator $G(q)$, possibly the same as used when calculating the reference, this gives

$$\hat{y}_k^1(t) = H(q)u_k(t) \qquad \hat{y}_k^2(t) = -H(q)u_k(t)$$

Then the error in iteration $k+1$ is approximately given by

$$\begin{aligned} e_{k+1}^1(t) &\approx e_k^1 - G(q)\delta u_k(t) \\ e_{k+1}^2(t) &\approx e_k^2 + G(q)\delta u_k(t) \end{aligned}$$

Definition of the vectors

$$\begin{aligned} \delta U_k &= [\,\delta u_k(0) \quad \delta u_k(h) \quad \dots \quad \delta u_k((n-1)h)\,]^T \\ E_k^1 &= [\,e_k^1(0) \quad e_k^1(h) \quad \dots \quad e_k^1((m-1)h)\,]^T \\ E_k^2 &= [\,e_k^2(0) \quad e_k^2(h) \quad \dots \quad e_k^2((m-1)h)\,]^T \end{aligned}$$

and the matrix

$$G = \begin{bmatrix} g(0) & 0 & \dots & 0 \\ g(h) & g(0) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g((n-1)h) & g((n-2)h) & \dots & g(0) \\ \vdots & \vdots & & \vdots \\ g((m-1)h) & g((m-2)h) & \dots & g((m-n)h) \end{bmatrix}$$

where $g(t)$ is the pulse response of $G(q)$ gives the system description

$$U_{k+1} = U_k + \delta U_k \tag{3}$$

$$E_{k+1}^1 \approx E_k^1 - G\delta U_k \tag{4}$$

$$E_{k+1}^2 \approx E_k^2 + G\delta U_k \tag{5}$$

Zero-order-hold sampling and normalization of the constraints (2) gives

$$\underbrace{\begin{bmatrix} 1 & 1 & \dots & 1 \\ n-\frac{1}{2} & n-\frac{3}{2} & \dots & \frac{1}{2} \end{bmatrix}}_{A} \delta U_k = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The update of the acceleration reference δU_k is now given as the solution to the following quadratic optimization problem

$$\min_{\delta U_k} \; {E_{k+1}^1}^T W_1 E_{k+1}^1 + {E_{k+1}^2}^T W_2 E_{k+1}^2 + {U_{k+1}}^T W_u U_{k+1}$$

subject to $A\delta U_k = 0$

All δU_k that satisfy the constraint are given by $K\theta$ where K is the kernel of A and θ is an arbitrary vector. Insertion of (3), (4), (5), $\delta U_k = K\theta$ and differentiation with respect to θ gives

$$\delta U_k = K\left[K^T(G^T(W_1+W_2)G + W_u)K\right]^{-1} \times K^T(G^T(W_1 E_k^1 - W_2 E_k^2) - W_u U_k) \quad (6)$$

The update law for the ILC is now given by (3) and (6) and can be written as.

$$U_{k+1} = QU_k + L_1 E_k^1 + L_2 E_k^2 \quad (7)$$

Where Q, L_1 and L_2 are matrices corresponding to time-varying filters in the update law given in (1).

5. Simulation results

The update law in (7) is evaluated using simulations on both a linear and a nonlinear process model to determine suitable weightings W_1, W_2 and W_u. The nonlinear process model is chosen to mimic some of the nonlinear behavior experienced in reality, see [6], but there is no direct physical meaning of the model.

The model used in the update law is given in continuous time by the transfer operator

$$G(p) = \frac{a}{2g}\frac{\omega_m^2}{p^2 + 2\zeta_m\omega_m p + \omega_m^2}$$

with $\zeta_m = 0$ and $\omega_m = \sqrt{\frac{g\pi}{a}\tanh\frac{b\pi}{a}} = 21.0$ rad/s where $a = 0.07$ m is the package width and $b = 0.2$ m is the liquid depth, see [6]. The system is sampled with sampling period $h = 0.01$ s giving the discrete time transfer operator $G(q)$ and the matrix G.

The surface elevation reference and initial acceleration reference are calculated using this model and the minimum energy approach described in [6, 8, 9]. The surface elevation reference is augmented with 20 zeros, see Figure 1 for plots of the initial acceleration reference and the surface elevation references.

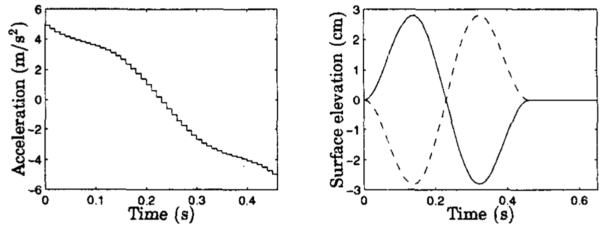

Figure 1 Initial acceleration reference $u_0(t)$ (left) and surface elevation references (right) $r^1(t)$ (solid) and $r^2(t)$ (dashed).

The movement time is $T = 0.46$ s and the movement distance is $d = 0.2$ m.

The following performance measures are introduced

$$\begin{aligned} S_k &= \max(|e_k^1(t)|, |e_k^2(t)|) \\ C_k &= \max(|u_k(t)|) \\ R_k &= \sum_{i=n}^{m-1} [e_k^1(ih)]^2 + [e_k^2(ih)]^2 \end{aligned} \quad (8)$$

to be used when evaluating different weightings. Here S is the maximum error, C is the maximum control and R is a measure of the residual slosh.

5.1 Linear process model

The linear process model is given in continuous time by the transfer operator

$$P(p) = \frac{1.1a}{2g}\frac{\omega_p^2}{p^2 + 2\zeta\omega_p p + \omega_p^2}$$

with $\omega_p = 0.9\omega_m$ and $\zeta = 0.01$. The measurements are given by

$$\begin{aligned} y_k^1(t) &= P(p)u_k(t) + v_k^1(t) \\ y_k^2(t) &= -P(p)u_k(t) + v_k^2(t) \end{aligned}$$

where v_k^* is white noise.

First the influence of W_u on the control input is examined. The weights are chosen as $W_1 = W_2 = I$ and $W_u = \varphi I$ and the iteration is continued until the output has converged. Figure 2 shows the performance measures in (8) for different values of φ. The figure shows that the maximum error and the residual slosh decreases fast when φ is between 10 and 0.1 and then there is only a small decrease as φ is decreased further. In the opposite manner the maximum control signal increases slowly for φ in the range 10 to 0.1 and faster for even smaller values of φ. This indicates the 0.1 is a suitable choice of φ giving a good tradeoff between error and maximum control signal.

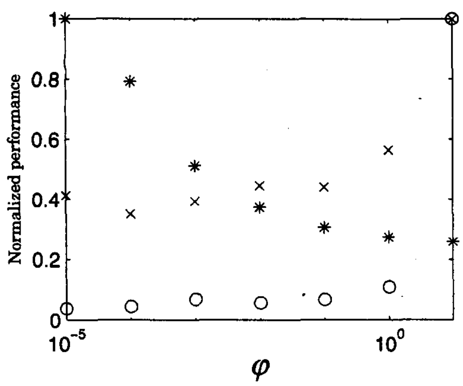

Figure 2 Normalized performance as a function of φ with the weights $W_1 = W_2 = I$ and $W_u = \varphi I$ for the linear process model when the iteration has converged, S ($\times$), C ($*$) and R ($\circ$). Notice that the maximum control signal increases rapidly for small values of φ while the maximum error is only slightly reduced.

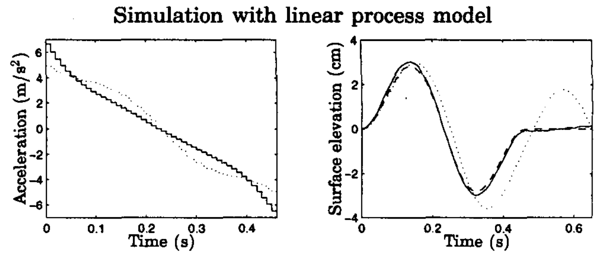

Figure 3 Acceleration reference (left), after five iterations (solid) and initial (dotted), and simulated surface elevation on the backward side of the container $y^1(t)$ (right), after five iterations (solid), initial (dotted) and reference (dashed), with the linear process model and the weights $W_1 = W_2 = I$ and $W_u = 0.1I$. Note that the surface elevation on the forward side is the same as on the backward side with opposite sign except for the measurement noise. The method is successful in reducing the residual slosh due to the model error.

Figure 3 shows the acceleration reference and the surface elevation after five iterations with $\varphi = 0.1$. The figure shows that the ILC algorithm successfully finds an acceleration reference that fulfills the specifications. It is possible to reduce the residual slosh even further by increasing the weight on the 20 last samples. Figure 4 shows the surface elevation for the weights $W_1 = W_2 = I$ and $W_1 = W_2 = \text{diag}(I_{46},\ 10I_{20})$ with $W_u = 0.1I$, where I_n is the $n \times n$ identity matrix. The figure shows that the residual slosh is reduced.

5.2 Nonlinear process model

The nonlinear process model is given in continuous time by the state space description

$$
\begin{aligned}
\dot{x}_1(t) &= -2\zeta\omega_m x_1(t) - \omega_m^2 \frac{\tanh 30x_2(t)}{30} + \frac{a\omega_m^2}{2g} u_k(t) \\
\dot{x}_2(t) &= x_1(t) \\
y_k^1(t) &= \frac{4}{5} x_2(t) + 7x_2^2(t) + v_k^1(t) \\
y_k^2(t) &= -\frac{4}{5} x_2(t) + 7x_2^2(t) + v_k^2(t)
\end{aligned}
$$

where ζ, ω_m, a and v_k^* is the same as earlier in the paper. The term $\frac{\tanh 30x_2(t)}{30}$ will give an amplitude dependent oscillation frequency and the quadratic term in the output equations will give asymmetric oscillation, the peaks are higher than the crests are deep. These are behaviors that have been observed in real slosh, see [6], they can also be motivated mathematically, see [12]. This model is only designed to mimic these phenomena and has no direct physical meaning.

Figure 5 shows the surface elevation after five iterations with the nonlinear process model and the weights $W_1 = W_2 = \text{diag}(I_{46},\ 10I_{20})$ and $W_u = 0.1I$. The figure shows that the residual slosh is small but there are large differences between the resulting surface elevation and the reference.

With the nonlinear process model the peaks are higher than the crests are deep and it is not possible to follow the reference completely. There is a coupling between the surface elevation on the forward and the backward side. Therefore, if the peak is lowered on the backward side the crest will be raised on the forward side which will make the quadratic error larger. Since it is more important to lower the maximum surface elevation one way to deal with the asymmetric behavior is to set the W_1 and W_2 to zero when the reference is negative.

Figure 6 shows the surface elevation after five iterations with the nonlinear process model and the weights $W_1 = \text{diag}(I_{23},\ 0_{23},\ 10I_{20})$, $W_2 =$

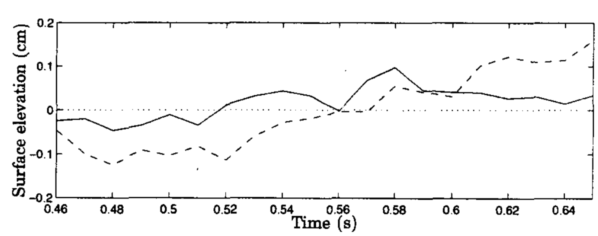

Figure 4 Simulated surface elevation on the backward side of the container $y^1(t)$ after five iterations, reference (dotted), with the linear process model and the weights $W_1 = W_2 = I$ (dashed) and $W_1 = W_2 = \text{diag}(I_{46},\ 10I_{20})$ (solid) and $W_u = 0.1I$. Notice that the residual slosh is reduced when the W_1 and W_2 are increased in the last 20 samples.

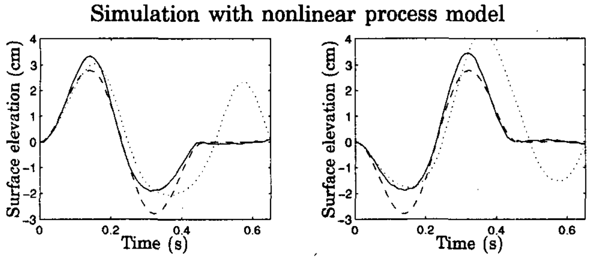

Figure 5 Simulated surface elevation on the backward side of the container $y^1(t)$ (right) and on the forward side $y^2(t)$ (left), after five iterations (solid), initial (dotted) and reference (dashed), with the nonlinear process model and the weights $W_1 = W_2 = \text{diag}(I_{46},\ 10I_{20})$ and $W_u = 0.1I$. The method successfully decreases the residual slosh but the maximum slosh is much larger than the reference.

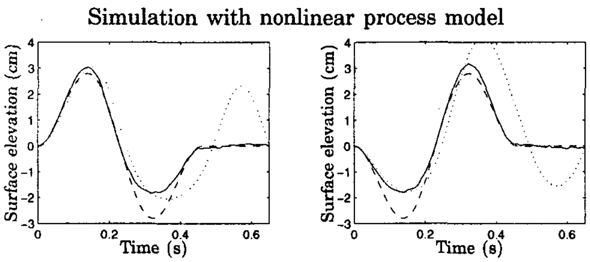

Figure 6 Simulated surface elevation on the backward side of the container $y^1(t)$ (right) and on the forward side $y^2(t)$ (left), after five iterations (solid), initial (dotted) and reference (dashed), with the nonlinear process model and the weights $W_1 = \text{diag}(I_{23},\ 0_{23},\ 10I_{20})$, $W_2 = \text{diag}(0_{23},\ I_{23},\ 10I_{20})$ and $W_u = 0.1I$. Compared with Figure 5 the maximum slosh is reduced due to the modified weights.

$\text{diag}(0_{23},\ I_{23},\ 10I_{20})$ and $W_u = 0.1I$, where 0_n is a $n \times n$ zero matrix. The figure shows that the error is reduced for the positive surface elevation and that the maximum surface elevation is decreased.

6. Experimental results

The experimental evaluation was performed in an industrial testbed constructed by Tetra Pak Research & Development AB, Lund, Sweden. The testbed consists of a carriage mounted on a rubber belt driven by a servo motor. The container is a paper carton used for packaging beverages and the carton holder is the same as used in the packaging machine. The package is filled with tap water dyed with white paint.

The surface elevation is measured on one side of the package with an infrared laser displacement sensor. There is only one sensor on one side of the package. Therefore two experiments are needed in each iteration, one experiment moving the package forward and another moving it backwards. For more details about the testbed see [6].

The motion control system is implemented in the Matlab/Simulink environment on a PC running the Linux operating system, see [11].

First experiments are done with the nominal model in the update law with $\omega_m = 21.0$ rad/s and $\zeta_m = 0$ and the weights $W_1 = \text{diag}(I_{23},\ 0_{23},\ 10I_{20})$, $W_2 = \text{diag}(0_{23},\ I_{23},\ 10I_{20})$ and $W_u = 0.1I$. The surface elevation after ten iterations is shown in Figure 7. The figure shows that the method is successful in reducing the residual slosh.

In [6, 8, 9] it is shown that the performance of the minimum energy acceleration reference can be increased by modifying the model parameters ω_m and ζ_m. It is shown through experiments that $\omega_m = 19.1$ rad/s and $\zeta_m = 0.013$ gives the minimum amount of residual slosh for this particular movement. The surface elevation after two iterations with the modified model in the update law and the same weights as before is shown in Figure 8.

The resulting surface elevation on both sides of the package is very close to the ones obtained with the nominal model, the only difference is the number of iterations two, for the modified model, versus ten, for the nominal model. On the other hand, about 40 experiments were performed to obtain the modified model parameters.

7. Conclusions

An optimization approach to constrained iterative learning control has been developed. The resulting update law is simple and consists of three time-varying linear filters. The filters are calculated once using a model of the system and the weighting matrices.

The method is evaluated in simulations using both a linear and a nonlinear process model and in real experiments. The simulations give insight into how the weights should be chosen and confirm that the method is successful in reducing the residual slosh if there are differences in the process model and the model used in the update law, even if the process model is nonlinear. The method is also successful in reducing the residual slosh in experiment performed in the industrial testbed.

Acknowledgments

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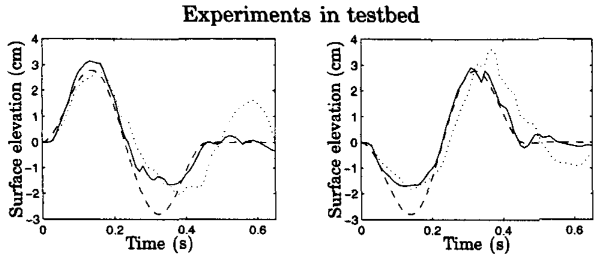

Figure 7 Measured surface elevation on the backward side of the container $y^1(t)$ (right) and on the forward side $y^2(t)$ (left), after ten iterations (solid), initial (dotted) and reference (dashed), with the weights $W_1 = \text{diag}(I_{23}, 0_{23}, 10I_{20})$, $W_2 = \text{diag}(0_{23}, I_{23}, 10I_{20})$ and $W_u = 0.1I$, and reference model parameters $\omega_m = 21.0$ rad/s and $\zeta_m = 0$.

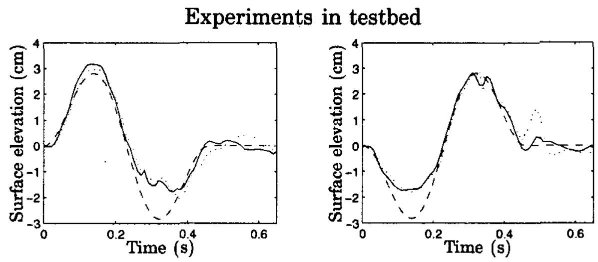

Figure 8 Measured surface elevation on the backward side of the container $y^1(t)$ (right) and on the forward side $y^2(t)$ (left), after two iterations (solid), initial (dotted) and reference (dashed), with the weights $W_1 = \text{diag}(I_{23}, 0_{23}, 10I_{20})$, $W_2 = \text{diag}(0_{23}, I_{23}, 10I_{20})$ and $W_u = 0.1I$, and reference model parameters $\omega_m = 19.1$ rad/s and $\zeta_m = 0.013$. The results are close to the results obtained with the nominal model showed in Figure 7 after ten iterations.

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